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THE EFFECTS OF TAXATION, PRICE CONTROL AND GOVERNMENT CONTRACTS IN OLIGOPOLY AND MONOPOLISTIC COMPETITION

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Many government contracts with or policies towards oligopolistic sectors essentially involve private firms selling a given proportion (θ), or quantity, of output to the government at a fixed price (p_R) with the remainder being sold on the open-market. Often this is combined with consumer rationing. Examples include cement and sugar in India, and health, housing and defence in many countries. The paper investigates the effects of these schemes (including sales and excise taxation) on prices, output and household welfare under oligopoly and monopolistic competition. Less government control (reduced θ) may raise prices and tax shifting can be above or below 100 percent.

1. Introduction

Government policy towards price, output and profits in oligopolistic industries can take many forms. However, formal analyses in public finance of the consequences of government action have tended to focus not only on the case of perfect competition (with occasional reference to monopoly) but also on a rather narrow range of policy tools, particularly excise or proportional sales taxes. The main purpose of this paper is to examine the effects on price, output and profits of changes in some of the schemes which occur in practice for a variety of different market structures. Some of the

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results will be surprising and we shall see that they depend quite sensitively on the precise form of the government intervention and on market structure.

A spectrum of possible market structures and outcomes, including perfect competition, monopolistic competition, Cournot-oligopoly and monopoly will be represented in a simple way using the conjectural-variations model of oligopoly in partial equilibrium, with and without free entry. The policies to be examined will be represented using a simple two-parameter framework which has a number of interesting interpretations. During the formal analysis we shall focus on a single one of them, which originally motivated this study, and that is the Indian system of dual pricing. Under this scheme a firm must sell a fixed proportion of its output to the government at a fixed price. The requisitioned output is then sold by the government to consumers at controlled prices. The scheme has been important for the major oligopolistic industries of cement and sugar.

Our concentration is on positive questions concerning the direction and magnitude of the effects of changes in policy. The welfare analysis of the schemes in terms of the optimum policy will, in the models studied, be obvious. A government concerned with efficiency (rather than distribution) should take over the market and charge price equal to marginal cost – a policy which is a limiting case for most of the examples here. Our concentration for welfare questions will rather be on marginal reform, i.e. on the effects of small movements in one of the policy variables from a given initial position. There will be many reasons, which we shall discuss briefly, outside the model, why a full movement to the optimum described may not be desirable or feasible. One should be interested, however, in setting the net benefits calculated within the model of a small movement from an initial position against any judgement concerning those welfare effects which might operate through the omitted factors. We shall also show how distributional judgements can be brought into an analysis of the schemes.

The different interpretations of the two-parameter framework should be kept in mind throughout and when the model is described we shall note explicitly how they apply. These interpretations include black markets and certain forms of profits taxation. Other examples of the specific industries and types of contract or control where the model would be relevant include the following. Major government defence contracts are often placed in oligopolistic industries where a substantial quantity or fraction of the output is bought at a fixed price leaving additional production to be sold on the open (world) market. A health authority might leave a doctor free to practice privately for a certain proportion or amount of his time provided the remainder is allocated to the public sector. It is quite common in housing developments in a number of countries for a government to insist that a certain proportion of dwellings be let at regulated prices (often to low income groups). In all these cases it will be seen both that government action results in two prices in the market and that there may be important elements of oligopoly or monopolistic competition.

The plan of the paper is as follows. In the next section we examine the basic model of oligopoly and monopolistic competition without dual pricing and investigate the effects of specific taxes. This has three purposes (1) to assemble in a succinct form the basic requirements for the subsequent comparative static analysis in a model which caters for several different structures, (2) to aid the interpretation of results on dual pricing – the analogy of elements of the scheme with a cost subsidy implies that an understanding of the effects of specific taxes is necessary, (3) to provide an extension of the results of Seade on oligopoly to monopolistic competition. The important series of papers by Seade (1980a, 1980b, 1987) on the conjectural variations model provides a most useful background to this analysis. However his analysis of tax shifting is confined to specific taxes and the oligopoly model and we are concerned with a much broader class of policy tools, with free entry and monopolistic competition and with welfare.

The policy schemes are presented formally in section 3, using the language of dual pricing. This will include as a special case, the analysis of a proportional sales tax. In section 4 we look at effects on household welfare and in section 5 we examine an alternative version of the scheme where the retention quantity rather than the proportion is fixed. This allows considerable expansion in the scope of interpretations. The penultimate section contains a discussion of a number of possible extensions to the analysis concerning income distribution, more complex versions of the scheme and of the demand structure. In the final section we draw out the conclusions of the analysis both in terms of general lessons concerning the analysis of public finance questions in this type of model and particular aspects of the type of scheme.

Examples of the results include the following. The effect of increasing the proportion of output taken by the government on open-market price can be either positive or negative. The crucial determinants are the relation between the government purchase price and marginal cost in oligopoly and, in monopolistic competition, the level of the elasticity of the elasticity of demand also comes into the picture; the explicit conditions are characterised in the paper (these two factors also determine what happens to profits). Thus, for example, a move towards deregulation in the sense of decreasing the proportion taken by the government can *increase* open-market prices in sharp contrast to the competitive model. Increases in government purchase price will lower open-market price but will raise or lower profits according as the elasticity of the elasticity of demand is high or low. As we have noted the full optimum (ignoring distribution) involves complete government takeover and price equal to marginal cost but movements in that direction from a given initial position may not be welfare improving.

2. The model and the effects of excise taxes

We shall make extensive use of an approach to the theory of oligopoly which, following Stigler (1964) and Cowling and Waterson (1976) incorporates an explicit model of the conduct of firms based on conjectural variation. This model has been thoroughly investigated in an important series of papers by Seade (1980a, 1980b, 1987). The treatment here is based on Dixit and Stern (1982). We shall extend the model to one of monopolistic competition by introducing free entry. The model is used to bring out the range of possible outcomes and the role of some key parameters and there is no claim that is the only oligopoly model deserving consideration. We examine the effects on prices, profits and the number of firms (in the case of monopolistic competition) of changes in excise or specific taxes. Proportional sales taxes are examined as a special case of dual pricing in the next section.

Output of firm i is x_i, market share s_i, and costs $K_i+c_ix_i$ where K_i represents a fixed cost and c_i a marginal cost. Total output is X and price $p=\phi(X)$. A firm i conjectures that the reactions of other firms to a small change in its output satisfies

$$\frac{\partial(X-x_i)}{\partial x_i}=\alpha\left(\frac{X-x_i}{x_i}\right), \tag{1}$$

i.e. if its own output goes up by 1 percent, that of the rest of the market will go up by α percent. The case $\alpha=0$ corresponds to the Cournot–Nash assumption, and $\alpha=1$ to the firms' conjecturing that they will not change the market share by changing output.[1] A necessary condition for the maximisation of profits by the ith firm at a positive level of output given its conjectures, is that perceived marginal revenue be equal to marginal cost. Using (1) this may be written, where ε is the elasticity of demand (taken as a positive number)

$$p\left\{1-\frac{(\alpha+(1-\alpha)s_i)}{\varepsilon}\right\}-c_i=0. \tag{2}$$

Adding and dividing by the number of active firms, n,

$$p\left(1-\frac{\gamma}{\varepsilon}\right)-\bar{c}=0, \tag{3}$$

where $\bar{c}=(\sum_i c_i)/n$ and $\gamma=\alpha+[(1-\alpha)/n]$. We suppose $\gamma\geqq 0$. Where the total

[1] We shall, for the most part, assume $0\leqq\alpha\leqq 1$ although this is not required for (1) to retain interest – for example the competitive model can usually be recovered as a special case corresponding to conjectured $\mathrm{d}p/\mathrm{d}x_i$, and thus $\mathrm{d}X/\mathrm{d}x_i=0$ thus involving negative α.

number of firms in the market is fixed we have a 'generalised-Cournot' equilibrium. For simplicity we shall in what follows assume that all firms in the market have positive output (i.e. are active). To ease notation we replace $\bar{c}$ by c and assume identical marginal costs but the results for this case extend readily on replacing $\bar{c}$ by c.

