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THE THEORY OF REFORM AND INDIAN INDIRECT TAXES

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Given a set of value judgements, an initial state, and a model of the economy, one can ask whether some feasible tax change would increase welfare. We do this by defining the marginal cost in terms of welfare of raising an extra rupee from the ith good. The inverse optimum problem is the calculation of non-negative welfare weights on households which imply that the initial state is optimum. If no such welfare weights exist, then a Pareto improvement is possible. We illustrate the concepts and results using data from the Indian economy for 1979–80. Directions of tax reform for a number of specific social welfare functions and for Pareto improvements are presented.

1. Introduction

There are a number of ways to evaluate a tax system. One is to specify a model of the economy and its initial equilibrium together with value judgements, embodied in some social welfare function, and then ask whether it is possible to reform taxes so as to increase social welfare. Obviously if we are at an optimum with respect to the social welfare function, then no improving reform is feasible. A second approach is to ask whether there is a set of value judgements under which, given the model of the economy, the initial state of affairs would be deemed as optimum. That is the inverse optimum problem. The value judgements may then be used in a number of ways. One might infer that these are indeed the value judgements of the government and use them in appraising other decisions. Or if the computed value judgements were seen as objectionable, then they could be employed to criticise the existing state of affairs. in the sense that it could be seen as optimum only with respect to disagreeable values. Thirdly, we can seek to discover Pareto improvements in order to avoid using a, possibly controversial, social welfare function.

The purpose of this paper is to present theory in a way which shows how these approaches may be implemented and to display the interrelations between them. We shall then illustrate our methods with an empirical discussion of the possibilities for tax reform in India. We deal entirely with marginal reforms in this paper. This has some considerable advantages from the point of view of data requirements (see below) but is, of course, limited in scope.

In the next section we develop the theory. We first demonstrate how, given a social welfare function, directions of improving reform may be found. For each good we define the marginal cost in terms of social welfare of raising an extra unit of revenue from increasing the tax on that good. If these marginal costs differ across goods, then we increase welfare at constant revenue by reducing taxes on a good with a higher marginal cost and increasing them on a good with a lower marginal cost. Away from the optimum there will be many marginal costs of public funds, depending on the source.

At an optimum all these marginal costs must be equal, thus giving us for each tax a first-order condition for optimality. In the inverse optimum problem we use these first-order conditions to solve for social welfare weights on increments in income to each household *assuming* existing taxes are optimum. In this discussion (subsection 2.3) we draw on our work on the Indian tax system presented in Ahmad and Stern (1981, 1983a, 1985) and discuss the interpretation of this type of approach. The data requirements for implementing the approach are discussed in subsection 2.2.

There is no guarantee that the welfare weights calculated in the inverse optimum will be non-negative. If some are negative this suggests that at our initial point the utility possibility frontier (which will be tangential at an optimum to social indifference curves in utility space) is upward sloping in some directions. In other words, a Pareto improvement is possible. One can show (see subsection 2.4), using the Minkowski–Farkas lemma, that *either* a Pareto improvement is possible *or* there exists a solution to the inverse optimum problem with non-negative welfare weights. In subsection 2.5 we show how Pareto improvements or inverse optima may be calculated using simple linear programming techniques.

Results on improving directions of reform for Indian taxes are presented in section 3 together with those for inverse optima and Pareto improvements. We also discuss robustness of results to some of the assumptions and methods in section 4. In section 5 we offer some concluding remarks.

2. Theory

We begin this section with the problem of finding a beneficial direction of reform given that we have a social welfare function. We then examine the problem of inferring the welfare function (or at least its local properties) on the assumption that the existing state of affairs is optimum, the inverse-optimum problem, and then examine the relation between this latter problem and that of finding a Pareto improving reform. From one point of view, the discussion proceeds from the particular to the general, since one can see the analysis of Pareto improvements and the characterisation of Pareto optima as including the case where we have a social welfare function [see Guesnerie

(1977)]. However, the language of the social welfare function, and in particular, of the social marginal utility of income, will be central in understanding what follows and we therefore introduce it from the outset.

Throughout the formal analysis of this paper we shall be concerned with the direction of marginal reforms, i.e. with the direction of small movements from the status quo. This provides considerable theoretical advantages and economises on the assumptions and data, as we shall see. It is, however, limiting in scope in that it says nothing about the *size* of the reform and should be set side-by-side a non-marginal discussion — see, for example, King (1983). Furthermore, if there is *one* welfare improving direction of reform there will in general be many, and the question of which direction to choose is left open. With wider choice or further structure in the problem, one could integrate the choice of the direction and the size of step. Indeed, this is what is done when we characterise the optimum. The choice of a reform rather than an optimising problem raises some interesting issues, but they will not be pursued here. In this analysis we leave ambiguous the related problems of step size and choice amongst improving directions.

This ambiguity could be seen as problematic but could also prove advantageous in that much is no doubt left out of any model of taxation, and the choice of step length and amongst directions might be governed by these omitted, but possibly important, considerations. In the formal sense, however, we do not integrate such additional considerations into the choice amongst improving directions where there are many.

2.1. *Welfare reform*

We concentrate in this paper on consumer welfare and the government revenue constraint. We therefore adopt a simple model of the production side of the economy. It is most straightforward to suppose that producer prices are fixed and there are constant returns to scale, so that tax increases are reflected as consumer price increases and there are no pure profits. We think of the government as requiring certain resources, and thus revenue, for some given activities (these activities will be fixed throughout our analysis and thus there will be no effect of changes in them). With our model it is straightforward to show that constraints on production feasibility may be represented by the government budget constraint.[1] For more complicated models it is convenient and natural to introduce shadow prices to summarise the welfare effects of meeting demand changes, i.e. maintaining feasibility. These issues are discussed in Drèze and Stern (1983) and applications

[1]See, for example, Guesnerie (1977, p. 185). One does not need to assume fixed producer prices but the absence of pure profit is important — this may be imposed by constant returns to scale or 100% profits taxation.

described in Ahmad, Coady and Stern (1984). The assumptions of this paper essentially involve shadow prices equal to producer prices.

If factor incomes are fixed (for example there is inelastic supply of a single factor which is untaxed) then we may write household behaviour and utility simply as functions of consumer prices q. There are n goods indexed by i and t in a vector of specific taxes. Under our assumptions:

$$q = p + t, \tag{1}$$

where p is the fixed producer price vector. Thus, we may speak interchangeably of changes in, and derivatives with respect to, q and t. There are H households indexed by $h = 1, 2, \ldots, H$.

Given prices q, the demand of household h, $x^h(q)$, maximises utility, $u^h(x^h)$, subject to the household budget constraint. Then $v^h(q)$, the indirect utility function, gives the maximum utility possible at prices q and

$$v^h(q) = u^h(x^h(q)). \tag{2}$$

We suppose that the social welfare function is of the Bergson–Samuelson variety which may be written:

$$W(u^1, u^2, \ldots, u^H). \tag{3}$$

We also write social welfare as a function of prices, $V(q)$, where

$$V(q) = W(v^1(q), v^2(q), \ldots, v^H(q)). \tag{4}$$

The aggregate demand vector $X(q)$ is given by

$$X(q) = \sum_h x^h(q), \tag{5}$$

and government tax revenue R is

$$R = t \cdot X = \sum_i t_i X_i. \tag{6}$$

The tax problem is then:

R: Find a vector of tax changes $\mathrm{d}t$ such that $\mathrm{d}V \geqq 0$ and $\mathrm{d}R \geqq 0$ with one of the inequalities holding strictly. Thus, we wish to find a tax change which yields a welfare improvement but which does not decrease tax revenue. One could use methods similar to those described below to discuss reforms in the level of R, but they will not be considered here.

We can find welfare improvements as in **R** if the marginal cost, λ_i, in terms of social welfare of an extra rupee raised via the ith good exceeds that for the jth good. Then we increase welfare at constant revenue by increasing taxes on the jth good by an amount sufficient to raise one rupee and decreasing taxes on the ith good by an amount sufficient to lose one rupee. To raise an extra rupee on the ith good we have to increase the ith tax by $1/(\partial R/\partial t_i)$. Thus:

$$\lambda_i = -\frac{\partial V}{\partial t_i}\bigg/\frac{\partial R}{\partial t_i}, \tag{7}$$

since $\partial V/\partial t_i$ is the response of social welfare to a tax change and we have the minus sign to denote the marginal *cost*. It is obviously central in what follows that away from the optimum the λ_i will differ. Notice that this means there is a marginal cost of public funds corresponding to each source and that one should not speak of *the* marginal cost of public funds without specifying the source.

A sufficient condition for a welfare improvement to be possible is therefore:

C: There exists i and j such that $\lambda_i \neq \lambda_j$.

There will of course in general be many welfare improving directions. Indeed, there will be a whole cone of directions given by the intersection of the two half-spaces defined by

$$\mathrm{d}V = v \cdot \mathrm{d}t \geqq 0 \tag{8}$$

and

$$\mathrm{d}R = r \cdot \mathrm{d}t \geqq 0. \tag{9}$$

where v_i is $\partial V/\partial t_i$ and r_i is $\partial R/\partial t_i$.

This is illustrated in the two-dimensional case by the shaded area AOB in fig. 1, where AA' is the line of constant revenue, $\mathrm{d}R=0$, BB' is constant welfare, $\mathrm{d}V=0$, and the initial position is the point O. We confine attention to marginal reforms so that we may treat v and r as constant vectors and the lines AA' and BB' as straight. The condition for the absence of a local improving reform is that the two lines lie along each other. This is precisely that the vector v should be proportional to r or that $\lambda_i = \lambda_j$ for all i and j.

