Determinants of Shadow Prices in Open Dual Economies
Author(s): Avinash Dixit and Nicholas Stern
Source: *Oxford Economic Papers*, New Series, Vol. 26, No. 1 (Mar., 1974), pp. 42-53
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/2662487
Accessed: 09/09/2013 07:13

DETERMINANTS OF SHADOW PRICES IN OPEN DUAL ECONOMIES[1]

By AVINASH DIXIT and NICHOLAS STERN

I. Introduction

COST-BENEFIT analysts in underdeveloped countries are making increasing use of shadow prices. If we are to have accurate shadow prices, we must know how sensitively they depend on various factors, so that we can try to obtain tighter estimates of the critical parameters. Dixit [**2**] has recently carried out such an analysis for a two-sector, two-good model where the main concern was the sensitivity of the shadow wage. He argued that its sensitivity with respect to the supply and demand elasticities for food was larger than that with respect to the marginal product of agricultural labour. Most of his paper studied a closed economy. Here we extend the analysis for a similar model to the case of an open dual economy. Newbery [**6**] has recently discussed the open dual economy in greater detail, and we carry out sensitivity analysis along similar lines.

We concentrate attention on the two determinants of the shadow wage just mentioned. There are two other important aspects of the shadow wage which have recently received much attention—the appropriate social demand price for investment in terms of marginal consumption increments to different groups[2] and the role of the urban wage in urban unemployment.[3] We assume here that the first question has been answered, and consider only an extreme case—the criterion in our model is the maximization of investment. This may not be a bad approximation to the objectives of some governments and it greatly simplifies the analysis. We mention briefly some effects of relaxing this assumption. We have thus assumed that some rough approximation to an intertemporal analysis has been carried out and has yielded a very large price for investment. The static analysis performed here is to be viewed in the context of this intertemporal model. This procedure is the same as in Dixit [**2**].

There is no unemployment in our model. It is the actual wage that is equal in the two sectors, as opposed to the expected wage of the urban unemployment models. We suspect that the introduction of expected-wage equalization would not alter our main conclusions provided the gap between

[1] This paper was written while we were visiting the Massachusetts Institute of Technology. We are grateful to their Department of Economics for its hospitality, and the National Science Foundation for financial support. Our interest in this work was stimulated by David Newbery's paper [**6**].

[2] See, e.g., Sen [**7**], Dixit [**1**], Stern [**9**], and Newbery [**5**].

[3] See, e.g., Todaro [**11**] and Stiglitz [**10**].

urban wages and rural incomes is reasonably small. Our model would not be applicable to those underdeveloped economies where this gap is large.[1]

We examine sensitivity of the shadow wage in three circumstances: first where the government sets no tariffs or taxes but chooses only advanced sector employment, second where tariff and employment are policy variables, and finally where employment, tariff, and indirect tax on the agricultural sector can all be chosen.

II. The free trade case

Most of the notation is the same as in Dixit [**2**] and Newbery [**6**] but the technique is different. We use the expenditure function, and mention the new notation this introduces in an appendix. The two sectors produce different goods: manufactures, indexed 1, and food, indexed 2. The production function for sector i is $Y_i(L_i)$, and a_i and m_i are the average and marginal products respectively and each is a function of L_i. The manufactured product is the numéraire and p denotes the price of food. All consumers are identical and the representative individual's demand for food is $f(y,p)$ when his income is y. The manufacturing wage is denoted by w. The government owns the manufacturing sector and can invest all the surplus after wages are paid. The agricultural sector is owned by peasants who cannot be taxed directly. Employment in the ith sector is L_i, with L_1+L_2 fixed and normalized to unity.

The simplest case of the open dual economy is where the government does not interfere with trade. The domestic price of food is thus equal to the world price p_0. The problem becomes

choose L_1 to maximize $\qquad I = Y_1 - wL_1$

subject to $\qquad L_1+L_2 = 1,$

$$w = a_2 p_0.$$

The constraints are conditions for equilibrium in the labour market. In an open economy, the only other condition is trade balance, which is used up in simplifying the maximand.

