Training and Search during Unemployment*

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November 17, 2010

Abstract

Displaced workers often experience large losses in earnings, even a long time after reemployment. Training programs during unemployment mitigate these losses but also affect the unemployed's willingness to search. This paper analyzes how mandatory training programs affect the optimal design of unemployment insurance and how the training intensity should evolve during the unemployment spell. I find that the optimal path of unemployment consumption may be reversed when introducing training programs and that even the long-term unemployed should be incentivized to find employment despite the depreciation of human capital during unemployment. Targeting training programs towards the long-term unemployed is optimal only if the fall in human capital upon displacement is small relative to the depreciation during unemployment.

Keywords: Unemployment, Optimal Insurance, Human Capital, Training

JEL Classification Numbers: H21, J62, J64

*I would like to thank Arthur Campbell, Peter Diamond, Florian Ederer, Jesse Edgerton, Bengt Holmström, Henrik Kleven, Gerard Padró i Miquel, Frans Spinnewyn, seminar participants at MIT, Warwick University and the EEA-ESEM meetings and in particular Ivan Werning for many helpful comments and suggestions.

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1 Introduction

Optimal unemployment insurance trades off the provision of incentives to search for employment and the provision of insurance against the consequences of unemployment. The natural consequence of unemployment is the foregone wage while unemployed. However, even after returning to work, many workers still have substantially lower wages than before displacement. The losses in earnings may be large and long lasting (Jacobson, LaLonde and Sullivan 1993, von Wachter, Song and Manchester 2009). In the US, one fourth of the re-employed have wages that were at least 25 percent lower than in their previous job (Kling 2006). At the source of these earnings losses may be the loss of human capital; displaced workers lose human capital at the moment they lose their job and their human capital continues to depreciate during unemployment. Unemployment insurance should insure the unemployed against both the loss of current earnings and the expected loss of future earnings. At the same time, providing incentives for search is more important if finding a job avoids further depreciation of human capital. By countering the losses in human capital, training programs during unemployment could mitigate future earnings losses and thus play an important role for unemployment policy.

This paper analyzes how mandatory training programs should be incorporated in the design of unemployment insurance. Like the unemployment benefits and taxes, the intensity of the training programs is allowed to vary with the length of the unemployment spell. The paper makes two important contributions. First, the paper shows how the introduction of training programs affects the trade-off between the provision of insurance and incentives for search, reversing previous conclusions in the optimal unemployment insurance literature. Second, the paper relates the optimal timing of training programs to two different sources of human capital loss, upon job loss and during unemployment. This sheds light on the practice of targeting training programs towards the long-term unemployed and on the potential value of training programs as an integral part of unemployment policy.

Despite their importance in practice, training programs have received little attention in the literature on optimal unemployment insurance. Spending on labor market programs, active and passive, averages about 3 percent of GDP in the OECD countries. The proportion of spending on active labor market programs rather than on unemployment benefits has increased to 40-50 percent in most European countries, of which on average 40 percent is spent on training programs. The impact of training programs has been estimated in the empirical literature. While there is a lot of heterogeneity in impact across different programs (Heckman et al. 1999), a meta-analysis of recent work by Card, Kluve and Weber (2009) suggests that training programs do have a positive long-run effect.

This paper considers an agent at the start of an unemployment spell in an infinitehorizon model. As long as the agent has been unsuccessful in finding employment, he chooses how much search effort to exert. The agent's human capital, however, depreciates during the unemployment spell. This depreciation lowers his productivity upon reemployment, but can be countered by training efforts during unemployment. The unemployed worker bears the cost of these training efforts imposed by the social planner. The training and search efforts interact as in the multi-task setting by Holmström and Milgrom (1991). If training efforts increase the marginal cost of search, the required participation to training programs implies a negative *lock-in* effect with low exit rates when programs are intensive. I characterize the optimal unemployment insurance contract, specifying the consumption levels during unemployment and upon reemployment and the training level as a function of the length of the unemployment spell.

A large literature has analyzed the optimal timing of unemployment benefits and taxes - trading off insurance and incentives for search - in isolation from other unemployment policies. This paper focuses on the same trade-off but integrates the use of training programs in the analysis. I make strong modeling assumptions about the nature of training programs to sharpen this focus and keep the analysis tractable. First, I assume that training efforts increase the worker's output when reemployed and thus are an effective tool to counter the exponential depreciation in an unemployed's worker human capital. Second, I assume that training efforts are imposed by the social planner, while the search efforts to find a job are chosen by the unemployed worker. In practice, the participation to training programs is often required to remain eligible for unemployment benefits. Job search monitoring are becoming more important as well though, with job seekers being asked to provide evidence of their job search efforts. From a theoretical perspective, it also seems more interesting to analyze the unobservability of search efforts than training efforts, as unemployment insurance inherently distorts the incentives to leave unemployment. Unemployment benefits do not distort the incentives to exert training efforts when the unemployed capture the resulting increase in productivity when employed. Third, I assume that the same training technology is not available on the job. This assumption speaks more to an environment where training is necessary to make the unemployed 'employable' or where the opportunity cost of training is much higher on the job than during unemployment. Finally, I also ignore the use of burdensome training programs to screen between different types of job seekers (Besley and Coate 1992) or to target unemployment benefits (Akerlof 1978).

I find that given the optimal unemployment policy - combining monetary transfers and training - the unemployed worker is in one of three states depending on the level of his human capital:

- In the *training state*, the level of human capital is so low that no search is induced. Training efforts are imposed to increase the level of human capital. Since no incentives are needed, the social planner can fully smooth the unemployed's consumption.

- In the *search-and-training state*, human capital is sufficiently high so that it is optimal to induce search efforts by providing incomplete insurance. The depreciation of human capital increases both the value of insurance and value of search. By mitigating the depreciation, training reduces the need to induce high levels of search and allows the social planner to focus more on consumption smoothing. The trade-off between providing insurance and inducing search is relaxed.[1]

- In the *stationary state*, the social planner makes the unemployed participate in training programs to maintain the same level of human capital. At the same time, they are given incentives to search for a job.

Without effective training technology, the unemployed worker can only be in one of two states: either human capital is sufficiently high such that it is worth to induce the unemployed to search for employment, or human capital is too low such that the social planner puts the unemployed on social assistance, paying constant welfare payments and not inducing active participation (Pavoni and Violante 2007).

The contrast depending on the presence of an effective training technology is also reflected in the optimal consumption profile. Without training, the optimal consumption profile is decreasing for the short-term unemployed and constant for the long-term unemployed. With training, the optimal consumption profile may be constant for the short-tem unemployed and decreasing for the long-term unemployed. This relates to two important results in the literature on optimal unemployment insurance without training facility. First, Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) show that consumption should be strictly decreasing with the length of the unemployment spell, as long as the unemployed agent is not on social assistance and no search is induced. With training, the training state may preceed the search-and-training state during the unemployment spell. In the the training state, the social planner provides temporarily no incentives for search, but focuses on consumption smoothing only. Hence, the optimal consumption path is constant before it starts decreasing. Second, Pavoni and Violante (2007) and Pavoni (2009) show that with human capital depreciation the social assistance state is absorbent and reached after a finite time in unemployment. With training, the human capital of the long-term unemployed converges to a positive, stationary level. The social planner never stops inducing the long-term unemployed to search, whatever the length of the unemployment spell. Hence, the optimal consumption path remains decreasing for the long-term unemployed.

I perform numerical simulations for CARA preferences with monetary costs of efforts, for which the state space of the recursive problem becomes one-dimensional. The numerical simulations show that the training state, if it exists, always preceeds the

[1] Even when no training is imposed at the start of the unemployment spell for unemployed agents with very high levels of human capital, job seekers who are unsuccessful in finding employment will eventually be required to start training and are induced to search at the same time.

search-and-training state. The level of human capital converges during the unemployment spell to a unique, stationary level. This convergence is monotone and global. The immediate policy implication is that the difference between this stationary level and the level of human capital at the start of the unemployment spell determines the optimal timing of training. If the initial level of human capital is lower, training is more intensive towards the start of unemployment. If the initial level of human capital is higher, training becomes more intensive throughout unemployment. At the same time, the simulations suggest that the welfare gain from introducing training programs in the unemployment policy design is substantial when the initial level of human capital is low, while it is negligible when the initial level of human capital is high.

The human capital level at the start depends on the fall in human capital upon displacement, while the stationary level depends on the depreciation rate of human capital during unemployment. Upon displacement, the unemployed may lose firm-specific or industry-specific human if they are re-employed in a different firm or industry (Neal 1995, Ljungqvist and Sargent 1998). These losses are more likely to matter in declining industries or industries shifting production abroad. During unemployment, human capital may decrease due to the explicit depreciation of skills or as a process of "unlearning by not doing" (Coles and Masters 2000). If unemployment spells persist for a long time, unemployed workers can get detached from the labor market, lose work habits and confidence in the own skills (Falk et al. 2006). The empirical evidence for the depreciation of human capital during unemployment is mixed.[2] The decrease in human capital, upon displacement and during unemployment, has nevertheless been central in explaining the persistence of unemployment and the European unemployment dilemma (Pissarides 1992, Ljungqvist and Sargent 1998, Machin and Manning 1999), as well as the negative duration dependence of exit rates (Blanchard and Diamond 1994, Acemoglu 1995).

In practice, training programs are often targeted to the long-term unemployed. The results of this analysis put this practice into question. It is only optimal to target the long-term unemployed if the depreciation in human capital is sufficiently important relative to the fall upon displacement. In this case, training programs may add a negligible value to an optimally designed insurance system. Training programs are particularly valuable when the fall in human capital upon displacement is significant. In this case, however, participation to training programs should be required from the start of the unemployment spell.

This paper builds on a recent literature on optimal unemployment insurance that departs from stationary models by introducing depreciating human capital. Shimer and Werning (2006) analyze the optimal timing of benefits in a McCall search model,

[2] For instance, Frijters and van der Klaauw (2006) find that the re-employment wage distribution deteriorates significantly, in particular during the first six months of unemployment. On the other hand, Card et al. (2007) and Van Ours and Vodopivec (2006) find no significant effect of an increase in unemployment duration on either the wage or the duration of employment in the new job.

assuming that savings are not observable. Human capital depreciation reduces the arrival rate of job offers or deteriorates the distribution of the wages being paid on the job. Pavoni (2009) analyzes the optimal unemployment insurance contract when the unemployed agent has the binary choice to exert costly search effort or not. The depreciation of human capital reduces the output upon re-employment and the probability to become employed if searching. In this paper, I assume that human capital only determines the output, but search effort is a continuous variable. The probability to become employed thus endogenously decreases during the unemployment spell if no training facility is available. Pavoni and Violante (2007) introduce costly job monitoring as an alternative to the provision of incentives and analyze the optimal sequencing of three unemployment policies (standard unemployment insurance, job monitoring and social assistance) and the optimal consumption profile given these policies. Wunsch (2009) also focuses on the optimal sequencing of policies, but adds job search assistance as a policy.[3]

The paper is organized as follows. In Section 2, the model is presented. In Section 3 and 4, I set up the social planner's problem and I characterize the optimal unemployment insurance contract. In Section 5, I show how the recursive problem simplifies for CARA preferences with monetary costs of efforts. In section 6, I calibrate the model to perform some numerical simulations, focusing on the optimal path of training and consumption. I also calculate the welfare gain from incorporating training programs as an unemployment policy. The last section concludes. The proofs are in appendix.

