The Transmission of Monetary Policy through Redistributions and Durable Purchases*

Vincent Sterk  
University College London

Silvana Tenreyro  
London School of Economics

November 2016

Abstract
This paper studies a redistribution channel for the transmission of monetary policy. Using a tractable OLG setting in which the government is a net debtor, we show that standard open market operations (OMO) conducted by Central Banks have significant revaluation effects that alter the level and distribution of wealth in the economy and the real interest rate. Expansionary OMO generate a negative wealth effect (the private sector as a whole is a net creditor to the government), increasing households’ incentives to save for retirement and pushing down the real interest rate. This, in turn, leads to a substitution towards durables, generating a temporary boom in the durable good sector. With search and matching frictions, a form of productive investment is added to the model and the fall in interest rates causes an increase in labour demand, raising aggregate employment. The mechanism can mimic the empirical responses of key macroeconomic variables to monetary policy interventions. The model shows that different monetary interventions (e.g., OMO versus helicopter drops) can have sharply different effects on activity and that the fiscal policy stance plays a key role in the transmission mechanism.

JEL Codes: E1, E52, E58, E32, E31.

Keywords: Open Market Operations, Durables, Heterogeneous Agents

*For helpful comments, we would like to thank Marios Angeletos, Marco Bassetto, Francesco Caselli, Larry Christiano, Carlos Garriga, Jordi Gali, Mark Gertler, Bernardo Guimaraes, Patrick Kehoe, Amir Kermani, Pete Klenow, Per Krusell, David Laibson, Francesco Lippi, Andy Neumeyer, Christopher Palmer, Michael Peters, Monika Piazzesi, Morten Ravn, Ricardo Reis, Martin Schneider, Rob Shimer, Harald Uhlig, Ivan Werning, Jaume Ventura, Francesco Zanetti, and seminar participants at LSE, Stanford, Berkeley, CREI, UAB, UCL, Manchester, Mannheim, Sveriges Riksbank, Southampton University, EIEF, HECER, SED, Normac 2013, the Northwestern-UCL conference 2014, the 2015 CEP - St. Louis Fed - WUSTL Workshop, the 2015 annual conference of De Nederlandsche Bank, and the RES 2016.
1 Introduction

A central question in monetary economics is how monetary policy interventions transmit to the real economy. This paper contributes to the literature by quantitatively studying a redistribution channel for the transmission of monetary policy. Using a tractable quantitative model, the paper shows that this channel can account for a significant fraction of the empirical responses of key macroeconomic aggregates to monetary policy interventions.

An important element for the transmission channel we emphasize is the rather uncontro-

versial assumption (applicable to the United States and other industrialized countries) that the government is a big net debtor in the economy (while households as a whole are net creditors\(^1\)). Overlapping generations of households consume durable and non-durable goods and work and save for retirement through bonds, money holdings, and durable goods.\(^2\) A temporary expansion in monetary policy carried out through open market operations (OMO), whereby the central bank purchases government bonds, pushes down the nominal interest rate and leads to a temporary increase in inflation. This price adjustment, needed to close the gap between money supply and demand, causes a downward revaluation of the government debt, generating a negative wealth effect for the household sector.\(^3\) The fall in private wealth induces households to save a larger fraction of their income, as they seek to restore their retirement savings, pushing down the real interest rate. This in turn leads to a substitution towards durable goods, generating a boom in the durable good sector. With search and matching frictions in the labor market, job vacancies are a form of productive investment, as they create durable employment matches. The decline in the real interest rate thus increases the demand for both durables and productive investment, leading to an increase in aggregate employment and output.

The emphasis on durable goods in the model is motivated by the empirical finding that the response of activity to monetary policy is largely driven by the response of the durable goods sector. The introduction of search and matching frictions, while not necessary for the qualitative results, adds realism and generates significant persistence in the responses of

\(^1\) US households tend to hold bank deposits, while banks hold government bonds; we implicitly assume that competitive banks fully pass-through their losses to households and accordingly, in the model, we merge the household and banking sectors.

\(^2\) Bond and money holdings are imperfect substitutes.