The shadow wage is defined to be equal to the advanced sector marginal product when employment is optimum. The proportionate difference between the shadow wage and the market wage is derived in Dixit [**2**], and is given by

$$(m_1-w)/w = (1-s_2)L_1/L_2 \tag{1}$$

where $s_i = m_i/a_i$ the competitive share of labour.

[1] Examples are Uganda, Puerto Rico, and Iran; see Stern [8], pp. 104–10. We suspect that equilibrium analyses of the kind done here and in expected-wage equalization models will not be very helpful in these cases.

This does not involve food supply and demand elasticities at all, and is moderately sensitive to s_2. The derivative of the left-hand side with respect to s_2 is $-L_1/L_2$, which will typically be about $-0{\cdot}25$.

III. Sensitivity with an optimum tariff

We next suppose that the government can impose a tariff to separate the world price p_0 from the domestic price p but that all domestic consumers face the same price. The equilibrium wage in domestic prices is $w = a_2 p$. Following Newbery, we write the outcome of valuing its consumption components at world prices as

$$w_0 = [a_2 - f(w,p)]p + f(w,p)p_0. \tag{2}$$

The problem becomes:

choose L_1, p to maximize $\quad I = Y_1 + p_0 Y_2 - w_0$

subject to

$$L_1 + L_2 = 1, \tag{3}$$

$$w = a_2 p. \tag{4}$$

The maximand is the difference between output and consumption, both valued at world prices; the constraints are conditions for equilibrium in the labour market. Using (3), we can take p and L_2 to be the control variables.

The optimum choice of p requires

$$\partial w_0/\partial p = 0. \tag{5}$$

We can rewrite the expression for w_0 as

$$w_0 = (p - p_0)M/L_2 + p_0 a_2,$$

where M is the marketed surplus,

$$M(p, L_2) = Y_2 - pf(a_2 p, p).$$

Using this, (5) becomes

$$(p_0 - p)/p = 1/\epsilon, \tag{6}$$

where $\epsilon = (p/M).\partial M/\partial p$ is the elasticity of supply of the marketed surplus. It is easy to show that

$$\epsilon = \delta(\theta - \lambda)/(1 - \delta) \tag{7}$$

where θ and λ are the price and income elasticities of demand for food and δ is the share of food in the budget. We assume the stability condition $\theta > \lambda$, as in Dixit [**2**].

The first-order condition with respect to L_2 gives

$$\begin{aligned} m_1 &= p_0 m_2 - \partial w_0/\partial L_2 \\ &= p_0 a_2 s_2 - [1 + (p_0 - p)f_w]p\,\partial a_2/\partial L_2. \end{aligned}$$

Using (6), this becomes

$$m_1 = a_2 p_0 \left(s_2 + \frac{1 - s_2}{L_2} \frac{\lambda\delta + \epsilon}{1 + \epsilon} \right). \tag{8}$$

Equation (6) is derived by Dixit [**2**], although he used δ, θ, and λ instead of ϵ. He also gave a variant of (8). Newbery [**6**] noted that (6) takes the simpler form using ϵ. He also derived (5) (somewhat differently) and recommended that calculations be made in terms of w_0 (Dixit had worked with w). He thus gave a variant of (8) with $(m_1-w_0)/w_0$ on the left-hand side (see [**6**], equation (5)).

The values of the parameters used by Dixit were $L_1 = 0{\cdot}2$, $L_2 = 0{\cdot}8$, $s_2 = 0{\cdot}5$, $\theta = 0{\cdot}9$, $\lambda = 0{\cdot}7$, and $\delta = 0{\cdot}7$, which give $\epsilon = 0{\cdot}47$, $p/p_0 = 0{\cdot}32$, and $(m_1-w)/w = 1{\cdot}85$. Dixit showed that $(m_1-w)/w$ was rather more sensitive to θ and λ than to s_2.