To compute λ_i we must examine $\partial V/\partial t_i$ and $\partial R/\partial t_i$; for the former we use the definition of the indirect social welfare function (4), and for the latter the equation for revenue (6). Now if we increase the ith tax the hth household is worse off in money terms by the amount consumed, x_i^h, or in terms of utility,

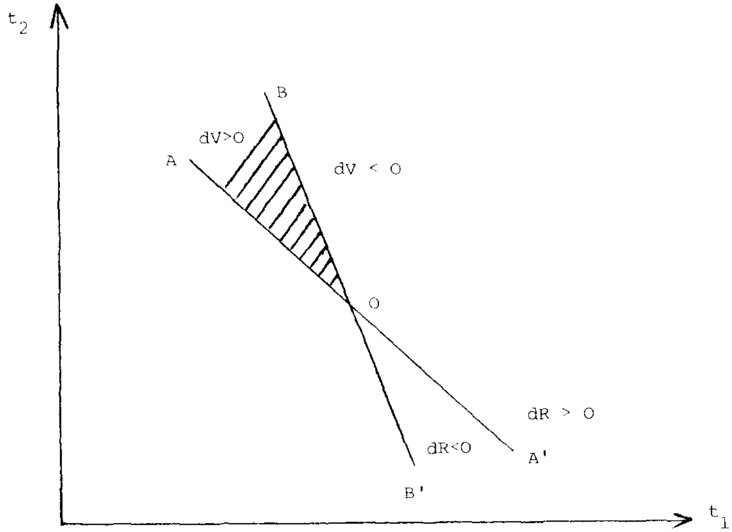

Fig. 1. (*Note: AOB* is the region of feasible welfare improvement. O is the original position. Marginal changes only are considered so that AA' and BB' are straight lines.)

by $\alpha^h\ x_i^h$, where α^h is the (private) marginal utility of income. That is the familiar Roy's identity:

$$\frac{\partial v^h}{\partial q_i} = -\alpha^h x_i^h. \tag{10}$$

Thus, using (10), (4) and the constancy of producer prices:

$$\frac{\partial V}{\partial t_i} = -\sum_h \beta^h x_i^h, \tag{11}$$

where

$$\beta^h = \frac{\partial W}{\partial u^h}\alpha^h. \tag{12}$$

is the social marginal utility of income of household h, or the welfare weight. And using (6) we have:

$$\frac{\partial R}{\partial t_i} = X_i + \sum_k t_k \frac{\partial X_k}{\partial t_i}. \tag{13}$$

We can now calculate λ_i by using (7), (11) and (13). These expressions can be used to provide decompositions of λ_i and special cases, which are useful in understanding certain features and in interpreting results. From (7), (11) and (13) we have:

$$\lambda_i = \frac{\sum_h \beta^h x_i^h}{X_i + \sum_k t_k \left(\frac{\partial X_k}{\partial t_i}\right)}. \tag{14}$$

Eq. (14) may be examined in a similar manner to that familiar from the literature on optimum commodity taxation. For example, in the (implausible) case where $\beta^h = 1$ (all h) and cross-price effects may be ignored, the marginal cost of funds λ_i is given by:

$$\lambda_i = \frac{1}{1 - \varepsilon_i (t_i / q_i)} \tag{14a}$$

where ε_i is the own price elasticity and t_i/q_i is the tax as a proportion of the consumer price.

It should be noticed, however, that for applied work, it is not necessary to decompose gross price responses into income and substitution effects, as one would in the derivation of the many-person Ramsey rule [see, for example, Atkinson and Stiglitz (1980)]. What matters for the calculation in practice is the matrix of *aggregate* price responses (see subsection 3.5 below for a full discussion).

Decompositions are most conveniently analysed using the inverse of λ_i:

$$\frac{1}{\lambda_i} = \frac{X_i}{\sum_h \beta^h x_i^h} + \frac{\sum_k t_k \frac{\partial X_k}{\partial t_i}}{\sum_h \beta^h x_i^h}. \tag{15}$$

The number $1/\lambda_i$ is the revenue cost on the margin of generating an extra unit of welfare via a reduction in the ith tax (essentially it is $\Delta R/\Delta V$). As we see from (15), it may be decomposed into two components, the first of which involves only household demands and welfare weights, and the second, in addition, taxes and aggregate demand responses. The first term on the right-hand side of (15) is the reciprocal of the 'distributional characteristic' of the good [see, for example, Feldstein (1972) and Atkinson and Stiglitz (1980)]. With strong aversion to inequality this term will play an important role in the ranking of $1/\lambda_i$ across goods, since the dominant contribution to it would be the reciprocal of the share in total consumption of good i by the poorest groups. Thus, taking the effect of this term only, the highest λ_i, or lowest

$1/\lambda_i$, would be for the good with the highest consumption share by the poor in its total. Notice that with equal welfare weights (say, of unity) this term will be one for all goods and thus will not in this sense contribute to the ranking of λ_i.

An alternative way of expressing the combination of distributional and revenue effects in λ_i is to write

$$\lambda_i = D_i \Bigg/ \left[\frac{t_i}{t_i X_i} \frac{\partial}{\partial t_i} (t \cdot X) \right], \tag{15a}$$

where D_i is the distributional characteristic and the denominator is a tax elasticity.

The second term in (15) involves the effect of demand responses on revenue and it is clear that it must in general play a role. There is a special case, however, that of uniform proportional taxes, where the first and second terms in (15) are proportional. If t_k equals αq_k, then we have:

$$\sum_k t_k \frac{\partial X_k}{\partial t_i} = \alpha \sum_k q_k \frac{\partial X_k}{\partial q_i}, \tag{16}$$

since with fixed producer prices $dt_i = dq_i$. And from the household budget constraints we have, from the standard adding-up conditions, that

$$\sum_h q_k \frac{\partial X_k}{\partial q_i} = -X_i. \tag{17}$$

Hence, for uniform proportional taxes

$$\frac{1}{\lambda_i} = (1-\alpha) \frac{X_i}{\sum_h \beta^h x_i^h}, \tag{18}$$

and the directions of improving reform are given entirely by the distributional characteristic. From the uniform position we should raise taxes on goods with low distributional characteristics and reduce them on goods with high distributional characteristics. Where all the welfare weights are equal, then all the λ_i will be equal, and there will be no improving marginal reform. This is similar to the result for a one-consumer economy that uniform taxation is optimum, provided the necessary revenue does not exceed the lump-sum income of the consumer, since in that case proportional taxation at the appropriate rate acts just like the optimum lump-sum tax.

Notice that the numerator in the second term in (15) may be seen as a weighted sum of the aggregate demand derivatives with the taxes as weights. We know in general from (17) that weighting the demand derivatives by the prices q_k and adding will lead to the number $-X_i$, *whatever* the demand derivatives happen to be, provided that they are consistent with the adding-up property. The condition (17) therefore acts as a constraint on the sensitivity of λ_i and $1/\lambda_i$ to variations in assumptions on the aggregate demand system. In this sense proportional taxes, $t_k = \alpha q_k$, are a limiting case in that the ranking of the λ_i is completely insensitive to the demand system. It is important to remember that we are looking only at indirect taxes here. Both reform and optimality questions are sensitive to the combination of the range of policy tools available and to functional forms [see Stern (1984) and section 4 below for further discussion].

2.2. *Data implications*

There are four necessary items of information: the household demands x_i^h, the taxes t_k, the aggregate demand derivatives $\partial X_k/\partial t_i$, and the welfare weights β^h. The first of these is available if we have household expenditure data. The second may be derived from a study of the indirect tax system. In practice this may require considerable effort, particularly if there are taxes on intermediate goods. Notice that our taxes, t_k in the preceding analysis, are taxes on final consumption goods. Therefore the application is greatly simplified if taxes on intermediate goods can be translated into taxes on final goods. This is possible for certain models of the production process, and we can calculate the 'effective tax' or the tax element in the price taking into account taxation of inputs, inputs into inputs and so on. For the Indian case we have carried out substantial work on this problem. Much of this is presented in Ahmad and Stern (1983a) and will not be discussed in detail here, but we report briefly in section 3. The demand derivatives can be obtained from estimates of aggregate demand systems, and the β^h are explicit value judgements which are introduced exogenously. One is, of course, interested in experimenting with different possible sets of value judgements.

The theory we have derived is therefore in a form which can be readily applied. In practice our estimates of the various quantities and parameters will not be entirely reliable and it will be important to carry out sensitivity analyses. Notice, however, that the list of information we require does not contain items which would be necessary for the analysis of non-marginal reforms. In particular, we do not need estimates of demand and utility functions for *individual* households. For a marginal reform all the household information that is necessary is the consumptions since these tell us what the utility consequences of marginal changes would be. Notice that these quantities would give us utility changes in money terms whether a good is chosen optimally [as in (10)] or it is rationed. In the rationed case, the ration quantity is obviously a measure of the money loss from a unit price increase. Furthermore, the information on aggregate demand and taxes that is necessary may be combined in the n numbers $\partial R/\partial t_i$ as given in (13). In practice one would calculate $\partial R/\partial t_i$ using taxes and demand derivatives, but it may be convenient to carry out sensitivity analysis in terms of $\partial R/\partial t_i$.

2.3. *The inverse optimum problem*

As we have seen, a necessary condition for optimality is that all the λ_i

should be equal. If we call the common value λ, then the condition is:

$$\frac{\partial V}{\partial t_i}+\lambda\frac{\partial R}{\partial t_i}=0. \tag{19}$$

Clearly, this is the first-order condition we should find if we considered the problem:

$$\text{maximise} \quad V(t) \quad \text{subject to} \quad R(t)\geqq \bar{R},$$

where we take a Lagrange multiplier λ for the constraint and form the Lagrangian:

$$\mathscr{L}=V+\lambda(R-\bar{R}). \tag{20}$$

Using (11), writing $r_i=\partial R/\partial t_i$ and dividing by λ we have:

$$\sum_h \frac{\beta^h}{\lambda} x_i^h = r_i \tag{21}$$

or

$$\beta' C = r', \tag{22}$$

where the hth component of β is β^h (where convenient and appropriate we shall set $\lambda=1$ by choice of scale for V), and C is the $H\times n$ consumption matrix with hith element x_i^h. The inverse optimum problem is **I**

I: Find β satisfying (22).

The inverse optimum is familiar to economists in the form of demand analysis where we make inferences concerning the preferences of individuals from their decisions concerning demand. It may seem unusual or peculiar when applied to government decisions, but in some ways the spirit is similar. We say that if the individual (government) takes this decision in this environment then it is behaving *as if* his (its) objective were described by this utility function (this set of welfare weights). There are, of course, important differences. A government may not behave as a single rational decision-maker or the environment may be perceived much less clearly or accurately than for the individual. These considerations lend a different perspective to the inverse optimum problem for governments and can suggest different versions of the problem. Thus, for example, one can see the solution to the inverse optimum problem as part of a commentary on government policy: for example, this policy would only be optimum for a rational decision-maker in this environment if he had these very peculiar values. Alternatively, we could say that given certain values then this policy could only be optimum if the government had the following view of the environment.

