On the Biological Foundation of Risk Preferences[*]

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Abstract

This paper considers a continuous-time biological model in which the growth rate of a population is determined by the risk attitude of its individuals. We consider choices over lotteries which determine the number of offspring and involve both idiosyncratic and aggregate risks. We distinguish between two types of aggregate risk: *environmental shocks* and *natural disasters*. Environmental shocks influence the death and birth rates, while natural disasters result in instantaneous drops in population size.

Our main result is a utility representation of the evolutionary optimal behavior. The utility is additively separable in the two types of aggregate risk. The term involving environmental shocks is a von Neumann-Morgenstern utility which induces the same attitude towards both idiosyncratic and aggregate risk. The term involving disasters cannot be interpreted as an expected utility maximization and induces less tolerance towards aggregate risk.

1 Introduction

Most models in economics take preferences as given and derive the choices induced by those preferences. This paper does just the opposite. We entertain the hypothesis that choice behaviours are genetically determined and shaped by natural selection. The underlying individual preferences are then merely the representations of those evolutionary optimal choice behaviours. We work from the basic premise that in the long-run context of evolution, only the fastest-growing genes survive. As this paper focuses on risk preferences, we consider choices over lotteries that affect the reproductive value of individuals. Our main result is a utility representation of the optimal choice behaviour.

The corner stone of our analysis is a characterization of the long run dynamics of a population that inhabits a risky environment. Two types of aggregate risk are present, which we refer to as

environmental shocks and natural disasters. Environmental shocks influence the birth and death rates of individuals, determining the rate of increase of the population size. Meanwhile, natural disasters cause discrete drops in population size, affecting the level as opposed to the slope. The reproductive fitness of an individual is also subject to idiosyncratic risk, that is, conditional on an environmental shock or a natural disaster, the birth rate and the survival probability are still random variables. Because the population is affected by aggregate shocks, it does not grow at a constant rate. Our key result consists of a characterization of the asymptotic growth rate as a function of the ergodic distributions of the various types of risk. The main theorem of this paper states that this function is the utility representation of the evolutionary optimal choice behaviour over lotteries in an environment where the risk is determined by individuals' choices.

In order to better explain our contribution and to contrast our results with those in existence, we first describe Robson's (1996a) seminal paper on the evolution of risk preferences. Time is discrete and individuals live for one period. The number of offspring an individual has is determined by the realization of a lottery. A lottery, L, is described by a triple, $\left(\Omega, G, \{F(\cdot|\omega)\}_{\omega \in \Omega}\right)$, where Ω denotes the set of possible states of the world, G is the ergodic distribution on Ω, and $F(\cdot|\omega)$ is the distribution on the number of offspring if the state of the world is ω. Conditional on ω, the realization of $F(\cdot|\omega)$ is independent across individuals, so this element represents the idiosyncratic component of the risk. The distribution G represents the aggregate risk, since ω determines the distribution of reproductive values in the population. Robson (1996) shows that the asymptotic growth rate of the population is

$$u(L) = \int \log \left(\int \eta dF(\eta|\omega) \right) dG(\omega). \tag{1}$$

Since the gene inducing the choice of the lottery L grows at rate $u(L)$ in the long run, Robson (1996a) interprets u as the utility representation of the evolutionary optimal behaviour. That is, the surviving gene chooses L over L' if and only if $u(L) \geq u(L')$. The main implication of (1) is that the evolved attitude towards risk depends strongly on whether the risk is idiosyncratic or aggregate. In fact, individuals will be relatively less tolerant of aggregate risk, as compared to idiosyncratic risk. To illustrate this observation, consider two lotteries, L_1 and L_2. Suppose that both lotteries generate the same distribution of offspring, H. However, the risk induced by L_1 is purely idiosyncratic while L_2 involves only aggregate risk.[1] By (1), $u(L_1) = \int \eta dH(\eta) > \int \log \eta dH(\eta) = u(L_2)$. That is, even though L_1 and L_2 induce the same risk from the viewpoint of a single individual, L_1 is strictly preferred to L_2.

In contrast to Robson (1996a), we consider a continuous time model, which allows us to distinguish between two types of aggregate risk: environmental shocks and natural disasters. Recall that environmental shocks determine the rate of increase of the population size, while natural disasters cause instantaneous changes in the level. In discrete time, any aggregate shock necessarily results

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[1] Formally, $L_1 = \left(\Omega, G, \{F_1(\cdot|\omega)\}_{\omega \in \Omega}\right)$ where $F_1(\cdot|\omega) \equiv H(\cdot)$ and $L_2 = \left(\mathbb{R}_+, H, \{F_2(\cdot|\omega)\}_{\omega \in \Omega}\right)$ where $F_2(\eta|\omega) = 0$ if $\eta < \omega$ and one otheriwise.

in a discrete change in the population size. It then becomes natural to ask whether the results obtained by Robson (1996a) apply only to natural disasters, or to aggregate risk in general. Our somewhat surprising answer is that his results do not hold for environmental shocks: the evolutionary optimal behaviour induces the same attitude towards both idiosyncratic risk and aggregate risk due to environmental shocks.

In order to state our representation theorem, let us describe our setup in detail. Time is continuous and population dynamics are determined by a lottery $L = (\Omega, G, \lambda, F_n(\cdot|\omega), F_e(\cdot|\omega))$. The set Ω denotes the set of states of the world and G is the ergodic distribution on Ω. The function $\lambda : \Omega \to \mathbb{R}$ is the arrival rate of a disaster. The c.d.f. $F_n(\cdot|\omega)$ denotes the distribution of the survival probability at the moment a natural disaster occurs. $F_e(\cdot|\omega)$ denotes the distribution of the net birth-rate (birth-rate minus death-rate) of an individual conditional on ω. Our main result is that the asymptotic growth rate, $U(L)$, is

$$U(L) = \int \int \varepsilon dF_e(\varepsilon|\omega) dG(\omega) + \int \lambda(\omega) \log\left(\int \eta dF_n(\eta|\omega)\right) dG(\omega). \tag{2}$$

In order to show that the function U can indeed be interpreted as a utility function of the evolutionary optimal choice behaviour in this context, we introduce the stochastic arrival of choice problems over lotteries. A gene is defined to be a choice behaviour, that is, a mapping from choice sets into choices. At each moment in time, the dynamics of the population are determined by the lottery chosen in the most recent choice problem. We show that only those genes which behave as if they were maximizing the function U will survive.