2 Model

I consider an agent at the start of an unemployment spell. The agent has human capital θ, which determines his production upon re-employment $y(\theta)$, with $y' > 0 \geq y''$ and $y(0) = 0$. During each period of unemployment, the agent exerts efforts in two dimensions, search and training. Search effort s increases the probability $\pi(s)$ to find employment with $\pi' > 0 \geq \pi''$. Once the unemployed agent has found a job, he remains employed forever.[4] Training effort t increases the unemployed's human capital θ and thus output upon re-employment. Both search and training efforts are costly, but the marginal cost of search may increase with the level of training. I assume a convex cost function $\psi(s, t)$ and allow effort to interact as in the multi-tasking setting considered

[3] Pavoni and Violante (2005) and Wunsch (2008) considered a training technology in the numerical simulations of earlier versions of their papers. They assumed that training efforts cannot be imposed, but are induced by rewarding the unemployed for high values of observable human capital with higher unemployment benefits. They also assumed that training and search efforts are extreme rivals and cannot be both exerted in the same period.

[4] I only model incentive problems during one unemployment spell. Wang and Williamson (1996), Zhao (2000) and Hopenhayn and Nicolini (2009) analyze incentive problems on the job and multiple unemployment spells.

by Holmström and Milgrom (1991).[5] A positive cross-derivative $\psi_{s,t}(s,t)$ gives rise to a lock-in effect from mandating training efforts. I assume that training efforts are imposed by the social planner, but all costs are borne by the unemployed and captured by the cost function $\psi(s,t)$.[6]

Preferences The per-period utility during unemployment and employment are denoted by $u(c, \psi(s,t))$ and $u^e(c^e)$ respectively, where c and c^e are the consumption levels during unemployment and upon re-employment respectively. The expected lifetime utility for an agent starting in unemployment equals

$$u(c_0, \psi(s_0, t_0)) + \sum_{\tau=1}^{\infty} \beta^{\tau}[\pi_{\tau}^{e} u^{e}(c_{\tau}^{e}) + (1 - \pi_{\tau}^{e}) u(c_{\tau}, \psi(s_{\tau}, t_{\tau}))],$$

where the probability to be employed in period τ equals $\pi_{\tau}^{e} = \pi_{\tau-1}^{e} + \pi(s_{\tau-1})(1 - \pi_{\tau-1}^{e})$. I focus on two standard types of preference specifications.

Specification 1 (Additive preferences)

$$u(c, \psi(s,t)) = u(c) - \psi(s,t) \text{ and } u^e(c^e) = u(c^e)$$

Specification 2 (CARA preferences with monetary effort cost)

$$u(c, \psi(s,t)) = -\exp(-\sigma(c - \psi(s,t)) \text{ and } u^e(c^e) = -\exp(-\sigma c^e)$$

Human Capital Human capital θ decreases during unemployment. First, human capital falls immediately when the agent loses his job. This fall may capture the loss of firm- or industry-specific human capital upon displacement. Second, human capital continuously depreciates during unemployment. This depreciation may capture the loss of job-skills, self-confidence or work habits, discriminatory preferences of employers or even the foregone learning-by-doing. I only model the depreciation in human capital explicitly, but characterize the optimal contract as a function of the level of human capital at the start of the unemployment spell.

Training Technology Training increases human capital. An unemployed agent with human capital θ_τ, exerting training effort t_τ in period τ, will have human capital at $\tau + 1$ equal to $\theta_{\tau+1} = m(\theta_\tau, t_\tau)$. Human capital depreciation implies $m(\theta_\tau, 0) \leq \theta_\tau$, but training may counter the depreciation $m_t(\theta_\tau, t_\tau) > 0$. I assume exponential

[5] I ignore efforts during employment. With monetary costs of efforts, this is only a rescaling of the net-output produced during employment.

[6] In practice, organizing training programs may be costly for the government or even profitable when for instance trainees are temporarily employed in public jobs. Of course, the social planner can compensate the unemployed agent or be compensated through the monetary transfers.

depreciation with a linear training technology,

$$m(\theta, t) = \theta(1-\delta) + t.$$

Both the foregone income and the decrease in expected future income due to unemployment are increasing in the level of human capital. Without training, the human capital of long-term unemployed converges to 0 for which there is no added-value of being employed. I assume that no training technology is available during employment such that the level of human capital remains constant once re-employed.

3 Social Planner's Problem

The social planner has three instruments at his disposal as a function of the length of the unemployment spell τ: unemployment consumption $\{c_\tau\}$, employment consumption $\{c_\tau^e\}$ and training $\{t_\tau\}$. I assume that the level of human capital at the start of unemployment θ_0 is known to the social planner and I characterize the optimal consumption and training profile conditional on this starting level.[7] I follow the dual approach and minimize the expected cost of the insurance scheme providing a given level of expected life-time utility V_0 to the agent with human capital level θ_0, as in Spear and Srivastava (1987). Rather than writing the optimal contract as a function of the length of the unemployment spell, I write the optimal contract recursively. Since the starting level of human capital is known and the training technology is deterministic, the recursive problem has only two state variables: the unemployed agent's current level of human capital θ and the expected discounted utility promised last period to the unemployed agent V. These two state variables summarize all relevant aspects of the agent's unemployment history. Given this recursive problem, I characterize the optimal contract $\{c(V,\theta), V^e(V,\theta), V^u(V,\theta), s(V,\theta), t(V,\theta)\}$ that assigns expected life-time utility level V to the unemployed individual with human capital θ by , where $V^e(V,\theta)$ and $V^u(V,\theta)$ denote the promised utilities when respectively employed and unemployed in the next period.[8] Notice that the implementation of the optimal consumption profile depends on the extent to which the unemployed have access to savings and whether these savings are observable or not (Werning 2002, Shimer and Werning 2008). If savings could be restricted to zero, the consumption profile can be implemented by setting unemployment benefits equal to c_τ and taxes upon re-employment equal to $y(\theta_\tau) - c_\tau^e$. If savings cannot be restricted, the optimal consumption profile may not be implementable. The relevant trade-off is then between the provision of incentives

[7] I thus ignore unobservable heterogeneity in human capital across job seekers in this analysis. If the social planner was restricted to one plan, he would focus on the starting human capital level of some representative agent.

[8] The optimal contract as a function of the spell length τ can be found by calculating the optimal sequence of state variable pairs $(V_{\tau+1}, \theta_{\tau+1}) = (V^u(V_\tau, \theta_\tau), (1-\delta)\theta_\tau + t(V_\tau, \theta_\tau))$ starting from (V_0, θ_0) and evaluating the optimal consumption levels and training efforts for each of these vectors.

for search and *implementable* consumption smoothing. The analysis suggests that the main channel through which training programs affect the optimal contract is through the search incentives, as training efforts change both the value and cost of search efforts. I thus expect these insights to be robust for the unobservability of savings and ignore this issue here to simplify the analysis.

The optimal contract solves

$$C(V,\theta)=\min_{c,V^e,V^u,s,t} c+\beta\left[\pi(s)C^e(V^e,m(\theta,t))+(1-\pi(s))C(V^u,m(\theta,t))\right]$$

such that

$$u(c,\psi(s,t))+\beta[\pi(s)V^e+(1-\pi(s))V^u]\geq V \qquad (PC)$$

$$s\in\arg\max u(c,\psi(s,t))+\beta\left[\pi(s)V^e+(1-\pi(s))V^u\right]. \qquad (IC)$$

The expected total cost for the social planner consists of the cost this period and the expected cost from the next period on. The cost this period is equal to the unemployment consumption level c. The expected cost from tomorrow on depends on whether the agent finds work today, the respective promised utilities V^e and V^u and the level of human capital. The social planner is constrained to offer a contract for which the agent's expected utility is higher than V. This is captured by the promise-keeping constraint (PC). The search efforts of the unemployed agent cannot be observed. The agent chooses the search level that maximizes his expected utility given the contract, which is captured by the incentive compatibility constraint (IC). The social planner refrains from providing full insurance and creates a wedge between V^e and V^u to give incentives for search. To guarantee the incentive compatibility of the contract, the first order condition of the agent's optimization problem is sufficient if $u(c,\psi(s,t))$ is concave in s,

$$u_\psi(c,\psi(s,t))\psi_s(s,t)+\beta\pi'(s)\left[V^e-V^u\right]=0.$$

Notice that when deciding how intensively to search, the unemployed job seekers care about the consumption levels and not about their potential productivity $y(\theta)$ or the taxes and subsidies separately.

The expected cost for the social planner to assign V^e to the agent after he has found employment, equals

$$C^e(V^e,m(\theta,t))=\min_{c^e,\hat{V}^e} c^e-y(m(\theta,t))+\beta C^e(\hat{V}^e,m(\theta,t))$$

such that

$$\frac{u^e(c^e)}{1-\beta}\geq V^e.$$

Since there is no asymmetric information once the agent is re-employed, it is optimal to keep promised utility constant and give the same level of consumption in every future period. The social planner's problem during unemployment simplifies to

$$C(V,\theta) = \min_{c,V^u,V^e,s,t} c + \beta \left[\pi(s)\frac{(u^e)^{-1}((1-\beta)V^e) - y(m(\theta,t))}{1-\beta} + (1-\pi(s))C(V^u, m(\theta,t))\right]$$

such that

$$V - u(c,\psi(s,t)) - \beta[\pi(s)V^e + (1-\pi(s))V^u] \leq 0 \qquad (\lambda)$$

$$u_\psi(c,\psi(s,t))\psi_s(s,t) + \beta\pi'(s)[V^e - V^u] \leq 0. \qquad (\mu)$$

I proceed under the assumption that $C(V,\theta)$ is convex for the relevant pairs (V,θ).[9] The first order conditions and the two envelope conditions are in appendix.

4 Optimal Insurance Contract

In this section, I characterize analytically how training and consumption optimally evolve during unemployment in three different states: the training state, the search-and-training state, and the stationary state. I do not establish the optimal sequence of states during the unemployment spell formally. However, the numerical simulations in Section 6 suggest a simple and robust sequence. The search-and-training state always converges to the stationary state and is preceeded by the training state if the human capital level at the start of the unemployment spell is sufficiently low.

4.1 Training State

If the level of human capital at the start of the unemployment spell is too low, the unemployed are required to participate to training programs to increase their human capital before being induced to search. The output on re-employment would be very low, making the transition to employment undesirable when an employer is not willing or not able to provide similar training. Since the optimal level of search is zero, no incentives for search are provided. The Lagrange multiplier on the incentive compatibility constraint μ equals zero. The first order conditions for the consumption levels coincide with those in the first best. The Lagrange multiplier on the promise-keeping constraint λ equals the shadow cost of the promised utility V and remains constant during the unemployment spell as long as no search is induced.[10]

Proposition 1 *In the training state ($s_\tau = 0$), $\Delta_{\lambda,\tau} = \lambda_\tau - \lambda_{\tau+1} = 0$.*

With $\mu = 0$, λ equals the inverse of the marginal utility of consumption. The marginal utility of consumption thus remains constant during unemployment in the

[9] In the numerical simulations, I find that for $y(\theta)$ sufficiently concave the value function is indeed convex.

[10] From the first-order conditions, it also immediately follows that $C^e_V(V^e,m) = C_V(V^u,m)$. This implies that the (net) consumption level is also the same upon re-employment. This is only relevant if a job seeker can become employed without searching ($\pi(0) > 0$).

training state.

Corollary 1 *In the training state, unemployment consumption c remains constant for additive preferences.*

Corollary 2 *In the training state, unemployment net-consumption $c - \psi$ is constant for preferences with monetary costs of efforts.*

Since no incentives for search are given, the social planner can completely insure the unemployed agent, as in the first best. The optimal path of consumption is thus constant during the training state. Hence, with the introduction of training, the optimal consumption profile can be temporarily constant. This is in contrast with the result by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). Without training, they find that as long as the unemployed are in the labor force, search should be encouraged and thus unemployment consumption and consumption upon re-employment should be strictly decreasing with the length of the unemployment spell.