The last formulation is a different version of the inverse optimum problem and one which we investigated in Ahmad and Stern (1981). Thus, as well as considering problem **I** we asked, given welfare weights β, and consumption patterns C, what would perceived revenue responses r have to be in order to satisfy (22)? These calculated r could be used to infer implicit government assumptions about certain demand responses. Or. given assumptions about demand responses, they could be used to calculate the tax element t in the price. Notice that this is *not* a calculation of what optimum taxes should be. One is taking given current prices, given demands, and given demand responses and asking what would the tax component in the price have to be in order to be described as optimum. This was interesting in our context because it was calculated in advance of our calculation of effective taxes so we did not then know what were appropriate assumptions about the tax element in the price. The inverse optimum calculation of taxes gave us a benchmark with which to compare our subsequent calculations.

In this paper we shall concentrate on the inverse optimum problem in the form of **I**, that is we try to find welfare weights which satisfy the first-order conditions for the optimum given by (22). It is clear that the number of tax instruments, here goods relative to the number of households, will be of considerable importance: (22) gives us n equations, one for each good, and we are seeking H unknowns, the components of β. If we group the households so that $H=n$ and the matrix C is invertible, then

$$\beta' = r'C^{-1}. \tag{23}$$

If, on the other hand, there are more goods than households, then it would not in general be possible to find β^h so that all n equations in (22) hold simultaneously. We might then try to choose β^h to minimise some measure of the deviations between the two sides. A similar approach arises if we describe β as depending on a number of household characteristics which is less than n [see, for example, Christiansen and Jansen (1978)].

The most common state of affairs in practice, however, will be when the number of households exceeds the number of tax instruments. One would expect then that there would be many possible vectors of β's satisfying (22). But there is no guarantee that any of these vectors will be non-negative, i.e. that we can find non-negative welfare weights on households which would yield the current state of affairs as an optimum. Indeed, in our earlier experiments on Indian data for 1974/75 when we grouped households so that $H=n$, we found that some welfare weights in our solution to (23) were negative. As explained in the introduction, negative welfare weights in the inverse optimum lead one to suppose that Pareto improvements are possible. It is to the identification of such improvements and their relation with the inverse optimum problem that we now turn.

2.4. Pareto improvements and the inverse optimum

The change in utility du^h of the hth household in response to tax changes dt is (recall α^h is the private marginal utility of income):

$$du^h = -\alpha^h \sum_i x_i^h \, dt_i. \tag{24}$$

The condition for the tax change not to decrease revenue is:

$$dR = r \cdot dt \geqq 0,$$

where as before $r_i = \partial R / \partial t_i$. Thus, we have a feasible Pareto improving change if we can find dt such that the $H+1$ inequalities,

$$C dt \leqq 0 \quad \text{and} \quad r \cdot dt \geqq 0, \tag{25}$$

are satisfied with at least one of them holding with strict inequality.

We illustrate this in figs. 2(a) and (b) with diagrams analogous to fig. 1. In fig. 2(a) we present an example where a Pareto improving change is possible — within the region AOB both individuals are better off and revenue increases. However, in fig. 2(b) it is not possible to find a Pareto improvement. In each diagram the revenue constraint is represented by the condition that we be on or above AA'. The condition that the utility of the first person should not increase is that we be on or below the line BB', and the condition that the utility of the second person should not decrease is that we be on or below the line CC'. In fig. 2(a) these three conditions are compatible for points in the cone AOB. However, in the second case, fig. 2(b), there are no points, other than the initial position O, that meet this condition. Notice that in fig. 2(b) the constant revenue line AA' lies *between* the BB', and CC', i.e. AA' is a *non-negative* linear combination of BB' and CC'.

Geometrically we require, for a Pareto improvement to be possible, that the intersection of the $H+1$ half-spaces defined by (25) should contain some point other than the origin. The intersection of these half-spaces will either be a convex cone, as in fig. 2(a), or the single point O as in fig. 2(b). Apart from special cases, the cone will be of full dimension, so that if a Pareto improvement is possible, then one will be able to find a direction which makes everyone better off and increases revenue, i.e. we can find a direction lying strictly off the hyperplanes AA', BB', etc. An example of a boundary case where strict improvement for everyone would not be possible would be if, in fig, 2(a), BB' actually lay along AA'. Then the only Pareto improving directions would be along AA' and this would, in the appropriate direction (OA), increase the utility of the second individual but leave that of the first individual unchanged.

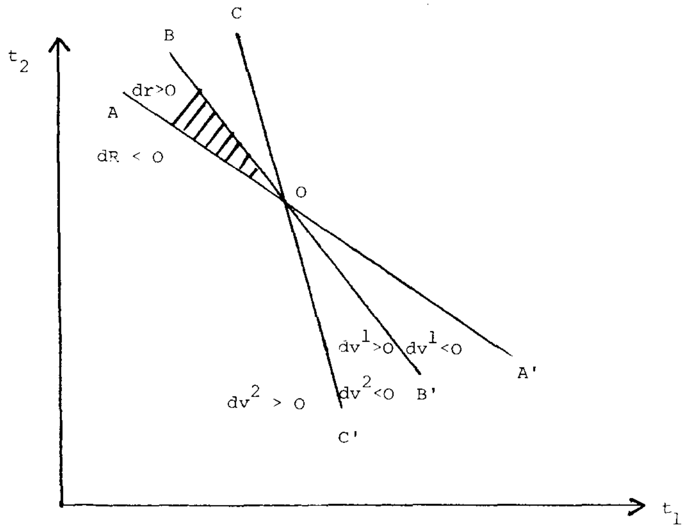

Fig. 2(a)

AOB is the region of Pareto improving changes. O is the original position.

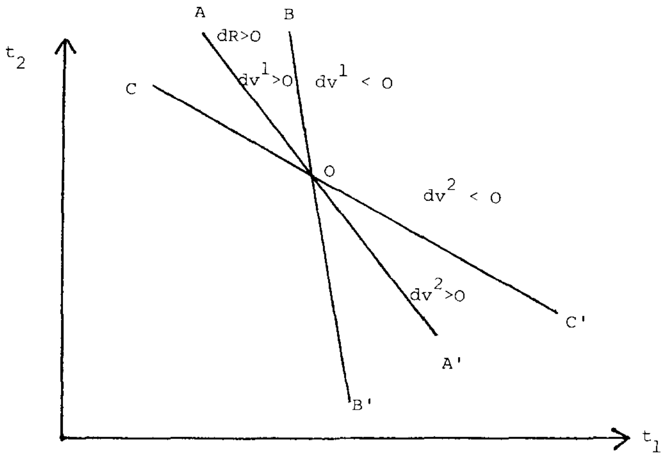

Fig. 2(b)

To be specific, let us present the problem of finding a Pareto improvement as seeking a tax change which strictly increases revenue without making anyone worse off (one would then, for example, make everyone better off by reducing the tax on a good which everyone consumes by an amount which is sufficiently small to keep the revenue increase positive). Thus, we pose the problem as finding $\mathrm{d}t$ such that

Q: $C\mathrm{d}t \leqq 0$ and $r \cdot \mathrm{d}t > 0$.

Now the Minkowski–Farkas lemma [see, for instance, Intrilligator (1971, p. 92)] tells us that *either* there exists a dt satisfying **Q** *or* r is a non-negative linear combination of the rows of C, i.e. there exists an H-vector y, with $y^h \geqq 0$, such that

$$r' = y'C. \tag{26}$$

The either/or is strict in the sense that one and only one of the situations applies.

The Minkowski–Farkas lemma therefore tells us that *either* a feasible Pareto improvement exists, *or* a solution to the inverse optimum problem, with *non-negative welfare weights*, exists.

The reason is that (26) is exactly of the form of (22) and therefore y plays exactly the role of the vector β in the inverse optimum problem **I**. Notice that in fig. 2(b), where no Pareto improvements exists, the revenue constraint AA' is a non-negative linear combination of the constant utility lines BB' and CC' exactly as the Minkowski–Farkas lemma suggests.

The view of the inverse optimum problem as that of finding non-negative weights y which express r as linear combinations of rows of C suggests a natural piece of intuition concerning the possibility of finding such weights. For there is one row of C for each household and therefore, as we increase the number of households (for example by taking finer disaggregations), we increase the likelihood of being able to express r as a linear combination of the rows of C. By the Minkowski–Farkas lemma another way of expressing this is to say that, as we increase the number of households, it will become more and more difficult to find a Pareto improvement.

The simple but obvious either/or result is clearly of considerable importance for the analysis of tax reform in practice. For we can first find out whether a Pareto improvement is possible by trying to solve problem **Q**. If the only possible change in taxes which does not lower anyone's utility and does not lower revenue is the trivial, or no change, $dt = 0$, we can then solve problem **I** confident that non-negative welfare weights for the inverse optimum problem are possible.

There is a fairly large (and quite technical) literature on Pareto improving tax reform following the work of Guesnerie (1977) [see, for example, Dixit (1979), Diewert (1978), Weymark (1979, 1981), and Tirole and Guesnerie (1981)]. The apparent complexity of the literature lies sometimes in the way the mathematics are expressed, sometimes in the study of a sequence of changes in a more general production structure, and sometimes in a concern with the question of the possibility of a Pareto improvement requiring production inefficiency. And where many of the complications are present together, more sophisticated mathematics may be necessary. The intuition behind the main results is, however, to a substantial extent that embodied in the simple discussion of this section. The result we have just derived using the Minkowski–Farkas lemma is essentially a special case of Guesnerie (1977, proposition 4) which is the main finding of that literature.