When free entry is considered we need for simplicity to assume firms are identical so that each potential entrant foresees the same profit, which would in turn be equal to post-entry profits of existing firms. We then have in addition to (3) the zero-profit condition

$$(p-c)X-Kn=0. \tag{4}$$

We can interpret this condition as one of perfect equilibrium where entering firms pay an entrance fee (or fixed cost) K, foreseeing accurately the conduct of the game which will take place after entry. Conditions (3) and (4) give the equilibrium in monopolistic competition. They are the basic equations of the model and should be very familiar as 'marginal revenue equals marginal cost' and 'average revenue equals average cost'.

The existence of a solution to (3), the condition for positive outputs, requires

$$\varepsilon>\gamma. \tag{5}$$

Stability of the generalised-Cournot equilibrium in the sense of Seade (1980a) requires

$$F>1-\frac{\varepsilon}{\gamma}, \tag{6}$$

where $F=p\varepsilon'/\varepsilon$ and ε is the elasticity of demand, $\varepsilon=-pX'/X$, where the prime denotes the derivative with respect to price. In the adjustment process firms are assumed to move their output towards the level which would satisfy (2) given the output of the other firms.[2] Given (5) we know that (6) is satisfied for $F\geqq 0$ (including the isoelastic case $F=0$). It should be clear that the role of F is important for stability since, for example, if F were negative and large in magnitude then an increase in output (and fall in price) could increase ε and lead firms to raise output still further. The second-order condition for profit maximisation of a given firm around the equilibrium

[2]For comparison of our analysis with that of Seade (1987), it should be noted that he uses the elasticity (E) of the slope of demand ($E=-Xp''/p'$) rather than the elasticity of the elasticity F adopted here – our results come out more neatly using F. $F=1+\varepsilon-\varepsilon E$ so that Seade's condition $E>2$, which plays an important role in his analysis, becomes $1-\varepsilon>F$.

(where $s_i = 1/n$) for identical firms is

$$F/\varepsilon > \left(\frac{1}{\varepsilon} - \frac{1}{\gamma}\right) - \frac{(1-\gamma)(\gamma-\alpha)}{\gamma^2}. \tag{7}$$

Hence the stability condition implies the second-order condition since $1 \geqq \gamma \geqq \alpha$.

Monopoly is the special case of generalised-Cournot with $n=1$ and perfect competition is a special case of generalised-Cournot with $\gamma=0$ and $K=0$ or of monopolistic competition with $K=0$, $\alpha=0$, $n=\infty$. Hence the two equations (3) and (4), either singly using (3), or as a pair, cover in a convenient way a wide range of market structures.

Writing the l.h.s. of (3) as $f(c,p,n)$ and the l.h.s. of (4) as $g(c,p,n)$ the comparative statics are derived from

Generalised-Cournot:

$$\frac{\partial p}{\partial c} = -f_c/f_p \tag{8}$$

$$\frac{\partial \Pi}{\partial c} = g_p \frac{\partial p}{\partial c} + g_c \tag{9}$$

where Π are industry profits

Monopolistic Competition:

$$\frac{\partial p}{\partial c} = \frac{f_c g_n - f_n g_c}{f_n g_p - f_p g_n} \tag{10}$$

$$\frac{\partial n}{\partial c} = \frac{f_p g_c - f_c g_p}{f_n g_p - f_p g_n}. \tag{11}$$

In generalised-Cournot n is fixed and in monopolistic competition Π is zero. We treat n as a continuous variable. Results are displayed in table 1.

The stability condition (6) may be rewritten as $f_p = 1-(\gamma/\varepsilon)+(F\gamma/\varepsilon)>0$. This tells us that the graph of f against p intersects the horizontal axis from below or, more familiarly we have the analogue in this model of the result in monopoly that the marginal revenue curve should intersect the marginal cost curve from above (and, with monopoly, conditions (6) and (7) are identical). Since f_n and g_p are positive and g_n is negative, the denominator in (10) and (11) is positive. Thus from (8)–(11) the sign of $\partial \Pi/\partial c$ and $\partial n/\partial c$ is the same i.e. an increase in marginal cost increases the number of

Table 1

Effects of cost (tax) increases on prices, profits and number of firms.

Generalised-Cournot

(i) $$\frac{\partial p}{\partial c}=\frac{1}{\left(1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}\right)}\gtrless 1 \quad \text{as} \quad F\lessgtr 1.$$

which is positive from (6).

(ii) $$\frac{\partial \Pi}{\partial c}=\frac{-X\gamma\left[1-\frac{1}{\varepsilon}+\frac{F}{\varepsilon}\right]}{1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}}.$$

Thus $\partial\Pi/\partial c<0$ if and only if $F>1-\varepsilon$.
(iii) The number n of firms is given.
(iv) For shifts in $n[\mathrm{d}(\Pi/n)/\mathrm{d}n]<0$ (using (7)).

Monopolistic competition

(i) $$\Delta\frac{\partial p}{\partial c}=\frac{pX(1-\alpha)}{\varepsilon n^2}+K \quad (>0),$$

(ii) $$\Delta\frac{\partial n}{\partial c}=X(1-\gamma)-X\left(1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}\right),$$

where Δ is positive (and equal to $f_n g_p - f_p g_n$, see eqs. (10) and (11)).

(iii) $$\frac{\partial p}{\partial c}>\frac{1}{1-\gamma} \quad \text{if and only if} \quad F<1-\varepsilon,$$

$$\frac{\partial n}{\partial c}<0 \quad \text{if and only if} \quad F>1-\varepsilon.$$

(iv) Profits are zero.

In the special case of linear demand curve:

Generalised-Cournot

$$\frac{\partial p}{\partial c}=\frac{1}{1+\gamma}<1,$$

$$\frac{\partial \Pi}{\partial c}=\frac{-2\gamma X}{1+\gamma}<0;$$

Monopolistic competition

$$\frac{1}{1-\gamma}\geq\frac{\partial p}{\partial c}\geq\frac{1}{1+\gamma},$$

with equality on the r.h.s. when α and γ are 1 (and note that when $n\to\infty$ and $\alpha\to 0$, as for perfect competition, then $\gamma\to 0$ and $\partial p/\partial c\to 1$).

$$\frac{\partial n}{\partial c}<0.$$

Note

The results in the table follow in a straightforward way using (8) to (11) and the definition of $f(\ )$ and $g(\ )$ as the l.h.s. of the equilibrium conditions (3) and (4). The term $(1-(\gamma/\varepsilon)+(F\gamma/\varepsilon))$ is f_p (see eq. (3)) and may be considered as the effect on 'perceived marginal revenue' of a marginal increase in price, thus embodying F, the elasticity of the elasticity. It is positive from the stability condition (6).

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firms in monopolistic competition if and only if it would increase profits in generalised-Cournot given the number of firms.

