A Biological Theory of Social Discounting

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Abstract

We consider a growth model in which intergenerational transfers are made via stocks of private and *public* capital. Private capital is the outcome of individuals' private savings while decisions regarding public capital are made collectively. We hypothesize that private saving choices evolve through individual selection while public saving decisions are the result of group selection. The main result of the paper is that the equilibrium return to private capital is much more than twice as larger as the return to public capital if the two types of capital are complements. In other words, social choices involving intertemporal trade-offs exhibit much more patience than individual choices do.

1 Introduction

Policy issues often involve intergenerational trade-offs. Perhaps, the most publicized one is global warming. Nordhaus (1994) found that there was no basis for draconian policies now to reduce CO_2 emissions; Stern (2006) suggested the situation was more dire. The difference can be traced almost entirely to the pure rate of time preference used by these two authors—Nordhaus used a few percent, based on estimates deriving from private economic behavior; Stern used essentially zero, as a philosophical position. CO_2 would obviously then be an important problem if there were reasons to expect that the private rates of time preference were too high, so that public decisions should be made on the basis of a lower rate.

To elaborate further, consider the following hypothetical problem. A policy maker has to decide whether to invest in research into an alternative energy source, which yields some return for the next generation. The internal rate of return of the project is R while the rate of return to private capital is r. The positive view of Nordhaus (1994) is based on the idea that the implemented policies must be in line with the preferences of the current generations. Since, by revealed preference, the current generation is willing to save at return r, any project will be implemented as long as $R \geq r$. But what if $R < r$? The Pareto criterion offers little guidance in this case. Depending on whether the

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policy maker weights the welfare of the next generation more or less heavily than does the current generation, he might find it desirable either to implement or to abandon it. Stern (2006) argues that the justification of a positive discounting is that the human race may be extuinguished. Since the probability of this event appears to be small, Stern (2006) suggests that the project should be implemented even if $R \approx 0$.

The goal of our paper is to provide a positive theory of private and social discounting. We consider a model where both private and collective decision problems are present. Our theory is positive in the sense that both of these decisions reflect the preferences of the members of the society. Our main result is that there is a discrepancy between the discount factor of an individual applied to private decisions and the one applied to collective decisions. We show that individuals exhibit more patience if decisions are made collectively. In particular, we show that the social discount factor is more than twice as large as the private factor. This result implies that one must be careful when using data on individual consumption-saving behaviour to predict the implementation of social policies even from a positive viewpoint. Our theory predicts that individuals might welcome the implementation of social projects with a rate of returns vastly lower than demanded for private projects.

We consider an evolutionary model where individuals face intertemporal tradeoffs. One tradeoff is made individually; a second is made collectively. We model this by having two different capital stocks in the production function; private and public. Each individual can decide how much to invest in private capital, but the contribution to public capital is the same for everybody and is decided collectively. Reproduction is sexual and the return from saving is equally shared by the offspring. If the members of a couple save more in total, their children receive more resources each but they are left with fewer resources to use to determine the number of offspring. In other words, consumption-saving decisions involve a quantity-quality trade-off. We hypothesize that individual choices evolve through individual selection and collective decisions evolve through group selection. To be more specific, private saving choices must be evolutionary stable, that is, they must be resistant to invasion by rare mutants. Collective decisions must generate the largest possible population growth subject to the requirement that private saving decisions are evolutionary stable.

Our main result is that the equilibrium gross return to public capital is less than half as large as that of the private capital. In other words, individuals are significantly more patient towards collective decisions than towards private choices. The key observation is that sexual reproduction implies that rare mutants are only a half related to their children. As a consequence, the individually selected saving behaviour maximizes its own growth rate as if its frequency was zero in the population. To see this, note that mutants grow fastest initially if they maximize their growth rate taking their zero frequency into account. Mutants can also match the growth rate of the rest of the population by mimicing their behavior. Hence, the only way to ensure that the mutants do not invade the population is for the saving behavior of the original population to maximize the growth rate of the associated gene as if its frequency were zero.[1]