Guesnerie (1977) raised the problem that in certain cases a Pareto improvement may actually require production inefficiency. The possibility may be represented straightforwardly using fig. 2. The cone of Pareto improvements might be *BOC* (if, for example, the first individual consumes negative amounts of the two goods) in which case any Pareto improvement requires $dR>0$. Less artificial examples would be available in higher dimensions. Notice that an additional tax tool for returning revenue to consumers would usually dispose of this problem since we could then reduce revenue back to zero using the extra tool and make consumers better off [see Smith (1983)].

We hope our exposition will make the results more accessible and illustrate the advantage of being able to work with the revenue constraint in the more simple contexts. Furthermore, the language of the inverse optimum, in our judgement, adds considerably to the intuition. Our particular motivation here, however, is to show how the theory can be applied. We therefore turn to the question of how the Pareto improvement, or alternatively, the inverse optimum, may be calculated.

2.5. The computation of Pareto improvements and inverse optima

We wish to find a tax change, dt, such that $r\,dt>0$ and $C\,dt\leqq 0$, i.e. to increase revenue subject to making no person worse off. One way of doing this would be to maximise the increase in revenue subject to no person being made worse off. Thus we can try to see whether the following linear programme has solutions, where τ stands for dt:

$$\text{maximise} \quad r\tau \quad \text{subject to} \quad C\tau\leqq 0. \tag{27}$$

We must clearly impose some bounds on τ since the programme is linearly homogeneous in τ, so that if there exists a Pareto improvement, and thus some τ_0 with $r\tau_0<0$, then (27) as stated implies that we could multiply τ_0 by any positive number and yield an improvement as large as we please. Furthermore, we wish, in any case, to confine attention to small changes and therefore are interested only in directions. As explained at the beginning of section 2, the choice of the size of the step, and the relation between this and the appropriate direction, is left open in this analysis. Accordingly we shall impose bounds on τ which say that we cannot increase or reduce any tax by more than a certain amount.

It is instructive to rewrite (27) in a form which allows comparison with our earlier analysis of reform in section 2.1. To do this we consider not simply increases in taxes $\mathrm{d}t$ but increases by an amount sufficient to raise one rupee. Thus, we change the variable in (27) from τ to δ with δ_i being the extra revenue raised from increasing taxes on the ith good. Thus

$$\delta_i = r_i \tau_i . \tag{28}$$

Then we write (27) together with bounds on τ (and thus δ) as:

$$\begin{aligned} &\text{maximise } \iota \cdot \delta \\ &\text{subject to } \quad L\delta \leqq 0 \\ &-1 \leqq \delta_i \leqq +1, \end{aligned} \tag{29}$$

where the elements of the vector ι are unity, L is the matrix with hith element λ_i^h where

$$\lambda_i^h = x_i^h \bigg/ \frac{\partial R}{\partial t_i}, \tag{30}$$

and we have bounded tax changes with the requirement that we cannot increase or reduce any tax by an amount which changes revenue by more than one rupee. The λ_i^h, and thus problem (29), have a very natural interpretation: λ_i^h is the marginal cost in money terms to the hth household of increasing the tax on the ith good by an amount sufficient to raise one rupee. The constraints corresponding to households in problem (29) are simply that the sum of the marginal costs from the tax changes to each household should be negative. Results from solving (29) using Indian data are presented in the next section.

The λ_i^h corresponds to λ_i, as defined in subsection 2.1 [see eqs. (7) and (11)] with β^h being the unit vector with 1 in the hth place and zeros elsewhere. Thus, where the β's are specified exogenously:

$$\lambda_i = \sum_h \beta^h \lambda_i^h \tag{31}$$

or

$$\lambda' = \beta' L. \tag{32}$$

In general if there is one direction of Pareto improvement, then there will be many. We have in the above concentrated on maximising revenue gain subject to leaving no household worse off. Alternatively, we could maximise the utility increase for one household, say the poorest, whilst neither reducing revenue nor the utility of any other household. Formally we consider the programme:

$$\begin{aligned} &\text{Maximise} \quad -\delta\lambda^h \\ &\text{subject to} \quad \iota\delta \geqq 0 \\ &\text{and} \quad L_{-h}\delta \leqq 0, \qquad -1 \leqq \delta_i \leqq +1, \end{aligned} \tag{33}$$

where λ^h is the hth row of L (thus with ith component λ_i^h) and L_{-h} is the $(H-1)\times n$ matrix corresponding to L with the hth row deleted.

If no solution to the problem of finding a Pareto improvement exists, then the maximum for programmes (29) and (33) will be zero, given by $\delta=0$. We can then, by the Minkowski–Farkas lemma, solve the inverse optimum problem with non-negative welfare weights, i.e. there exists a set of $y^h \geqq 0$ solving

$$r' = y'C' \tag{26}$$

or

$$\iota' = y'L. \tag{34}$$

As we remarked above [see (23)], the solution to (26) is straightforward if $H=n$ and C is invertible, for then

$$y' = r'C^{-1}. \tag{35}$$

This suggests that when $H<n$ we try post-multiplying (26) by the generalised inverse of C, i.e. $C'(CC')^{-1}$ if C is of rank H to give

$$r'C'(CC')^{-1} = y'. \tag{36}$$

Thus, we could find a solution to (26) by regressing r on the rows of C and reading y off as the regression coefficients. With $H<n$ it is likely that there will be a unique set of welfare weights satisfying (26) and by the Minkowski–Farkas lemma they will be non-negative.

More generally and for the case $H>n$ we could find a solution with non-negative components by solving the programme which, say, maximises the weights on the first household:

$$\begin{aligned} &\text{Maximise} \quad y^1 \\ &\text{subject to} \quad r' = y'C \\ &\text{and} \quad y^h \geqq 0 \quad \text{and} \quad \sum_h y^h \leqq 1. \end{aligned} \tag{37}$$

If the maximum value is greater than zero, then we know there exists a set of welfare weights giving positive weight to the first household and with respect to which the status quo is optimum. If the maximum is zero, then we know that even though the inverse optimum is soluble it must reflect zero concern with the welfare of the first household. Alternatively, we might maximise the weight on the hth household using a maximand y^h. To be sure of finding a set of weights in a single programme we would maximise $\sum_h y^h$. There will in general be many sets of non-negative welfare weights for $H>n$ if no Pareto improvement exists and procedures of the kind described in (37) are simply a convenient way of finding one of the sets.

We have in this section described a clear strategy for computation in the problems of Pareto improvement and the inverse optimum. We first establish whether or not an improvement is possible by explicitly trying to compute one as in (29) or (33). If it is possible, we can display several different directions. If a Pareto improvement is impossible, then we can compute non-negative welfare weights for the inverse optimum using a procedure such as (37).

3. Results

We begin in subsection 3.1 with a discussion of the data requirements that were identified in subsection 2.2. Directions of tax reform given specific exogenous welfare weights are presented briefly in subsection 3.2 using Indian data for 1979/80. In subsection 3.3 we examine the inverse optimum problem and in subsection 3.4 we look at some of the Pareto improvements that are possible. At the end of this section, in subsection 3.5, we examine the robustness of the analysis to the specification of the model and parameters.

3.1. Data

We saw in subsection 2.2 that four main sets of data are required: household consumption, taxes, aggregate demand derivatives, and welfare weights. Of these it was the availability of the necessary information on taxes that led to the choice of 1979/80 as the year for which the analysis was carried out. This was the base year for the Indian Sixth Plan, and also one for which an input–output table was available [see Government of India (1981)]. The input–output table was essential because of the substantial taxation of intermediate goods in India, and this permitted the estimation of effective taxes for the 89-commodity groups used in the input–output table. By the effective tax on a good we mean the extra government revenue arising from an exogenous increase in final demand for the good. With taxes on intermediate goods this becomes, in a linear model, the element of tax in the market price of a good. Thus, in this case the marginal concept of the effective tax as defined coincides with the average.

The commodity-wise allocations of revenue for different taxes, and subsidies, as well as the resulting estimates of effective taxes for India for 1979/80, are presented in Ahmad and Stern (1983a). These will not be discussed further here. Note, however, that India has a fairly complicated federal system of taxation, with the central government controlling essentially tariffs and excise duties on domestic production, and the tax revenues of the state governments being largely composed of sales taxes and excises on alcohol. The all-India vector of effective indirect tax derived from Ahmad and Stern (1983a) and used in this paper is limited in a number of respects. It does not include that part of the tax element in the price of commodities arising from the taxation of assets. This issue and experiments with alternative pricing rules are discussed in greater detail in Ahmad and Stern (1985).

Although the tax data refer to 1979/80, the latest available consumer expenditure matrix (household by commodity) was for the year 1973/4 [see Government of India (1977)]. We therefore make adjustments to consumer expenditure data for compatibility with 1979/80 aggregates. We take as given the 89×1 vector of private consumption expenditure at market prices from the 1979/80 input–output table, and the distribution of total expenditure across 14 rural and 14 urban groups from the published 1973/74 expenditure survey. We thus have row and column totals for a 28×89 matrix of consumer expenditure for 1979/80. We apply the RAS method [see, for example, Bacharach (1971)] to the 1973/74 expenditure matrix using these row and column totals thereby generating the final consumer expenditure matrix for 1979/80. For further details see Ahmad and Stern (1983a, pp. 49–50).

Not all the tax information in our 89×1 effective tax vector and the 28×89 consumer expenditure matrix can be utilised for the analysis of tax reform described in earlier sections. This is because demand elasticities are not available for the 89 sectors. One of the latest available estimates of aggregate own and cross price elasticities for India is by Radhakrishna and Murty (1981). These are based on an extended version of the linear expenditure system estimated for nine commodity groups.[2] We are, therefore, constrained in our choice of commodity groups, and have aggregated the consumer demand matrix and tax vectors to match the nine-commodity dimensions of the demand elasticity matrix.

The number of commodity groups and the form of the demand function used are likely to have an important influence on the results. It is well known that optimum taxes are sensitive to the specification of the demand system [see Atkinson (1977), Atkinson, Stern and Gomulka (1980), and Deaton (1981)] and the issue of the robustness of the results is discussed further below (section 4).

[2]These are: cereals; milk and milk products; edible oils; meat, eggs and fish; sugar and gur; other food; clothing; fuel and light and other non-food. For details see Radhakrishna and Murty (1981). In further work we shall be carrying out our own estimates.