It is also clear that a decrease in the number of firms in oligopoly (generalised-Cournot) will raise the price (f_p and f_n are both positive). Hence $\partial p/\partial c$ under generalised-Cournot is lower than $\partial p/\partial c$ in monopolistic competition if and only if an increase in costs decreases profits in generalised-Cournot.

For the most part we shall be dealing with a general demand curve. There are two examples we shall use at a number of points: the isoelastic case (constant ε) and the linear case (constant slope of the demand curve). Results for the former are given by putting $F=0$ in the formulae which follow. For the linear case, $X=a-bp=b(p^*-p)$, where $p^*=a/b$ is the price which gives zero demand, we have $\varepsilon=p/(p^*-p)$ and $F=p^*/(p^*-p)$; thus ε goes from zero to infinity as p goes from 0 to p^* and F goes from 1 to infinity. For the linear case (3) takes the convenient form

$$p=\frac{1}{1+\gamma}c+\frac{\gamma}{(1+\gamma)}p^*, \tag{3a}$$

i.e. price is a weighted average of the marginal cost (or $\bar{c}$ more generally) and the price at which demand is zero, p^*. We shall assume $p^*>c$. Where $n=1$, γ is unity and the weights are equal. In the linear case profits must fall in generalised-Cournot when c increases (since $F>1-\varepsilon$), hence in monopolistic competition, the number of firms decreases and the effects of a marginal cost increase on price is higher than for generalised-Cournot.

The results on $\partial p/\partial c$ and $\partial \Pi/\partial c$ for the generalised-Cournot model are special cases of those in Seade (1980a), where we assume here that all firms are identical. The derivation is particularly straightforward for this case in contrast to that of heterogeneous firms when some subtlety is necessary [Seade (1987)]. The result on $\mathrm{d}(\Pi/n)/\mathrm{d}n$ is contained in Seade (1980b). Seade does not treat monopolistic competition, does not treat the linear case (an interesting one) explicitly and does not consider the broader class of policy tools in the next section.

We can interpret an increase in c as an increase in a specific tax on the commodity. As we can see the effect on price of a unit tax increase can be greater or smaller than one. If F is high (see linear case) then an increase in the tax has a small effect on the price because the increase in price, increases the elasticity which dampens (through (3)) the effect of the price increase. On the other hand where F is low the increase in price can be greater than one (and if low enough, i.e. $<1-\varepsilon$, profits increase). These results are in sharp contrast to the standard analysis of the competitive case where the effect of a unit tax increase on consumer price is (for small taxes) $\eta/(\varepsilon+\eta)$ and

$0 \leqq \eta/(\varepsilon+\eta) \leqq 1$ where η is the elasticity of supply. We have seen that where markets are not competitive 'full tax shifting' or $((\partial p/\partial c)=1)$ is *not* a polar case. It is possible that it provides a sensible middle choice for some applied work.

3. The positive effects of dual pricing

As we remarked in the introduction, the policy framework we shall study has many different interpretations in terms of different practical policies. To avoid switching back and forth between possible interpretations we shall set up and discuss the model in terms of the Indian system of dual pricing but once the scheme has been explained we shall show explicitly how the different interpretations arise.

Dual pricing involves the compulsory sale of a fixed proportion $(1-\theta)$ of the output of each firm to the government at a 'retention price' p_R. We shall examine in this section the effects on prices, output and profits of introducing, or varying, such a scheme in the different market structures described above. We suppose that all the quantity sold by firms to the government is made available to consumers through a rationing system at the retention price p_R. We shall assume all consumers are identical. In section 6 we consider heterogeneous consumers and the possibility that the issue price may differ from the retention price.[3]

To keep things simple, we shall assume that total market demand, i.e. purchases from ration shops plus those from the open market, is a function of the market price, p, only. In order to justify this we may first assume that the effect on demand of the effective increase in lump-sum income associated with the ration is negligible. This effective increase is $\bar{x}(p-p_R)$ the difference between open market and retention price. Secondly we assume either that the ration $\bar{x}$ is resaleable at p or that each household consumes more than its ration i.e. it makes open-market as well as ration-shop purchases: in each case the opportunity cost to the household of the marginal unit is the open market price p. The relaxation of some of these assumptions is discussed in section 6.

The conjectural variation by the firms when the scheme is operating is assumed to be the same as without, i.e. given by (1) where x_i is the total output for firm i (including that sold to the government). If each firm knows that all the other firms are being forced to sell the fixed proportion to the

[3]The Indian scheme therefore involves a combination of regulation or taxation of firms and rationing of consumers. Whilst the literature on the theory of quantity and points rationing is fairly extensive, see for example Neary and Roberts (1980) and Tobin (1952) there appears to be rather little on dual pricing. In the Indian context see Mukherji et al. (1980), and an interesting programme of work by Professor V.K. Chetty entitled 'Project on Price and Distribution Controls in India' at the Indian Statistical Institute, Delhi.

government in the same manner as itself, then this seems the natural assumption.

The equilibrium conditions (3) and (4) are replaced by

$$f(\theta, p_R, c, p, n) \equiv p\theta\left(1-\frac{\gamma}{\varepsilon}\right) - c + (1-\theta)p_R = 0, \tag{12}$$

$$g(\theta, p_R, c, p, n) \equiv (p\theta - c + (1-\theta)p_R)X - Kn = 0. \tag{13}$$

We may now consider the various possible interpretations of the scheme. First, its effects can be seen as a sales tax at rate $(1-\theta)$ together with a per unit production subsidy of $(1-\theta)p_R$; subsidies of various kinds to specific industries often coexist with sales taxes or VAT so examples of this type akin to the scheme must be common. Secondly the tax liability of the firm is $T_i = (1-\theta)(p-p_R)x_i$. This is like a tax on profits before fixed cost with a marginal cost decreed by the government to be p_R. Given that costs are often unobservable and distinctions between fixed and marginal cost are blurred in accounting, this may not be far away from some profits taxes in practice.

As we suggested in section 1 there are a number of further possible interpretations of the system involving particular policies adopted in certain industries. Thus, thirdly we could think of medical doctors being allowed to work a proportion θ of their time in private practice if the remaining $(1-\theta)$ is available to the state at a fixed price. Or, fourthly, one could imagine a market price controlled at p_R but where a proportion θ of sales are on the black market – θ then could be influenced by enforcement. One could think of θ as the black market proportion for each firm, or the probability of any given firm being 'raided' and forced to sell its entire output to the government at the requisition price. A fifth reinterpretation concerns housing developments where a certain fraction of the housing must be sold at regulated prices – examples like this exist in the U.S.A. and probably elsewhere. And a sixth important further use of the model could be in the analysis of defence contracting where the government undertakes to take a fixed proportion, or quantity (see section 5), leaving the industry free to sell the remainder on the open market. Note that the positive aspects of the scheme in our model (i.e. effects on market price and output) do not depend on the consumers ration scheme or the ration price – and see section 6. A seventh interesting case could be where a public sector monopoly is instructed to maximise profits but to ensure that everyone has a basic minimum at a regulated price. This could take the form of a fixed quantity or proportion of output made available at price p_R (the analysis is easily modified to the case of fixed retention quantity – see section 5 below).

The different interpretations will suggest different parameters for comparative statics. We work in terms of θ and p_R following the example which originally motivated the scheme. The first reinterpretation, for example, would lead one to focus on a sales tax and a specific tax (or cost subsidy). These can be obtained from our results very easily: a simple sales tax corresponds to $p_R=0$ (with the tax as $1-\theta$) and the specific tax was examined in the preceding section.