Newbery used the same parameters to obtain $(m_1-w_0)/w_0 = 0{\cdot}14$, and made two points using the calculations with w_0. First, that m_1 is close to w_0 although it is not close to w, and second, that such difference as there is between m_1 and w_0 is mostly accounted for in expressions involving the production function in agriculture rather than the marketed surplus elasticity. He concluded that the former was more important in determining the shadow wage.

It seems to us, however, that this approach can be misleading and that questions of relative importance are best looked at in terms of thorough sensitivity analysis. In the model under discussion the relevant magnitude is m_1, the level of the shadow wage. We should like to know which parameters to concentrate on if we are to have accurate estimates of the shadow wage. If it turns out that m_1 is close to w_0 for a broad range of the parameters of the problem, this is helpful to someone who knows w_0 and wants to make a quick calculation of m_1. But such a result is *not* helpful to the decision maker who has to set the policy variables that will determine both m_1 and w_0. He must find how the parameters affect m_1 directly. This is clearly the case here; as p is a policy variable, w_0 is endogenous and cannot be assumed to be known in advance and used as a first approximation to m_1.

To analyse sensitivity properly, we examine the expression (8) for m_1. We regard a_2 and L_2 as fixed and known with sufficient accuracy. Then

$$\frac{\partial m_1}{\partial s_2} = a_2 p_0\left\{1-\frac{1}{L_2}\frac{\lambda\delta+\epsilon}{1+\epsilon}\right\} \tag{9}$$

and

$$\frac{\partial m_1}{\partial \epsilon} = a_2 p_0\left\{\frac{1-s_2}{L_2}\frac{1-\lambda\delta}{(1+\epsilon)^2}\right\}. \tag{10}$$

Using the parameter values given earlier, we get $\partial m_1/\partial s_2 = 0{\cdot}19\, a_2 p_0$ and $\partial m_1/\partial \epsilon = 0{\cdot}14\, a_2 p_0$. Sensitivity with respect to ϵ is greater if we use lower test values of s_2, i.e. if we believe more in surplus labour theories. Since the uncertainty about the values of s_2 and ϵ is likely to be similar in the sense that, for many situations, a prior probability distribution for the two parameters would cover a range of similar measure and have a similar shape, it seems fair to conclude that m_1 is equally sensitive to the two parameters. The measure of the plausible range for both parameters might be 0·5, for we should probably be prepared to guess that, in a particular situation, s_2 would lie within a range of length $\frac{1}{2}$ and ϵ would lie between 0 and 0·5.[1]

It is possible to interpret the model as one where the modern sector is privately owned, and then the control variable that interests the government is the wage tax or subsidy. We can write (8) as

$$m_1/w = [s_2(1+\epsilon)+(1-s_2)(\lambda\delta+\epsilon)/L_2]/\epsilon. \tag{8'}$$

The reader can easily check that the tax is rather more sensitive to ϵ than to s_2 for the values of the parameters described. We do not regard such an interpretation of the model as very desirable, since the objective function becomes maximization of profits. Issues of wage and profit taxation are rather more complicated than the considerations embodied in this model, and it seems more reasonable to regard the advanced sector as publicly owned and the government as interested in m_1.

There is another possibility, namely that the modern sector is publicly owned but the operation of it is decentralized. In this case, the project managers will find it more convenient to apply a correction directly to w, which they can observe in the market, than to look at the consumption components of w, value them at world prices to get w_0, and then apply another correction.

We have been discussing the sensitivity of just one of the policy variables, m_1 (or L_1), to ϵ and s_2. There is another policy variable in the model, namely p, and its optimum value is clearly very sensitive to ϵ and does not depend on s_2.

IV. Sensitivity with tariff and indirect tax

We now suppose that the government can set a price of food to workers in the manufacturing sector different from the price received by peasants. This might be achieved by a marketing board which buys from the peasants and then sells to the workers. Alternatively, we can think of the government setting a tariff and then imposing an indirect tax on the peasants' marketed surplus.