The two additive terms in the utility representation (2) have very different economic interpretations. The first term, which is associated with environmental shocks, is a standard von Neumann-Morgenstern representation with Bernoulli utility as the identity function.[2] This implies that the choice induced by U depends only on the expected value of the net birth-rate; it doesn't matter whether the source of the risk is aggregate or idiosyncratic. The second term, which is associated with natural disasters, is analogous to (1) of Robson (1996a). Indeed, if $\lambda(\omega) \equiv 1$, which corresponds to the frequency of discrete changes in Robson (1996a), the two expressions coincide. Let us again point out that this term is not a von-Neumann Morgenstern representation of risk attitude. Interestingly, this term is formally identical to a smooth ambiguity-averse preference representation where the ambiguity is determined by G and the ambiguity aversion is determined by the logarithm function. For a discussion and axiomatic characterization of smooth ambiguity averse preferences, see Klibanoff et al. (2005).

Let us emphasize that in our model, environmental shocks do not by any means have to be associated with smooth changes. In fact, these shocks might have profound instantaneous effects

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[2]Since the lotteries directly determine net birth-rates, the Bernoulli utility is the identity function, and hence, these preferences exhibit risk-neutrality. However, it is not hard to generate risk-aversion in our model. If the lotteries were to determine reproductive resources instead of net birth-rates and these resources could be transformed into offspring according to a reproduction function, the Bernoulli utility would become the reproduction function. The concavity of such a reproduction function would induce risk-averse preferences.

on death rates. The spread of a viral disease, for example, could triple the death-rate and wipe out a significant fraction of the population within a short period of time. Dramatic changes in weather conditions could also be modelled as environmental shocks. A long lasting drought reduces food sources, and a large proportion of the population might starve to death within a summer. What these examples have in common is that the increased number of deaths occur through a shift in the rate, and do not occur at a single point in time. Of course, the evolutionary optimal behaviour tries to eliminate the risk of increased death rates associated with such environmental shocks. However, our model predicts that this type of risk is not treated any differently from idiosyncratic risk.

There are, undoubtedly, events which cause the instantaneous death of many individuals at the same time. Such events might include earthquakes, forest fires and floods. It is less clear, however, that these events should be considered more important than environmental shocks in determining the evolution of risk preferences. At the end, we do interpret our representation theorem as, at least a partial, evolutionary justification for von Neumann-Morgenstern preferences.

Literature Review

Numerous papers use evolution to at least partially explain preferences. The first is probably Becker (1976), who adopts an evolutionary argument to explain altruism. Overviews of the theories on the relationship between biology and economic behaviour can be found in Robson (2001 and 2002). Here we review only selected papers concerned with the evolution of risk preferences.

Our continuous time assumption is not the only difference between our paper and Robson (1996a). Robson (1996a) considers finite populations and derives conditions under which the population does not go extinct with probability one. We consider continuum populations and, by assumption, they never go extinct. The assumption regarding the continuum population enables us to integrate out the idiosyncratic risk and sidestep problems related to extinction. We note that if a population grows at a constant rate asymptotically, it either goes extinct or it becomes very large. The former case is uninteresting from the evolutionary viewpoint and in the context of the latter one, the continuum assumption does not seem far-fetched.

Robson (1996b) analyses a strategic biological model. First, each male chooses a lottery. The outcome of the lottery is the male's income. After the outcomes of the lotteries are realized, each female decides which male to mate with. A male can mate with multiple females but not vice versa. The number of offspring of a female is a function of the male's income per females who chose him. This model provides a rationale for a particular form of risk taking. Males are willing to take fair bets which result in small losses with large probability and large gains with small probability. The intuition is that a large gain might enable the male to mate with more females. Small losses, on the other hand, might reduce slightly the number of offspring per female but unlikely to reduce the number of females the male can mate with. Under reasonable assumptions on the reproduction function, the positive effect on the number of females dominates the negative effect on the number of offspring per female.

Dekel and Scotchmer (1999) also analyze evolutionary optimal choices over lotteries in a strate-

gic model. In their model, individuals participate in contests within groups. The outcome of a contest is determined by the realizations of the lotteries picked by the group members. The realizations of the lotteries are independent across individuals and the winner is the individual with the largest realization. The key assumption is that only the winner reproduces. Since the winner takes all, the evolutionary optimal behaviour will necessarily be risk-loving in some sense. However, the authors show that the evolutionary optimal behaviour cannot be represented by rational preferences.

Unlike Robson (1996b) and Dekel and Scotchmer (1999), our model does not involves strategic interactions; individuals only solve decision problems. Indeed, it is not clear to us how one might identify preferences and strategic components from equilibrium behaviour. It is, of course, not to say that the strategic nature of the environment does not influence the evolution of preferences.

Finally, the difference between the effects of idiosyncratic and aggregate risks on preferences are also emphasized by Robson and Samuelson (2009). The authors consider the evolution of intertemporal preferences in an age-structured biological model. Among other things, they show that if the effects of aggregate shocks on an individual do not depend on the age of the individual, then aggregate risk slows down population growth. It is not hard to show, that our main result imply that this conclusion is only true for natural disasters but not for environmental shocks.

2 Model

Time is continuous and is indexed by $t \in \mathbb{R}_+$. At each moment, a continuum of individuals make up the population. The population dynamics are governed by the lottery

$$L = \left(\Omega, G, \{\lambda(\omega)\}_{\omega \in \Omega}, \{F_e(\cdot|\omega)\}_{\omega \in \Omega}, \{F_d(\cdot|\omega)\}_{\omega \in \Omega} \right). \tag{3}$$

$\Omega \subset \mathbb{R}$ is the set of possible states of the world. Let ω_t denote the state of the world at time t. The dynamic process $\{\omega_t\}_{t \in \mathbb{R}}$ is an ergodic[3] Markov process with ergodic distribution G. We assume that ω_t is almost surely continuous in t almost everywhere.