Optimal Training If the agent remains unemployed with certainty, the Euler equation characterizing the training profile simplifies for both additive and CARA preferences with monetary costs to

$$\psi_t(0, t_{\tau-1}) = \beta\,(1-\delta)\,\psi_t(0, t_\tau).$$

When increasing training at $\tau - 1$ by one unit, training at τ can be decreased by $1-\delta$ units without changing the level of human capital at $\tau + 1$. With no prospects for employment, deferring training is desirable because the effect of training depreciates over time and the cost of future training is discounted.

Proposition 2 *In the training state, the optimal level of training is increasing over time ($t_{\tau-1} < t_\tau$) for additive and CARA preferences and exponential decay.*

4.2 Search-and-Training State

When the level of human capital is sufficiently high, the social planner induces the unemployed to search for a job by providing incomplete insurance. The social planner changes both consumption during unemployment and upon re-employment to provide incentives.

The shadow price of the expected utility of the unemployed V decreases during unemployment in the search-and-training state. This follows immediately from the Euler equation. The intuition is that higher promised utility tomorrow relaxes the promise-keeping constraint today, but also decreases the incentives to search for a job today. At the optimum, the shadow price of promised utility tomorrow equals the shadow price of promised utility today minus its impact on incentives for search today,

$$\lambda_{\tau+1} = \lambda_\tau - \mu_\tau \frac{\pi'(s_\tau)}{(1-\pi(s_\tau))}. \tag{1}$$

Proposition 3 follows since the Lagrange multiplier on the IC constraint is positive when search is induced, regardless of the presence of training.

Proposition 3 *In the search-and-training state ($s_\tau > 0$), $\Delta_{\lambda,\tau} = \lambda_\tau - \lambda_{\tau+1} > 0$.*

The shadow price λ_τ is equal to $C_V(V_\tau, \theta_\tau)$ and thus depends on both V and θ. For both additive preferences and CARA preferences, the shadow price takes a simple form though.

With additive preferences, the shadow price equals the inverse of the marginal utility of unemployment consumption, $\frac{1}{u'(c_\tau)}$. Hence, if search is positive, the result by Shavell and Weiss (1979) that unemployment consumption is decreasing still holds.

Corollary 3 *In the search-and-training state, unemployment consumption c decreases for additive preferences.*

The presence of the training technology does not change the rationale for a decreasing consumption path, as long as search is induced. If unemployment consumption were constant, the social planner could increase consumption this period and decrease consumption next period such that the social planner's expected cost remains the same for a given level of search. Since the consumption levels and therefore the marginal utilities are initially the same, the change in the consumption pattern has only a second order effect on the expected utility of the unemployed agent. The reduction in tomorrow's unemployment consumption will induce a higher search level though and thus decreases the expected payments to be made by the social planner. Interestingly, although the threat of future training requirements induces job seekers to search harder today, the social planner does not stop using the threat of lower future consumption levels for long-term unemployed job seekers.

With CARA preferences with monetary costs, the shadow price equals an inverse function of the promised utility V, independent of the level of human capital.[11]

Corollary 4 *In the search-and-training state, the promised utility during unemployment V decreases for CARA preferences with monetary costs of efforts.*

[11] Notice that if preferences are not additive, the Shavell and Weiss argument for decreasing consumption may not hold for two reasons. First, if efforts are changing over time, equality of unemployment consumption may not imply equality of marginal utility with respect to consumption. Second, the decrease in consumption next period may increase the marginal cost of search next period. The implied decrease in search next period may outweigh the increase in search this period. With monetary costs of efforts, equality of net-consumption does imply equality of marginal utilities. However, a decrease in net-consumption still increases the marginal cost of search.

Spreading the incentives for current search efforts over all future periods of unemployment allows providing more insurance ex ante. The presence of training does not change this. However, since it mitigates the depreciation in human capital, the social planner may prefer to give less incentives for search and focus more on insurance. If the introduction of training decreases the value of $\mu_\tau \frac{\pi'(s_\tau)}{1-\pi(s_\tau)}$ in condition (1), the social planner will actually smooth consumption more. The numerical simulations in Section 6 suggest that the decrease in net-consumption during unemployment is less pronounced in the optimal scheme with training compared to the optimal scheme without training, in particular when the level of human capital is low and training is thus intensive.

The social planner also adjusts consumption upon re-employment to improve the trade-off between incentives and insurance. From the first order conditions with respect to V^e and V^u, I find that

$$C_V(V,\theta) = \pi(s)C_V^e(V^e, m(\theta,t)) + (1-\pi(s))\, C_V(V^u, m(\theta,t)). \qquad (2)$$

With additive preferences, this simplifies to the Rogerson condition (or inverse Euler equation),

$$\frac{1}{u'(c_\tau)} = \pi(s_\tau)\frac{1}{u'\left(c^e_{\tau+1}\right)} + (1-\pi(s_\tau))\,\frac{1}{u'(c_{\tau+1})},$$

which implies that consumption upon re-employment must exceed unemployment consumption. The successful job seeker is immediately rewarded with higher consumption.[12] This reward upon re-employment, however, is made dependent on the duration of the unemployment spell, spreading again incentives over all future states, also during employment. Hopenhayn and Nicolini (1997) find conditions for additive preferences, without human capital decay and training, under which consumption upon re-employment is decreasing for long-term unemployed. Corollary 5 provides a generalization of their result

Corollary 5 *Consumption upon re-employment cannot remain constant or be always increasing with the length of the unemployment spell, both for additive preferences and CARA preferences with monetary costs.*

The presence of training affects the degree of consumption smoothing, but does not change the nature of the conditions characterizing the optimal consumption levels in the search-and-training state. The resulting changes in human capital, however, do affect the implementation of the optimal consumption path. The social planner wants to provide insurance against the loss of human capital, disconnecting the consumption levels during unemployment and upon re-employment from the (potential) productivity

[12] This again does not generalize for CARA preferences with monetary cost of efforts, since high unemployment consumption reduces the marginal effort cost of search.

of the agent. Taxes upon re-employment that increase with the length of the unemployment spell do not necessarily imply decreasing consumption levels. If human capital decreases during the unemployment spell, taxes may be lower after long unemployment spells to smooth consumption. If human capital increases with the length of the unemployment, taxes may be higher after long unemployment spells to preserve incentives for search. In general, optimal taxes upon re-employment can thus be positive or negative and depend non-monotonically on the length of the unemployment spell. Similarly, the optimal unemployment replacement rate with respect to the potential wage does not necessarily evolve monotonically during unemployment.

Optimal Training Human capital depreciation increases both the value of insurance and the value of inducing search. However, the consumption schedule cannot provide more incentives for search without reducing insurance and vice versa. By countering the depreciation, training is a valuable alternative. The value of training depends on how much the unemployed decide to search, which depends on how much they are required to train. Training and search are complements with respect to the expected value of finding employment, but they are substitutes with respect to the costs, as training increases the marginal cost of search. When the social planner requires more intensive training efforts, it can still fully control the private gains from finding employment by adjusting the consumption levels, but it cannot avoid that additional search is more costly. The value of training and search efforts depends on the level of human capital as well. Conditional on search, the return to training is higher for low levels of human capital if $y''(\theta) \leq 0$. However, conditional on training, the return to search is higher for high levels of human capital since $y'(\theta) > 0$. Compared to the training state, the impact of training on the incentives for search and on potential output in the next period becomes relevant. For additive preferences, the Euler equation equals

$$\frac{\psi_t(s_{\tau-1}, t_{\tau-1})}{u'(c_{\tau-1})} + \mu_{\tau-1}\psi_{s,t}(s_{\tau-1}, t_{\tau-1}) =$$
$$\beta\pi(s_{\tau-1})\frac{y'(\theta_\tau)}{1-\beta} + \beta(1-\pi(s_{\tau-1}))[\frac{\psi_t(s_\tau, t_\tau)}{u'(c_\tau)} + \mu_\tau\psi_{s,t}(s_\tau, t_\tau)]\,(1-\delta)\,.$$

For CARA preferences with monetary costs, the Euler equation equals

$$\psi_t(s_{\tau-1}, t_{\tau-1}) + \mu_{\tau-1}u'(c_{\tau-1} - \psi_{\tau-1})\psi_{s,t}(s_{\tau-1}, t_{\tau-1}) =$$
$$\beta\pi(s_{\tau-1})\frac{y'(\theta_\tau)}{1-\beta} + \beta(1-\pi(s_{\tau-1}))\left[\psi_t(s_\tau, t_\tau) + \mu_\tau u'(c_\tau - \psi_\tau)\psi_{s,t}(s_\tau, t_\tau)\right](1-\delta)\,.$$

For both types of preferences, increasing $t_{\tau-1}$ increases the cost of training efforts, but changes also the incentives by increasing the marginal cost of search if $\psi_{s,t} > 0$. On the other hand, the increase in training at $\tau - 1$ increases human capital at τ. This increases the output when a job is found. When no job is found, this increase allows to decrease training at τ to bring human capital back to the same level at $\tau + 1$ if $t_{\tau-1}$ had not been increased.[13] In section 6, I use numerical simulations to get more insights in the value of training and its timing during the unemployment spell. Notice that when the job seeker is given the choice how much to train rather than how much to search, his choice of training efforts would be efficient if he receives the resulting increase $y'(\theta_\tau)$ in output when re-employed.[14]

4.3 Stationary State

If training programs are effective, the long-term unemployed may converge to a stationary state with positive human capital θ^*. In such a stationary state, the same level of human capital is maintained with constant training effort $t^* = \delta\theta^*$. The unemployed exit unemployment with positive probability, otherwise the social planner would not impose costly training efforts. Hence, under convergence to such a state, the social planner never gives up on the unemployed and continues to induce them to find employment. This is in stark contrast with the optimality of *social assistance* when no training facility is available (Pavoni and Violante 2007). Individuals on social assistance receive welfare payments and are not encouraged to find employment. Without training, the depreciation of human capital causes the potential production to be too small compared to the cost of inducing search after a finite time of unemployment.[15] Hence, the unemployed enter social assistance within finite time and once they have entered social assistance, they never leave again.

The introduction of training thus changes the optimal consumption path for the long-term unemployed as well. Without training facility, the unemployment consumption path becomes constant in finite time, as the unemployed enter social assistance. With training facility, search continues to be induced as human capital converges to a positive level. Hence, Proposition 3 still applies.

Corollary 6 *In a stationary state with $\theta^* > 0$, unemployment net-consumption and consumption upon re-employment are decreasing with the length of the unemployment spell for CARA preferences with monetary cost of effort.*

The corollary does not prove convergence. The numerical simulations in section 6, however, show that human capital indeed converges to a positive stationary level

[13] Both Euler equations assume that the optimal level of training effort is positive. If the level of human capital is sufficiently high, it may be optimal not to require any training at all at the start of the unemployment spell, but only start training the unemployed when human capital has sufficiently depreciated, and provide incentives for search simultaneously.

[14] However, by also allowing employment consumption to depend on the length of the unemployment spell, which is standard in the literature since Hopenhayn and Nicolini (1997), the incentives for training, conditional on search, are no longer perfectly aligned between the social planner and the unemployed.

[15] This requires that without training technology, human capital converges to a sufficiently small level. Pavoni and Violante (2007) assume human capital converges to 0.

if training is sufficiently effective. Moreover, the convergence is monotone and global. Regardless of the level of human capital at the start, human capital converges to this unique level.

If the unemployed can train to maintain their human capital, there is no reason to stop providing incentives for search, even after very long unemployment spells. This intuition naturally generalizes to other preference specifications. That this results in ever decreasing expected utility for the long-term unemployed, and thus ever decreasing consumption for CARA preferences, of course assumes that there is no lower bound on the expected utility (see Pavoni 2007), for instance coming from limited liability or political constraints.