Suppose p_1 and p_2 are the prices faced by the manufacturing and agricultural sectors respectively. Equilibrium in the labour market requires that the utility obtainable in each sector should be the same. This can no longer be expressed in terms of the wage rate and we have to use some form of the utility function. It turns out to be easiest to work in terms of the expenditure function. (Readers unfamiliar with the expenditure function

[1] These are crude impressionistic values used in absence of any hard statistical evidence, and are similar to the numbers used by earlier writers.

will find a brief description of its properties in the appendix.) Equilibrium in the labour market is now expressed by

$$a_2 p_2 = E(1, p_2; u), \tag{11}$$

$$w = E(1, p_1; u), \tag{12}$$

$$L_1 + L_2 = 1 \tag{13}$$

where E is the expenditure function, and u the common utility level.[1] Manufactures being numéraire, their price, one, is inserted just as a reminder in the arguments of E. Given government policy[2] (p_1, p_2, L_1), these equations fix L_2 (and thus a_2), u, and w.

Now investment equals the surplus in industry plus the government's profits (or tax revenues) from food transactions. If it buys an amount M of marketed surplus from each peasant at price p_2, sells an amount D to each individual worker at price p_1, and achieves material balance by trading with the rest of the world at price p_0 with a balanced trade account, we have, adding the three sources of government revenue,

$$I = (Y_1 - wL_1) + (p_0 - p_2) M L_2 + (p_1 - p_0) D L_1. \tag{14}$$

We can find expressions for M and D easily from the expenditure function

$$D = E_p(1, p_1; u), \tag{15}$$

$$M = a_2 - E_p(1, p_2; u). \tag{16}$$

We now reserve δ for the expenditure share of food in the agricultural sector, i.e. $\delta = (a_2 - M)/a_2$.

The first-order conditions for a maximum of I are $\partial I/\partial p_1 = 0 = \partial I/\partial p_2$ and $\partial I/\partial L_1 = \partial I/\partial L_2$,[3] where we must remember that w, u, M, and D are functions of (p_1, p_2, L_2) by (11), (12), (15), and (16), and by the dependence of a_i, m_i on L_i. Then, using (14), the first-order conditions can be written as

$$-L_1 \frac{\partial w}{\partial p_1} + (p_0 - p_2) L_2 \frac{\partial M}{\partial p_1} + D L_1 + (p_1 - p_0) L_1 \frac{\partial D}{\partial p_1} = 0, \tag{17}$$

$$-L_1 \frac{\partial w}{\partial p_2} - M L_2 + (p_0 - p_2) L_2 \frac{\partial M}{\partial p_2} + (p_1 - p_0) L_1 \frac{\partial D}{\partial p_2} = 0, \tag{18}$$

$$(m_1 - w) + (p_1 - p_0) D = -L_1 \frac{\partial w}{\partial L_2} + (p_0 - p_2)\left(M + L_2 \frac{\partial M}{\partial L_2}\right) + (p_1 - p_0) L_1 \frac{\partial D}{\partial L_2}. \tag{19}$$

The first of these simplifies very easily, for (12) and (15) give $D = \partial w/\partial p_1$ and (11) and (16) give $\partial M/\partial p_1 = 0$. With $L_1 > 0$ and $\partial D/\partial p_1 = E_{pp}(1, p_1; u) < 0$ by Slutsky–Hicks theory, (17) becomes

$$p_1 = p_0. \tag{20}$$

[1] It turns out to be easier to keep u as an independent variable than to compound (11) and (12) using the indirect utility function.

[2] Actually any one of L_1, L_2 and w could be thought of as the employment policy variable. We usually use L_2, for convenience.

[3] A slight abuse of notation is involved here. See (19) for the explicit statement.

The simplest way to implement this policy would be to have the modern sector open to free trade and impose a tax on transactions with the backward sector. The optimality of this policy was stated by Newbery [**6**]. The wage at domestic prices is then equal to that at world prices. It is easy to see that the result $p_1 = p_0$ generalizes to the maximand $I+u/q$ where q is the social demand price of investment in terms of manufactured consumer goods, for (11) shows that u is determined by p_2 and L_2 alone, leaving the condition (17) unchanged. The result *does* depend on L_1 and p_2 being chosen at their optimum values, i.e. on the over-all framework of policy.