Environmental shocks.— The net birth-rate of an individual at time t is $\varepsilon_t = b_t - \delta_t$, where b_t denotes her birth rate and δ_t denotes her death rate. The variable $\varepsilon_t \in \mathbb{R}$ is distributed according to the conditional c.d.f. $F_e(\varepsilon_t|\omega_t)$ and it is measurable in t almost surely. We allow the net birth rate of an individual, ε_t, to change over time even if ω_t stays constant. We assume that $\mathbb{E}(\varepsilon|\omega)$ is a bounded function of ω, and $F_e(\varepsilon|\omega)$ is uniformly continuous in ω. The realizations of $\varepsilon_t's$ are assumed to be independent across individuals conditional on ω_t. We assume that the Law of Large

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[3] This implies that whenever $\int_A 1 dG(\omega) > 0$ then for each $\omega' \in \Omega$, conditional on $\omega_0 = \omega'$:

$$\int I(\omega_t \in A) \, dt = +\infty,$$

almost surely. That is, conditional on any initial condition, the process spends an infinite amount of time in every positive G-measure set with probability one (see e.g. Glynn (1994) and Duffie and Glynn (2004)).

Numbers holds, and hence, at time t the population grows at rate

$$r\left(\omega_t\right) \equiv \int \varepsilon dF_e\left(\varepsilon|\omega_t\right). \tag{4}$$

Natural disasters.— Natural disasters hit the population stochastically according to a non-homogenous Poisson process. The arrival rate of the process at time t is $\lambda\left(\omega_t\right) \in [0, \infty)$ where $\lambda : \Omega \to \mathbb{R}_+$ is a bounded, measurable function in $L^1\left(\Omega, G\right)$. If a natural disaster occurs at time t, an individual survives with probability $\eta_t \in [0, 1]^4$, and $F_n\left(\cdot|\omega_t\right)$ denotes the distribution of η_t conditional on ω_t. We assume that $F_n\left(\eta|\omega\right)$ is uniformly continuous in ω. We also assume that $\mathbb{E}\left(\eta|\omega\right)$ is uniformly bounded away from zero, that is, the population never goes extinct. Again, the realization of η_t is independent across individuals conditional on ω_t. This assumption essentially implies that the survival probability η_t is re-drawn at each moment a natural disaster occurs, and hence, the distribution of η_t is indeed independent of the previous history of disasters. We appeal to the Law of Large Numbers once more and assume that the fraction of the population that survives a natural disaster is

$$j\left(\omega_t\right) \equiv \int \eta dF_n\left(\eta|\omega_t\right). \tag{5}$$

We further assume that for each individual, the shocks ε_t and η_t are independent at each time t conditional on ω_t.

In what follows, we describe the simple case in which each random variable is binary and all changes in the environment arrive according to Poisson distributions.

Example. Suppose that $\Omega = \left\{\omega^H, \omega^L\right\}$ and the state of the world switches according to a Poisson process with arrival rate $\mu > 0$. This process is assumed to be independent of all other random variables. So, the ergodic distribution of ω is given by $\Pr\left(\omega^H\right) = \Pr\left(\omega^L\right) = 1/2$.

The net birth rate of an individual is also binary, $\varepsilon_t \in \left\{\varepsilon^l, \varepsilon^h\right\}$ and its distribution is described by

$$\varepsilon_t = \begin{cases} \varepsilon^h & \text{with probability } p_e\left(\omega_t\right), \\ \varepsilon^l & \text{with probability } 1 - p_e\left(\omega_t\right), \end{cases}$$

where $\varepsilon^h > \varepsilon^l$ and $p_e\left(\omega^H\right) > p_e\left(\omega^L\right)$. These inequalities imply that if the state is ω^H, an individual reproduces at a higher rate in expectation. The net birth rate of an individual is constant over time unless the state of the world changes. When the state changes, the net birth rate of each individual is instantaneously redrawn according to the distribution described above.

Natural disasters arrive according to a Poisson process with rate one, that is, $\lambda\left(\omega^H\right) = \lambda\left(\omega^L\right) = 1$. The survival probability of an individual, η_t, can take one of two values from $\left\{\eta^l, \eta^h\right\}$ and its distribution is described by

$$\eta_t = \begin{cases} \eta^h & \text{with probability } p_n\left(\omega_t\right), \\ \eta^l & \text{with probability } 1 - p_n\left(\omega_t\right), \end{cases}$$

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[4] For our mathematical results to hold, we do not need η_t to be less than one. We only make this assumption for the sake of interpretation.

where $\eta^h > \eta^l$ and $p_n\left(\omega^H\right) > p_n\left(\omega^L\right)$. These inequalities imply that if the state is ω^H, each individual is more likely to survive a natural disaster. At the moment when the state, ω_t, changes or a natural disaster occurs, the survival probability of each individual is redrawn according to the distribution described above. Otherwise, the survival probability of each individual is constant.

Finally, the aggregate net birth rates and the average survival rates in disasters are described by

$$r\left(\omega\right) = \varepsilon^h p_e\left(\omega\right) + \varepsilon^l\left[1 - p_e\left(\omega\right)\right], \tag{6}$$

$$j\left(\omega\right) = \eta^h p_n\left(\omega\right) + \eta^l\left[1 - p_n\left(\omega\right)\right], \tag{7}$$

respectively.

3 Results

In this section we analyze the speed of population growth. Let y_t denote the size of the population at time t and let y_0 be normalized to one. If there is a natural disaster at time t, y_t denotes the size of the surviving population. The basic difficulty is that, due to aggregate shocks, the population does not grow at a steady state rate. Nevertheless, it is possible to characterize a growth rate, g, such that if t is large enough, the size of the population is approximately the same as if it were growing at a constant rate g, that is,

$$y_t \approx e^{gt}. \tag{8}$$

Such a growth rate g is called the *continuously compounded growth rate* of the population and is formally defined in Section 3.2.