This behaviour distorts the quantity-quality trade-off involving children when compared to asexual reproduction. Indeed, if reproduction was asexual, children and grandchildren are identical. Therefore, the saving decision which maximizes fitness equates the marginal return to saving measured in the number of grandchildren with the marginal return of consumption in terms of offspring. If reproduction is sexual, children are more valuable than grandchildren because they are twice as related. As a consequence, individuals prefer to sacrifice some grandchildren by investing less in their offspring in order to increase the number of their offspring. Therefore, couples have a preference for too many children with too low quality. This induces a slower growth rate in the equilibrium steady state, with fewer children than in the case of asexual reproduction. Equivalently, this private investment is made with too much impatience. The decision that is made on the public level is technologically analogous to the private investment. However, this decision is made collectively and must be followed by each member of the community. Hence, this choice is guided by the appropriate lower rate of time preference. Indeed, the public decision may involve still greater investment to partially offset the distortion created by too low a level of private investment.

We have in mind a rather general message here—there might be a depletable resource such as big game, for example—or there might be a problem with polluting the local environment. Considering our specific growth model with two capital stocks and the steady state merely facilitates the analysis. The basic conclusions seem bound to hold in a variety of models involving intertemporal tradeoffs. Indeed, in a final section, we sketch an application of this analysis to the modern debate concerning global warming.

Related Literature

There is a small but steadily growing literature investigating the relationship between biology and economic behavior. Further, some of these papers examine how time preferences evolved through individual selection, see for example Rogers (1994), Robson and Szentes (2008), and Robson et. al. (2012). The basic conclusion of these papers is that the evolutionarily stable choice behaviour can be represented by time-separable utility. The discount rate is the sum of the mortality rate and the population growth rate, and the overall utility function is reproductive value. Robson and Samuelson (2009) shows that this conclusion crucially depends on the assumption that all the risk is idiosyncratic. They argue that if there is aggregate uncertainty the evolutionary stable behaviour no longer has an expected utility representation. In all these papers, all the decisions are made at the individual level. Our primary goal here is to highlight a discepancy between preferences for individual and collective choices.

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[1] Bergstrom (1995) analyzes games played by siblings. He concludes that the strategy that cannot be invaded by a mutant chooses an action as if the opponent mimics this action with probability half. This is the same principle as here. That is, although, in equilibrium, siblings have the same genes and take the same actions, they behave as if this probability was only a half.

There are also papers that consider macroeconomic phenomena in a biological context. These involve standard growth models with the twist that the reproduction technology is specified instead of the preferences of individuals. The goal then is to characterize the choices that induce the largest population growth rate. Hansson and Stuart (1990) consider a neoclassical growth model in which clans compete for shares of resources. A clan's production depends on the stock of clan-specific capital and on the total population of all clans. The clans indeed face a carrying capacity constraint, meaning that per capita production decreases as total population rises. An individual is active for only one period, and each clan faces a consumption-saving decision. Higher consumption results in higher immediate population growth but a smaller capital stock, and hence smaller output, in the next generation. The carrying capacity constraint means that each clan's size must be constant in the long-run equilibrium. As a result, evolution selects clans with a zero rate of time preference. Agents maximize the per capita steady state consumption of current and future generations.

Robson and Wooders (1997) also considers a macroeconomic model where total output depends on different types of capital and labor. The per capita income of each type of labor determines its growth rate. The authors show that when the balanced growth rate is maximized, income must be distributed across individuals in accordance with marginal product pricing. In these papers, reproduction is asexual and all the choices are made collectively. In other words, they are similar to our model without the private capital.

Finally, there is a literature in macroeconomics which considers both private and public choices—for example Perotti (1992 and 1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), and Milanovic (2000). In these models, individuals are heterogenous in wealth and collective decisions are made regarding either income redistribution or public capital. (The aggregate production function in Alesina and Rodrik (1994) is essentially identical to ours.) The key assumption of these papers is that the collective decision is made by the median voter. The authors investigate the relationship between economic grwoth and income inequality. The typical conclusion of this literature is that large income inequality is harmful for economic development.

2 The Model

Time is discrete and there is a continuum of individuals.[2] In each period, adult individuals are randomly matched, allocate resources to reproduction and capital investment, and die. The newborns become the adults of the next period, and the return on investment is then available. We next describe the technologies for production and reproduction.