5 CARA Preferences with Monetary Costs

In this section, I show that for CARA preferences with monetary costs the value function is additive in V and θ efforts, $C(V,\theta) = h(V) - g(\theta)$. I guess and verify $h(V)$, as in Werning (2002) and Shimer and Werning (2008), which only leaves $g(\theta)$ to be approximated numerically.

With CARA preferences, the optimal response to an increase in promised utility V is to increase all consumption levels by the same amount, regardless of the level of human capital. Increasing the consumption levels equally, today and in the future, while employed and unemployed, leaves the margins for search and training unchanged. For search, this is clear from the incentive compatibility constraint and the properties of CARA preferences. Since $u(x+y) = -u(x)u(y)$ and $u(x) = -\frac{u'(x)}{\sigma}$, the promised utilities V^e and V^u and marginal utility $u'(c-\psi)$ are all rescaled by $-u(\varepsilon)$ after an ε-increase in all consumption levels. Hence, the incentive compatibility constraint,

$$\beta\pi'(s)[V^e - V^u] = u'(c-\psi)\psi_s(s,t),$$

remains binding after an equal increase in all consumption levels. The fact that an equal increase in all consumption levels is an optimal response to an increase in V implies that the optimal promised utilities V^e and V^u and current-period utility $u(c-\psi)$ are proportional to life-time utility V, for a given level of human capital θ. I can rewrite the optimal contract as $\{\alpha_u(\theta), \alpha_{V^e}(\theta), \alpha_{V^u}(\theta), s(V,\theta), t(V,\theta)\}$ with

$$V^e(V,\theta) = \alpha_{V^e}(\theta)V \quad (3)$$

$$V^u(V,\theta) = \alpha_{V^u}(\theta)V \quad (4)$$

$$u(c-\psi) = \alpha_u(\theta)V. \quad (5)$$

Given exponential utility, the optimal increase in the consumption levels in response to an increase in V is thus independent of the level of human capital and does not interact with search or training either. The value function is additive in θ and V.

Proposition 4 *For CARA preferences with monetary cost of efforts, we have*

$$C^{e}(V^{e},\theta)=-\frac{\ln(-V^{e}(1-\beta))}{\sigma(1-\beta)}-\frac{y(\theta)}{1-\beta}$$

and

$$C(V,\theta)=-\frac{\ln(-V(1-\beta))}{\sigma(1-\beta)}-\frac{g(\theta)}{1-\beta}$$

for some unknown function $g(\theta)$.

I rewrite and simplify the Bellman equation for $C(V,\theta)$ in terms of $g(\theta)$ in appendix, using the expressions in Proposition 4. In the following section, I solve numerically for $g(\theta)$ and the optimal policy variables as a function of human capital θ. In a stationary state, however, human capital θ^* and thus training t^* are constant. From equations (3), (4) and (5), it follows that the relative change in per-period utility is constant during the stationary state

$$\frac{u\left(c_{\tau}-\psi\left(s_{\tau},t^{*}\right)\right)}{u\left(c_{\tau+1}-\psi\left(s_{\tau+1},t^{*}\right)\right)}=\frac{u\left(c_{\tau}^{e}\right)}{u\left(c_{\tau+1}^{e}\right)}=cst.$$

Like in a model without human capital and training (see Werning 2002 and Spinnewijn 2009), this implies that consumption during unemployment and upon re-employment is constantly decreasing in the length of the unemployment spell and the level of search effort exerted is constant as well. The optimal stationary level of search and training (and thus human capital) can be characterized without knowing $g(\theta)$. In appendix, I derive that the stationary state must satisfy

$$\begin{aligned}\beta\pi'(s^*)[\frac{y(\theta^*)-\psi(s^*,\delta\theta^*)+\kappa/(1-\beta)}{1-\beta(1-\pi(s^*))}]+\frac{\partial\kappa}{\partial s}/(1-\beta) &= \psi_s(s^*,\delta\theta^*)\\ \beta\pi(s^*)\frac{y'(\theta)/(1-\beta)}{1-\beta(1-\pi(s^*))(1-\delta)}+\frac{\partial\kappa}{\partial t}/(1-\beta) &= \psi_t(s^*,\delta\theta^*).\end{aligned}$$

The expected gain of search is the increased probability to produce output $y(\theta)$ rather than to search and train at cost $\psi(s,\delta\theta)$. The expected gain from training is the increase in production upon re-employment $y'(\theta)$. The difference in discounting between search and training comes from the fact that training efforts add to a depreciating stock of human capital, whereas search efforts vanish every period. The influence of the incentive compatibility is completely reflected in the function $\kappa(s,\delta\theta,\frac{\alpha_{V^u}}{\alpha_u})$, which equals 0 in the first best and is described in appendix.

6 Numerical Simulations

In this section, I calibrate the dynamic model to calculate the optimal unemployment insurance contract numerically. I analyze the optimal timing of training and search during the unemployment spell and the interaction with the optimal path of consumption. The numerical methodology is based on value function iteration with discretization of the state space. I restrict attention to CARA preferences with monetary effort costs which essentially makes the state space one-dimensional as shown in Proposition 4. The calibration exercise closely follows the previous literature when possible. In contrast with the previous literature, I explicitly model the returns and costs of continuous search and training efforts. I calibrate the search parameters to match the empirical estimates of the exit rate and the elasticity with respect to the unemployment benefit level, evaluated at the current US unemployment insurance scheme. I take the same parameters for the training effort costs, but I vary the parameter values capturing the effectiveness of training efforts and the cost rivalry with search efforts.

6.1 Calibration

CARA Preferences The unit of time is set to be one month and the monthly discount factor $\beta = 0.996$ to match an annual discount factor of 0.95. The unemployed individual has CARA preferences $u(c, \psi(s,t)) = -\exp(-\sigma(c - \psi(s,t)))$ with CARA coefficient $\sigma = 2$.

Human Capital Depreciation Human capital depreciates exponentially at a monthly depreciation rate $\delta = 0.0135$, following Pavoni and Violante (2007). Human capital determines output upon re-employment, $y(\theta) = \theta^{\omega}$ with $\omega < 1$.

Search Costs and Returns The probability to find a job as a function of search effort s equals $\pi(s) = 1 - \exp(-\rho s)$, following Hopenhayn and Nicolini (1997). I assume a convex monetary cost of search $\psi_0 s^{\psi_1}$.

The parameters ω, ρ, ψ_0 and ψ_1 are chosen as follows. First, the calibrated model - evaluated for an insurance contract approximating the current US unemployment insurance system (i.e. a replacement rate of 50 percent for 6 months) - matches two empirical moments; the simulated exit rate equals 0.2, matching the sample exit rate in Spinnewijn (2009), the simulated elasticity in the exit rate with respect to a change in benefits equals 0.5, matching the empirical evidence summarized in Krueger and Meyer (2002). Second, the stationary level of human capital in the optimal scheme is normalized to 1 and the stationary search level when unemployed corresponds to a monetary cost equal to one third of output when employed. This calibration exercise implies $\omega = 4/5$, $\rho = 1/8$, $\psi_0 = 1/4$ and $\psi_1 = 6/5$.

Training Costs and Returns I assume the same monetary cost of training $\psi_0 t^{\psi_1}$ and introduce a linear interaction term $\psi_{s,t} st$ in the cost function,

$$\psi(s,t) = \psi_0 s^{\psi_1} + \psi_0 t^{\psi_1} + \psi_{s,t} st.$$

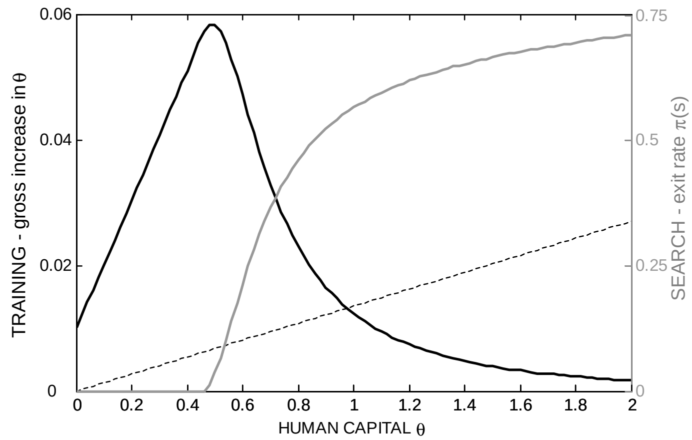

Figure 1: Policy functions: training and search efforts as a function of human capital θ. Training is expressed as the gross increase in θ ($z \times t$). Search is expressed as the exit rate ($\pi(s)$). The depreciation in human capital is presented by the dotted line.

I assume a linear training technology such that the next month's level of human capital equals $m(\theta, t) = (1 - \delta)\,\theta + zt$. I show results for different values of z and $\psi_{s,t}$, capturing the effectiveness of training and the complementarity with search respectively. Given the optimal contract and the standard parameter specification ($\psi_{s,t} = 0.02; z = .003$), the long-term unemployed spend half as much effort on training relative to search.

6.2 Optimal Policy Functions

Figure 1 presents the optimal level of training and search efforts as a function of the level of human capital. The training and search efforts are represented by respectively the resulting gross increase in human capital $z \times t$ and the exit rate $\pi(s)$ respectively. The training efforts are imposed by the social planner, while the search efforts are chosen by the job seeker. The depreciation in human capital $\delta\theta$ is presented by the dotted line.

The unemployed agent is in the training or the search-and-training state, depending on whether his human capital is lower or higher than the cut-off level $\underline{\theta} = .46$. For levels below $\underline{\theta}$, the level of human capital is too low to induce the unemployed to search for work. The unemployed agent only exerts effort to increase the level of human capital. The training intensity is increasing in human capital, in line with Proposition 2. For levels above $\underline{\theta}$, the level of human capital is sufficiently high for the social planner to induce the unemployed to search for work at the expense of complete insurance. The induced search efforts are increasing in the level of human capital. The exponential depreciation of human capital makes that the foregone output and the expected loss in future output are increasing with the level of human capital. The social cost of remaining unemployed is thus higher, the higher the level of human capital. In the search-and-training state, the unemployed is not only induced to search for work, but

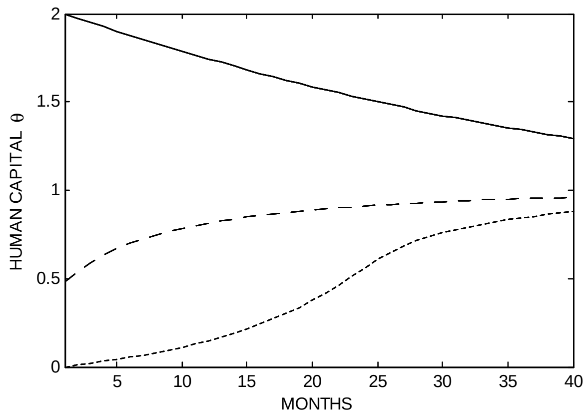

Figure 2: Human capital θ during the unemployment spell, starting with high human capital ($\theta_0 > \theta^*$ - solid), low human capital ($\theta_0 < \theta^*$ - dashed) and no human capital ($\theta_0 = 0$ - dotted).

at the same time obliged to participate in training programs.

The training and search policy function have opposite slopes. This is first of all driven by the rivalry in the cost of effort function. Time spent on training cannot be spent on search. Hence, the marginal cost of search is likely to be higher when the unemployed is required to participate in training programs (i.e. $\psi_{s,t} > 0$). Second, the value of training and search efforts interact with the level of human capital. Search is more valuable when human capital is high, whereas training is more valuable when human capital is low. These two effects dominate the complementarity between training and search efforts coming from their interaction in determining the next period's expected output $\pi(s)\,y(m(\theta,t))$.