Next, we derive an expression for the law of motion of the population along a realized path of the random variables. In Section 3.2, we use this expression to prove our key result, which is a characterization of the continuously compounded growth rate in terms of the lottery (3). Finally, in Section 3.3, we introduce stochastically arriving choice problems. Each choice problem is a set of lotteries, and each lottery is just like L in (3). We show that only those genes which behave as if they were maximizing the continuously compounded growth rate will survive.

3.1 Law of motion of the population

Let $N\left(t\right)$ denote the number of natural disasters which occurred between time zero and time t. In addition, let $\tau_i \in [0, t]$, $i \in \{1, ..., N\left(t\right)\}$, denote the arrival time of the ith natural disaster.

Proposition 1 *The size of the population at time t is given by:*

$$y_t = \exp\left[\int_0^t r\left(\omega_s\right) ds\right] \prod_{i=1}^{N(t)} j\left(\omega_{\tau_i}\right), \tag{9}$$

where $\prod_{i=1}^{N(t)} j\left(\omega_{\tau_i}\right)$ is defined to be one if $N\left(t\right) = 0$.

A notable property of the expression on the right-hand side of (9) is that it is multiplicatively separable in natural disasters and environmental shocks. To illustrate this observation, let us consider a population that grows at a constant rate for a unit amount of time. Suppose that this population is hit by a disaster and half of the population dies. Then, irrespective of the exact time of the disaster, the population at the end of the time period will be half as large as it would have been if the disaster had not occurred. Of course, this argument presumes that the disaster leaves the net birth rate of the surviving population unaffected. In our model, this assumption is satisfied because ε_t and η_t are independent conditional on ω_t.

Proof. We prove the proposition by induction with respect to the number of natural disasters, $N(t)$. Suppose first that $N(t) = 0$. Then, by (4), the law of motion of the population is described by the following differential equation between time zero and time t:

$$\dot{y}_\tau = y_\tau r(\omega_\tau). \tag{10}$$

The solution of this differential equation is[5]

$$y_t = \exp\left(\int_0^t r(\omega_s)\, ds\right),$$

which is just the statement of the proposition for $N(t) = 0$.

Suppose that the statement of the proposition is true for all t whenever $N(t) \leq k$ and let us assume that $N(t) = k + 1$. By the inductive hypothesis,

$$\lim_{\substack{\tau \to \tau_{N(t)} \\ \tau < \tau_{N(t)}}} y_\tau = \exp\left\{\int_0^\tau r(\omega_s)\, ds\right\} \prod_{i=1}^{N(t)-1} j(\omega_{\tau_i}).$$

At time $\tau_{N(t)}$ there is a natural disaster and, by (5), only a fraction $j\left(\omega_{\tau_{N(t)}}\right)$ of the population survives. Hence,

$$y_{\tau_{N(t)}} = \exp\left\{\int_0^\tau r(\omega_s)\, ds\right\} \prod_{i=1}^{N(t)} j(\omega_{\tau_i}). \tag{11}$$

The law of motion of the population for $\tau \in [\tau_{N(t)}, t]$ is again described by the differential equation (10) and initial condition (11). The solution is

$$y_t = y_{\tau_{N(t)}} \exp\left(\int_{\tau_{N(t)}}^t r(\omega_s)\, ds\right) = \exp\left\{\int_0^t r(\omega_s)\, ds\right\} \prod_{i=1}^{N(t)} j(\omega_{\tau_i}),$$

where the second equality follows from (11). ∎

3.2 Continuously compounded growth rate

Motivated by (8), we first provide a formal definition for the continuously compounded growth rate.

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[5] The solution exists because r is continuous and bounded, see Coddington and Levinson (1955), Chapter 2.

Definition 1 *We call the number $g \in \mathbb{R}$ the continuously compounded growth rate of the population if*

$$\lim_{t \to \infty} \frac{\log y_t}{t} = g$$

almost surely.

In order to see that the continuously compounded growth rate is indeed a useful object in the evolutionary context, consider two populations, y_t^1 and y_t^2, with corresponding compounded growth rates g_1 and g_2. We show that if $g_1 > g_2$ then, asymptotically and with probability one, y_t^1 is going to be infinitely large relative to y_t^2. To see this, note that, by Definition 1,

$$\lim_{t \to \infty} \frac{\log \frac{y_t^1}{y_t^2}}{t} = \lim_{t \to \infty} \frac{\log y_t^1}{t} - \lim_{t \to \infty} \frac{\log y_t^2}{t} = g_1 - g_2 > 0$$

almost surely. But this can only be the case if y_t^1 / y_t^2 converges to infinity as t goes to infinity with probability one.

Next, we show that the continuously compounded growth rate exists and characterize it in terms of the lottery, L. This is our main result.

Theorem 1 *For almost all $(\{\omega_t\}, \{N(t)\})_{t \in \mathbb{R}}$,*

$$\lim_{t \to \infty} \left(\frac{\log y_t}{t} \right) = \int \int \varepsilon \, dF_e(\varepsilon|\omega) \, dG(\omega) + \int \lambda(\omega) \log \left(\int \eta \, dF_n(\eta|\omega) \right) dG(\omega). \qquad (12)$$

Let us explain the basic idea of the proof. By Proposition 1,

$$\lim_{t \to \infty} \frac{\log y_t}{t} = \lim_{t \to \infty} \frac{\int_0^t r(\omega_s) \, ds}{t} + \lim_{t \to \infty} \frac{\sum_{i=1}^{N(t)} \log j(\omega_{\tau_i})}{t}, \qquad (13)$$

where the second term in the right-hand side is defined to be zero if $N(t) = 0$. Since ω_t is an ergodic process, both $r(\omega)$ and $\log j(\omega)$ are also ergodic. As a consequence, the limit on the left-hand side of (12) is the sum of the time averages of two ergodic variables. Birkhoff's Ergodic Theorem states that, under certain conditions, the time average of the realization of an ergodic variable converges to the expected value of the variable with probability one, where the expectations are formed according to the ergodic distribution. In the proof, we argue that Birkhoff's Ergodic Theorem is applicable, and show that the time average of $r(\omega)$ and $\log j(\omega)$ converge to the first and second terms in the right-hand side of (12), respectively.