However, tax reform procedures are generally less demanding in informational requirements and probably less sensitive to model specification than optimum tax calculations [see below and Deaton (1984)]. The demand system used by Radhakrishna and Murty was the linear expenditure system, as modified by Nasse (1970) to allow minimum consumption levels to depend on prices. This was estimated separately for five urban and five rural groups classified by expenditure category. The data were the time series of cross-sections published in the reports of the National Sample Survey Organisation for rounds 2 to 25 (1947–48 to 1970–71). Aggregate demand elasticities were computed from calculations of demand derivatives at group means for each of the ten groups. The elasticities given by Radhakrishna and Murty are for 1970–71.

We require elasticities of total demand for each good for 1979/80 for this analysis and we must ensure that these elasticities satisfy the condition imposed by the budget constraints. Accordingly, the Radhakrishna–Murty elasticity matrix has been adjusted so that it satisfies the 1979/80 shares of consumer expenditure [for further details see Ahmad and Stern (1983a)]. This matrix is presented in the Appendix table, and we write the (adjusted) elasticity of the demand for good j with respect to the price of good i as ε^*_{ji}.

The final requirement for computations of tax reform is the set of value judgements. These are discussed in the next subsection.

3.2. *Tax reform*

A central element in our analysis is to estimate the welfare loss from increasing the tax on the ith good by an amount sufficient to raise one rupee of government revenue. We called this loss λ_i, and showed that under the assumptions of subsection 2.1 it was given by eqs. (7), (11) and (13). We can rewrite the denominator (13) to obtain:

$$\lambda_i = \frac{\sum_h \beta^h p_i x_i^h}{p_i X_i + \sum_j \frac{t_j^e}{p_j} p_j X_j \varepsilon^*_{ji}}, \tag{38}$$

where the $p_i X_i$ are derived from our estimated consumer expenditure matrix, ε^*_{ji} from the adjusted elasticity matrix. and t_j^e/p_j is the overall effective tax vector as a proportion of the consumer prices.

The welfare weights β^h may be specified in a number of ways. As an example we can generate them using the function

$$U^h(I) = k\frac{I^{1-e}}{1-e}, \qquad e \neq 1,\ e \geqq 0$$

$$= k \log(I), \qquad e = 1,$$

where I^h is the total expenditure per capita of the hth household, and $e \geqq 0$ for concavity. We have $\beta^h = U'(I^h)$, and we choose a normalisation for β^h, by choice of k, so that the welfare weight for the poorest household is unity. Under these assumptions we have:

$$\beta^h = (I^1/I^h)^e. \tag{39}$$

This representation of β^h should be seen as a convenient local approximation given current prices and incomes rather than an exact expression holding for all prices and incomes [see Roberts (1980), for the problems that arise under the second interpretation]. With these assumptions and with the viewpoint we have suggested, β^h represents the marginal social value of a unit of expenditure to individual h relative to a unit to individual 1. The perception is that of the commentator, or the government, if it can be said to have one. With $e > 0$, $\beta^h < 1$ so that increments of expenditure to the poor are seen as more valuable than those to the rich. The ratio $\beta^h/\beta^{h'}$ increases with e for $I^h < I^{h'}$ and thus e may be thought of as an 'inequality aversion parameter'. In this exercise five levels of e have been chosen: 0, 0.1, 1, 2 and 5. A value for e of 0 implies that the policy-maker values Rs 1 of expenditure for the poorest individual as equivalent to Rs 1 for the richest. A value of e of 1 says that a marginal unit to h is worth half as much as a marginal unit to individual 1 if the expenditure of h is twice that of individual 1. Values of e in excess of 2 give a very much greater weight to the poorest, and 5 and above begin to approach the 'maxi-min' or Rawlsian utility function, by considering the welfare only of the poorest individuals (a marginal unit to the poorest is worth 32 times a unit to someone with twice the expenditure).

The isoelastic function β^h given in (39) represents just one commonly used method for generating the welfare weights. As defined they depend only on per capita expenditure levels. It is a straightforward matter, with sufficient data, to allow them to depend on other characteristics of the household, for example household size and composition, urban versus rural, region, caste and so on. There have been a number of discussions of the 'appropriate' level of e in the literature [see, for example, Stern (1977)]. It is interesting to note that after, one supposes, considerable introspection but prior to his being Chancellor of the Exchequer, Dalton suggested that e was greater than 1 and possibly around 2 [see Dalton (1939, pp. 97–99)].

As we saw above, the general form for the n-vector λ as a function of the H-vector β is given by (32). Accordingly, we present the matrix L' for the 9-household case in table 1(b) (the 9-household groups are aggregated from the 28). Column h of L' represents the welfare loss to the hth household from increasing the taxes on each of the goods by an amount sufficient to raise one rupee. The column therefore provides the λ_i corresponding to a social welfare function which gives a weight of 1 to the hth household and zero to

Table 1

(a) Effective taxes and λ's (28-household case)

Commodities	Effective taxes t^e	$e=0$: λ	Rank	$e=0.1$: λ	Rank	$e=1$: λ	Rank	$e=2$: λ	Rank	$e=5$: λ	Rank
Cereals	−0.052	1.0340	8	0.8845	7	0.2377	2	0.0665	2	0.0047	2
Milk and milk products	0.009	1.0037	9	0.8320	9	0.1703	9	0.0353	9	0.0008	9
Edible oils	0.083	1.0672	6	0.9002	6	0.2163	4	0.0542	4	0.0027	5
Meat, fish, eggs	0.014	1.0532	7	0.8844	8	0.2058	6	0.0508	5	0.0028	4
Sugar and gur	0.069	1.0892	5	0.9156	5	0.2124	5	0.0508	5	0.0021	6
Other foods	0.114	1.1352	4	0.9561	3	0.2293	3	0.0588	3	0.0037	3
Clothing	0.242	1.2450	1	1.0204	1	0.1920	7	0.0378	7	0.0010	8
Fuel and light	0.247	1.1632	2	0.9887	2	0.2562	1	0.0708	1	0.0054	1
Other non-food	0.133	1.1450	3	0.9328	4	0.1729	8	0.0356	8	0.0014	7

(Column heading in the original: Levels of inequality aversion e.)

(b) L' matrix for 9-commodity and 9-household groups

Household groups (Rs per capita)	(0–25)		(25–32)		(32–40)		(40–45)		(45–55)		(55–70)		(70–90)		(90–150)		(150+)	
Commodities	λ_i^h	Rank	λ_i^h	Rank	λ_i^h	Rank	λ_i^h	Rank	λ_i^h	Rank	λ_i^h	Rank	λ_i^h	Rank	λ_i^h	Rank	λ_i^h	Rank
Cereals	0.0027	2	0.0188	2	0.0211	2	0.0459	2	0.1036	2	0.1919	2	0.2257	2	0.3454	9	0.0790	9
Milk and milk products	0.0002	9	0.0024	9	0.0039	9	0.0102	9	0.0364	8	0.1023	7	0.1792	7	0.4496	2	0.2198	3
Edible oils	0.0012	5	0.0107	3	0.0129	4	0.0308	4	0.0757	4	0.1650	4	0.2152	5	0.4089	7	0.1467	7
Meat, fish, eggs	0.0018	4	0.0094	5	0.0120	5	0.0273	5	0.0684	5	0.1476	6	0.2016	6	0.4122	5	0.1730	5
Sugar and gur	0.0009	6	0.0076	6	0.0104	6	0.0257	6	0.0675	6	0.1572	5	0.2171	4	0.4408	3	0.1621	6
Other foods	0.0022	3	0.0137	4	0.0136	3	0.0356	3	0.0844	3	0.1707	3	0.2196	3	0.4095	6	0.1858	4
Clothing	0.0003	8	0.0034	8	0.0053	8	0.0131	8	0.0350	9	0.0878	9	0.1698	8	0.5519	1	0.3784	2
Fuel and light	0.0035	1	0.0207	1	0.0230	1	0.0471	1	0.1050	1	0.1952	1	0.2361	1	0.4002	8	0.1325	8
Other non-food	0.0007	7	0.0050	7	0.0060	7	0.0141	7	0.0387	7	0.0901	8	0.1480	9	0.4287	4	0.4137	1

Note: The welfare loss for commodity i in table 1(a), λ_i, represents the effects of an increase in the tax on the ith good sufficient to raise one rupee of government revenue on all groups, using welfare weights expressed in terms of units of per capita expenditure to the poorest group. In table 1(b) we present the corresponding figure but where λ_i^h represents the loss to the hth household in terms of its own expenditure.

the others. The matrix L' therefore gives polar cases of social welfare functions as well as providing the information to generate the λ_i corresponding to any particular set of value judgements. We invite the reader to carry out his own experiments using (32) and table 1(b). It could be used, perhaps unattractively, to analyse tax changes which would benefit groups of special concern to the government.

One interesting question concerning some given tax change which did not lose revenue would be the following. For which sets of value judgements (as expressed through the β's) would the change be regarded as an improvement? Formally, given δ such that $\sum \delta_i \geqq 0$, for which β do we have $\beta' L\delta < 0$? Intuitively we work out the money measure of net loss to each household and ask which welfare weights would result in the (weighted) sum being negative.

In table 1(a) we present the 9×1 vector of effective commodity taxes for India for 1979/80, based on Ahmad and Stern (1983b), and the λ_i's associated with various levels of inequality aversion, e. The influence of e is immediately apparent. With $e=0$, or no inequality aversion, λ_i for 'cereals' ranks lower than that for all other commodity groups with the exception of 'milk and milk products'. Thus, one would suggest an increase in the tax on 'cereals' relative to the other j groups (where $\lambda_i < \lambda_j$), if one valued one rupee to all groups equally. In this case one would also suggest a decrease in the tax on the 'clothing' group relative to all others.

As we showed in subsection 2.1, with $\beta^h = 1$ and uniform taxes all the λ_i are equal. Thus, one might be tempted to suppose that away from uniformity one should increase taxes which are above average and reduce taxes which are below average. There is no general presumption, however, that $t_k > \bar{t} > t_j$ implies $\lambda_k > \lambda_j$ (where $\bar{t}$ is an average of the taxes). It is possible to show that where (optimum) uniformity arises from the assumptions of (i) households identical except for wage rates, (ii) linear expenditure systems, and (iii) the optimum lump-sum poll-subsidy, then an improving reform results from decreasing taxes that are above average [see Deaton (1984)]. The result, of limited practical significance, should not be confused with the assumption that $\beta^h = 1$ for all h, which is being discussed here.