6.3 Optimal Path of Training and Human Capital

The change of human capital during the unemployment spell depends on difference between the gain in human capital due to the training efforts and the loss in human capital due to depreciation. Figure 1 shows that the training program exactly offsets the deprecation at the stationary level $\theta^* = 1$. Below this level ($\theta < \theta^*$), human capital increases. Above this level ($\theta > \theta^*$), human capital decreases. Figure 2 shows how the unemployed's human capital evolves with the length of the unemployment spell for three different levels of human capital θ_0 at the start; high ($\theta_0 > \theta^*$), low ($\theta_0 < \theta^*$) and no human capital ($\theta_0 = 0$). The convergence is global. Regardless of this level, human capital converges to the stationary level θ^*. Once the stationary level is reached, the social planner imposes training efforts such that human capital remains constant. This implies that the social planner never gives up on the unemployed. Even the long-term unemployed are trained to remain employable. This policy conclusion is in stark contrast with the optimality of social assistance when no training facility is available

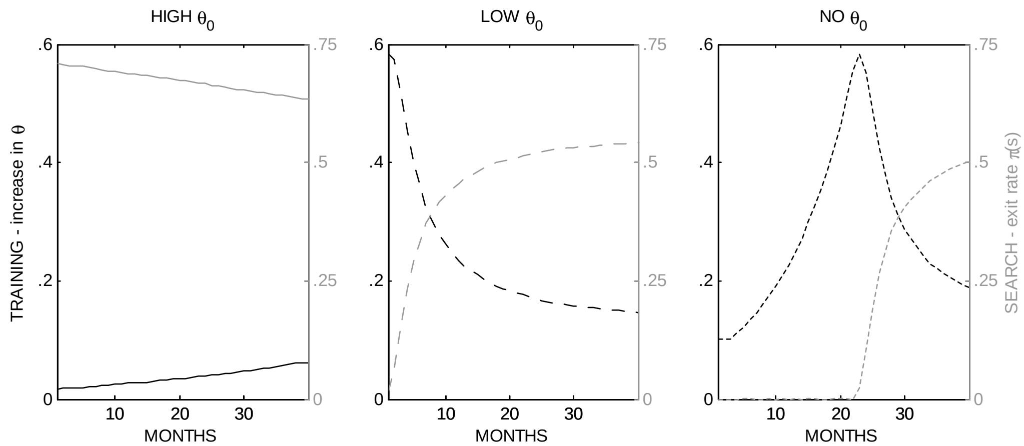

Figure 3: Training and search during the unemployment spell, starting with high human capital ($\theta_0 > \theta^*$ - solid), low human capital ($\theta_0 < \theta^*$ - dashed) and no human capital ($\theta_0 = 0$ - dotted).

(Pavoni and Violante 2007).

The convergence of human capital is also monotone and the stationary level θ^* is unique. Two unemployed individuals who are identical except for the level of human capital at the start of the unemployment spell converge to the same level of human capital. This has straightforward implications for the average training intensity and the timing of the training programs during the unemployment spell. First, the level of training is decreasing in the level of human capital for human capital levels $\theta > \underline{\theta}$, as shown in Figure 1. The monotone and global convergence thus implies that training is more intensive at any point during the unemployment spell for the unemployed who start the unemployment spell with a lower level of human capital, unless they start in the training state ($\theta < \underline{\theta}$).[16] Second, the difference between the human capital level at the start of the unemployment spell θ_0 and in the stationary state θ^* determines the optimal timing of training throughout the unemployment spell. Figure 3 shows the optimal training path during unemployment for different starting levels of human capital θ_0. If human capital is high at the start ($\theta_0 > \theta^*$), training is less intensive at the beginning of the unemployment spell and becomes more intensive during the unemployment spell. If human capital is low at the start ($\underline{\theta} < \theta_0 < \theta^*$), training is very intensive at the beginning of the unemployment spell and becomes less intensive during the unemployment spell. Finally, for very low human capital at the start ($\theta_0 < \underline{\theta}$), the unemployed agent starts in the training state. Training will be increasing at the beginning of the unemployment spell, but starts decreasing once θ passes $\underline{\theta}$. When the unemployed starts without any valued human capital, this training stage takes more than twenty months, as shown in the right panel of Figure 3. The panels also show

[16] During the training state, a job seeker with lower human capital will be subject to lower training intensity because of the convexity of training efforts and the depreciation of the returns to training (see Section 4.1).

the duration-dependence of the exit rates. As discussed before, the optimal levels of training and search follow opposite trends during the unemployment spell.

The difference between θ_0 and θ^* can be linked to the two sources for the loss of human capital after job loss. Human capital falls upon job loss and depreciates during unemployment. The fall in human capital when losing a job is reflected in the θ_0. Two identical agents will have different levels of human capital at the start of the unemployment spell, if the firm-specific or industry-specific capital they lose when losing their job is different. The depreciation in human capital is reflected in θ^*. Increasing the rate of depreciation decreases the stationary level of human capital. Therefore, if the fall in human capital upon displacement becomes more important, relative to the depreciation of human capital during unemployment, it is more likely that training should be targeted more to the short-term unemployed than to the long-term unemployed.[17]

In practice, training requirements are mostly imposed on the long-term unemployed. Only after some time in unemployment, one needs to enroll in particular training programs to remain eligible for unemployment benefits. Recent examples are the 'New Deal' in the United Kingdom and the 'Activation of Search Behavior' in Belgium. Such programs are more desirable if the long-term unemployed are particularly unemployable because of the depreciation of job skills during unemployment, alienation from the job market or the lack of on-the-job training. Not all programs have this particular focus on the long-term unemployed. In some countries, training is subsidized from the start of the unemployment spell and the unemployed are allowed to refuse jobs offered by the Public Employment Service if they enroll in these training programs. Some programs are aimed at young people or focus on large groups of workers who have been displaced as a result of industrial restructuring (e.g. public training programs in Germany after the unification). In general, targeting training programs to the short-term unemployed becomes more important if firm-specific and industry-specific human capital are significant and job displacement causes a big drop in human capital.

6.4 Optimal Path of Consumption

The use of training programs affects the optimal path of consumption during unemployment and upon re-employment. The introduction of training generally changes the trade-off between the provision of insurance and incentives for search. It even reverses previous results regarding the optimal consumption path by changing whether it is optimal to induce search at different times during the unemployment spell.

The introduction of (effective) training programs makes that the social planner never stops inducing the long-term unemployed to search for work. The optimal con-

[17]Notice that although the stationary level of human capital θ^* is decreasing in δ, the stationary training level $t^* = \delta\theta^*$ may be increasing in δ. However, whether t is higher for the short-term unemployed or the long-term unemployed only depends on the sign of $\theta_0 - \theta^*$ because of the monotone, global convergence.

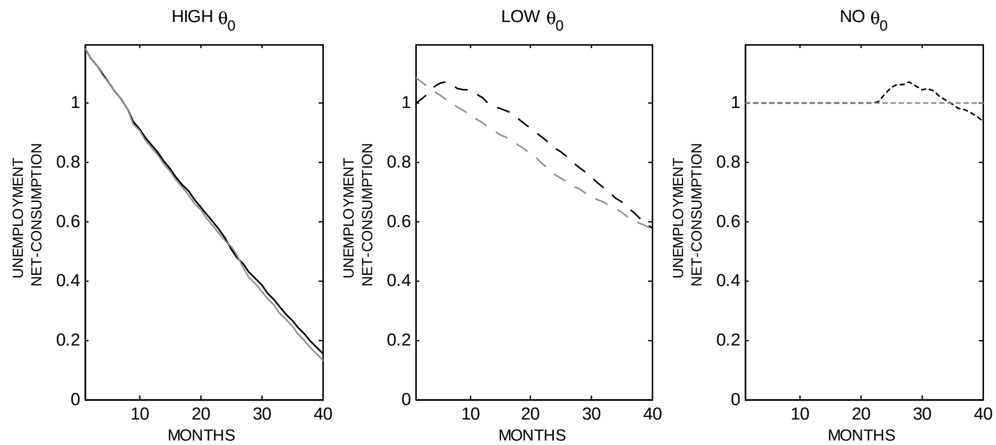

Figure 4: Net-consumption during the unemployment spell with training technology (black) and without training technology (grey), starting with high human capital ($\theta_0 > \theta^*$ - solid), low human capital ($\theta_0 < \theta^*$ - dashed) and no human capital ($\theta_0 = 0$ - dotted).

sumption path necessarily becomes downward sloping for long unemployment spells, as shown in Corollary 4. This is different when no training facility is available, as considered by Pavoni and Violante (2007). The long-term unemployed end up with social assistance in which they are not induced to search. Without training, the optimal consumption path necessarily becomes constant for long unemployment spells (right panel of Figure 4). The opposite pattern may occur at the start of the unemployment spell. If human capital is sufficiently low, the social planner only imposes training efforts and induces no search. Since no incentives for search are needed, there is no need to give up perfect consumption smoothing. Hence, consumption is constant during the training state of the unemployment spell (right panel of Figure 4). However, with no training facility, the optimal contract induces search by having strictly decreasing net-consumption levels as long as human capital is not too low at the start (Shavell and Weiss 1979, Hopenhayn and Nicolini 1997). The consumption scheme is the social planner's only instrument to insure the unemployed and provide incentives for search. Since training mitigates the effect of human capital depreciation, it reduces the need for search for a given level of human capital. This generally allows the social planner to focus more on insurance and smooth the marginal utility of consumption. Hence, net-consumption will not be as rapidly decreasing in the beginning of the unemployment spell, particularly when it is optimal to impose intensive training as shown in the center panel of Figure 4. Due to the interactions between consumption and the search and training efforts, the path may be even upward sloping . This suggests that training as an active labor market policy is more complementary to a continental European unemployment insurance scheme with low incentives for search (high and slowly decreasing benefits) than to the US unemployment insurance scheme with high incentives for search (low and rapidly decreasing benefits).

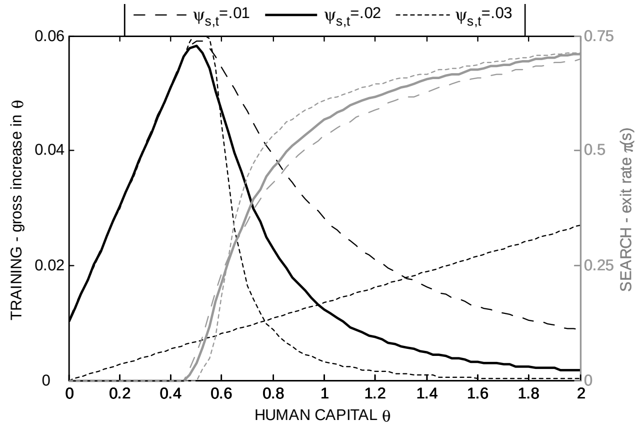

Figure 5: Effort substitutability: training and search policy functions for different parameter values for the cross-derivative of the effort cost function $\psi_{s,t}$.

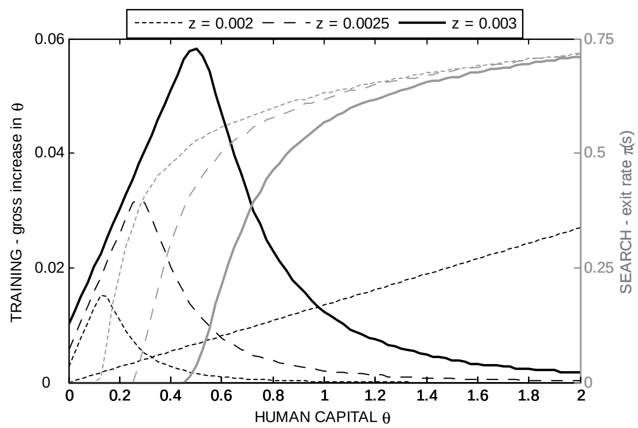

Figure 6: Training effectiveness: training and search policy functions for different parameter values for the impact of training z.