Further analysis of (18) and (19)—which will determine the optimum p_2 and L_1—requires evaluation of the derivatives of w, D, and M. To do this efficiently, we write the expressions for small proportional changes, denoted by hats over the corresponding variables. We use (11) and (12) to eliminate $\hat{u}$. Then, using notation explained below, we get

$$\hat{w} = -\beta(1-s_2)\hat{L}_2+\delta_1\hat{p}_1+\beta(1-\delta)\hat{p}_2, \tag{21}$$

$$\hat{D} = -\beta\lambda_1(1-s_2)\hat{L}_2-\eta_1\hat{p}_1+\beta(1-\delta)\lambda_1\hat{p}_2, \tag{22}$$

$$\hat{M} = -\frac{1-\lambda_2\delta}{1-\delta}(1-s_2)\hat{L}_2+\frac{\delta}{1-\delta}(\theta_2-\lambda_2)\hat{p}_2 \tag{23}$$

where δ_1 is the share of food in the budget of manufacturing workers, i.e. $p_1 D/w$; θ_i, λ_i, η_i are respectively the price, income, and compensated-price (Slutsky–Hicks) elasticities of demand for food in sector i (see appendix); $\beta = \xi_1/\xi_2$ where $\xi_i = uE_u(1, p_i; u)/E(1, p_i; u)$. We can read off derivatives directly from (21)–(23) and substitute in (18) and (19) to obtain equations (24) and (25) which will determine p_2 and L_1 (and therefore L_2):

$$\epsilon(p_0/p_2-1) = 1+\beta\frac{L_1}{L_2}\frac{E(1, p_0; u)}{E(1, p_2; u)}, \tag{24}$$

$$\frac{m_1-w}{w} = \beta(1-s_2)\frac{L_1}{L_2}+\left[\frac{E(1,p_2; u)}{E(1,p_0; u)}+\frac{\beta L_1}{L_2}\right]\frac{(1-\delta)-(1-\lambda_2\delta)(1-s_2)}{\epsilon}. \tag{25}$$

To proceed with sensitivity we must now use an explicit form of the expenditure function. We use a CES form, i.e.

$$E(1,p; u) = (1+bp^{\alpha})^{1/\alpha}g(u) \tag{26}$$

where $\alpha < 1$ (so that E is concave in p) and $g(u)$ is increasing. It is easy to check, using the results of the appendix, that $\delta = bp^{\alpha}/(1+bp^{\alpha})$, $\theta = 1-\alpha(1-\delta)$, $\lambda = 1$, and $\epsilon = -\alpha\delta$.[1] Finally, $\beta = 1$, which is the reason we chose this form. Note that β does not depend on the form of the function g. The Marshallian demand function for food which arises from (26) is $f(y,p) = ybp^{\alpha-1}/(1+bp^{\alpha})$.

[1] The stability conditions will be met, and ϵ will be positive, provided $\alpha < 0$. This will be the case in all that follows.

We may choose units of food such that the world price p_0 is one. Using this and (26), the equations for p_2 (the subscript is dropped from now on) and m_1 become

$$\epsilon(1-p)/p = 1+\frac{L_1}{L_2}[(1+bp^\alpha)/(1+b)]^{-1/\alpha}, \tag{24'}$$

$$\frac{m_1-w}{w} = (1-s_2)\frac{L_1}{L_2}+\frac{(1-\delta)s_2}{\epsilon}\{[(1+bp^\alpha)/(1+b)]^{1/\alpha}+L_1/L_2\}. \tag{25'}$$

Now using $\epsilon = -\alpha\delta$, $\delta = bp^\alpha/(1+bp^\alpha)$, and (24′) we can, given ϵ, δ, and L_1/L_2, solve for α, b, and p. In fact, substituting for b in terms of p in (24′) we have one equation in one unknown, p, which is easy to solve numerically. Given s_2, (25) can then be used to find $(m_1-w)/w$.