For moderate levels of inequality aversion, $e=1$ say, the λ_i switch around quite drastically. Now λ_i for 'cereals' is greater than that for all commodities except 'fuel and light', and that for 'clothing' has dropped to the third lowest position. For higher levels of inequality aversion, say $e=5$, this pattern is accentuated, with λ_i's for 'fuel and light' and 'cereals' ranking highest and 'clothing' dropped to the second lowest position. Compare the λ_i with the first column of table 1(b) and we see that $e=5$ is close to maxi-min in its ranking of the λ_i's.

There are numerous directions of welfare improving reform between the commodity groups, but we have seen that these are very sensitive to the specification of the judgements concerning inequality. The existing tax structure is not optimum for any of the levels of e chosen — at each one can identify $\lambda_i > \lambda_j$ for some pairs.

We can see from table 1(b) that it will often be the case that welfare improvements as judged using specified β^h's and the λ_i are not necessarily Pareto improvements. Thus, for the value judgement represented by $e=1$ increasing the tax on 'clothing' by an amount sufficient to raise one rupee and decreasing taxes on 'cereals' by a corresponding amount increases social welfare. Yet from table 1(b) we can see $\lambda_7^h > \lambda_1^h$ for household groups 8 and 9, so they are made worse off. Obviously if no Pareto improvement is possible this will be the case for *any* improvement as judged through specific β^h's.

3.3. The inverse optimum

If the tax system is not optimum with respect to the β's chosen above, then it would be reasonable to enquire whether one could identify some β's such that the tax vector in table 1 is optimum. This is the inverse optimum problem discussed in subsection 2.3 above. Eq. (23) is

$$\beta' = r'C^{-1},$$

where C is the $H \times n$ expenditure matrix for H household groups and n commodities with $H = n$, and where we assume C is invertible. To examine this case we have collapsed our matrix of consumer expenditures into a 9×9 matrix of 9 households and 9 commodity groups by aggregating the 28 household groups to 9. These household and commodity groups correspond to those in the price elasticity matrix and to table 1. The vector r represents $\partial R/\partial t_i$, and is 9×1. Table 2 presents the solution to equation (23).

Table 2
The inverse optimum, β's.

Household group: per capita expenditure (Rs/month)	β
0–25	−126.37
25–32	171.36
32–40	−53.94
40–45	−438.41
45–55	241.01
55–70	33.54
70–90	−80.68
90–100	18.90
above 150	−4.32

As was the case with our earlier work for 1973/74 [see Ahmad and Stern (1981)], we find that the only welfare weights satisfying (23) with the existing tax vector and $\partial R/\partial t_i$ contain negative elements. From subsection 2.4 we can deduce that Pareto improvements are feasible and we explore these possibilities below.

It is possible that given substantial inequality some people or governments may not have Paretian social welfare functions. Thus, negative social welfare weights on, for example, increments to the very rich, may genuinely reflect preferences. This would correspond to social indifference curves in utility space which were upward sloping for part of the range (e.g. far from equality). It does however seem unlikely that the welfare weights in table 2 do represent the judgements of any reasonable person since there is a large negative weight on the poorest. The conclusion would be that, given the model of the economy, the existing taxes are not optimum by anybody's standards.

3.4. Pareto improvements

The negative welfare weights in the previous subsection imply that directions of Pareto improving reform are possible. These can be investigated in a number of ways. As suggested in subsection 2.5 above, one could maximise revenue gain subject to making no household worse off, or alternatively one might seek to maximise the welfare of some given household group subject to constant revenue and keeping all other households no worse off. There are many such combinations, but we shall illustrate here the maximisation of revenue gain in one set of exercises, and that of the welfare of the poorest (rural) household in another. The analysis will first be carried out for the reduced 9-household matrix. We will then revert to the larger 28-household grouping (14 urban and 14 rural groups). The first set of calculations (maximisation of revenue gain) corresponds to the linear programme described in eq. (29):

$$\text{tax change A}\begin{cases}\text{maximise } \iota\delta\\ \text{subject to } L\delta \leqq 0\\ -1 \leqq \delta_i \leqq +1,\end{cases}$$

where $\delta_i = r_i \tau_i$, which are tax changes constrained to revenue changes for each good of no more than one rupee. L is the matrix with hith element λ_i^h ($\lambda_i^h = x_i^h / r_i, r_i = \partial R/\partial t_i$), and λ_i^h is the marginal cost to the hth household of raising the ith tax to raise one rupee of revenue.

The second set of programmes are formalised by (33):

$$\text{tax change B}\begin{cases}\text{maximise } -\delta\lambda^h\\ \text{subject to } \iota\delta \geqq 0\\ \text{and } L_{-h}\delta \leqq 0\\ -1 \leqq \delta_i \leqq +1,\end{cases}$$

where λ^h is the hth row of L, L_{-h} is the $(H-1)\times n$ matrix corresponding to L with the hth row deleted. Here the welfare of the hth household is being maximised, subject to no revenue loss and all other households at least as well off as before.

Table 3(a) lists the results of tax changes A and B. For tax change A, there is a revenue gain of Rs 0.32, thus $\Delta R>0$. Also, $\Delta V>0$ for any Paretian social welfare function, as household groups 4, 5 and 8 are made better off, with no other household group worse off. For tax change B, there is an improvement in the position of the poorest household group, but $\Delta R=0$. Note also that all households, apart from the two upper expenditure groups, are made better off by this change. Thus, there are many Pareto- improvements possible with the present r_i. In the two cases considered, the consequential tax changes are different. In order to maximise revenue one would increase the tax on 'cereals' and reduce that on 'sugar and gur', by Rs 1, respectively. However, in order to improve the position of the poorest household group, one would reduce the tax on 'cereals' to lose Rs 0.84, but compensate by increasing the tax on 'sugar' to raise Rs 1.

The solutions to the programme for finding an improving direction may be illustrated with a diagram, fig. 3, which builds on fig. 2(a). As in fig. 2(a) we have two tax instruments and two households. The revenue constraint is represented by the line AA', and the lines of constant utility for households 1 and 2 are given by BB' and CC'. We confine attention to small changes so the three lines are straight. The concern to keep the changes small can be represented in a number of ways, each of which would confine us to a small area in (t_1, t_2) space around O. We have chosen to do this by placing a bound on the extra revenue raised from taxation of each good — specifically we assume that we may not raise more than one extra rupee via each good. We are thus constrained to a rectangle centred on the point O, the original position with horizontal dimension $2/r_1$ and vertical dimension $2/r_2$.

The point of maximum revenue increase subject to the constraints is given by P. The feasible set is that part of the cone AOB [see fig. 2(a)] of Pareto improvements, without revenue loss, which lies inside the rectangle. The objective of raising extra revenue involves the maximisation of the distance normal to AA' the line of constant revenue. This gives the point P. The second tax is raised by the maximum permitted and the first tax is reduced

Table 3

(a) Linear programme tax changes (9 household groups)

Commodities	Tax change A (Rs)	Tax change B (Rs)
Cereals	+1.00	−0.84
Milk and milk products	+1.00	+1.00
Edible oils	+0.60	+1.00
Meat, fish, eggs	+1.00	+0.47
Sugar and gur	−1.00	+1.00
Other food	−1.00	−1.00
Clothing	−1.00	−1.00
Fuel and light	−1.00	−1.00
Other non-food	+0.72	+0.37
Revenue gain	+0.32	0

(b) Changes in household welfare (Rs)

Household		Tax	Tax
Group	(Rs)/capita	change A	change B
(1)	0–25	0	0.005
(2)	25–32	0	0.030
(3)	32–40	0	0.020
(4)	40–45	0.01	0.040
(5)	45–55	0.01	0.090
(6)	55–70	0	0.090
(7)	70–90	0	0.050
(8)	90–150	0.04	0
(9)	150+	0	0

Notes:

1. The entry in the table gives the revenue gain in rupees associated with altering taxes on the given good.
2. For tax change A we maximise revenue gain subject to leaving no household worse off [see (29)] and for tax change B we maximise the welfare increase for the poorest household subject to not losing revenue and making no household worse off. The programmes A and B are set out in the text.
3. We have imposed the constraint that the absolute value of the revenue change associated with each good should not exceed one rupee. Thus, for example, for programme A the price of milk has been increased by the maximum permitted and that of clothing reduced by the maximum permitted.

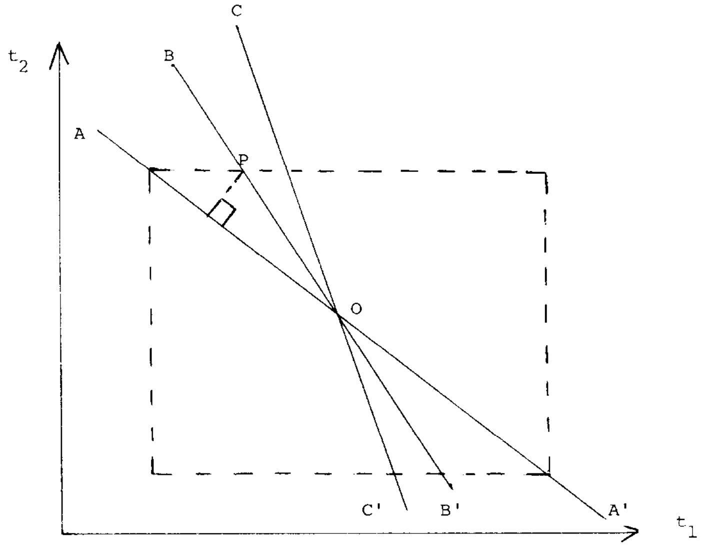

Fig. 3

by an amount which leaves the first individual just as well off as before. The second individual is strictly better off, since P is below CC'. From the directional point of view it is the constraint not to reduce the welfare of the first individual that is binding on extra revenue and we move as far along that constraint as is permitted by the constraints we have imposed on maximum movements. One can readily see how the solution would change for different constraints in (t_1, t_2) space on maximum movements.