Proof. First, consider the time average of $r(\omega_t)$. Since the state of the world has an ergodic distribution and r is continuous, Birkhoff's Ergodic Theorem implies that

$$\lim_{t \to \infty} \frac{\int_0^t r(\omega_s) \, ds}{t} = \int r(\omega) \, dG(\omega)$$

almost surely[6]. Substituting the definition of the function r in (4), we obtain

$$\lim_{t \to \infty} \frac{\int_0^t r(\omega_s) \, ds}{t} = \int \int \varepsilon \, dF_e(\varepsilon|\omega) \, dG(\omega). \qquad (14)$$

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[6] See, for instance, Doob (1953), Chapter XI, for a version of Birkhoff's Ergodic Theorem for continuous-time processes.

We now turn to the second expression on the right-hand side of (13) and rewrite it as

$$\frac{N\left(t\right)}{t}\frac{\sum_{i=1}^{N(t)}\log j\left(\omega_{\tau_i}\right)}{N\left(t\right)}.$$

If $\int\lambda\left(\omega\right)dG\left(\omega\right)=0$ then $\lambda=N\left(t\right)=0$ almost surely, so (14) implies (12). In what follows we restrict attention to the case when $\int\lambda\left(\omega\right)dG\left(\omega\right)>0$. We first show that

$$\lim_{t\to\infty}\frac{N\left(t\right)}{t}=\int\lambda\left(\omega\right)dG\left(\omega\right) \tag{15}$$

almost surely. To this end, assume that $t\in\mathbb{N}$ for simplicity and define $X_i=N\left(i\right)-N\left(i-1\right)$ for all $i\in\mathbb{N}$. Since $N\left(t\right)$ is a non-homogenous Poisson process, the random variables $X_1,X_2,...$ are independent conditional on the realization of $\{\omega_t\}_{t\in\mathbb{R}_+}$, and $\mathbb{E}X_i=Var\left(X_i\right)=\int_{i-1}^{i}\lambda\left(\omega_\tau\right)d\tau$. By Kolmogorov's Strong Law of Large Numbers[7]

$$\lim_{t\to\infty}\frac{N\left(t\right)-\int_0^t\lambda\left(\omega_\tau\right)d\tau}{t}=\lim_{t\to\infty}\frac{\sum_{i=1}^{t}\left[X_i-\int_{i-1}^{i}\lambda\left(\omega_\tau\right)d\tau\right]}{t}=0 \tag{16}$$

almost surely conditional on $\{\omega_t\}_{t\in\mathbb{R}_+}$. Finally, notice that Birkhoff's Ergodic Theorem implies that

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t\lambda\left(\omega_s\right)ds=\int\lambda\left(\omega\right)dG\left(\omega\right) \tag{17}$$

almost surely. From (16) and (17) the equation in (15) follows.

Next, we show that

$$\lim_{t\to\infty}\frac{\sum_{i=1}^{N(t)}\log j\left(\omega_{\tau_i}\right)}{N\left(t\right)}=\int\log j\left(\omega\right)\frac{\lambda\left(\omega\right)}{\int\lambda\left(\omega'\right)dG\left(\omega'\right)}dG\left(\omega\right) \tag{18}$$

almost surely. Recall that τ_i denotes the time of the ith natural disaster. For all $i\in\mathbb{N}$, define $Z_i=\log j\left(\omega_{\tau_i}\right)$. By (15) and $\int\lambda\left(\omega\right)dG\left(\omega\right)>0$ it follows that there are infinitely many disasters with probability one, so these variables are well-defined. The discrete time process $\{Z_i\}_{i\in\mathbb{N}}$ is also an ergodic Markov process and its ergodic distribution, $\hat{G}$, is given by[8]

$$d\hat{G}\left(\omega\right)=\frac{\lambda\left(\omega\right)dG\left(\omega\right)}{\int\lambda\left(\omega'\right)dG\left(\omega'\right)}.$$

Therefore, Birkhoff's Ergodic Theorem implies that[9]

$$\lim_{n\to\infty}\frac{\sum_{i=1}^{n}Z_i}{n}=\int\log j\left(\omega\right)d\hat{G}\left(\omega\right)$$

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[7] This theorem is indeed applicable since (See e.g. Feller (1950), Chapter X.7):

$$\sum_{i=1}^{\infty}\frac{Var\left[N\left(s_i\right)-N\left(s_{i-1}\right)\right]}{i^2}=\lim_{i\to\infty}\frac{1}{i^2}\int_0^i\lambda\left(\omega_\tau\right)d\tau=\lim_{i\to\infty}\frac{1}{i}\int\lambda\left(\omega\right)dG\left(\omega\right)=0<\infty.$$

[8] See Proposition 2 in Duffie and Glynn (2004).

[9] See Corollary 1 of Proposition 2 of Duffie and Glynn (2004).

with probability one. Substituting the definitions of Z_i and $\hat{G}$ into the previous equation yields (18). Finally, notice that from (15) and (18) it follows that

$$\lim_{t \to \infty} \frac{\sum_{i=1}^{N(t)} \log j\left(\omega_{\tau_i}\right)}{t} = \int \lambda\left(\omega\right) \log j\left(\omega\right) dG\left(\omega\right) = \int \lambda\left(\omega\right) \log \int \eta dF_n\left(\eta | \omega_t\right) dG\left(\omega\right) \qquad (19)$$

almost surely, where the second equality is just (5).

From equations (14) and (19) the statement of the theorem follows. ∎

Example (Continued). We apply Theorem 1 to the example presented in Section 2. Recall that in this example, the ergodic distribution on $\Omega = \left\{\omega^H, \omega^L\right\}$ is given by $\Pr\left(\omega^H\right) = \Pr\left(\omega^L\right) = 1/2$. Then substituting (6) and (7) into (12) we get

$$\begin{aligned}
\lim_{t \to \infty} \left(\frac{\log y_t}{t}\right) &= \frac{1}{2}\left[r\left(\omega^H\right) + r\left(\omega^L\right)\right] + \frac{1}{2}\left[\log j\left(\omega^H\right) + \log j\left(\omega^L\right)\right] \qquad (20) \\
&= \frac{1}{2} \sum_{\omega \in \{\omega^H, \omega^L\}} \left[\varepsilon^h p_e\left(\omega\right) + \varepsilon^l\left(1 - p_e\left(\omega\right)\right)\right] \\
&\quad + \frac{1}{2} \sum_{\omega \in \{\omega^H, \omega^L\}} \log\left[\eta^h p_n\left(\omega\right) + \eta^l\left(1 - p_n\left(\omega\right)\right)\right]
\end{aligned}$$

almost surely.