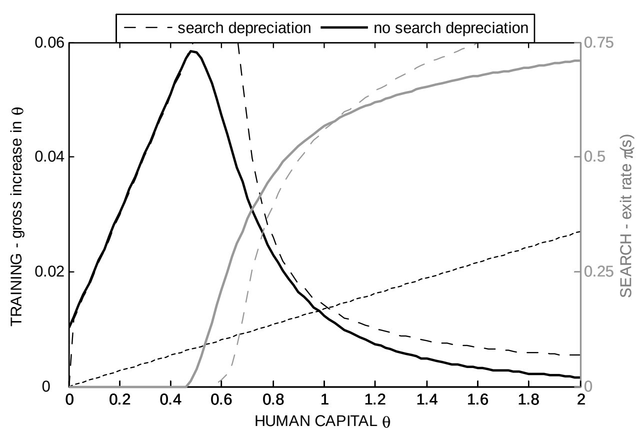

Figure 7: Search skill depreciation: training and search policy functions with and without search skill depreciation.

6.5 Robustness

I illustrate how the policy functions change when changing the effort substitutability, the effectiveness of the training programs and the returns to search. An increase in the cross-derivative $\psi_{s,t}$ increases the rivalry between training and search efforts. When subject to a given training program, the job seeker will search less when $\psi_{s,t}$ is higher. Figure 5 shows the policy functions for a higher and lower value of $\psi_{s,t}$. The higher $\psi_{s,t}$, the more likely it is that the social planner either imposes intensive training programs or induce intensive search. The level of human capital is lower in the stationary state. The social planner requires less training, but induces more search.

The optimal training intensity depends on the effectiveness of the training programs. Empirical studies suggests that the impact of training programs is very heterogeneous (Heckman et al. 1999). Figure 6 shows the optimal policy functions for different values of z in the human capital accumulation process,

$$m(\theta, t) = \theta\,(1 - \delta) + zt.$$

If the impact of training z decreases, the desired level of training is lower. If training has no impact on output, the social planner will not impose or threaten to impose any training. Training plays no deterring role here. The social planner can discourage the unemployed from remaining unemployed by lowering future unemployment benefits instead and actually increase its revenues. The impact of training effectiveness on the expected unemployment duration is ambiguous. For a given level of human capital, the optimal level of search is higher, since leaving unemployment is the only way to avoid the depreciation of human capital. However, since human capital decreases more rapidly with less effective training, the long-term unemployed search less. Notice also that the cut-off level $\underline{\theta}$ below which the unemployed start in the training state is lower when the effectiveness of training is smaller. The search policy function not only shifts up, but also to the left, as shown in Figure 6.

The depreciation of human capital only affects the output upon re-employment. This naturally implies that the exit rate depends negatively on the duration of the unemployment spell. I now introduce this negative dependence exogenously as well through *search depreciation*, following Shimer and Werning (2006). I assume

$$\pi(s, \theta) = 1 - \exp(-\rho\theta s)$$

such that the marginal return to search depends directly on the level of human capital ($\pi_{s,\theta} > 0$). Training does not only increase the output upon re-employment, but also the transition rates. Figure 7 compares the policy functions. The optimal exit rate is more responsive to human capital. The optimal level of training is higher overall, particularly for low levels and high levels of human capital in the search-and-training

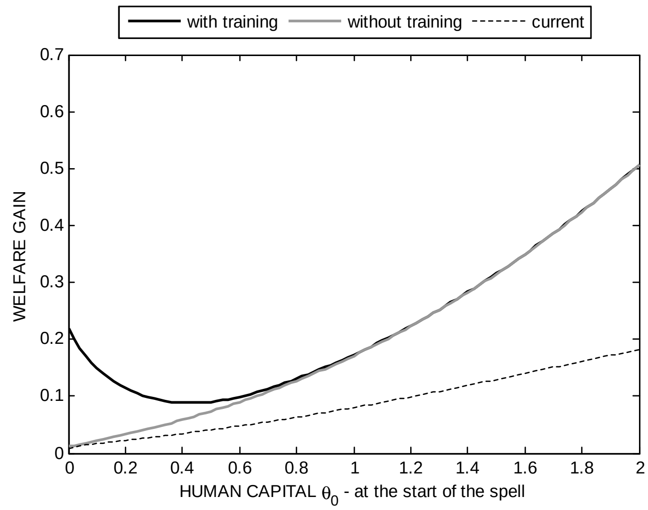

Figure 8: Welfare Comparison: welfare gain in terms of per-period consumption for the optimal policy with training, the optimal policy without training and the current policy, as a function of the level of human capital at the start of the unemployment spell.

state.

6.6 Welfare Gains

To evaluate the advantages of training programs for unemployment policies, I compare the welfare gains from the optimal schemes with and without training programs. I calculate the welfare gain as a function of the level of human capital at the start of the unemployment spell. I set the expected cost for the two schemes equal to the calibrated expected cost of the current US unemployment insurance scheme, which depends on the starting level of human capital. The current scheme provides a monthly unemployment benefit for a maximum duration of six months of about 50 percent of the pre-unemployment earnings. I set the monthly pre-unemployment earnings equal to 1, which is the output level to which the long-term unemployed converge given the optimal contract. The welfare gains are expressed in terms of the per-period consumption an unemployed job seeker without insurance is willing to pay for the unemployment insurance scheme.

Figure 8 shows the welfare gain as a function of the level of human capital at the start of unemployment for three different schemes: the optimal scheme with training, the optimal scheme without training and the current scheme. By construction, the optimal scheme which allows for the use of training dominates the optimal scheme without training, and both schemes dominate the current scheme. However, the relative welfare gains are very different depending on the initial level of human capital. Although the effectiveness of training remains unchanged, the additional welfare gain from introducing training is negligible for high starting levels of human capital, while it is very large for low starting levels of human capital. The gain increases exponentially up to 0.2 of monthly consumption as θ_0 decreases. The pattern is opposite for the welfare gain from financial insurance. Both the value of the current scheme and the additional value of the optimal insurance scheme without training are increasing in the starting level of human capital. As human capital is higher, the output upon re-employment $y(\theta)$ is higher. This increases both the willingness to pay for unemployment benefits and the scope for consumption smoothing.

The welfare analysis suggests that training is complementary as an unemployment policy, creating high value when financial insurance falls short. This results in the U-shape for the optimal policy's welfare gain as a function of human capital. A related implication is that if the conditions apply under which it is optimal to increase the training intensity during the unemployment spell, the introduction of training programs may add very little value to an unemployment insurance scheme that is optimally designed. This questions the focus on the long-term unemployed of many training policies implemented in practice.

7 Conclusion

The secular trend of increasing production mobility, technological innovations and shifts in consumer demand forces workers to switch jobs (and industries) more frequently. Job mobility does not only involve the risk of unemployment, but also the risk of wage loss (Low, Meghir and Pistaferri 2010). Displaced workers are often reemployed at lower wages and the persistent nature of this shock makes insurance against wage risk imperative. I approach training programs as a complementary unemployment policy to deal with the loss in wages. If the skill set of a displaced worker becomes redundant, incorporating training programs in unemployment policies is particularly valuable and these programs should be targeted towards the recently displaced. Only if the depreciation of human capital during the unemployment spell is sufficiently important, training should be targeted towards the long-term unemployed, as we often observe in practice.

The model has focused on the provision of insurance under moral hazard, ignoring political constraints and unobservable heterogeneity, two potential explanations for the focus on long-term unemployed. Requiring training programs only for the long-term unemployed helps *deterring* job seekers from remaining unemployed (Besley and Coate 1992, Black, Smith, Berger and Noel 2003) and allows the job seekers with high human capital to leave unemployment before the start of the costly programs. Notice that without political constraints, the social planner would always prefer to deter the unemployed by threatening to lower the monetary transfers rather than by imposing ineffective training programs. Moreover, if a menu of schedules could be offered, requiring intensive training from the start in the schedule designed for the job seekers with low human capital is not likely to encourage job seekers with high human capital to pretend they have low human capital.

The introduction of training programs also changes the design of the consumption scheme and may reverse previous optimality results. The integrated approach of the different aspects of unemployment policies is thus crucial. Firms may play an important role as well. The loss of skills increases the importance of inducing firms to internalize the costs of displacing workers. Firms could also be subsidized by governments to hire and train low-skilled workers who are not employable otherwise. The analysis here has shed light on the subsidy governments should be willing to pay for firms to play this role.

References

[1] Acemoglu, D., 1995. Public Policy in a Model of Long-term Unemployment. Economica 62, 161-78.

[2] Akerlof, G., 1978. The Economics of "Tagging" as Applied to the Optimal Income Tax, Welfare Programs, and Manpower Planning. American Economic Review 68(1), 8-19.

[3] Besley, T. and Coate, S., 1992. Workfare versus Welfare: Incentive Arguments for Work Requirements in Poverty-Alleviation Programs. American Economic Review 82(1), 249-261.

[4] Black, D., Smith, J., Berger, M., Noel, B., 2003. Is the Threat of Reemployment Services More Effective Than the Services Themselves? Evidence from Random Assignment in the UI System. American Economic Review 93(4), 1313-1327.

[5] Blanchard, O., Diamond, P., 1994. Ranking, Unemployment Duration and Wage Determination. Review of Economic Studies 61(3), 417-434.

[6] Card, D., Chetty, R., Weber, A., 2007. Cash-on-Hand and Competing Models of Intertemporal Behavior: New Evidence from the Labor Market. Quarterly Journal of Economics 122(4), 1511-1560.

[7] Card, D., Kluve, J., Weber, A., 2009. Active Labor Market Policy Evaluations. CESifo working paper 2570.

[8] Coles, M., Masters, A. 2000. Retraining and long-term unemployment in a model of unlearning by not doing. European Economic Review 44(9), 1801-1822.

[9] Falk, A., Huffman, D., Sunde, U., 2006. Do I have What It Takes? Equilibrium Search with Type Uncertainty and Non-Participation. IZA Discussion Paper 2531.

[10] Frijters, P., van der Klaauw, B., 2006. Job Search with Nonparticipation. The Economic Journal 508, 45-83.

[11] Heckman, J., LaLonde, R., Smith, J. 1999. The Economics and Econometrics of Active Labor Market Programs. In: Ashenfelter, O., Card, D. (Eds.), Handbook of Labour Economics, Vol. 3. Amsterdam: North-Holland, pp. 1865-2097.

[12] Holmström, B., Milgrom, P., 1991. Multi-Tasking Principal-Agent Analyses: Linear Contracts, Asset Ownership and Job Design. The Journal of Law, Economics & Organization 7, 24-52.

[13] Hopenhayn, H., Nicolini, J., 1997. Optimal Unemployment Insurance. Journal of Political Economy 105(2), 412-438.

[14] Hopenhayn, H., Nicolini, J., 2009. Optimal Unemployment Insurance and Employment History. Review of Economic Studies 76, 1049-1070.

[15] Jacobson, L., LaLonde, R., Sullivan, D., 1993. Earnings Losses of Displaced Workers. American Economic Review 83(4), 685-709.

[16] Kling, J., 2006. Fundamentally Restructuring Unemployment Insurance: Wage-loss Insurance and Temporary Earnings Replacement Accounts. Hamilton Project Discussion Paper 2006-05.

[17] Krueger, A., Meyer, B., 2002. Labor Supply Effects of Social Insurance. In: Auerbach, A., and Feldstein, M. (Eds.), Handbook of Public Economics, Vol. 4. Amsterdam: North-Holland, pp. 2327-2392.