From fig. 3 we can see that certain important features of tables 3(a) and (b) (initial columns) are illustrated. For example, some households have no improvement for tax change A in table 3(b), whereas some taxes in table 3(a) are increased by the maximum amount while others are not.

Both intuition and the Minkowski–Farkas lemma suggest that as the number of households is increased it will become more and more difficult to find a Pareto improvement. However, for our Indian data, increasing the number of households to the original 28 groups still allows the possibility of Pareto improvements and this is illustrated in tables 4(a) and 4(b).

Tax change A for the 28 household case is very much like that for the 9-household case, although the overall revenue gain is lower. Again in this case, $\Delta V > 0$ for any Paretian social welfare function; some middle expenditure households, mainly in rural areas, are being made better off. For tax change B, the welfare of the poorest rural household group is maximised. We have that $\Delta R = 0$, utility does not decrease for any household, and increases for most rural households and several higher income urban house-

Table 4

(a) Linear programme tax changes (28 household groups)

Commodities	Tax change A (Rs)	Tax change B (Rs)
Cereals	+1.00	−0.53
Milk and milk products	+1.00	−1.00
Edible oils	+0.05	−0.85
Meat, fish, eggs	+1.00	+1.00
Sugar and gur	−1.00	−1.00
Other food	−1.00	+1.00
Clothing	−0.32	+0.35
Fuel and light	−0.61	+1.00
Other non-food	+0.09	+0.03
Revenue gain	+0.21	0

(b) Changes in household welfare

Group (rural)	Per capita mean expenditure (Rs)	Tax change (A)	Tax change (B)	Group (urban)	Per capita mean expenditure (Rs)	Tax change (A)	Tax change (B)
R1	17.1	0	0.002	U1	13.7	0	0
R2	22.6	0	0	U2	22.3	0	0
R3	27.2	0	0.01	U3	27.5	0	0
R4	31.8	0	0.01	U4	31.6	0	0
R5	35.1	0	0.01	U5	36.8	0	0
R6	42.1	0.01	0.02	U6	42.4	0	0
R7	49.9	0.01	0.04	U7	50.4	0	0
R8	62.1	0.01	0.04	U8	62.3	0.01	0.01
R9	78.5	0	0.02	U9	79.1	0	0
R10	102.8	0	0	U10	103.5	0	0
R11	137.9	0.01	0	U11	138.8	0	0.01
R12	192.9	0.002	0	U12	195.1	0	0.03
R13	274.7	0	0.01	U13	277.2	0	0.02
R14	460.2	0	0.01	U14	464.0	0	0.03

Notes:

1. The entry in the table gives the revenue gain in rupees associated with altering taxes on the given good.
2. For tax change A we maximise revenue gain subject to leaving no household worse off [see (24)] and for tax change B we maximise the welfare increase for the poorest household subject to not losing revenue and making no household worse off.

holds. As in the 9-household case, several directions of Pareto improvement are seen to be possible.

4. Sensitivity of results

The elements of our analysis of reform have been the $H \times n$ matrix of consumer demands C, the aggregate demand derivatives $\partial X_j/\partial t_i$, the tax vector t and, where we use an explicit social welfare function, the welfare weights β^h. We shall discuss assumptions concerning the matrix C in subsection 4.1. Revenue responsiveness, the r_i, is discussed in subsection 4.2 and represents a combination of assumptions concerning the $\partial X_j/\partial t_i$ and the tax vector t. Since the aggregate demand derivatives enter the reform calculations only through r_i, we shall consider them in subsection 4.2. In subsection 4.3 we discuss the possible treatment of different assumptions concerning production and market structure.

Sensitivity to the welfare weights has been examined and emphasised in our discussion of table 1(a). As one should expect, value judgements do matter and the more egalitarian the values the greater the role of the distributional characteristic in influencing the ranking of the λ_i's [see eq. (15)].

4.1. The consumer demand matrix

The variation of the consumer demand matrix in this paper has been with respect to aggregation across consumers. Thus, we have used each of 9 and 28 household groups at various points in our discussion. We have, however, kept the aggregate consumption vector constant as given in the base year of the Sixth Plan, 1979/80. We saw that for both cases of 9- and 28-household groups Pareto improvements are possible. However, one would not in general expect the existence of Pareto improvements to be insensitive to the disaggregation across households. As one considers more and more household groups with differing consumption patterns the constraint that we find a reform that makes each of them better off becomes increasingly stringent. Obviously, however, if we were to merely divide into subgroups with identical consumption patterns it would be no more difficult to find Pareto improvements (although this is not what we did here in moving from 9 to 28 groups).

The direction of reform which maximises the revenue increase subject to making no households worse off was fairly insensitive to the disaggregation across households: compare the results of tax change A in tables 3(a) and 4(a). We suspect that this insensitivity may sometimes be present and sometimes not, and the reason may be illustrated using fig. 2(a) where a Pareto improvement is possible. Small movements in the constraint that the second household be made no worse off are represented by rotations of the line CC'. Clearly, this will make no difference to the line of steepest increase in revenue which we suppose lies along OB. Thus, replacing the second household by two households will not change the result for maximisation of revenue provided that neither of the two new households has a consumption pattern associated with a vector which lies anti-clockwise from BB'.

On the other hand, we would expect the direction of reform which maximises the welfare increase of the poorest household to be sensitive to the specification of its consumption pattern. If we disaggregate amongst households previously aggregated into a broad group, then we would indeed expect the consumption pattern of the poorest households to change and this was observed by fairly sharp differences in the relative x_i^h and λ_i^h for the poorest households as between the 9 and 28 group case. This sensitivity is illustrated in the difference in the results for tax change B between tables 3(a) and 4(a).

The sensitivity of the λ_i, when we use explicit value judgements β^h, to the degree of disaggregation will depend on the aversion to inequality. With no aversion to inequality, i.e. all the β^h equal, the directions of reform in this model are completely insensitive to the degree of aggregation. Aggregates are all that matter and the message is likely to be simply to increase taxes lower than the average and reduce those which are higher. However, for higher values of aversion to inequality as measured by e we would expect some sensitivity, since disaggregation amongst households may change the relative standard of living between the poorest and, say the average quite sharply. Thus, as we disaggregate we would, for the higher values of e, expect the criteria to approach more closely the maxi-min which will itself now be reflected by the consumption pattern of the new poorest group.

The level of aggregation is the only issue of sensitivity with respect to the consumption matrix that we have presented here. Further questions concern mistakes by consumers and mismeasurement of consumption. If the actual consumption patterns of households reflect disturbance by some random influences, then, if taxes are changed marginally, the consumption patterns from our survey may not give us money measures of welfare changes since households may not be disturbed or be disturbed differently in the new situation. One might be tempted to associate fitted values from previously estimated demand systems with the intended values and think of the difference between actual and fitted values as errors which are unlikely to be repeated. However, random factors in cross-section econometric estimates are usually very large and we would feel uneasy in regarding the fitted values as 'true' values, since the random terms are to a large extent an offshoot of deficiencies in modelling. It therefore seems more reliable to work with measured rather than fitted consumption. Notice that if the hth consumer is *rationed* at x_i^h for some good i, then x_i^h will still reflect the money measure of his loss from a (unit) price increase of i. Thus, we are treating the measured volume as portraying what the individual intended to consume given the prices, his budget constraint and any ration he faced.

One must recognise, of course, that the consumption measurements are themselves imperfect. If some external information on mis-recording is available then one might attempt to adjust consumption patterns using measured household characteristics. Moreover, this possibility might mitigate against excessive disaggregation across household groups.

In subsequent papers we shall present some comparisons of calculations using fitted and actual values. Our preliminary results suggest, not surprisingly, that much depends on how the fitted values are calculated. If, for example, one used a demand system which incorporates none or few differences across households and linear Engel curves one would expect reform calculations using fitted values to be more likely to suggest moves towards uniformity than if many household differences are included and non-linearity is allowed for. Given this type of problem and the large amount of information on consumption patterns which is thrown away in using fitted values (cross-section fits are generally poor) we would argue strongly in favour of using actual values.

4.2. *Tax responsiveness*

The issues of the measurement of taxes t_i and aggregate demand responses $\partial X_j/\partial t_i$ are combined in our approach in the parameters r_i, the responsiveness of revenue with respect to tax changes. Thus, changes in t_i and $\partial X_j/\partial t_i$ which yield the same r_i have no effect whatsoever on the results. Further, changes in assumed demand responses amongst subgroups in the population have no effects on results if aggregate demand responses are unchanged. It must be emphasised that we are considering here changes in demand responses rather than the demand *levels*. We are therefore considering only revenue responsiveness or the denominator, (13), or λ_i. If changes in our demand estimates change the numerator, (11), of λ_i which involves the demand levels, then sensitivity would be increased. We have already expressed the view, however, that changing the demand levels when we change our demand estimates would be an act of faith we should be reluctant to follow in practice. This is because fitted values are a reflection of our ignorance since they usually differ so much from actual values in a cross-section with, typically, very low R^2, or goodness-of-fit.

It should be emphasised that our treatment of sensitivity to demand specification is rather different from that which is standard in discussions of optimum commodity taxation. In that literature [see Atkinson (1977) and Deaton (1981)] one is changing the specification of demand levels as well as responses and thus, in the language of our reform discussion, both numerator and denominator of λ_i. For example, there is a result which states that if there is an identical Linear Expenditure System (LES) with households differing only in earnings, then with an optimum poll-subsidy, the optimum commodity tax is uniform [see Atkinson (1977)]. This does not apply in our case, for a number of reasons. First, governments in developing countries do not typically have the option of an optimal poll-tax or subsidy. Second, the LES was estimated for different household classes and was not identical across individuals. And, as remarked above, we have used actual demands rather than fitted values.

The above discussion suggests that the choice of specification of the demand system is less important (although still important) for the analysis of reform than it is for the calculation of optima. One needs aggregate demand derivatives only and these for the point at which we find ourselves. For the optimum it is necessary to specify the demand responses for each household and for an extended range. And one is forced to use fitted values for configurations away from the starting point. A similar way of expressing the relative sensitivity of reform and optimality calculations to demand assumptions was put to us by Roger Guesnerie. Optimum taxes depend on derivatives of any order of utility functions (supposing that they are analytical), whilst tax reform depends only on second derivatives. The estimate of second derivatives through econometric methods should be less sensitive to specifications than the estimate of derivatives of high order — the choice of specification may be seen as a substitute for estimates of high order derivatives.