There is a single aggregate output which is produced from public capital, M, private capital, K, and labor, L. The aggregate output available at time $t+1$ is given by the function $G(M_t, K_t, L_t)$ where M_t, K_t and L_t are the levels at time t of public capital, private capital, and labour, respec-

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[2] Presumably, one half of the individuals are male and one half female. There is, however, no need for a formal distinction in the model between the sexes.

tively. (More precisely, L_t is the number of adults at time t.) For mathematical convenience, both types of capital are "circulating," so there is 100% depreciation.

We assume the production function G exhibits constant return-to-scale. Defining $m = M/L$ and $k = K/L$ as per capita public and private capital, respectively, we then have $G(M, K, L) = LG(m, k, 1) = Lg(m, k)$, say, where $g(m, k)$ is per capita output. We assume that the function g is three times continuously differentiable, satisfies Inada conditions in each input, is strictly concave, and has the inputs as complements.[3]

The number of offspring a couple has depends on their total consumption, where consumption is the residual resources left after investment. Suppose that one individual in a couple has resources w_1 and privately saves k_1, whereas the other has resources w_2 and saves k_2. Then, if the per-capita public capital is fixed at m, they consume $c_1 = w - k_1 - m$ and $c_2 = w - k_2 - m$, respectively. The expected number of offspring of a couple then depends on $c_1 + c_2$, and is given by $2f(c_1 + c_2)$. The reproduction function f is continuously differentiable, where $f'(c) > 0$, for all $c \geq 0$, and $f(0) = 0$. We assume that each individual has access to the production technology g. Hence, if the parents invest k_1 and k_2 in private capital and the per-capita public capital is m then each of their offspring receive resources

$$\frac{g(m, k_1) + g(m, k_2)}{2f(c_1 + c_2)}.^{4} \tag{1}$$

We assume that choices regarding private saving evolve through individual selection while the decisions regarding public capital are shaped by group selection. Consider many isolated communities, each of which is characterized by a per-capita level of public capital m. Private saving behavior is genetic; each offspring inherits the choice behavior of one of her parents with probability half. Private savings behavior is the result of individual selection if it is resistant to invasion by any mutant, so that no mutant can grow faster than the original community. Such behavior is an evolutionarily stable strategy, ESS, and we show that, for each m, there is a unique ESS denoted by $\bar{k}(m)$. Finally, a per-capita level of public capital m^* is the result of group selection if the community characterized by $\overline{m}$ grows faster than any other community subject to the constraint that the private saving is an ESS in every community.[5] More formally, we will

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[3] That is, more precisely:

i) $g(0, k) = g(m, 0) = 0$, for all $k, m \geq 0$,

ii) $g_m(m, k) > 0, g_k(m, k) > 0$, for all $k, m > 0$,

iii) $g_m(m,k) \to \begin{cases} \infty, \text{ as } m \to 0, \text{ for all } k > 0 \\ 0, \text{ as } m \to \infty, \text{ for all } k \geq 0 \end{cases}$ and $g_k(m,k) \to \begin{cases} \infty, \text{ as } k \to 0, \text{ for all } m > 0 \\ 0, \text{ as } k \to \infty, \text{ for all } m \geq 0 \end{cases}$,

iv) $g_{mm}(m, k) < 0, g_{kk}(m, k) < 0$ and $g_{mm}(m, k)g_{kk}(m, k) - (g_{mk}(m, k))^2 > 0$ for all $m, k > 0$,

v) $g_{mk}(m, k) > 0$ for all $m, k > 0$.

[4] Alternatively, we could have assumed that individuals can save in a competitive market for capital. That is, if an individual saves k' and the population average of private capital is k then his offspring receive $k'g_k(m, k)$ resources. Such an assumption has no impact on our results but seems less plausible in an evolutionary model.

[5] The idea behind this concept is that larger groups displace smaller groups. This notion of "group selection" is controversial for non-human species in general. It is less so here. In the first place, private capital is chosen in

identify the per-capita public capital $m = \overline{m}$ which is feasible and maximizes the population growth subject to the constraint that the private saving is $\bar{k}(m)$.