The examples indicate that versions of the scheme arise in industries where perfect competition may not be the appropriate model. However the analysis under perfect competition is very straightforward and it is useful to have it as a reference point. We suppose that there are no fixed costs so that free entry results in a net price received per unit, $\theta p+(1-\theta)p_R$, equal to c. Alternatively one may think of c as the minimum average cost with a U-shaped cost-curve. Then the single equilibrium condition becomes

$$\theta p+(1-\theta)p_R=c. \tag{12'}$$

The implications for price and output of changes in θ and p_R are immediate. An increase in the open-market proportion θ at given p_R will lower price and increase output (since we require $p>p_R$). And an increase in p_R, at constant θ, will also lower market price and increase output. Note that there is no equilibrium with $p>p_R>c$.

The comparative statics of the scheme under oligopoly and monopolistic competition can be derived in an analogous manner to (8)–(11). The results for price, profits and the number of firms are given in table 2. The industry profits Π are now the l.h.s. of (13) (denoted by $g(\ )$). Notice that (12) can be written as

$$p\left(1-\frac{\gamma}{\varepsilon}\right)-\hat{c}=0, \tag{12a}$$

where $\hat{c}$ is $p_R+(c-p_R)/\theta$, which is analogous to (3). Hence in generalised-Cournot the effects on price of changes in θ and p_R follow straightforwardly from $\partial p/\partial c$ in table 1 on multiplying by $\partial\hat{c}/\partial\theta$ and $\partial\hat{c}/\partial p_R$. Hence the factor $\theta[1-(\gamma/\varepsilon)+(F\gamma/\varepsilon)]$ denoting the effect on 'perceived marginal revenue' of an increase in price (it is f_p where f is now the l.h.s. of (12)) again features prominently (see note to table 1). Once the effect on price has been calculated the effect on industry profits follows straightforwardly using (13). This describes the calculation of the expressions for the first four derivatives in table 2. Once they are calculated they may be signed fairly readily and the results are given in table 2 immediately following the derivatives.

The effects of changes in θ and p_R for monopolistic competition are derived as in (10) and (11) with θ and p_R respectively substituted for c. Thus the determinant $(f_n g_p-f_p g_n)$ of the matrix of partial derivatives with respect to the endogenous variables (which occurs in the denominator of (10) and

Table 2

Effects of changes in open market proportion, θ, and retention price, p_R, on prices, profits and number of firms.

Generalised-Cournot

Price: $$\frac{\partial p}{\partial \theta}=\frac{1}{\theta^2}\frac{(p_R-c)}{\left(1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}\right)},\qquad \frac{\partial p}{\partial p_R}=\frac{-(1-\theta)}{\theta}\frac{1}{\left(1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}\right)};$$

Profits: $$\frac{\partial \Pi}{\partial \theta}=\frac{\gamma X(p-p_R)(F-\sigma)}{\varepsilon\left(1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}\right)},\qquad \frac{\partial \Pi}{\partial p_R}=\frac{\gamma(1-\theta)X}{\left(1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}\right)}\left(1-\frac{1}{\varepsilon}+\frac{F}{\varepsilon}\right);$$

where

$$\sigma=(1-\varepsilon)-\frac{(1-\gamma)}{\left(1-\frac{p_R}{p}\right)}<0\quad(\text{since }\varepsilon>\gamma).$$

Number of firms is fixed.

$$\frac{\partial p}{\partial \theta}>0\quad\text{if and only if } p_R>c,\qquad \frac{\partial p}{\partial p_R}<0,$$

$$\frac{\partial \Pi}{\partial \theta}>0\quad\text{if}\quad F>\sigma,\qquad \frac{\partial \Pi}{\partial p_R}>0\quad\text{if and only if}\quad F>1-\varepsilon,$$

$$\frac{\partial \Pi}{\partial \theta}>0\quad\text{if}\quad p_R>c.$$

Monopolistic competition

Price: $$\Delta'\frac{\partial p}{\partial \theta}=K\left(\frac{p_R-c}{\theta}\right)-(p-p_R)X\cdot p\frac{\theta(1-\alpha)}{\varepsilon n^2},$$

$$\Delta'\frac{\partial p}{\partial p_R}=-(1-\theta)K-(1-\theta)K-(1-\theta)X\cdot p\frac{\theta(1-\alpha)}{\varepsilon n^2},$$

Number of firms: $$\Delta'\frac{\partial n}{\partial \theta}=(p-p_R)X\theta\left(1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}\right)+(p_R-c)X(1-\gamma),$$

$$\Delta'\frac{\partial n}{\partial p_R}=\theta\left(1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}\right)(1-\theta)X-\theta(1-\gamma)(1-\theta)X,$$

where

$$\Delta'=\frac{p\theta(1-\alpha)}{\varepsilon n^2}\cdot\theta X(1-\gamma)+\theta K\left(1-\frac{\gamma}{\varepsilon}+\frac{F\gamma}{\varepsilon}\right)>0.$$

Profits are zero.
Hence

$$\frac{\partial p}{\partial \theta}\begin{cases}>0 & \text{at}\quad p_R=p,\\ <0 & \text{for}\quad p_R\leqq c,\end{cases}\qquad \frac{\partial p}{\partial p_R}<0,$$

$$\frac{\partial n}{\partial \theta}>0\quad\text{if}\quad F>\sigma,$$

$$\frac{\partial n}{\partial \theta}>0\quad\text{if}\quad p_R>c,\qquad \frac{\partial n}{\partial p_R}>0\quad\text{if and only if}\quad F>1-\varepsilon.$$

Note

The results are based on eqs. (8)–(11) as applied to (12) and (13) with θ and p_R being respectively substituted for c. Thus $f_p=\theta[1-(\gamma/\varepsilon)+(F\gamma/\varepsilon)]$ appears prominently for generalised-Cournot and similarly $f_n g_p-f_p g_n$ $(=\Delta')$ in monopolistic competition. For interpretation, see text.

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(11) and is called Δ' in the lower part of table 2) appears throughout. Again once the expressions have been derived the derivatives may be signed in a straightforward manner.

Intuition on the results in table 2 may be guided by noting (i) that the scheme acts in part like a cost subsidy $(1-\theta)p_R$, (ii) what happens to 'perceived marginal revenue' (f_p is positive from stability) and (iii) in the case of monopolistic competition that an increase in profits for a given number of firms will increase the number of firms and lower prices. Thus an increase in p_R will lower the price in each of generalised-Cournot and monopolistic competition (notice that the sign of $\partial p/\partial c$ in table 1 is positive and given the stability condition does not depend on F). As before the effect will be greater in magnitude in monopolistic competition if and only if a decrease in costs (increase in p_R) increases profits and the number of firms. An increase in θ increases price if $p_R > c$ under generalised-Cournot, as should be clear from (12a) since it increases $\hat{c}$. On the other hand it will usually increase profits for a given number of firms (the condition is $F > \sigma$ where σ is negative, see table 2), and thus the number of firms with free entry, hence any price increase from an increase in θ in the case $p_R > c$ will be lower in monopolistic competition.