We give an example of the sensitivity of results to the estimates of tax responsiveness. Consider a small change in r_i $(=\partial R/\partial t_i)$, caused, say, by the effective tax vector being differently estimated from that presented in table 1. Suppose we now estimate effective taxes as uniform but yielding the same revenue as before. The r_i we have used should be replaced by, say, r_i^1. Using this r_i^1 we can investigate whether a Pareto improvement is possible following the procedure described in subsection 3.4 and eq. (29). We find that no Pareto improving tax is possible, which in turn implies, by the Minkowski–Farkas lemma, that non-negative welfare weights for the inverse optimum problem exist. Solving for y in eq. (35) in the 9×9 case we observe that positive welfare weights do indeed exist and are given in table 5(b). The ratios r_i^1/r_i are given in table 5(a). Note that the welfare weights are equal for all household groups in this case, and this is equivalent to the case where $e=0$, or there is no inequality aversion on the part of the government. As we saw above, if the policy-makers have no inequality aversion, then the optimum tax is a uniform one.

The changes in r_i from table 1 to table 5 (see column 2, table 5a) do not appear to be very large, yet the change in results is striking — from the existence to the non-existence of Pareto improvements. However, the change in taxes we have postulated is a very large one. Given our estimates of

Table 5

(a) Changes in $\partial R/\partial t_i$ $(=r_i)$

Commodity	t^{e^1}	r_i^1/r_i
Cereals	0.0833	0.9479
Milk and milk products	0.0833	0.9204
Edible oils	0.0833	0.9783
Meat, fish, eggs	0.0833	0.9655
Sugar and gur	0.0833	0.9985
Other food	0.0833	1.0407
Clothing	0.0833	1.1413
Fuel and light	0.0833	1.0663
Other non-food	0.0833	1.0496

(b) The inverse optimum for t^{e^1}

Household groups		
Group	(Rs) per capita	β
(1)	0–25	0.9167
(2)	25–32	0.9167
(3)	32–40	0.9167
(4)	40–45	0.9167
(5)	45–55	0.9167
(6)	55–70	0.9167
(7)	70–90	0.9167
(8)	90–150	0.9167
(9)	150+	0.9167

effective taxes, or the component of tax in price as shown in table 1 and our discussion in Ahmad and Stern (1983a), it does seem unlikely that the effective taxes are actually uniform. Given the complexity of the calculation and the diverse sources one cannot, however, easily provide standard errors for effective taxes and thus for errors in r_i from this source. On the other hand it would in principle be possible to provide the standard errors associated with specific estimates of demand elasticities. As important would be to estimate sensitivity of elasticities and thus the r_i with respect to different types of demand system. We tried some different sources [e.g. estimates of the Rotterdam system by Murty (1980)] in our earlier work and did not find great sensitivity of the rankings of λ_i. This is no doubt in part due to the fact that the terms in $\partial X_j/\partial t_i$ are aggregated in the expression for r_i by forming the sum $\sum_j t_j(\partial X_j/\partial t_i)$. If, for example, taxes are proportional to prices, this sum will be independent of demand estimates provided they satisfy the adding-up criterion. We would not, of course, argue that effective taxes are proportional in our example.

A feature of the demand estimates which may be of some importance is the level of aggregation across commodities. Obviously there are great data and specification problems in estimating cross-elasticities in highly disaggregated demand systems and they are in general therefore estimated using broad groups of commodities (groups between 5 and 20 in number are typical). However, from the public finance viewpoint this might conceal many valuable reforms which depend on the detail of cross-elasticities for their effect. We would also expect these cross-elasticities often to be higher in more disaggregated systems.

4.3. Production and market structure

Our model of the production side in terms of markets and production possibilities has been very simple. This has allowed us to ignore, amongst other things, three related features which are potentially of considerable importance: (1) differences between market prices for producers and shadow prices; (2) tax shifting which may differ from 100 percent; and (3) effects on technology and the demand and price of inputs and factors. We comment, briefly, on how the analysis might be extended.

If market prices and shadow prices differ, then we should, in addition to the tax revenue and consumption effects of reform, consider the change in the value of production at shadow prices. This may be incorporated by considering the difference between producer and shadow prices as a fictitious tax. Thus, an analysis in the same spirit can be pursued although it is obviously more complex [for further details see Drèze (1982), Drèze and Stern (1983), and Ahmad, Coady and Stern (1984)].

For tax shifting less than 100 percent, we would need a specific alternative model. This is very complex in a general equilibrium context, although some progress for partial equilibrium models is possible [see Stern (1982)]. One must recognise that shifting may be above as well as below 100 percent and that full forward shifting may not be an unreasonable assumption in many cases. For other than full forward shifting, the effect on consumers of tax changes can be calculated as above, provided we know the extent of the consumer price changes. We also need to add to the analysis the welfare effects of the *income* changes for households associated with the changing profits of firms and enterprises, consequent on the tax change.

Finally, we have to allow for the possibility of flexible coefficients and changes in demand for inputs and factors. For marginal changes the envelope theorem allows us to ignore the effect of changing input coefficients: input–output coefficients allow us to compute the marginal effects of changes in taxes on consumer prices. The calculation of the effects of changing factor incomes, with changing factor prices, may, however, be complicated. These would in turn have to be followed through to household incomes to calculate changes in welfare from this source. A number of these considerations may be subsumed in the shadow prices although their calculation would in principle embody these general equilibrium repercussions.

5. Conclusions

Our discussion has emphasised the advantages of dealing with marginal reforms. This procedure allows us to work with actual demand data and does not impose the difficulties of using fitted values. Furthermore, we need only aggregate demand elasticities and not those for individual households. Thus, the data demands for marginal analysis are much less severe, and as such its conclusions are probably more robust, than for non-marginal reforms.

The disadvantage of marginal analysis is that it deals only with directions of reform and thus not with specific recommendations for substantial changes or proposed reforms. We have, in Ahmad and Stern (1983a), investigated such questions. One is then forced to invoke indicators of welfare change for particular groups, and we adopted the commonly used equivalent variation based on specific demand and utility functions. The respective disadvantages and advantages of marginal and non-marginal analysis in this context suggest to us that they are complementary and both should be used in the analysis of reform.

We do not argue that our methods are robust with respect to parameter estimates and model specification, and one should not expect them to be so. The analysis of tax reform obviously concerns distributional judgements and estimates of demand responses, and these play a central role in determining the results. What we must try to do is learn how to incorporate these elements into policy analysis and understand the sensitivity of our conclusions to model specification and to parameter estimates.

Central to our discussion is the marginal cost in terms of social welfare of raising an extra rupee of government revenue from taxing a given good. If this marginal cost, λ_i, is lower for good i than it is for good j, then we increase welfare at given revenue by raising one more rupee from the ith good and one less from the jth. If the λ_i are unequal, there are of course many beneficial reforms. The calculation of λ_i requires specific distributional value judgements and we showed the sensitivity of the results to these value judgements. Not surprisingly a greater concern for the welfare of the poor leads one to be less attracted by raising taxes on the goods they consume.

The use of explicit social welfare functions is a useful way of approaching the problem. One can also ask whether Pareto improvements are possible and we showed how one can write down a linear programme to resolve the issue. If a Pareto improvement is not possible, then the Minkowski–Farkas lemma tells us that a solution, with non-negative welfare weights, to the inverse optimum problem exists. In other words we can find a social welfare function with respect to which the current state of affairs is optimum. We also displayed a method of calculating this function. The result is tidy in the sense that exactly *one* of the problems of finding a Pareto improvement and finding a solution to the inverse optimum (with non-negative weights) is soluble. Furthermore, we have provided a simple procedure using linear programmes for establishing which problem can be solved.

Finally, we discussed at some length the robustness of our results and issues for further research. The theory, and its application, of marginal reform is more robust than for non-marginal reform or computations of the optimum in the sense that it requires fewer assumptions. Nevertheless the results will be sensitive to the main constituents: the distributional value judgements and the responsiveness of revenue to tax changes. The methods we have suggested can be readily applied in countries that have consumer expenditure surveys and estimates of aggregate demand systems. In addition, there are many theoretical developments which are important and which seem feasible, particularly concerning more flexible descriptions of production and shadow pricing. It is a fruitful area for both theoretical and empirical research.

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Appendix: Adjusted price elasticity matrix

Commodities	Cereals	Milk and milk products	Edible oils	Meat, fish, eggs	Sugar and gur	Other food	Clothing	Fuel and light	Other non-food
Cereals	−0.3636	−0.0333	−0.0116	−0.0297	−0.0166	−0.0060	−0.0082	0.0144	−0.0150
Milk and milk products	−0.4948	−1.1310	−0.1526	0.1475	0.0162	0.1223	−0.0324	−0.0414	−0.0379
Edible oils	0.2562	−0.1861	−0.4769	0.0498	0.0563	0.1365	−0.0152	−0.0221	−0.0209
Meat, fish, eggs	−0.4899	0.3787	0.0580	−0.8460	0.0098	0.0902	−0.0162	−0.0268	−0.0198
Sugar and gur	−0.3712	0.0638	−0.0833	0.0023	−0.5950	−0.1603	−0.0233	−0.3933	−0.0313
Other food	−0.0382	0.1064	0.0352	0.0115	−0.0283	−0.8214	−0.0151	0.0033	−0.0199
Clothing	−0.2373	−0.0368	−0.0255	−0.0216	−0.0190	−0.0985	−0.7385	−0.0298	−0.0290
Fuel and light	−0.0993	0.0001	−0.0087	−0.0073	−0.0035	−0.0333	−0.0112	−0.4825	−0.0140
Other non-food	−0.0488	−0.0287	−0.0190	−0.0354	−0.0299	−0.0960	−0.0320	0.0110	−0.9546

Source: Radhakrishna and Murty (1981).
Note: See subsection 3.1 for details.

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