Tables Ia and Ib show how p and $(m_1-w)/w$ respectively vary with ϵ and s_2 given $\delta = 0{\cdot}7$. As before, p does not depend on s_2. Looking down the column $\epsilon = 0{\cdot}24$ and across the row $s_2 = 0{\cdot}5$ in Table Ib, it is probably fair to say that the sensitivity of $(m_1-w)/w$ to ϵ and s_2 is fairly similar. The sensitivity with respect to ϵ increases with s_2 and that with respect to s_2 decreases as ϵ increases.

TABLE Ia

Epsilon	0·0400	0·0800	0·1200	0·1600	0·2000	0·2400	0·2800	0·3200	0·3600	0·4000
Price	0·0033	0·0178	0·0386	0·0615	0·0850	0·1082	0·1309	0·1528	0·1739	0·1942

TABLE Ib

Epsilon	0·0400	0·0800	0·1200	0·1600	0·2000	0·2400	0·2800	0·3200	0·3600	0·4000
Share										
0·0000	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500
0·1000	0·4292	0·3462	0·3189	0·3044	0·2950	0·2882	0·2830	0·2787	0·2752	0·2722
0·2000	0·6085	0·4424	0·3877	0·3588	0·3400	0·3265	0·3159	0·3075	0·3004	0·2943
0·3000	0·7877	0·5386	0·4566	0·4132	0·3851	0·3647	0·3489	0·3362	0·3256	0·3165
0·4000	0·9669	0·6348	0·5255	0·4676	0·4301	0·4029	0·3819	0·3649	0·3508	0·3387
0·5000	1·1462	0·7310	0·5943	0·5220	0·4751	0·4411	0·4149	0·3936	0·3759	0·3609
0·6000	1·3254	0·8272	0·6632	0·5765	0·5201	0·4794	0·4478	0·4224	0·4011	0·3830
0·7000	1·5046	0·9234	0·7321	0·6309	0·5651	0·5176	0·4808	0·4511	0·4263	0·4052
0·8000	1·6839	1·0196	0·8010	0·6853	0·6102	0·5558	0·5138	0·4798	0·4515	0·4274
0·9000	1·8631	1·1158	0·8698	0·7397	0·6552	0·5940	0·5467	0·5085	0·4767	0·4496
1·0000	2·0423	1·2120	0·9387	0·7941	0·7002	0·6323	0·5797	0·5373	0·5019	0·4717

The computations were repeated for different values of δ. Lower values of δ increase sensitivity with respect to s_2 without much effect on sensitivity to ϵ. The results for $\delta = 0{\cdot}5$ are shown in Tables IIa and IIb. In any particular situation the range of uncertainty about s_2 would not be from 0 to 1 (see Section III), so it still remains fair to say that sensitivity to ϵ and s_2 is roughly similar.

We note that comparisons of the results of this section with those of Section III are not likely to be misleading even though the demand functions are different. The reason is that the values of the important parameters s_2, ϵ, δ, and L_1/L_2 have been kept similar. This section used $\lambda = 1$ instead of

$\lambda = 0{\cdot}7$ of Section III but, of the relevant equations, only (25) contains λ and in a manner such that the difference will not be important.

We note also that keeping L_1/L_2 constant is unlikely to be misleading even though it is an endogenous variable which we should calculate given production functions in each sector. For we can regard (18) and (19) after elimination of p_2 as one equation relating m_1 and L_1. A production function in the advanced sector gives us another relation, so determining the optimum value of each. We can always choose this function so that L_1/L_2 is approximately $\frac{1}{4}$, and since we feel that such an optimum value of L_1/L_2 is plausible we should choose such a function.