3 Results

We first characterize the biologically efficient steady state saving decisions. That is, we identify the public and private capital levels which are feasible and which maximize the growth rate of the population. This serves as a benchmark and allows us to identify the biological inefficiency due to individual selection.

Suppose that k, m and c are constant over time. For feasibility, the triple (m, k, c) has to satisfy the budget constraint that

$$\frac{g(m,k)}{f(2c)} = m + k + c, \tag{2}$$

where the left-hand-side is the inheritance of an offspring from her parents, as in (1), and the right-hand-side is the allocation of these resources to investments and consumption. Lemma 1 of the Appendix shows that each pair $m, k \geq 0$ uniquely determines the feasible level of consumption, $c(m,k) \geq 0$, say. If each couple has $2f(2c)$ offspring, the growth factor of this population is $f(2c)$. Since f is strictly increasing in c, the problem of maximizing the growth factor of the population is simply to

$$\max_{m,k \geq 0} c(m,k).$$

The key observation in the characterization of the solution to this is as follows. Replacing c by $c(m,k)$ in Equation (2) and differentiating both sides by m yields

$$2f'(2c)\, c_m(m,k)\,(c+m+k) + f(2c)\,(c_m(m,k)+1) = g_m(m,k)\,.$$

If (m,k) induces the largest possible consumption then $c_m = 0$, so the previous equation simplifies to $f(2c) = g_m(m,k)$. An analogous argument shows that $f(2c) = g_k(m,k)$. We have:

Theorem 1 *There is a unique pair $m, k > 0$ which induce the largest feasible consumption $c > 0$.*[6] *This pair is characterized by the first-order conditions*

$$g_m(m,k) = g_k(m,k) = f(2c).$$

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accordance with "individual selection." See Boyd and Richerson (1990) for the appeal of group selection in such circumstances. In the second place, humans are exceptional since culture makes it possible to ensure that individuals have the selfish incentives to produce rather arbitrary social outcomes. We rely on this argument to underly the choice of public capital.

One can explicitly model such a competition between groups by introducing a carrying capacity constraint, see for example Hansson and Stuart (1990). This would entail induce a zero growth rate in equilibrium.

[6] We abuse notation slightly by using m, k to denote the general values of these variables, as well as the specific optimal choices of them.

Proof. See the Appendix for a rigorous detailed proof. ■

These optimality conditions are familiar in that they equate the marginal product of each type of capital to the growth factor of population, $f(2c)$.[7] The only "wrinkle" is that the growth factor of population is endogenous, indeed it is the objective to be maximized. In particular, the marginal returns from public and private capitals are equal.

We turn now to formulating the problem with sexual reproduction. For each m, we compute the unique steady state investment in private capital which is immune to invasion by a rare mutant. Suppose that, in the candidate steady state equilibrium, each non-mutant adult allocates m to investment in public capital, $\bar{k}$ to private capital, leaving $\bar{c}$ to promote population growth. Consider then a rare mutant that must also allocate m to public capital, but allocates k to private capital, leaving c to promote growth.

Since mutants are rare, essentially all the matches involving mutants have one mutant and one non-mutant. We then calculate the maximum steady state growth factor for the mutants in this situation, so the mutant attains such steady state growth while remaining rare. The growth factor of the mutant population is $f(\bar{c}+c)$. That is, each mutant-non-mutant couple has $2f(\bar{c}+c)$ offspring, one half of whom are mutants, on average. By (1), the budget constraint of a mutant is

$$m+k+c=\frac{g\left(m,\bar{k}\right)+g\left(m,k\right)}{2f\left(\bar{c}+c\right)}. \tag{3}$$

Lemma 2 of the Appendix shows that, although there may exist values of k for which (3) cannot be satisfied, there must be some values for which it can be satisfied. The problem of maximizing the growth rate of the mutants is then:

$$\max c$$
$$\text{subject to } \exists k \in \mathbb{R}_+ \text{ such that (3) holds.}$$

Lemma 2 also shows that, when (3) can be satisfied, there is a unique solution for c given by $c(k)$. Proceeding informally, then, it follows from (3) that

$$2f'\left(\bar{c}+c\right)c'\left(k\right)\left(m+k+c\left(k\right)\right)+2f\left(\bar{c}+c\left(k\right)\right)\left(1+c'(k)\right)=g_k\left(m,k\right).$$