3.3   Endogenous growth and utility representation

As previously discussed, if the population corresponding to a particular gene has a higher continuously compounded growth rate than that of another population, it will eventually grow to be infinitely larger than that other population. Therefore, if the level of environmental risk is determined by choices made by individuals, and that choice behaviour is genetic, the continuously compounded growth rate is the evolutionary optimal decision criterion. That is, only those genes which generate the largest continuously compounded growth rate survive in the long run. This leads us to interpret

$$U\left(L\right) = \int \int \varepsilon dF_e\left(\varepsilon | \omega\right) dG\left(\omega\right) + \int \log \int \eta dF_n\left(\eta | \omega\right) \lambda\left(\omega\right) dG\left(\omega\right) \qquad (21)$$

as the utility representation of the evolutionary optimal choice behaviour. One concern which might arise with respect to our analysis thus far pertains to our implicit assumption that the choice of a lottery is made once and for all, and determines the growth rate of the gene forever. We have not yet demonstrated that the same utility criterion is used to solve individual choice problems if the overall risk is determined by a combination of various decisions. Next, we take our analysis one step further and formalize the claim that the function U can indeed be interpreted as a utility function of the evolutionary optimal choice behaviour in this context.

To this end, we introduce the stochastic arrival of choice problems. Each choice problem is a set of lotteries. Each of these lotteries is like L defined in (3). We define a gene to be a choice

behaviour, that is, a mapping from choice sets into choices. We then consider the dynamics of the population corresponding to a certain gene. At each moment in time, the dynamics of the population are determined by the lottery chosen in the most recent choice problem. We show that only those genes which behave as if they were maximizing the function U defined in (21) will survive.

Let $\mathcal{L}(\Omega, G)$ denote the set of lotteries of the form (3) with the same (Ω, G). The pair $(\{l_k\}_{k=1}^n, \{\mu_k\}_{k=1}^n)$ is called an *environment* where $l_k \subset \mathcal{L}(\Omega, G)$, $|l_k| < \infty$ and $\mu_k (> 0)$ is the Poisson arrival rate of l_k $(k = 1, ..., n)$. A gene, c, in the environment $(\{l_k\}_{k=1}^n, \{\mu_k\}_{k=1}^n)$ is a mapping from choice sets to elements of these sets, that is, $c : \{l_1, .., l_n\} \to \mathcal{L}(\Omega, G)$, such that $c(l_k) \in l_k$ for all $k = 1, ..., n$. Note that a gene is defined to be simply a choice behaviour on a restricted domain.

In what follows, we investigate the population growth corresponding to a certain gene, c. The population dynamics in the environment $(\{l_k\}_{k=1}^n, \{\mu_k\}_{k=1}^n)$ are defined as follows. At $t = 0$, the population is normalized to one and a choice problem is drawn, l_k with probability $\mu_k / \Sigma_{i=1}^n \mu_i$. If $t > 0$ and there is an $s < t$, such that (i) no choice problem arrived during the time interval $(s, t]$ and (ii) a choice problem l arrived at s, and $c(l) = L$ then the dynamics of the population at time t are determined by the lottery L. We say that a gene survives evolution if its induced continuously compounded growth factor is at least as large as that of any other gene.

Theorem 2 *For all (Ω, G) and for each environment $(\{l_k\}_{k=1}^n, \{\mu_k\}_{k=1}^n)$, the surviving gene, c^*, satisfies*

$$c^*(l_k) \in \arg\max_{L \in l_k} U(L), \tag{22}$$

for all $k = 1, .., n$.

Let us emphasize that the decision criterion in (22) is robust in the sense that the utility of a lottery is independent of the environment. That is, it depends neither on the other potential choice problems, nor on their frequencies of arrival.

We have not allowed for idiosyncratic uncertainty in the arrival of choice problems. However, it would not be difficult to introduce some such heterogeneity, and allow for the possibility that at some points in time different individuals of the same population face different problems. We might allow, for example, the population to consist of many colonies and the choice problems to arrive independently across colonies. In this case the size of the total population is simply the sum of the sizes of the colonies, so Theorem 2 remains valid.

Proof. Fix an environment, $(\{l_k\}_{k=1}^n, \{\mu_k\}_{k=1}^n)$, and an arbitrary gene c. Let

$$L_k = \left(\Omega, G, \left\{ \lambda^k(\omega) \right\}_{\omega \in \Omega}, \left\{ F_e^k(\cdot | \omega) \right\}_{\omega \in \Omega}, \left\{ F_d^k(\cdot | \omega) \right\}_{\omega \in \Omega} \right)$$

denote $c(l_k)$, and let y_t denote the size of the population corresponding to c at time t with the normalization $y_0 = 1$. Corresponding to (4) and (5), for each L_k, let $r^k(\omega)$ and $j^k(\omega)$ denote the

average net birth rate and the survival population, respectively, conditional on ω. We show that the continuously compounded growth rate of the gene is

$$\sum_{k=1}^{n} \frac{\mu_k}{\mu_1 + ... + \mu_n} U(L_k). \tag{23}$$

In other words, the asymptotic growth rate of the gene is the average of the continuously compounded growth rates induced by the chosen lotteries weighted by the frequencies of these choices. Note that the statement of the theorem follows from (23), because the fastest growing gene maximizes the summation in (23) pointwise, that is, it satisfies (22).