Table IIa

Epsilon	0·0400	0·0800	0·1200	0·1600	0·2000	0·2400	0·2800	0·3200	0·3600	0·4000
Price	0·0144	0·0377	0·0626	0·0874	0·1116	0·1348	0·1572	0·1786	0·1991	0·2186

Table IIb

Epsilon	0·0400	0·0800	0·1200	0·1600	0·2000	0·2400	0·2800	0·3200	0·3600	0·4000
Share										
0·0000	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500	0·2500
0·1000	0·7168	0·5314	0·4599	0·4196	0·3929	0·3736	0·3587	0·3469	0·3371	0·3289
0·2000	1·1835	0·8127	0·6698	0·5892	0·5359	0·4971	0·4674	0·4437	0·4242	0·4079
0·3000	1·6503	1·0941	0·8797	0·7588	0·6788	0·6207	0·5762	0·5406	0·5114	0·4868
0·4000	2·1171	1·3754	1·0895	0·9284	0·8217	0·7443	0·6849	0·6374	0·5985	0·5658
0·5000	2·5839	1·6568	1·2994	1·0981	0·9646	0·8679	0·7936	0·7343	0·6856	0·6447
0·6000	3·0506	1·9381	1·5093	1·2677	1·1076	0·9914	0·9023	0·8312	0·7727	0·7237
0·7000	3·5174	2·2195	1·7192	1·4373	1·2505	1·1150	1·0110	0·9280	0·8598	0·8026
0·8000	3·9842	2·5008	1·9291	1·6069	1·3934	1·2386	1·1197	1·0249	0·9470	0·8816
0·9000	4·4510	2·7822	2·1390	1·7765	1·5363	1·3622	1·2285	1·1217	1·0341	0·9605
1·0000	4·9177	3·0636	2·3489	1·9461	1·6793	1·4857	1·3372	1·2186	1·1212	1·0394

In this section we have been examining sensitivity of $(m_1 - w)/w$, and it makes no difference whether we evaluate the manufacturing wage at domestic or world prices. The interest in using w has been discussed in Section III. However, w is still an endogenous variable, depending on ϵ, δ, and s_2, even though L_1/L_2 is fixed. As in the previous section, we also looked directly at the sensitivity of m_1. The results are very much like those found there from (9) and (10); the sensitivities to ϵ and s_2 were similar, and that with respect to ϵ was greater when s_2 was smaller.

Some other features of interest emerged from the numerical calculations. For, in addition to telling us about sensitivity, the tables tell us the directions of change and bounds for policy as the parameters change. For low values of ϵ, p is quite low. As ϵ increases, so does p; and in the limit as ϵ goes to ∞, p goes to 1. This dependence is the same as that found by Hornby [**3**]. As ϵ increases, the government's monopoly power in food decreases, and in the limit, free trade becomes optimum. Further, as ϵ increases, m_1

increases but $(m_1 - w)/w$ decreases going in the limit to the free trade case of Section II. Then we have

$$m_1 > w = a_2. \tag{27}$$

For the range of ϵ investigated, we find

$$a_2 > m_1 > w > a_2 p \tag{28}$$

(remember that p_0 has been set equal to one), which of course cannot be valid for high ϵ because of the limiting result (27).

V. Conclusions

We have compared, for an open dual economy, the sensitivity of the shadow wage to variations in the price elasticity of marketed surplus (ϵ) with that to changes in the imputed share of labour in agriculture (s_2). The model of Section III, where the domestic price of food must be uniform but a tariff is possible, has previously been analysed by Dixit [**2**] but the sensitivity comparisons were sketchy and were not carried through in terms of ϵ. Since Newbery [**6**] has pointed out that all the effects of demand elasticities were channelled through ϵ, analysis of sensitivity in terms of ϵ seemed desirable. It turns out that the shadow wage is roughly equally sensitive to ϵ and s_2.

We have also argued that the level of the shadow wage itself is the appropriate concept for sensitivity analysis and not the difference between the shadow wage and the wage evaluated at world prices, the magnitude discussed by Newbery [**6**]. The reason is that the wage at world prices is itself endogenous to the model.

The analysis of sensitivity when indirect taxation of agriculture is also possible is more complicated, and is carried out in Section IV using a specific expenditure function. The conclusion concerning sensitivity is similar.

We also prove the result stated by Newbery that the price of food to manufacturing workers should be equal to the world price provided that manufacturing employment is being chosen optimally.