If k generates the largest possible level of consumption, then $c'(k)=0$, so the previous equation becomes

$$2f(c(k)+\bar{c})=g_k(m,k).$$

To complete the informal argument, note that mutants match the growth rate of the original population if they invest $\bar{k}$ in private capital, but they maximize their growth rate by choosing k. So, unless $k=\bar{k}$, the mutants can invade the original population. In short, $\left(\bar{k},\bar{c}\right)$ is an evolutionarily stable strategy, ESS, if and only if $k=\bar{k}$ and $c=\bar{c}$. We have:

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[7] The assumption that all capital depreciates 100% means that that marginal product of capital must equal the growth *factor*, rather than just the growth *rate*.

Theorem 2 *For each $m > 0$, there exists a unique (pure strategy) ESS $\left(\bar{k}, \bar{c}\right)$ which satisfies*

$$g_k(m, \bar{k}) = 2f(2\bar{c}).^{8}$$

Proof. Again, see the Appendix for a more rigorous proof. ■

The equilibrium condition determining $\bar{k}$ equates the marginal product of private capital to *twice* the population growth factor. Other things equal, then, the equilibrium choice of private capital would be much too low.

The effect of sexual reproduction is to substantially increase impatience and therefore reduce investment in private capital. The mutants try then to have a larger number of offspring by reducing the income these offspring will have in the next period. The cost of this would be a reduced potential number of grandchildren.[9] However, in the steady state, since income falls, the ultimate effect of the mutant is to reduce the number of children and the growth factor.

In a conceptual sense then the mutant is trading off immediate rewards (children) against a delayed reward (grandchildren) and leaning too much in favor of the immediate reward. Sex then in this model is an underlying cause of too much impatience.

Consider now how the optimal level of public capital, $\bar{m}$, say, is determined by group selection. Not only should the group choose on the basis of a lower and more appropriate rate of time discount, but the group should increase m still further in order to offset the deleterious effect of too low a value of k. The optimal public capital maximizes the growth rate subject to the feasibility constraint and to private saving being determined as above by individual selection. Formally, $\bar{m}$ is the solution of the problem[10]

$$\begin{aligned} & \max_m c \\ \text{subject to } f\left(2c\right)\left(m+k+c\right) &= g(m,k) \text{ and } g_k(m,k) = 2f(2c). \end{aligned}$$

Again, we content ourselves with an intuitive characterization of the solution, leaving a rigorous treatment to the Appendix. Suppose $\bar{c}\left(m\right)$ and $\bar{k}(m)$ satisfy the constraints, and differentiate both sides of the first constraint with respect to m to get

$$\begin{aligned} & 2f'\left(2\bar{c}\left(m\right)\right)\bar{c}'\left(m\right)\left(m+\bar{k}\left(m\right)+\bar{c}(m)\right)+f\left(2\bar{c}\left(m\right)\right)\left(1+\bar{k}'\left(m\right)+\bar{c}'\left(m\right)\right) \\ = \; & g_m\left(m,\bar{k}\left(m\right)\right)+g_k\left(m,\bar{k}\left(m\right)\right)\bar{k}'\left(m\right). \end{aligned}$$

If $\bar{m}$ maximizes $\bar{c}$ then $\bar{c}'\left(\bar{m}\right) = 0$. Using the second constraint as well, we then have

$$f\left(2\bar{c}\left(\bar{m}\right)\right)\left(1-\bar{k}'\left(\bar{m}\right)\right) = g_m\left(\bar{m},\bar{k}\left(\bar{m}\right)\right).$$

It follows that $\bar{k}'\left(\bar{m}\right) > 0$, since the two types of capitals are complements. We have:

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[8] We again abuse notation slightly by using $\bar{m}, \bar{k}$ to denote the values of these variables that might arise in the general population, as well as the specific ESS values of them.

[9] More precisely, what is reduced is the income available to children. This income can be used to produce new children, but also for investment to generate the income of the new children when they, in turn, become adults.

[10] For clarity, we spell out both relevant constraints, and avoid notation derived from satisfying one or both of them.