The result that an increase in open market proportion θ can *increase* price in oligopoly (where $p_R > c$) is the opposite from that of perfect competition (where the case $p_R > c$ does not arise in equilibrium) and deserves emphasis. We can gain intuition on the result from the interpretation of the scheme as a sales tax $(1-\theta)$ and cost-subsidy $(1-\theta)p_R$. If p_R is exactly equal to c then price or 'marginal revenue' and cost are affected in exactly the same proportion by an increase in θ and there is no effect on price. The position is analogous to a proportional profits tax. However if p_R exceeds c then from a given $MC = MR$ equilibrium 'marginal cost' is shifted above 'marginal revenue' by an increase in θ and price has to be increased to restore equality between 'marginal cost' and 'marginal revenue' (stability requires 'marginal revenue' to fall with quantity and increase with price). Notice that the crucial condition is whether or not retention price exceeds 'marginal cost' and the elasticity of the elasticity does not play a central role; we have placed 'marginal revenue' and 'marginal cost' in inverted commas here since the former refers to the industry equilibrium condition rather than that of a particular firm and both are modified to include the effects of the scheme.

4. Effects on welfare

The introduction of the scheme will affect the welfare of consumers through the open market price, p, and the value of the lump-sum transfers $(1-\theta)X(p-p_R)$. We assume initially that consumers are identical, and recall that government revenue from the scheme is zero since retention price and issue price are equal. Thus our discussion of welfare is in terms of the levels of profits and consumer surplus (generalised-Cournot) and the level of consumer surplus (monopolistic competition). In section 5 we comment on distribution amongst consumers and the case where issue and retention price may differ. As we have noted the optimum for a fixed number of firms, and where distribution is not an issue, is for the government to take over and charge marginal cost. This is a limiting case of the scheme. For monopolistic competition the first best with price equal to marginal cost is not feasible since profits must be zero. There is no solution in this case with θ and p_R interior to the constraints and the boundary solution involves a single firm in order to keep fixed costs as low as possible and get prices as close as possible to marginal cost subject to the constraint of zero profits. This analysis is presented in the appendix together with the verification of price equal marginal cost as the optimum for generalised-Cournot.

Changes in consumer surplus $\mathrm{d}W$ may be written as

$$\mathrm{d}W = \mathrm{d}[(1-\theta)X(p-p_R)] - X\,\mathrm{d}p, \tag{14}$$

$$\mathrm{d}W = -X(p-p_R)\mathrm{d}\theta - [\theta X - (p-p_R)(1-\theta)X']\,\mathrm{d}p - (1-\theta)X\,\mathrm{d}p_R. \tag{15}$$

We look at the effects of changes in θ and p_R for the generalised-Cournot in table 3. The calculations are based on (15) and use the expressions for $\partial p/\partial\theta$ and $\partial p/\partial p_R$ presented in table 2.

In interpreting the results in the table we should note that if p_R is set so that $p = p_R$ then

$$p = p_R = \frac{c}{1-(\theta\gamma/\varepsilon)}. \tag{16}$$

Note that a retention price equal to market price is not the same as abolishing the scheme (compare (16) and (3)) but rather results in lower prices than without the scheme – in effect it raises the elasticity of demand. With p_R chosen for any given θ so that $p = p_R$ the government essentially controls the market price through θ and can choose any price above c. In the limit as θ tends to zero, price tends to marginal cost. This obviously involves negative profits but in the usual way maximises $\Pi + W$ the sum of producer and consumer surplus.

There may be many reasons outside the model why the government does not adopt a complete take-over with price equal to marginal cost. There may be legal, political or administrative constraints on the degree of intervention.

Table 3

Effects on consumer welfare and profits: generalised-Cournot.

$$1-\varepsilon<F<1+\delta, \quad \frac{\partial \Pi}{\partial p_R}>0, \quad \frac{\partial W}{\partial p_R}>0,$$

$$F>1+\delta, \quad \frac{\partial \Pi}{\partial p_R}>0, \quad \frac{\partial W}{\partial p_R}<0,$$

$$F<1-\varepsilon, \quad \frac{\partial \Pi}{\partial p_R}<0, \quad \frac{\partial W}{\partial p_R}>0,$$

where

$$\delta=(\theta\gamma)^{-1}\left(1-\frac{p_R}{p}\right)(1-\theta)\varepsilon^2>0.$$

$$F>\sigma, \quad \frac{\partial \Pi}{\partial \theta}>0,$$

where $\sigma<0$ (see table 2).

If $c<p_R$, $\partial W/\partial\theta<0$.

If $c>p_R$ then $\partial W/\partial\theta<0$ at $\theta=1$ and $\partial W/\partial\theta>0$ at $\theta=0$ (for $F\geqq 0$).

Linear demand

$F=p^*/(p^*-p)$; $\varepsilon=p/(p^*-p)$ where p^* is a/b the price at which demand becomes zero. Thus $F>1$ and at $p=p_R$ (i.e. $\delta=0$) we have $\partial W/\partial p_R<0$ and $(\partial\Pi/\partial p_R>0)$.

As $p\to p^*$ then ε and $F\to\infty$ and $F/\varepsilon\to 1$. Hence for p close to p^*, $F<1+\delta$ and $\partial W/\partial p_R>0$.

Thus for a given θ, there exists a $\hat{p}_R$ which makes W a maximum; and $p_1<\hat{p}_R<p_2$ with p_2 given by (16) and p_1 satisfying $\theta p^*+(1-\theta)p_1=c$, so that if $p_R=p_1$ then $p=p^*$ (see (12)). Note that $p_1<c<p_2$.

Note

The derivatives of W may be calculated using (15) and price derivatives from table 2. The derivatives of Π are taken from table 2 and set alongside those for W for comparison. For interpretation, see text.

There might be other reasons concerning possible problems of management and control of cost or considerations of revenue. Thus the effects on consumer welfare of local changes in θ and p_R from some initial position, and given the model, are of interest since these may then be set alongside the factors omitted from the model to form a judgement concerning possible directions of reform. Suppose, for example, that a reduction in θ, or greater government intervention, led to an increase in marginal cost (so that c becomes a function of θ with $\partial c/\partial\theta<0$). This might be because more resources would be devoted to concealing incremental output if the government were to take a higher proportion. And if the government were to take over completely it might not be able to hold marginal costs down to levels which would exist under partial control. There could then be an interior optimum which would be characterised by $\partial W/\partial\theta$ equal to zero where, in addition to the terms already introduced we would add to $\partial W/\partial\theta$ the term $(\partial W/\partial c)(\partial c/\partial\theta)$. In this way the calculations provided here can be combined with those aspects which the model ignores. In many cases the combination could not be done formally and would have to be judgemental. It is possible, however, that these further considerations would require a complete recasting of the model where the effects of government policy take very different forms.

From table 3 we see that if $F<1+\delta$ then raising the retention (and ration) price at given θ yields a fall in market price p sufficient to compensate the consumers for the higher ration price. If F is also above $(1-\varepsilon)$ then profits rise too with an increase in p_R. On the other hand if $F>1+\delta$ then (locally) raising the retention price to the market price (at fixed θ) lowers welfare and increases profits (the case of linear demand provides an example). Welfare rises and profits fall when θ is lowered (for $F>0$ and $p_R>c$). Notice that a low F implies that an increase in p_R gives a big fall in market price (the price fall is not strongly dampened by an elasticity decrease) and thus works in favour of consumers. On the other hand, for similar reasons, a low F is less favourable for any profit increase arising from an increase in p_R.

