A note on ‘Mechanism games with multiple principals and three or more agents’ by T. Yamashita*

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1 Introduction

Yamashita (2009) considers a common agency model with many principals and many agents. The agents have private information and principals take actions. The author analyzes equilibria in a class of communication games and proves a folk theorem. Yamashita (2009) shows that an allocation is implementable if and only if it is incentive compatible and the payoff of each principal is higher than a threshold value for the principal. Unfortunately, these treshold values are not characterized in terms of the primitives of the model. Therefore, the paper has the same shortcoming as the majority of the literature on common agency models. Namely, the analysis of the author does not lead to a characterization of the equilibria.

Yamashita (2009) assumes that a contract of a principal is a function from message profiles of the agents to the action space of the principal. In particular, each message profile uniquely determines an action to be taken by the principal. This means that the principals are essentially forced to participate in the contracting game and delegate the choice of actions to the agents. This note argues that this assumption is controversial, and proves that relaxing it leads to a full characterization of the set of equilibria if the information is complete. To be more specific, we show that an allocation is implementable if and only if the payoff of each principal is higher than his pure minmax value.

2 The Model and the Result

There are $J\,(\geq 2)$ principals and $I\,(\geq 3)$ agents. The action space of Principal j is the finite set Y_j, and let $Y = \times_{j=1}^{J} Y_j$. The payoff function of Principal j is $v_j : Y \to \mathbb{R}$, and the payoff

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function of Agent i is $u_i : Y \to \mathbb{R}$. Each agent is endowed with a message space M, such that $|M| \geq J+1$. For simplicity we assume that $\{0, ..., J\} \subset M$. Define the contract space of Principal j as $C_j = \left\{c_j | \; c_j : M^I \to 2^{Y_j}\right\}$, that is, a mapping from message profiles to subsets of the actions space.

The game has three stages. First, principals offer contracts simultaneously. These contracts are publicly observable. Second, each agent send messages to each principal privately. Finally, principals take actions simultaneously from the subsets of their action spaces which are determined by the contracts and the messages. That is, if Principal j offers c_j and the message profile sent to Principal j is m then Principal j must take an action from the set $c_j(m)$. We characterize the set of pure-strategy Subgame Perfect Nash Equilibrium (SPNE) of this game. The existence of a SPNE is only guaranteed if mixing is allowed. In what follows, we assume that a pure-strategy SPNE exists.

The complete information model of Yamashita (2009) is identical to ours except that the author assumes that the contract space of Principal j is $D_j = \left\{c_j | \; c_j : M^I \to Y_j\right\}$. That is, the message profile of the agents uniquely determines an action. Hence, the principals make no strategic decisions at the final stage of the game. Next, we illustrate by an example the controversy related to this assumption.

Example. Suppose that $J = 2$ and $I = 3$. Assume that the principals are playing the Matching Pennies Game. That is, $Y_1 = Y_2 = \{H, T\}$, and the payoffs to the principals are defined by the following matrix:

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Finally, suppose that the payoff to each agent is identical to the payoff to Principal 1.

We show that the allocation (H, H) is a SPNE outcome in the game of Yamashita (2009). The equilibrium contract of Principal 1, d_1, is such that $d_1(m') = H$ for an $m' \in M^3$ and T otherwise. The contract of Principal 2, d_2, is such that $c_2(m) = H$ for all $m \in M^3$. Define the strategies of the agents in every subgame such that whenever they can send a message profile that generates (H, H) or (L, L), they will do so. In particular, on the equilibrium path they send m' to each principal. We only have to show that Principal 2 cannot profitably deviate. Notice that no matter what contract he offers, the agents can always send a message profile such that either L or H is taken by Principal 2. Since the range of c_1 is $\{H, L\}$ they can always send messages to Principal 1 so that either (H, H) or (L, L) is generated.

Principal 2 can be maxmined in this example because he is forced to offer a contract which takes away his flexibility of choosing an action at the final stage of the game. Indeed, Principal 2 would profitably deviate by offering $\overline{d}_2$where $\overline{d}_2(m) = \{H, L\}$ for all m and randomize between H and L at the final stage. In other words, the principals are forced to involve the agents in taking an action. We find this assumption hard to justify on economic grounds.

Theorem 1 *An allocation $y^* \in Y$ is implementable if and only if*

$$v_j(y^*) \geq \min_{y_{-j}} \max_{y_j} v(y_j, y_{-j}).$$

Notice that a centralized mechanism could implement the same allocations by deterministic rules as our set of implementable allocations. Also observe that whether or not an allocation is implementable does not depend on the preferences of the agents.

Let $y^q_{-q} \in \arg\min_{y_{-q}} \max_{y_q} v_q(y_q, y_{-q})$. That is, y^q_j is used by Principal j to punish Principal q. For each j, let y^j_j denote an arbitrary element of Y_j.

Proof. We start with the "if" part. Let m^j_i denote the message of Agent i sent to Principal j. Define the equilibrium contract of Principal j as follows:

$$c^*_j\left(m^j_1, ..., m^j_I\right) = \begin{cases} y^*_j & \text{if } \left|\left\{m^j_i : m^j_i = q\right\}\right| < I - 1 \text{ for all } q \in \{1, ..., J\}, \\ y^q_j & \text{if } \left|\left\{m^j_i : m^j_i = q\right\}\right| \geq I - 1. \end{cases}$$

That is, Principal j punishes Principal q if at least $I-1$ agent reported that Principal q deviated, and takes action y^*_j otherwise. Define the strategy of Agent i as follows. If $c_j = c^*_j$ for all j then $m^j_i = 0$ for all j. If $c_j = c^*_j$ for all $j \neq q$, and $c_q \neq c^*_q$ then $m^j_i = q$ for all $j \neq q$. That is, if Principal q is the only principal who deviated, Agent i reports it to each principal. Define the rest of the strategies such that they constitute equilibrium strategies in the subgames.[1] To see that these strategies constitute a SPNE, first notice that a deviation of a single agent has no effect on the outcome given that at least $J-1$ principals offered the proposed equilibrium contracts. Any deviation of Principal q at the contracting stage induces the action profile y^q_{-q} by the other principals, and hence, his payoff will be weakly smaller than his minmax value. Since $v_j(y^*) \geq \min_{y_{-j}} \max_{y_j} v(y_j, y_{-j})$, such a deviation is not profitable.

Next we prove the "only if" part. Suppose that $c^*_1, ..., c^*_J$ is an equilibrium contract profile supporting y^* and $v_j(y^*) < \min_{y_{-j}} \max_{y_j} v_j(y_j, y_{-j})$. Suppose that Principal j deviates at the contracting stage and offers c_j where $c_j(m) = Y_j$ for all m. Let y'_{-j} denote the action profile taken by the principals other than Principal j in the subgame generated by $\left(c_j, c^*_{-j}\right)$. Then Principal j can take an action in $\arg\max_{y_j} v_j\left(y_j, y'_{-j}\right)$ at the last stage of the game. Since

$$v_j(y^*) < \min_{y_{-j}} \max_{y_j} v_j(y_j, y_{-j}) \leq \max_{y_j} v_j\left(y_j, y'_{-j}\right),$$

this deviation is profitable, a contradiction. ■

References

[1] Yamashita, T. (2009): "Mechanism games with multiple principals and three or more agents", *Econometrica* (forthcoming).

[1] This is possible because we assumed that a SPNE exists.