Theorem 3 *Group selection for the level of public capital, given individual selection for the level of private capital, generates a level of public capital* $\bar{m} > 0$*, private capital* $\bar{k}(\bar{m}) > 0$ *and consumption* $\bar{c}(\bar{m}) > 0$ *which satisfy*

$$g_m(\bar{m}, \bar{k}(\bar{m})) = f(2\bar{c}(\bar{m}))\left(1 - \bar{k}'(\bar{m})\right) < f(2\bar{c}(\bar{m})) < 2f(2\bar{c}(\bar{m})) = g_k(\bar{m}, \bar{k}(\bar{m})).$$

Proof. See the Appendix. ■

That is, the marginal product of public capital is reduced below the population growth factor precisely because the marginal of private capital is above the population growth factor and increasing public capital increases the equilibrium level of private capital.

4 Discussion

Carrying Capacity—We have not explicitly modeled competition among groups. Instead, we simply assume that group selection generates collective choices that maximize population growth, with no explicit restriction on what this maximum rate might be. However, limited carrying capacity would force the equilibrium growth factor to be one (as in Hansson and Stuart (1990)). After all, any appreciable growth rate of a few percent, if maintained for the 1.8 million years of our existence, would lead to the absurd result that we would have long since exhausted standing room on earth and are standing on one anothers' shoulders, exploding into space at the speed of light. This consideration renders puzzling anthropological data for modern hunter-gatherers with growth rates of even a few percent.

One attractive resolution of this is as follows. Suppose that these modern observations on growth are representative of past hunter-gatherer societies. However, suppose that these periods of tranquil growth are interrupted by the occasional disaster during which the population is drastically reduced. (In the simplest case, these disasters bear equally on all age classes.) There is no shortage of candidates for such disasters—ice-ages being only the best-known. Recently, it has been suggested that a meteorite that hit the earth about one million years ago (ref?) caused a catastrophic drop in the human population.

Robson and Samuelson (2009) establish results in a similar setting to this in which mortality rates are subject to aggregate shocks. Rates of time preference are then those appropriate to the tranquil periods of steady growth, with the catastrophes merely serving to lower growth to a plausible level. The bottom line is essentially that the present results can be taken as is, with no constraint set by a zero growth rate.

Extrapolation—Could this analysis apply to the current debate about global warming? How might the current scenario concerning hunter-gatherers be translated to modern circumstances? We hypothesize that the private rate of time preference was, at least to some extent, hard-wired by evolution. We also hypothesize that public rates of time preference were similarly hard-wired, but at a lower level.[11] That is, individuals evolved to desire that public decisions be made on the basis of a lower rate of time preference than the rate used in private decisions.[12] Given the evolutionary basis of the public rate of time preference in maximizing population growth, the analysis might most convincingly be interpreted as a positive theory of public decisions, rather than an intrinsically ethical one. It predicts that public decisions about global warming would be made using a lower rate of time preference than used in private decisions, without having to be accepted as being the philosophical basis for such a lower rate.[13] Indeed, we hypothesize that the link between fertility and population growth, on the one hand, and the rates of time preferences used in public and private decisions, on the other, has been severed.[14] We emphasize that we do not claim that investment in public projects (like global warming) is technologically similar to our model. However, we believe that the aggregate CRS production function with complementarities is not far fetched from the evolutionary prespective. Then individuals would apply these preferences even to decision problems which are technologically different from the ones where the preferences evolved.

Cooperation—Parents here make their savings decisions simultaneously and non-cooperatively, and offspring are a public good, so it is not surprising that there is a free-rider problem. Indeed, there is undercontribution by each parent to the private capital that will generate income for each child. However, the issue is more subtle than this suggests. Undercontributing to the private capital of each child is achieved by producing too many children. What does too little investment in private capital imply? It limits the reproductive options of one's children. For simplicity, think of this as reducing the number of grandchildren. What is the basic reason for the distortion then? Although the effect of sex is to dilute one's concern with children by a factor of 1/2, the dilution of concern with grandchildren is even greater, involving a factor of 1/4. Hence sex creates a distortion from an attempt to favor the quantity of children over the quantity of grandchildren.