Finally, we hope to have shown that it is by no means difficult to see how the results of simple general equilibrium models depend on their assumptions. We have only to change the assumptions in the desired direction and compare the new general equilibrium model with the old one. Thus, when one of the questionable assumptions is that the economy is closed, we allow trade in some form and compare the equilibrium under trade with that under autarky. Using a partial equilibrium approach and 'trying to identify and incorporate the more important indirect interactions' can at best duplicate the results of an analysis which handles the whole simultaneity. The proper role of partial analyses when studying economies as a whole must be one of stopping gaps when a general analysis is intractable with the methods currently available. Even then, one must beware of pitfalls that await the partial equilibrium thinker, some of which can be more easily spotted by the logic of general equilibrium analysis. Realizing the importance of the endogeneity of w_0 in the model of this paper is an excellent illustration.

We should reiterate that even general equilibrium analysis may be inadequate in certain circumstances. One should make informed judgements about this, too, when selecting the proper model.

Balliol College, Oxford
St. Catherine's College, Oxford

REFERENCES

1. Dixit, A. K., 'Optimal development in the labour surplus economy', *Review of Economic Studies*, 35 (1), Jan. 1968, 23.
2. —— 'Short-run equilibrium and shadow prices in the dual economy', *Oxford Economic Papers*, 23 (3), Nov. 1971, 384.
3. Hornby, J. M., 'Investment and trade policy in the dual economy', *Economic Journal*, 78 (1), Mar. 1968, 96.
4. Karlin, S., *Mathematical Methods and Theory in Games, Programming and Economics*, Reading, Mass.: Addison–Wesley.
5. Newbery, D. M. G., 'Public policy in the dual economy', *Economic Journal*, 82 (2), June 1972, 567.
6. —— 'The robustness of equilibrium analysis in the dual economy', manuscript. Revised version in this issue, pp. 32–41.
7. Sen, A. K., *Choice of Techniques*, Oxford: Basil Blackwell, 1960.
8. Stern, N. H., *Location and the Rate of Development*, D.Phil. dissertation, Oxford University, 1971.
9. —— 'Optimum development in a dual economy', *Review of Economic Studies*, 39 (2), Apr. 1972, 171.
10. Stiglitz, J. E., 'Rural–urban migration, surplus labour, and the relationship between urban and rural wages', *Eastern Africa Economic Review*, 1 (2), Dec. 1969, 1.
11. Todaro, M. P., 'A model of labor migration and urban unemployment in less developed countries', *American Economic Review*, 59 (1), Mar. 1969, 138.

APPENDIX

We consider a two-good case with one of them as numéraire. The expenditure function $E(1,p;u)$ is defined as the minimum expenditure necessary to achieve utility level u when prices are $(1,p)$. It is clear that E is increasing in all arguments and concave in p; it is homogeneous of degree one in the vector $(1,p)$. The partial derivative $E_p(1,p;u)$ equals the utility-compensated demand for the second commodity. (For proofs of these results, see, e.g., Karlin [4], § 8.6.)

We then have $\delta = pE_p/E$, the expenditure proportion. The derivative of the compensated demand function is E_{pp}, so the compensated elasticity is $\eta = -pE_{pp}/E_p$ in numerical value. The Marshallian elasticity is found by writing

$$0 = dE = E_p\,dp + E_u\,du$$

to keep income constant, and subject to this

$$dE_p = E_{pp}\,dp + E_{pu}\,du$$

so
$$\left.\frac{\partial E_p}{\partial p}\right|_{E\ \text{constant}} = E_{pp} - E_{pu}\,E_p/E_u$$

or
$$\theta = -p(E_{pp} - E_{pu}\,E_p/E_u)/E_p.$$

The income elasticity is found from $dE_p = E_{pu}\,du$ and $dE = E_u\,du$ to get

$$\left.\frac{\partial E_p}{\partial E}\right|_{p\ \text{constant}} = \frac{E_{pu}}{E_u}$$

or
$$\lambda = EE_{pu}/(E_p E_u).$$

From these, we get the Slutsky–Hicks formula in elasticity form:

$$\theta = \eta + \delta\lambda.$$