[18] Ljungqvist, L., Sargent, T., 1998. The European Unemployment Dilemma. Journal of Political Economy 106(3), 514-550.

[19] Low, H., Meghir, C., Pistaferri, L. 2010. Forthcoming. Wage risk and employment risk over the life-cycle. American Economic Review.

[20] Machin, S., Manning, A., 1999. The Causes and Consequences of Long-Term Unemployment in Europe. In: Ashenfelter, O., Card, D. (Eds.), Handbook of Labour Economics, Vol. 3. Amsterdam: North-Holland, pp. 3085-3139.

[21] Neal, D. 1995. Industry-Specific Human Capital: Evidence from Displaced Workers. Journal of Labor Economics 13(4), 653-677.

[22] Pavoni, N., Violante, G., 2007. Optimal Welfare-to-Works Programs. Review of Economic Studies 74(1), 283-318.

[23] Pavoni, N. 2007. On Optimal Unemployment Compensation. Journal of Monetary Economics. 54, 1612-1630.

[24] Pavoni, N. 2009. Optimal Unemployment Insurance, with Human Capital Depreciation and Duration Dependence. International Economic Review 50(2), 323-362.

[25] Pissarides, C. 1992. Loss of Skill during Unemployment and the Persistence of Employment Shocks. Quarterly Journal of Economics 107, 1371-1391.

[26] Shavell, S., Weiss, L., 1979. The Optimal Payment of Unemployment Insurance Benefits Over Time. Journal of Political Economy 87, 1347-1362.

[27] Shimer, R., Werning, I., 2006. On the Optimal Timing of Benefits with Heterogeneous Workers and Human Capital Depreciation. NBER Working Paper 12230.

[28] Shimer, R., Werning, I., 2008. Liquidity and Insurance for the Unemployed. American Economic Review 98(5), 1922-1942.

[29] Spear, S., Srivastava, S., 1987. On repeated Moral Hazard with Discounting. Review of Economic Studies 53, 599-617.

[30] Spinnewijn, J. 2009. Unemployed but Optimistic: Optimal Insurance Design with Biased Beliefs. MIT mimeo

[31] Van Ours, J., Vodopivec, M., 2006. How Shortening the Potential Duration of Unemployment Benefits Affects the Duration of Unemployment: Evidence from a Natural Experiment. Journal of Labor Economics 24(2), 351-378.

[32] von Wachter, T., Song, J., Manchester, J., 2009. Long-Term Earnings Losses due to Mass Layoffs During the 1982 Recession: An Analysis Using U.S. Administrative Data from 1974 to 2004. Working Paper

[33] Wang, C., Williamson, S., 1996. Unemployment Insurance with Moral Hazard in a Dynamic Economy. Carnegie-Rochester Conference Series on Public Policy 44, 1-41.

[34] Werning, I., 2002. Optimal Unemployment Insurance with Unobservable Savings. MIT Working Paper.

[35] Wunsch, C., 2008. Optimal Use of Labour Market Policies. University of St. Gallen Working Paper.

[36] Wunsch, C., 2009. Optimal Use of Labor Market Policies: The Role of Job Search Assistance. CESifo Working Paper 2890

[37] Zhao, R., 2000. The Optimal Unemployment Insurance Contract: Why a Replacement Ratio? University of Chicago Working Paper.

Appendix A: First Order and Envelope Conditions

The social planner minimizes the expected costs of the insurance scheme given an individual rationality constraint and incentive compatibility constraint with Lagrange multipliers λ and μ respectively

$$C(V,\theta) = \min_{c,V^u,V^e,s,t} c + \beta \left[\pi(s)\tfrac{(u^e)^{-1}((1-\beta)V^e)-y(m(\theta,t))}{1-\beta} + (1-\pi(s))C(V^u, m(\theta,t))\right]$$

such that

$$\begin{aligned} V - u(c,\psi(s,t)) - \beta[\pi(s)V^e + (1-\pi(s))V^u] &\leq 0 && (\lambda) \\ u_\psi(c,\psi(s,t))\psi_s(s,t) + \beta\pi'(s)\left[V^e - V^u\right] &\leq 0. && (\mu) \end{aligned}$$

For an interior solution, the first order conditions are

$$\begin{aligned} 0 &= 1 - \lambda u_c - \mu u_{c,\psi}\psi_s && (FOC_c) \\ 0 &= C_V^e(V^e,m) - \lambda - \mu\frac{\pi'(s)}{\pi(s)} && (FOC_{V^e}) \\ 0 &= C_V(V^u,m) - \lambda + \mu\frac{\pi'(s)}{(1-\pi(s))} && (FOC_{V^u}) \\ 0 &= \beta\pi'(s)\left[C_V^e(V^e,m) - C_V(V^u,m)\right] - \mu(u_{\psi,\psi}\left(\psi_s\right)^2 && (FOC_s) \\ &\quad + u_\psi\psi_{s,s} + \beta\pi''(s)\left[V^e - V^u\right]) \\ 0 &= \beta\left[\pi(s)C_\theta^e(V^e,m) + (1-\pi(s))C_\theta(V^u,m)\right]m_t && (FOC_t) \\ &\quad - \lambda u_\psi\psi_t - \mu\left[u_{\psi,\psi}\psi_s\psi_t + u_\psi\psi_{s,t}\right] \end{aligned}$$

The envelope conditions are

$$\begin{aligned} C_V(V,\theta) &= \lambda && (EC_V) \\ C_\theta(V,\theta) &= \beta\left[\pi(s)C_\theta^e(V^e,m) + (1-\pi(s))C_\theta(V^u,m)\right]m_\theta && (EC_\theta) \end{aligned}$$

Appendix B: Proofs

Proof of Proposition 1

By the EC_V at $\tau+1$, we have

$$C_V(V_{\tau+1}, m(\theta_\tau, t_\tau)) = \lambda_{\tau+1}.$$

Since $\mu_\tau = 0$ if $s_\tau = 0$, I find from FOC_{V^u} at τ

$$C_V(V_{\tau+1}, m(\theta_\tau, t_\tau)) = \lambda_\tau.$$

Hence, $\lambda_\tau = \lambda_{\tau+1}$.$\square$

Proof of Proposition 2

With $s_\tau = 0$, EV_θ at τ simplifies to

$$C_\theta(V_\tau, \theta_\tau) = \beta C_\theta(V_{\tau+1}, \theta_{\tau+1})\,(1-\delta)\,.$$

Substituting for $C_\theta(V_\tau, \theta_\tau)$ and $C_\theta(V_{\tau+1}, \theta_{\tau+1})$ from the FOC_t at $\tau - 1$ and τ respectively, I find

$$\lambda_{\tau-1} u_\psi(c_{\tau-1}, \psi_{\tau-1})\psi_t(s_{\tau-1}, t_{\tau-1}) = \beta\,(1-\delta)\,\lambda_\tau u_\psi(c_\tau, \psi_\tau)\psi_t(s_\tau, t_\tau).$$

Since $\lambda_{\tau-1} = \lambda_\tau$ by Proposition 1 and $u_\psi(c_{\tau-1}, \psi_{\tau-1}) = u_\psi(c_\tau, \psi_\tau)$ for the respective preferences, I find

$$\psi_t(s_{\tau-1}, t_{\tau-1}) = \beta\,(1-\delta)\,\psi_t(s_\tau, t_\tau).$$

Since ψ is convex, $s_{\tau-1} = s_\tau = 0$ and $\beta\,(1-\delta) < 1$ imply $t_\tau > t_{\tau-1}$.$\square$

Proof of Proposition 3

From the FOC_{V^u} and EC_V, I find that

$$\Delta_{\lambda,\tau} \equiv \lambda_\tau - \lambda_{\tau+1} = \mu_\tau \frac{\pi'(s_\tau)}{(1-\pi(s_\tau))}.$$

From FOC_s, I find

$$\mu = \frac{\beta\pi'(s)\,[C^e(V^e, m(\theta,t)) - C(V^u, m(\theta,t))]}{(u_{\psi,\psi}\,(\psi_s)^2 + u_\psi\psi_{s,s} + \beta\pi''(s)\,[V^e - V^u])}.$$

First, $s_\tau > 0$ only if $C^e(V^e, m(\theta,t)) - C(V^u, m(\theta,t)) < 0$. Hence, the numerator is negative. Second, the denominator equals the second derivative of the agent's expected utility with respect to search. This derivative is negative. Therefore, we have that $\mu > 0$ and with $\pi'(s_\tau) > 0$ this implies $\Delta_{\lambda,\tau} > 0$.$\square$

Proof of Proposition 4

Since an equal increase in all consumption levels only rescales the utility levels, I expect only the utility ratio's to be dependent on human capital. Here, I rescale all promised utilities with this period's utility $u(c-\psi)$ which allows me explicitly solving the two side-constraints for $\bar{V}^e \equiv V^e/u$ and $\bar{V}^u \equiv V^u/u$. The expected cost of insuring an unemployed agent becomes

$$\begin{aligned} C(V,\theta) = \min c + \beta[\pi(s)C^e(\bar{V}^e u, \theta(1-\delta)+t) \\ + (1-\pi(s))C(\bar{V}^u u(c-\psi(s,t)), \theta(1-\delta)+t)] \end{aligned}$$

such that

$$1+\beta[\pi(s)\bar{V}^e+(1-\pi(s))\bar{V}^u] \leq \frac{V}{u} \qquad (\lambda)$$

$$\pi'(s)\beta[\bar{V}^e-\bar{V}^u] = \psi_s'(s,t)\frac{u'}{u}. \qquad (\mu)$$

Using the CARA properties, the IC constraint simplifies to

$$\pi'(s)\beta[\bar{V}^e-\bar{V}^u] = -\sigma\psi_s'(s,t).$$

The IC constraint does not depend on the level of consumption and the promise-keeping constraint can remain satisfied after an ε-increase in V by increasing $u(c)$ by ε (and therefore V^e and V^u with ε). The first order condition with respect to c equals

$$1+\beta\left[\pi(s)C_V^e u'\bar{V}^e+(1-\pi(s))C_V u'\bar{V}^u\right]-\lambda\frac{V}{u^2}u'=0$$

$\Leftrightarrow$

$$1+\beta\left[\pi(s)C_V^e u'\bar{V}^e+(1-\pi(s))C_V u'\bar{V}^u\right]=-\lambda\frac{\sigma V}{u}.$$

Notice also that the envelope condition with respect to V states that

$$C_V(V,\theta)=\frac{\lambda}{u}$$

or, using the first order condition,

$$C_V(V,\theta)=-\frac{1+\beta\left[\pi(s)C_V^e u'\bar{V}^e+(1-\pi(s))C_V u'\bar{V}^u\right]}{\sigma V}.$$

With CARA preferences,

$$C^e(V^e,\theta') = \frac{-\ln(-V^e(1-\beta))}{\sigma(1-\beta)}-\frac{\theta'}{1-\beta}$$

$$C_V^e(V^e,\theta') = \frac{-1}{\sigma(1-\beta)V^e}.$$

Substituting, I find

$$C_V(V,\theta)=-\frac{1+\beta\left[\frac{\pi(s)}{1-\beta}+(1-\pi(s))C_V(-\sigma u)\bar{V}^u\right]}{\sigma V}. \qquad (6)$$