There are mechanisms that would address this distortion. Perhaps a biological solution would be to forge a pair bond between parents; there are also familiar game-theoretic mechanisms for inducing cooperation in repeated interactions. To the extent that these solutions are biological, the effective change in preferences might be confined to settings involving children or might be imperfect anyway. To the extent that the solutions are game theoretic, they would work in the light of the inappropriate selfish preferences, and would not generate selection in favor of lower private rates of time preference. In either case, then, there would remain a divergence between private and public rates of time preference.

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[11] It is plausible that rates of time preference are also partly subject to enculturation, but this leads us still further from conventional economics.

[12] Individuals may vote for public choices from a less-than-perfectly-selfish perspective. Feddersen and Sandroni (2006) present a model in which voters are motivated by ethical obligation. We extend this notion to time preference.

[13] See Binmore (2005) for an argument that we should take evolution seriously as the foundation of our expressed ethical sense.

[14] Key here is the advent of modern birth control.

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5 Appendix-Proofs

Lemma 1 *For all $m, k \geq 0$, the equation (2) has a unique solution $c(m, k) \geq 0$. Further, $c(m, k)$ is continuously twice differentiable.*

Proof of Lemma 1. If $m = 0$ or $k = 0$ then the only solution is $c = 0$. If $m, k > 0$ then $(m+k+c)f(2c)$ is less than $g(m, k)$ at $c = 0$, greater than $g(m, k)$ if c is large enough, and strictly increasing in c. Hence there exists a unique solution $c(m, k) > 0$, which is clearly twice continuously differentiable.

Proof of Theorem 1. The solution cannot involve $m = 0$ or $k = 0$ since this implies $c = 0$ and $c > 0$ is feasible. Further, since $g_m(m, k) \to 0$ as $k \to \infty$, it follows that $g(m, k)/(m + k + c) \to 0$ as $k \to \infty$, so that $c \to 0$. Similarly, $c \to 0$ as $m \to \infty$. Hence there must exist an interior optimum with $m, k > 0$. Hence $c_m(m, k) = c_k(m, k) = 0$. It then follows from differentiating $(m + k + c)f(2c) = g(m, k)$ that $g_m(m, k) = g_k(m, k) = f(2c)$.

Further, there is only one solution to these first order conditions. It is straightforward to show that, for each $m > 0$ there exists a unique $k(m)$ such that $g_m(m, k(m)) = g_k(m, k(m))$. Further, $k(m) \to 0$ as $m \to 0$ and $k(m)$ is continuously twice differentiable. Define $\hat{c}(m) = c(m, k(m))$. Upon differentiating the constraint twice and then setting $c_m(m, k) = c_k(m, k) = 0$, we obtain, dropping most arguments for simplicity,

$$(1 + k'' + \hat{c}'')f(2\hat{c}) + 2(m + k + \hat{c})f'(2\hat{c})\hat{c}'' = g_{mm} + 2g_{mk}k' + g_{kk}(k')^2 + g_k k''.$$

Since g is strictly concave, $g_{mm} + 2g_{mk}k' + g_{kk}(k')^2 < 0$. Recalling that $g_k = f(2c)$, it follows that $\hat{c}'' < 0$ at any solution of the first order conditions. But this implies there can only be one such solution, so there is a unique optimum as well.

Lemma 2 *For given* $m, \bar{k}, \bar{c} > 0$, *the constraint*

$$2\,(m + k + c)\,f\,(\bar{c} + c) = g\left(m, \bar{k}\right) + g\,(m, k)$$

may have no solution for c *for some* $k \geq 0$. *However, the set of feasible* k*'s is a non-degenerate interval. On this interval, there is a unique solution* $c(k)$ *which is twice differentiable.*[15] *Further, given each* $m > 0$, *there is a unique (feasible) best reply,* $c(\bar{c})$, *say, for the mutant to each* $\bar{c} > 0$. *This is also the only solution to the first order condition*