Hence although maximisation of $\Pi+W$ requires the raising of p_R to p and the lowering of θ to 0, giving $\Pi<0$ eventually, this does not by itself tell us that consumer welfare will increase monotonically en route. For example, suppose we start with $p_R<p$ and $\theta>0$ and first increase p_R to p for given θ and then let θ tend to zero (adjusting p_R to keep it equal to p). Then in the linear case we have consumer welfare falling (if $p_R>\hat{p}_R$, see table 3) and profits rising as p_R is increased to p followed by movements in the opposite direction as θ is lowered.

We have emphasised however, that the full government takeover may not be feasible or desirable for reasons external to the model. In particular θ may be set exogenously by convention, statute or limitations on enforcement and the government may have effective control only over the retention/ration price. For example, under the interpretation of price control with black markets there will be constraints on enforcement which prevent $\theta=0$. Or in medical contracts the proportion θ may be the result of a long-standing agreement which it would be difficult to change, and so on. In such a case, we may be interested in the optimum p_R for given θ. In the linear example, and from the point of view of consumer welfare one would have an interior solution and would set p_R at $\hat{p}_R$, below the market price. On the other hand in the isoelastic case $F=0$ we would raise p_R to p to increase consumer welfare.

We also see from table 3 that if $c>p_R$ then there is a θ between zero and one which maximises W for given p_R. One might therefore ask whether there is an interior maximum (with $p<p_R$ and $0<\theta<1$) for W (as distinct from Π plus W which has already been discussed). The answer is negative: it is straightforward to show (see Appendix) that $\partial W/\partial p_R\leqq 0$ implies $\partial W/\partial\theta<0$.

Intuitively the explanation is as follows. The policy variables p_R and θ enter the model only through their effect on the price per unit $p\theta+(1-\theta)p_R$ with the exception that θ affects in addition the mark-up over marginal cost. Thus if p_R is set optimally with respect to W for given θ there is a potential gain from reducing θ and lowering this mark-up.[4]

Since $\partial W/\partial\theta<0$ at $\theta=1$ and $p=p_R$, and there is no interior solution, the relevant boundary is $\theta=0$. Hence the maximum for W is given by $\theta=0$ and $p_R=c$ (see (12)). Thus, in the model, consumer welfare is maximised by the government taking full control of the market and setting the ration price at marginal cost (the limit of (12) as θ tends to 0 acts as a floor preventing a lower ration price). As in the maximisation of $\Pi+W$ firms make a loss equal to the fixed cost.

Negative profits are ruled out in monopolistic competition, for which results are presented in table 4. For low F (<1) we can see that the optimum policy is to raise p_R to p and lower θ to the minimum consistent with zero profits. Since $\partial p/\partial\theta$ and $\partial n/\partial\theta$ are positive for $p=p_R$ this choice of θ (given the policy for p_R) has the effect of minimising the price and the number of firms.

Table 4

Effects on consumer welfare: Monopolistic competition.

$$\frac{\partial W}{\partial p_R}>0, \quad \text{if } F<1,$$

$$\frac{\partial W}{\partial p_R}<0, \quad \text{at } p=p_R \text{ for given } \theta \text{ and linear demand},$$

$$\frac{\partial W}{\partial \theta}<0, \quad \text{at } p=p_R.$$

These results are derived in a straightforward manner using (15) and the expressions for $\partial p/\partial p_R$ and $\partial p/\partial\theta$ provided in the second part of table 2.

For higher F (e.g. linear demand) we would wish (for any given θ) to lower p_R if it were equal to p. However one can show as before (see Appendix) that $\partial W/\partial p_R\leqq 0$ implies $\partial W/\partial\theta<0$ so that there is no optimum for both θ and p_R which is interior to the constraint $n\geqq 1$.

The role of F in these results is through the magnitude of the price response following a change in p_R or θ. Consider, for example, an increase in p_R for given θ. The effect of this is to lower the price and this contributes an increase in W through the second term on the r.h.s. of (14). However the

[4]The argument is analogous to one in the theory of policy towards crime where in certain models the optimum selection of penalty will imply that (costly) enforcement should be reduced; p_R and θ play similar roles to (the opposite of) penalties and enforcement [see Stern (1978)].

decrease in $(p-p_R)$ lowers the value of the lump-sum transfer and the net increase in welfare is positive only if the fall in p is large enough. But, the larger is F the greater the reduction in the elasticity of demand following a fall in price. And a reduction of the elasticity dampens the price fall. Hence the higher is F the less likely is the first effect to dominate and the less attractive is the scheme from the point of view of consumer welfare.

5. Fixed retention quantity

An alternative version of the scheme which is also of practical relevance is where the quantity requisitioned per firm is fixed. The case of fixed retention quantity may be appropriate when, for example, output is not observable. The only information required is the existence of a firm. It may be the more natural model for the interpretation of government contracts at fixed prices, e.g. the government may have a specific quantity of defence equipment in mind. And it may be appropriate for government contracts with the medical profession where a fixed proportion of time may in practice be a fixed amount with the proportion of time spent on the private market being flexible. The analysis can be presented in a similar manner to that for the fixed retention proportion in that we focus on the effect of small changes in the retention price and the retention quantity and look at shifts in the '$MR=MC$' and '$AR=AC$' conditions. We shall not provide full detail and we shall simply indicate how one can proceed and some results. There are, however, important differences in these results flowing essentially from the requisition being similar to a lump-sum tax on the firms in the fixed quantity case. The similarity is exact for changes in the retention price, although not for the quantity since an increase in the retention quantity reduces the free-market demand.

We suppose that the requisition quantity x_R is the same for each firm so that the total amount requisitioned X_R is nx_R. Writing X_F for the free-market quantity we have X equal to X_F+X_R, and X_F is the sum of x_i^F which is defined as x_i-x_R. We assume that conjectures concern the free-market quantity so that (1) is replaced by

$$\frac{\partial(X^F-x_i^F)}{\partial x_i^F}=\alpha\left(\frac{X^F-x_i^F}{x_i^F}\right), \tag{17}$$

and the perceived marginal revenue MR_i is given by (18)

$$MR_i=p\left(1-\frac{1}{\varepsilon}[(1-\alpha)s_i^F+\alpha\theta]\right), \tag{18}$$

where s_i^F is x_i^F/X. Adding across firms we have, where $\theta=X_F/X$,

$$p\left(1-\frac{\theta\gamma}{\varepsilon}\right)=\bar{c}, \tag{19}$$

which replaces (12). We may interpret θ as the choice variable in place of x_R since there is a one-to-one relationship between θ and x_R given p_R. We can establish readily from (19) and the stability condition (see below) that $\partial p/\partial\theta>0$ and hence that $\partial X/\partial\theta<0$. But $\theta=1-nx_R/X$ hence an increase in x_R is associated with an increase in X and a decrease in θ. We can now conduct the analysis as before using the pair of eqs. (19) and (13) instead of (12) and (13).

Notice that p_R does not enter (19) and thus does not affect the price in the generalised-Cournot case – a reduction in p_R acts, in this case, just like a lump-sum tax on profits. This feature provides the main difference between the system with fixed retention proportions and fixed retention quantities. Note, however, that a change in x_R will affect the open-market quantity. As we have just seen an increase in x_R will imply an increase in total market quantity and a reduction in price. The open market quantity X^F may, however, go up or down (it is easy to check that in the monopoly case, for example, the condition[5] for an increase is $\theta E-1>0$).

The existence and stability conditions become $\varepsilon>\theta\gamma$ and $F>1-\varepsilon/\theta\gamma$ respectively and it is straightforward to construct for this case the tables corresponding to tables 2, 3 and 4 on prices, profits and welfare.