The statement of Proposition 1 is still valid because it is about the population dynamics along realized paths and it does not matter whether these paths are determined by a single lottery or the combination of many lotteries. Recall that $N(t)$ denotes the number of natural disasters which occurred between time zero and time t and $\tau_i \in [0, t]$, $i \in \{1, ..., N(t)\}$, denotes the arrival time of the ith natural disaster. Let L_t denote the most recently chosen lottery at time t. Finally, $r(\omega_t)$ and $j(\omega_t)$ denote $r^k(\omega_t)$ and $j^k(\omega_t)$, respectively, if $L_t = L_k$. Then, by Proposition 1,

$$\log y_t = \int_0^t r(\omega_s) \, ds + \sum_{k=1}^{N(t)} \log j(\omega_{\tau_i})$$

$$\sum_{k=1}^{n} \left[\int_0^t I(L_t = L_k) r^k(\omega_s) \, ds + \sum_{k=1}^{N(t)} I(L_t = L_k) \log j^k(\omega_{\tau_i}) \right],$$

where I denotes the indicator function. Next, we decompose y_t to be the product of k different population sizes. To this end, let us define z_t^k for all $k = 1, ..., n$ and $t \in \mathbb{R}_+$ such that,

$$\log z_t^k = \int_0^t I(L_s = L_k) r^k(\omega_s) \, ds + \sum_{k=1}^{N(t)} I(L_{\tau_i} = L_k) \log j^k(\omega_{\tau_i}).$$

Intuitively, z_t^k would be the size of a population at time t if its growth is determined by L^k at time t if $L_t = L_k$ and stays constant in any other moment. It follows from the previous two displayed equations that

$$\log y_t = \sum_{k=1}^{n} \log z_t^k. \tag{24}$$

Notice that

$$\lim_{t \to \infty} \frac{\log z_t^k}{t} = \lim_{t \to \infty} \left[\frac{\int_0^t I(L_s = L_k) \, ds}{t} \right] \left[\frac{\log z_t^k}{\int_0^t I(L_s = L_k) \, ds} \right]. \tag{25}$$

Since choice problems arrive independently and according to the arrival rates $\mu_1, ..., \mu_n$,

$$\lim_{t \to \infty} \left[\frac{\int_0^t I(L_s = L_k) \, ds}{t} \right] = \frac{\mu_k}{\mu_1 + ... + \mu_n} \tag{26}$$

almost surely. Define $\tilde{z}_\tau^k$ to be $z_{T(\tau)}^k$ where the function T is defined by $\int_0^{T(\tau)} I(L_s = L_k) \, ds = \tau$ for all $\tau \in \mathbb{R}_+$. Intuitively, $\tilde{z}_\tau^k$ would be the size of a population at time τ if its growth just like z_t^k

13

except that time is measured by a clock which is stopped whenever $L_t \neq L_k$. Note that

$$\lim_{\tau \to \infty} \frac{\log \widetilde{z}_\tau^k}{\tau} = \lim_{\tau \to \infty} \frac{\log z_{T(\tau)}^k}{\int_0^{T(\tau)} I\left(L_s = L_k\right) ds} = \lim_{t \to \infty} \frac{\log z_t^k}{\int_0^t I\left(L_s = L_k\right) ds},$$

where the first equality follows from the definition of $\widetilde{z}_\tau^k$ and the second one from $\lim_{\tau \to \infty} T\left(\tau\right) = \infty$. Since the arrivals of the choice problems are independent of ω_t, the lottery determining the growth of $\widetilde{z}_\tau^k$ is identical L_k. Hence, Theorem 1 implies that

$$\lim_{\tau \to \infty} \frac{\log \widetilde{z}_\tau^k}{\tau} = U\left(L_k\right). \tag{27}$$

Plugging (26) and (27) into (25) we get

$$\lim_{t \to \infty} \frac{\log z_t^k}{t} = \frac{\mu_k}{\mu_1 + \ldots + \mu_n} U\left(L_k\right).$$

Hence, by (24),

$$\lim_{t \to \infty} \frac{\log y_t}{t} = \lim_{t \to \infty} \sum_{k=1}^n \frac{\log z_t^k}{t} = \sum_{k=1}^n \frac{\mu_k}{\mu_1 + \ldots + \mu_n} U\left(L_k\right),$$

which is just (23). ∎

Interpretation

The two terms on the right-hand side of (21) have very different behavioural implications. Observe that the lottery L can be viewed as the combination of two compound lotteries: $\left(\{F_e\left(\cdot|\omega\right)\}_\omega, G\right)$ and $\left(\{F_n\left(\cdot|\omega\right)\}_\omega, G\right)$, which are associated with environmental shocks and natural disasters respectively. Equation (21) implies that a decision maker who maximizes U reduces the compound lottery $\left(\{F_e\left(\cdot|\omega\right)\}_\omega, G\right)$ to a simple one. That is, conditional on the risk associated with natural disasters, choices will be based only on the expected rate of reproduction. Whether environmental shocks are aggregate or idiosyncratic is irrelevant. Therefore, the first term is simply a standard von Neumann-Morgenstern representation.

In sharp contrast, the second term does not correspond to expected utility maximization. In particular, the compound lottery $\left(\{F_n\left(\cdot|\omega\right)\}_\omega, G\right)$ is not reduced to a simple one in the utility function U. This term is formally identical to the representation of smooth ambiguity-averse preferences. In the context of ambiguity (Ω, G) corresponds to the subjective state space and beliefs, $F_n\left(\cdot|\omega\right)$ describes uncertainty, and ambiguity-aversion is determined by the logarithmic function. Since the logarithmic function is concave, this representation implies that the decision maker with utility function U is less tolerant towards aggregate risk than towards idiosyncratic risk.

One notable feature of the function U is its additive separability in the risks due to environmental shocks and natural disasters. This arises from the fact that y_t is multiplicatively separable in these two types of aggregate risk. As previously mentioned, this separability is due to our assumption that, for each individual, ε_t and η_t are independent conditional on ω_t. We could relax

this independence assumption and still obtain an additive representation similar to (21). However, the c.d.f. F_e would be replaced by an ergodic distribution of the birth rates which would depend on, among other objects, F_n and λ. So, while the utility representation would still be additive, the term corresponding to environmental shocks would depend on the risks faced due to disasters.