$$2f(\bar{c} + c) = g_k(m, k).$$

Proof of Lemma 2. The constraint $2(m + k + c)f(\bar{c} + c) = g(m, \bar{k}) + g(m, k)$ need not be capable of being satisfied for all $k \geq 0$. However, it always satisfied with $k = \bar{k} > 0$ and $c = \bar{c} > 0$. Over the domain where it can be satisfied, there is a unique $c(k)$, for each k. The function $c(k)$ is strictly quasiconcave, and twice continuously differentiable. At the edges of this domain either $k = 0$ or $c = 0$. It follows that the domain is a non-degenerate interval. If $k = 0$ is in the domain, it cannot be optimal since then $c'(0) \leq 0$ but $g_k(m, k) \to \infty$ as $k \to 0$. It follows that there is an interior best reply. Differentiating the constraint once and setting $c'(k) = 0$ yields that $2f(\bar{c} + c) = g_k(m, k)$. Differentiating the constraint twice and then setting $c'(k) = 0$ yields

$$2c''(k)f(\bar{c} + c) + 2(m + k + c)f'(\bar{c} + c)c''(k) = g_{kk}(m, k) \text{ so } c''(k) < 0.$$

It follows that there can only be one such solution of $c'(k) = 0$. Hence there is a unique best reply, $c(\bar{c})$, as well.

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[15] We suppress, for the moment only, that c is also a function of m.

Proof of Theorem 2. Consider the conditions characterizing the mutant best reply $2f(\bar{c}+c) = g_k(m,k)$ and $2\,(m+k+c)\,f\,(\bar{c}+c) = g\left(m,\bar{k}\right) + g\,(m,k)\,.$ If $\bar{c}=0$ then $2f(c) = g_k(m,k) > 0$ so $c > 0$. On the other hand, if $\bar{c}$ is large enough, the conditions cannot be satisfied. At the largest feasible $\bar{c}$, $c = 0$. Over the feasible domain, let $c(\bar{c})$ and $k(\bar{c})$ denote the solutions for c and k. Now differentiating the second condition yields $2(k'(\bar{c})+c'(\bar{c}))f(\bar{c}+c)+2(m+k+c)f'(\bar{c}+c)(1+c'(\bar{c})) = g_k k'(\bar{c})$. Using the first condition, we have that $c'(\bar{c}) < 0$. It follows that there exists a unique $\bar{c}$ such that $c(\bar{c}) = \bar{c} > 0$. This is then an ESS because $c = \bar{c}$ is the unique best reply to $\bar{c}$. Indeed, from the above construction, this is is the only ESS in pure strategies.

Proof of Theorem 3. If $m = 0$, then $c = 0$, which is not optimal since $m > 0$ implies $c > 0$. If $m \to \infty$, then $f(2c) = g(m,k)/\,(m+k+c) \to 0$, since $g_m(m,k) \to 0$. Hence $c \to 0$ as $m \to \infty$. It follows that there exists an interior solution. From Theorem 2, the constraints

$$\left(m+\bar{k}+\bar{c}\right) f\,(2\bar{c}) = g\left(m,\bar{k}\right) \text{ and } g_k(m,\bar{k}) = 2f(2\bar{c})$$

determine $\bar{k}$ and $\bar{c}$ as functions of m. Let these functions be $\bar{k}(m)$ and $\bar{c}(m)$, say, which are continuously differentiable. At any interior solution, $\bar{c}'(m) = 0$ so that, from the second constraint—

$$g_{km}(m,\bar{k})+g_{kk}(m,\bar{k})\bar{k}'(m) = 0 \text{ so that } \bar{k}'(m) = -\frac{g_{km}(m,\bar{k})}{g_{kk}(m,\bar{k})} > 0, \text{ since } g_{km}(m,\bar{k}) > 0 \text{ and } g_{kk}(m,\bar{k}) < 0.$$

The first constraint then yields

$$f(2\bar{c})\left(1+\bar{k}'(m)\right) = g_m(m,\bar{k}) + g_k(m,\bar{k})\bar{k}'(m)$$

Using $g_k(m,\bar{k}) = 2f(2\bar{c})$ again, it follows that, at the constrained best choice of $m = \bar{m}$,

$$g_m(\bar{m},\bar{k}(\bar{m})) = f(2\bar{c}(\bar{m}))\left(1-\bar{k}'(\bar{m})\right) < f(2\bar{c}(\bar{m})).$$