In the generalised-Cournot case $\partial W/\partial p_R=-\partial\Pi/\partial p_R=(1-\theta)X$ since increasing p_R acts simply like a lump-sum transfer from consumers to producers. Raising θ raises prices, as we have noted, i.e. greater government intervention in the form of lower θ must lower prices, where this was only a possibility in the case of fixed retention proportion. Consumers would always want the government to lower both θ and p. With monopolistic competition one can again show that $\partial W/\partial p_R\leqq 0$ implies $\partial W/\partial\theta<0$ so that there is no optimum for θ and p_R which is interior to the constraint $n\geqq 1$.

6. Some extensions

6.1. Income distribution

If we introduce different consumers, indexed by h, with welfare weights β^h then we can write

$$\mathrm{d}W=\sum_h\beta^h\,\mathrm{d}\left[(1-\theta)\frac{X}{H}(p-p_R)\right]-\sum_h\beta^h x^h\,\mathrm{d}p, \tag{20}$$

[5]See footnore 2.

if everyone has an equal ration. This can be approximated by

$$\mathrm{d}[(1-\theta)X(p-p_R)]-vX\,\mathrm{d}p, \tag{21}$$

where we normalise the welfare weights so that $(1/H)\sum\beta^h$ is one and v is the distributional characteristic

$$\left(\frac{1}{H}\sum_h \beta^{h^h}x^h\right)\Bigg/\left(\frac{1}{H}\sum_h \beta^h\right)\left(\frac{1}{H}\sum_h x^h\right).$$

The approximation lies in assuming the distributional characteristic may be treated as constant with respect to the changes being considered. We would expect to have $v<1$ since for normal goods β^h and x^h would be negatively correlated (if β^h decreases with income). For changes involving prices only, homotheticity is enough to guarantee constant v [see e.g. Roberts (1980)] but notice that there also exist effects on lump-sum income here. In practice the constancy of v is unlikely to be misleading.

Hence one way of considering income distribution is to place a greater weight on the lump-sum transfer term than on the consumer surplus term. Since we have a zero lump-sum transfer at $p=p_R$ it is clear that the weight on this term could be sufficiently high to guarantee the attractiveness of introducing the scheme where lump-sum transfers are not feasible in other ways.

One could also consider a criterion which was $\lambda\Pi+\mu W$ where the weights λ and μ may differ. In some cases (see e.g. table 3 and p_R) Π and W move in the same direction but in others (e.g. linear case with p_R to be chosen and θ fixed) trade-offs can arise. Usually an increase in θ will involve increasing profits but reducing consumer welfare. Where the weight λ on profits is low then the results would approximate those for the maximisation of W, thus where θ can be chosen this points towards θ tending to zero. However in other circumstances λ may be high, for example where some of the firms are government owned or where they are peasant co-operatives (e.g. food, cotton or sugar processing) or where the government wishes to avoid possible closures. Then one might expect an optimum θ between 0 and 1.

6.2. *Issue price different from retention price*

It is straightforward to make the issue price for the ration to households p_I, different from the retention price p_R. The equilibrium price depends only on p_R. The expression for welfare now contains p_I in place of p_R. Since p is independent of p_I we have, for example that $\partial W/\partial p_I=(1-\theta)X<0$. If $p_I>p_R$ the scheme makes a profit for the government.

6.3. *Effects of ration on demand*

It is possible to examine different assumptions concerning the effect of the ration on demand. If the ration is not resaleable and is greater than the amount desired (at the ruling open market price) for certain households, then they would make no open market purchases. The benefits of changes in the scheme for such households would be confined to the size and price of the ration. Thus an increase in p_R would reduce welfare since there would be no offsetting benefit from the reduction in open market price. An increase in the ration would increase welfare, although if the household has more than it would wish to buy at p the effect may not be large.

When the ration is less than the amount desired (or resaleable) then one can also consider the effects of relaxing the assumption that there is no income effect on demand arising from changes in the ration. Note that the welfare effects of the lump-sum transfer associated with the ration are already included. An increase in p_R, for example, would then lower demand. As we have seen (table 2) an increase in p_R lowers the market price and this would be reinforced by the effects operating through demand shifts from the income effect. Thus whilst the explicit consideration of the income effects would complicate the formal analysis one can guess at some of the consequences of introducing them into the model and it seems unlikely that there would be radical changes in the results.

7. Concluding comments

The main purpose of this paper has been the examination of the effects of government tax and regulation policies towards oligopolistic and monopolistically competitive industries. The possible government policies were captured using a simple two-parameter framework which we showed had a number of interpretations as different kinds of taxes, combinations of taxes and regulatory policies. We conducted the analysis and interpretation in terms of the scheme which originally motivated the study, the Indian system of dual pricing. A broad range of market structures and outcomes were characterised using the conjectural variations model together with free entry.

Our first task (section 2) was to present a succinct way of summarising these different structures and to study the effects of a simple excise tax system. Thus we introduced monopolistic competition as a perfect equilibrium in a model of conjectural variations oligopoly in which entry decisions are made with a view to post-entry profits. This allowed us to extend the analysis of the effects of simple excise taxes by Seade (1987) in oligopoly with a fixed number of firms to the case of monopolistic competition. The comparison with oligopoly shows that the price increasing effect of a tax will be higher in monopolistic competition if and only if taxes reduce profits for a given number of firms. Note, however, that excise taxes may increase profits if the elasticity of the elasticity of demand, F, is sufficiently low. The proportion shifted in oligopoly is lower the higher is F since with higher F an increase in market price causes a higher rise in the elasticity thus dampening the effect of the price increase more strongly. However, if F is sufficiently low then the equilibrium price increase can be large enough to raise profits and bring about entry where this is permitted. For example in the isoelastic case tax shifting in oligopoly will exceed 100 percent and will be still greater (lower) in monopolistic competition if the elasticity of demand is less (greater) than one. On the other hand the linear demand curve provides an interesting example where tax shifting is less than one in oligopoly but shifting is definitely increased by allowing free entry. Taken together these examples show that the simple analysis of tax shifting in perfect competition may be misleading and that one should allow for a much broader range of possibilities. Thus, for example, 100 percent shifting is certainly not the polar case which it would appear to be in a simple model of perfect competition.

The dependence of these results, and those concerning dual pricing, on the elasticity of the elasticity indicates that one must be careful in choosing functional form for demand functions in policy analysis [for a cautionary tale familiar in other contexts, see e.g. Atkinson and Stiglitz (1980, ch. 14)]. We would, in general, congratulate an econometrician who could produce a reliable estimate of a demand elasticity and here we find ourselves asking for an estimate of the elasticity of the elasticity. Functional forms which might be useful in practice are, for $F \neq 0$,

$$\log X = B - Ap^F, \tag{22}$$

which has constant elasticity of the elasticity (for $F = 0$ we have the familiar isoelastic form $\log X = B - A \log P$) or for $E \neq 1$,

$$p = B - AX^{1-E}, \tag{23}$$

which has constant elasticity E with respect to X of the slope $\mathrm{d}p/\mathrm{d}X$, of the inverse demand curve [see Seade (1980a, 1980b, 1987)] on the role of E; note $E = -(Xp''/p')$ and $F = p\varepsilon'/\varepsilon$ so that $F = (1 + \varepsilon - \varepsilon E)$).