4 On the limit of the model of Robson (1996a)

Theorem 1 implies that environmental shocks and natural disasters affect growth in very different ways. However, one might suspect that environmental shocks could be approximated arbitrarily well with a sequence of *small* natural disasters. Can the continuous time model be viewed as the limit of discrete time models as time intervals shrink? In order to answer this question, we revisit the model of Robson (1996a) in the context of a binary example. We shall consider two possible lotteries; a purely idiosyncratic one and purely aggregate one. Robson (1996a) shows that the population corresponding to the idiosyncratic lottery grows faster. In what follows, we show that as the length of a time interval goes to zero, the growth rates of the two populations converge.[10]

Suppose that time is discrete and indexed by $t \in \mathbb{N}$. Assume that $\Omega = \left\{ \omega^H, \omega^L \right\}$ and the ergodic distribution of ω is given by $\Pr \left(\omega^H \right) = \Pr \left(\omega^L \right) = 1/2$. Each individual lives for one period, and her number of offspring is the realization of a lottery. In each period t, the realization of the lottery is independent across individuals conditional on ω_t. We consider the following two lotteries

$$L_A = \left\{ \begin{array}{ll} H & \text{if } \omega = \omega^H, \\ L & \text{if } \omega = \omega^L \end{array} \right. \quad \text{and } L_I = \left\{ \begin{array}{ll} H & \text{with probability } 1/2, \\ L & \text{with probability } 1/2, \end{array} \right.$$

where $1 < L < H$. Note that the lottery L_A involves only aggregate risk and the lottery L_I involves only idiosyncratic risk. Also note that the risks induced by these two lotteries are the same from the viewpoint of a single individual; with probability one-half she produces H offspring, and with probability one-half she produces L. Let $g(L)$ denote the compounded growth factor of the population corresponding to the lottery $L (\in \{L_A, L_I\})$. The main result from Robson (1996a) implies

$$\log g \left(L_A \right) = \frac{\log H + \log L}{2} < \log \left(\frac{H + L}{2} \right) = \log g \left(L_I \right), \tag{28}$$

and hence, the population choosing L_I grows faster than the other.

In what follows, we shrink the length of the time intervals from one to Δ and examine the consequences on the speed of population growth as Δ approaches zero. If the time intervals are downscaled and individuals reproduce according to the lotteries L_A and L_I in each period, the populations grow faster, and they explode as Δ goes to zero. In order to compare growth rates

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[10]We are able to prove the same result in the general model of Robson (1996a) but we believe that the binary example is sufficient to illustrate our point.

across different Δ's, we redefine the lotteries as follows

$$L_A^\Delta = \begin{cases} H^\Delta & \text{if } \omega = \omega^H, \\ L^\Delta & \text{if } \omega = \omega^L \end{cases} \quad \text{and } L_I^\Delta = \begin{cases} H^\Delta & \text{with probability } 1/2, \\ L^\Delta & \text{with probability } 1/2. \end{cases}$$

The idea behind this rescaling is as follows. If the realization of the lottery is constant within a unit of time, the induced growth factor does not depend on Δ. To see why, note that if an individual produces H^Δ offspring in each Δ-long period, she will have $\left(H^\Delta\right)^{1/\Delta} = H$ genetic copies after a unit amount of time. Let $g_\Delta\left(L^\Delta\right)$ denote the compounded growth factor of the population corresponding to $L^\Delta\left(\in\left\{L_A^\Delta, L_I^\Delta\right\}\right)$ per Δ-long time period. Again, from Robson (1996a) it follows that

$$\log g_\Delta\left(L_A^\Delta\right) = \frac{\log H^\Delta + \log L^\Delta}{2} < \log\left(\frac{H^\Delta + L^\Delta}{2}\right) = \log g_\Delta\left(L_I^\Delta\right). \tag{29}$$

Next, we take Δ to zero and compare the growth factors per unit interval of the populations corresponding to L_A^Δ and L_I^Δ. Note that if the growth factor per Δ-long period is g_Δ then the growth factor per unit-interval is $(g_\Delta)^{1/\Delta}$. Consider first the population governed by the aggregate lottery L_A^Δ. By (29),

$$\lim_{\Delta\to 0} \log g_\Delta^{1/\Delta}\left(L_A^\Delta\right) = \lim_{\Delta\to 0} \frac{\log H^\Delta + \log L^\Delta}{2\Delta} = \frac{\log H + \log L}{2}. \tag{30}$$

Comparing this expression with the left-hand side of (28), we conclude that the population facing the aggregate lottery is unaffected by the time scaling. Consider now the population governed by the idiosyncratic lottery. Again by (29),

$$\lim_{\Delta\to 0} \log g_\Delta^{1/\Delta}\left(L_I^\Delta\right) = \lim_{\Delta\to 0} \frac{\log\left(\frac{H^\Delta + L^\Delta}{2}\right)}{\Delta}.$$

On the right-hand side, both the numerator and the dominator converge to zero. We apply L'Hopital's rule to obtain

$$\lim_{\Delta\to 0} \log g_\Delta^{1/\Delta}\left(L_I^\Delta\right) = \lim_{\Delta\to\infty} \frac{1}{\left(\frac{H^\Delta + L^\Delta}{2}\right)}\left(\frac{H^\Delta \log H + L^\Delta \log L}{2}\right) = \frac{1}{2}\left(\log H + \log L\right), \tag{31}$$

where the second equality follows from the observation that both H^Δ and L^Δ converge to one as Δ goes to zero. From (30) and (31), we conclude that, as Δ goes to zero, the population facing aggregate risk grows just as fast as the population facing only indiosyncratic risk.

5   Conclusion

We believe endogenizing preferences to be an important research agenda, and that adopting the biological approach, as we have done, might prove to be a fruitful enterprise. It seems reasonable to hypothesize that evolution did not influence only physical traits but also shaped choice behaviours. An advantage of this approach is that it has strong predictions about the relationship between

fertility and choices which, in principle, can be tested empirically. We do recognize that many choice problems faced by individuals in modern times were unlikely to be faced in evolutionary times. Yet, we hypothesize that preferences, at least in part, are hardwired and that many choices made today reflect the evolutionary optimal behaviour.

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