I guess and verify whether $C(V,\theta)=-\frac{\ln(-V(1-\beta))}{\sigma(1-\beta)}-\frac{g(\theta)}{\sigma(1-\beta)}$ for some function $g(\theta)$. For our guess, we have $C_V(V,\theta)=-\frac{1}{\sigma(1-\beta)V}$. When I plug this into (6), I get

$$C_V(V,\theta) = -\frac{1+\beta\left[\frac{\pi(s)}{1-\beta}+\frac{1-\pi(s)}{1-\beta}\right]}{\sigma V}$$

$$= -\frac{1}{\sigma(1-\beta)V}.$$

Since this holds for any pair (V,θ), our guess and verify method has pinned down the first term of the cost function. That is,

$$C(V,\theta)=\frac{\left(-\frac{1}{\sigma}\right)\ln(-V\,(1-\beta))}{1-\beta}-\frac{g(\theta)}{1-\beta}.$$

The effect on the cost for social planner of an increase in V does not depend on the level of human capital.□

Proof of Corollary 1

From FOC_c, $\frac{1}{u'(c)}=\lambda$. The result follows by Proposition 1.□

Proof of Corollary 2

With $\mu=0$, I find from FOC_c that $\frac{1}{u'(c-\psi)}=\lambda$. The result follows by Proposition 1.□

Proof of Corollary 3

From FOC_c, $\frac{1}{u'(c)}=\lambda$. The result follows by Proposition 3.□

Proof of Corollary 4

From EC_V, $C_V(V,\theta)=\lambda$. In Proposition 4, I will show that $C_V(V,\theta)=\frac{-1}{\sigma(1-\beta)V}$ for CARA preferences. The result follows by Proposition 3.□

Proof of Corollary 6

As long as $s_\tau>0$, Proposition 3 applies. In a stationary state with $\theta^*>0$ and $t^*=\delta\theta^*>0$, it cannot be optimal to have $s_\tau=0$ and thus $\pi(s_\tau)=0$. The social planner would not impose costly training efforts if the unemployed will never leave unemployment again. For CARA preferences, I find that

$$\begin{aligned} c^e &= (u^e)^{-1}\left((1-\beta)\alpha_{V^e(\theta)}V\right)\\ c-\psi &= u^{-1}\left(\alpha_u(\theta)\,V\right)\end{aligned}$$

(see Proposition 4), with the inverse functions $(u^e)^{-1}$ and u^{-1} strictly increasing and $\alpha_{V^e}(\theta)$ and $\alpha_u(\theta)$ positive. Since θ is constant in a stationary state and V does decrease by Corollary 4, the result immediately follows.□

Proof of Corollary 5

Using $\Delta_{\lambda,\tau}=\mu_\tau\frac{\pi'(s_\tau)}{(1-\pi(s_\tau))}$, I find from FOC_{V^e}

$$\frac{1}{u^{e\prime}(c^e_{\tau+1})}-\lambda_\tau=\Delta_{\lambda,\tau}\frac{1-\pi(s_\tau)}{\pi(s_\tau)}. \tag{7}$$

Evaluating (7) at τ and $\tau-1$, I find

$$\frac{\Delta_{\lambda,\tau-1}}{\pi(s_{\tau-1})}=\left[\frac{1}{u^{e\prime}(c^e_\tau)}-\frac{1}{u^{e\prime}(c^e_{\tau+1})}\right]+(1-\pi(s_\tau))\,\frac{\Delta_{\lambda,\tau}}{\pi(s_\tau)}.$$

Notice that $\Delta_{\lambda,\tau} \to 0$ for $\tau \to \infty$. Either $\Delta_{\lambda\,\tau} = 0$ or $\Delta_{\lambda,\tau} > 0$ by Proposition 1 and 3. If for CARA preferences $\Delta_{\lambda,\tau} > 0$, then $V_\tau > V_{\tau+1}$, since $\Delta_{\lambda,\tau} = \frac{-1}{\sigma(1-\beta)V_\tau} - \frac{-1}{\sigma(1-\beta)V_{\tau+1}}$. Either V converges to $\bar{V}$ or V converges to $-\infty$. In both cases, $\Delta_{\lambda,\tau} \to 0$. If for additive preferences $\Delta_{\lambda,\tau} > 0$, then $c_\tau > c_{\tau+1}$ since $\Delta_{\lambda,\tau} = \frac{1}{u'(c_\tau)} - \frac{1}{u'(c_{\tau+1})}$. Either c converges to $\bar{c}$ or c converges to $-\infty$. In both cases, $\Delta_{\lambda,\tau} \to 0$ as well. By integration,

$$\frac{\Delta_{\lambda,\tau-1}}{\pi(s_{\tau-1})} = \left[\frac{1}{u^{e\prime}(c^e_\tau)} - \frac{1}{u^{e\prime}(c^e_{\tau+1})}\right] + \sum_{k=1}^{\infty}\left(\prod_{l=0}^{k-1}(1-\pi(s_{\tau+l}))\right)\left[\frac{1}{u^{e\prime}(c^e_{\tau+k})} - \frac{1}{u^{e\prime}(c^e_{\tau+k+1})}\right].$$

Since $\Delta_{\lambda,\tau-1} > 0$ for $s_{\tau-1} > 0$, we cannot have that $c^e_{\tau+k} \leq c^e_{\tau+k+1}$ for all $k > 0$. Hence, whenever search is positive during unemployment, it must be that at some later time consumption upon re-employment is strictly decreasing with the length of the unemployment spell.□

Appendix C: CARA Preferences with Monetary Cost of Efforts

Bellman Equation for $g(\theta)$ I rewrite the Bellman equation for $C(V,\theta)$ in terms of $g(\theta)$. I use the expressions in Proposition 4 to substitute for $C^e(\alpha_{V^e}V, \theta(1-\delta)+t)$ and $C(\alpha_{V^u}V, \theta(1-\delta)+t)$ with

$$C(V,\theta) = \min c + \beta\left[\pi\frac{-\ln(-\alpha_{V^e}V(1-\beta))}{\sigma(1-\beta)} + (1-\pi)\frac{-\ln(-\alpha_{V^u}V(1-\beta))}{\sigma(1-\beta)}\right] - \beta\left[\pi(s)\frac{y(\theta(1-\delta)+t)}{1-\beta} + (1-\pi(s))\frac{g(\theta(1-\delta)+t)}{1-\beta}\right]$$

such that

$$\alpha_u + \beta[\pi(s)\,\alpha_{V^e} + (1-\pi(s))\alpha_{V^u}] = 1 \qquad (\lambda)$$
$$\alpha_{V^e} - \alpha_{V^u} \leq -\frac{\sigma\alpha_u\psi_s(s,t)}{\beta\pi'(s)}. \qquad (\mu)$$

The first two terms in the objective function can be rewritten to

$$\begin{aligned} & -\tfrac{\ln(-u)}{\sigma} + \psi(s,t) - \tfrac{\beta\ln(-V(1-\beta))}{\sigma(1-\beta)} + \beta\left[\pi\tfrac{-\ln(\alpha_{V^e})}{\sigma(1-\beta)} + (1-\pi)\tfrac{-\ln(\alpha_{V^u})}{\sigma(1-\beta)}\right] \\ = & -\tfrac{\ln u/V}{\sigma} - \tfrac{\ln(-V)}{\sigma} + \psi(s,t) + \tfrac{\ln(-V(1-\beta))}{\sigma} - \tfrac{\ln(-V(1-\beta))}{\sigma(1-\beta)} + \beta\left[\pi\tfrac{-\ln(\alpha_{V^e})}{\sigma(1-\beta)} + (1-\pi)\tfrac{-\ln(\alpha_{V^u})}{\sigma(1-\beta)}\right] \\ = & -\tfrac{\ln(-V(1-\beta))}{\sigma(1-\beta)} + \psi(s,t) - \frac{1}{\sigma(1-\beta)}\left[(1-\beta)\ln(\tfrac{\alpha_u}{1-\beta}) + \beta\pi\ln(\alpha_{V^e}) + \beta(1-\pi)\ln(\alpha_{V^u})\right]. \end{aligned}$$

Since

$$C(V,\theta) = -\frac{\ln(-V(1-\beta))}{\sigma(1-\beta)} - \frac{g(\theta)}{1-\beta},$$

I find that the Bellman equation for $g(\theta)$ equals

$$g(\theta) = \max_{\alpha,s,t} \beta \left[\pi(s)\, y(\theta(1-\delta)+t)) + (1-\pi(s))g(\theta(1-\delta)+t)\right] - (1-\beta)\psi(s,t) + \frac{1}{\sigma}\left[(1-\beta)\ln(\frac{\alpha_u}{1-\beta}) + \beta\pi(s)\ln(\alpha_{V^e}) + \beta(1-\pi(s))\ln(\alpha_{V^u})\right]$$

such that the IR and IC constraint hold.

Stationary State for CARA Preferences Assuming an interior solution, the value function $g(\theta)$ in the second best can be characterized by an unconstrained maximization after substituting in for the IR and IC constraint. That is,

$$\begin{aligned} g(\theta) &= \max \beta \left\{\pi(s) y\left(\theta\left(1-\delta\right)+t\right) + (1-\pi(s)) g\left(\theta\left(1-\delta\right)+t\right)\right\} \\ &\quad -(1-\beta)\psi(s,t) + \kappa(s,t,\tfrac{\alpha_{V^u}}{\alpha_u}) \end{aligned}$$

with

$$\begin{aligned} \kappa(s,t,\tfrac{\alpha_{V^u}}{\alpha_u}) = \frac{1}{\sigma}\{&\beta\pi(s)\ln(\tilde{\kappa}(s,t)+\tfrac{\alpha_{V^u}}{\alpha_u}) + \beta(1-\pi(s))\ln(\tfrac{\alpha_{V^u}}{\alpha_u}) \\ &- \ln(1+\beta[\pi(s)\,\tilde{\kappa}(s,t)+\tfrac{\alpha_{V^u}}{\alpha_u}]) - (1-\beta)\ln(1-\beta)\} \end{aligned}$$

and

$$\tilde{\kappa}(s,t) = -\frac{\sigma\psi_s(s,t)}{\beta\pi'(s)}.$$

Notice that only three control variables remain. With $g(\theta)$ concave, the first order conditions with respect to s, t and $\frac{\alpha_{V^u}}{\alpha_u}(=\frac{V^u}{u})$ are

$$\begin{aligned} \beta\pi'(s)[y\left(\theta\left(1-\delta\right)+t\right) - g\left(\theta\left(1-\delta\right)+t\right)] - (1-\beta)\,\psi_s(s,t) + \frac{\partial\kappa}{\partial s} &= 0 \\ \beta\left\{\pi(s)y'\left(\theta\left(1-\delta\right)+t\right) + (1-\pi(s))g'\left(\theta\left(1-\delta\right)+t\right)\right\} - (1-\beta)\psi_t(s,t) + \frac{\partial\kappa}{\partial t} &= 0 \\ \frac{\partial\kappa}{\partial\frac{\alpha_{V^u}}{\alpha_u}} &= 0. \end{aligned}$$

The envelope condition equals

$$g'(\theta) = \beta\left\{\pi(s)y'\left(\theta\left(1-\delta\right)+t\right) + (1-\pi(s))g'\left(\theta\left(1-\delta\right)+t\right)\right\}(1-\delta).$$

In a steady state θ^*, training equals the depreciation in human capital to maintain the same level, that is $t^* = \delta\theta^*$. Hence, from the objective function and envelope condition, I get

$$
\begin{aligned}
g(\theta) &= \frac{\beta\pi(s(\theta))y(\theta) - (1-\beta)\psi(s(\theta),\delta\theta) + \kappa(s,t,\frac{\alpha_V u}{\alpha_u})}{1-\beta(1-\pi(s(\theta)))} \\
g'(\theta) &= \frac{\beta\pi(s(\theta))y'(\theta)\,(1-\delta)}{1-\beta(1-\pi(s(\theta)))(1-\delta)}.
\end{aligned}
$$

Substituting this in the first order conditions, we find the expressions in the text.