The different government policies to be considered were described in section 3 using two parameters. In terms of the Indian dual-pricing scheme which was used for much of the discussion these parameters are the proportion, θ, of the output which is sold on the open market (the government retaining $1-\theta$) and the government retention price p_R. Several different interpretations of the policies were offered: a sales tax $(1-\theta)$ and prodyction subsidy $(1-\theta)p_R$; profits taxation at rate $(1-\theta)$ where per unit costs are deemed by the government to be p_R; contracts for medical practice where a specified proportion of time, $1-\theta$, has to be spent in the public service and the remainder can be in private practice; defence contracts; black marketing of a proposition θ of output in a market where all sales are supposed to be at regulated prices; and so on.

The effect of an increase in the retention price p_R is to lower the open market price. Since the role of p_R can be seen as a subsidy the magnitude of the reduction will depend on the elasticity of the elasticity in the manner described in section 2. Hence, for example, it is possible that a reduction in p_R would bring about a larger fall in p – we would require 'tax shifting' to exceed $(1-\theta)$. Again whether the effect is larger or smaller in monopolistic competition than in oligopoly depends on what happens to profits.

The effect on the open market price of an increase in the open market proportion, θ, depends on the relation between the retention price and marginal cost. Where the retention price is above marginal cost then an increase in the open market proportion will increase the open market price. The reason is that in this case (and thinking of the interpretation of sales tax and cost subsidy) marginal cost is shifted upwards, from the point of equality with marginal revenue, by more than marginal revenue which then has to rise to restore equilibrium, thus increasing the price. Thus, in this case, a shift towards the free market and deregulation *raises* prices.

This is the most striking of the results on the positive effects of changes in the scheme particularly when expressed in terms of the other possible interpretations. Hence, for example, a deliberate decrease in enforcement in a market with a controlled price could result in an increase in the black market price. A decrease in proportional profits taxation could increase price. Or allowing medical practitioners to spend a greater proportion of their time in private practice could increase the price of private medicine. These results are not really paradoxical if one considers them carefully in terms of market equilibrium but they do caution against simplistic arguments based on the perfectly competitive model.

We examined welfare economic aspects of the schemes in section 4 ignoring distribution and focussing on aggregate consumer welfare. If both the retention price and the open market proportion can be chosen then the optimum policy for all market sutructures and versions of the scheme is to set the open market proportion to the minimum possible: zero in generalised-Cournot and the lowest consistent with zero profits in monopolistic competition. The government essentially takes full control of the market. The limiting value of both p and p_R is the marginal cost c in the case of generalised-Cournot.

We argued, however, that full take-over may be ruled out or be undesirable from considerations outside the model; for example for legal or historical reasons or limitations on government management. Where the open-market proportion is fixed the optimum retention price, from the point of view of consumers, depends on the curvature of the demand curve and on the market structure. If the elasticity of the elasticity (F) is sufficiently low then one would increase the retention price to the market price. However, where F is higher there will be an optimum retention price below the market price (where F is higher there are lower gains to consumers from a reduction in market price following an increase in p_R, which itself acts like a tax or cost reduction). Note that this means that there is an efficiency gain from the scheme in the case of monopolistic competition, since then both profits and government revenue are zero so that consumer surplus is the relevant efficiency criterion.

We saw in section 5 that the versions of the scheme with fixed retention proportion and fixed retention quantity can be analysed in similar ways. There are many similar results but also important differences. In the latter case the retention price has no effect on open market price in the generalised-Cournot model but an increase in government intervention in the market through an increase in the retention quantity must now *lower* prices. Consumers unambiguously prefer decreases in both p_R and θ. With monopolistic competition and fixed θ interior solutions for p_R are possible.

We discussed in section 6 how the analysis might be generalised to include distribution, government surpluses or deficits from the scheme and different demand specifications. In particular we found that where the scheme provides for lump-sum transfers which are not available through other methods then there will generally be distributional arguments in its favour.

The models of oligopoly and the policies considered in this paper have been fairly straightforward. We hope, nevertheless, to have shown that many different markets and outcomes can be captured using this simple framework and that the analysis of a broader range of government policies than the standard tax tools, in markets which are not perfectly competitive, is important and can be fruitful.

Appendix

It is shown here that in the models examined in this paper there is no optimum for consumer welfare, if both p_R and θ are choice variables, which is interior to the inequality constraints in the problem. For the generalised-Cournot case it is clear that the optimum involves complete takeover and price equal to marginal cost; for monopolistic competition we also require a boundary solution.

We consider the problem

$$\max_{\theta, p_R} W,$$

subject to

$$f(\theta, p_R, c, p, n) \equiv p\theta\left(1-\frac{\gamma}{\varepsilon}\right) - c + (1-\theta)p_R = 0, \tag{12}$$

$$g(\theta, p_R, c, p, n) \equiv (p\theta - c + (1-\theta)p_R)X - Kn = 0, \tag{13}$$

and

$$p \geqq p_R \geqq 0; \quad 0 \leqq \theta \leqq 1; \quad X(p) \geqq 0; \quad n \geqq 1,$$

in the case of monopolistic competition. The derivatives of W are given by (15). For generalised-Cournot we drop (13) and $n \geqq 1$ (since n is fixed).

From (15) we have

$$\frac{\partial W}{\partial \theta} = -X(p-p_R) - [\theta X - (p-p_R)(1-\theta)X'] \frac{\partial p}{\partial \theta}, \tag{22}$$

$$\frac{\partial W}{\partial p_R} = -(1-\theta)X - [\theta X - (p-p_R)(1-\theta)X'] \frac{\partial p}{\partial p_R}. \tag{23}$$

Further

$$\frac{\partial p}{\partial p_R} = \frac{-f_{p_R}}{f_p} \quad \text{and} \quad \frac{\partial p}{\partial \theta} = \frac{-f_\theta}{f_p}, \tag{24}$$

in the case of generalised-Cournot and

$$\Delta' \frac{\partial p}{\partial \theta} = f_{p_R} g_n - f_n g_{p_R}, \qquad \Delta' \frac{\partial p}{\partial p_R} = f_\theta g_n - f_n g_\theta, \tag{25}$$

where $\Delta' = f_n g_p - f_p g_n$ in the case of monopolistic competition.

From (22) and (23)

$$(1-\theta)\frac{\partial W}{\partial \theta} - (p-p_R)\frac{\partial W}{\partial p_R} = -M\left[(1-\theta)\frac{\partial p}{\partial \theta} - (p-p_R)\frac{\partial p}{\partial p_R}\right] \tag{26}$$

where M is the expression in square brackets in (22) and (23).

For generalised-Cournot we then have the r.h.s. of (26) as

$$-\frac{M}{f_p}(1-\theta)\frac{p\gamma}{\varepsilon}. \tag{27}$$

Both M and f_p are positive (the latter by stability see (6)) so that (27) is positive and

$$\frac{\partial W}{\partial p_R} \leqq 0 \quad \text{implies} \quad \frac{\partial W}{\partial \theta} < 0 \quad \text{for} \quad \theta < 1. \tag{28}$$

It is easily checked using (25) that (28) applies to the case of monopolistic competition, and using (12) and (19) to the case of fixed retention quantity and monopolistic competition. Hence in all these cases we cannot simultaneously have $\partial W/\partial p_R = 0$ and $\partial W/\partial \theta = 0$ (for $\theta < 1$) and there is no interior solution. We can use (28) and the information in tables 3 and 4 to check the boundary at which the optimum will occur. This will involve $\theta = 0$ in generalised-Cournot and $n = 1$ in monopolistic competition. In the former case this implies $p_R = c$ (if p is bounded).

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