\(^3\) Though the intervention redistributes wealth from retired towards working-age households, we argue that the dominant effect is the redistribution away from the household sector and to the government.
economic variables to monetary policy, in line with the empirical evidence. (For expositional clarity, we study versions of the model with and without search and matching frictions.)

The redistributive channel in our model is motivated by Doepke and Schneider (2006a)’s empirical study, which shows that inflationary episodes can cause significant revaluations of assets and redistributive effects from wealthy, middle age, and old households towards the government (the main debtor) and poor, young households. Similar evidence is documented by Adam and Zhu (2014) for European countries and Canada. Despite the stark empirical findings, most DSGE models used for the quantitative monetary policy analysis rely on a representative agent formulation and thus abstract from redistributional effects. In this paper, we show that these redistributive effects can have a sizeable impact on real macroeconomic aggregates.

We proceed in two steps. First, building on Gertler and Karadi (2015)’s identification strategy, we show that following an unexpected monetary policy expansion, the real value of public debt falls and the price level increases. The results indicate a swift and significant response of the aggregate price level, without the so called “price puzzle” resulting from other identification strategies. These, in themselves, are novel findings that motivate the exploration of revaluation effects. Furthermore, we corroborate that the durable-good sector is the key driver of the response of real activity to monetary policy expansions, and show that nondurables and services display a relatively mild response. In the second step, we develop a tractable model to quantitatively study the aggregate effects caused by the revaluation of government liabilities due to monetary policy interventions. We show that the model can quantitatively account for most of the increase in durable expenditures, and a substantial part of the response in non-durables following a monetary policy expansion. A crucial element in the model is the presence of a government sector; despite playing a passive role, its presence is relevant as it leads to a redistribution of wealth away from the private sector—as well as across households—causing a fall in the real interest rate and a boom in durables.

An open issue is of course what the government does with its windfalls. Following

---

4Relatedly, Coibion, Gorodnichenko, Kueng and Silvia (2012) find that monetary expansions reduce inequality, as measured by Gini coefficients, suggesting a redistribution away from wealthier individuals.

5The swift (and positive) response of the price level was somehow glossed over by Gertler and Karadi (2015)’s study. We compare our results to theirs in the Appendix.

6An expansionary OMO improves the financial position of the government via two channels. First, an increase in prices reduces the real value of government debt. Second, the operation increases the Central Bank’s bonds holdings and consequently its stream of interest revenues, which are transferred to the Treasury.
standard assumptions in the literature, the government in our model is a passive agent; in particular, the model abstracts from government consumption and assumes that the Treasury follows a balanced budget policy, using the increased net income flows to finance a persistent reduction in (non-distortionary) taxes. While these tax cuts help to compensate households for their wealth losses, they do not undo the redistributive effects. In particular, retirees emerge as the biggest losers from the operation whereas future (unborn) generations benefit the most. In between these extremes are agents who are in the working phase of their lives when the shock hits. They suffer a negative revaluation of their retirement savings but do not receive full compensation from the Treasury once they retire. So, on net, living agents lose and this breakdown of the Ricardian equivalence (Barro, 1974) leads to the non-neutrality of money.

Our model highlights that the real effects of open market operations can be sharply different from the effects of “helicopter drops”, that is, tax cuts financed by an increase in the money supply, even though the effects of the two policies on nominal interest rates and prices are similar. Indeed, we show that an expansionary helicopter drop causes a counterfactual fall in durables and a decline in output and hours. The difference, as will become clear, is driven by the distributional effects that the two policies generate. Our analysis takes Doepke and Schneider (2006a)’s results one step further to show that the macroeconomic effects stemming from the revaluation of wealth will critically depend on how the policy is implemented.

We conclude by stressing that our model can complement the New Keynesian (NK) model and indeed help in addressing the criticism levied by Barsky, House, and Kimball (2007) against the NK mechanism. These authors integrate durable goods into an otherwise standard sticky-price framework and show that when durables’ prices are relatively flexible (as appears to be the case in the data), the model generates a counterfactual decline in durables following an expansionary monetary shock. By allowing for redistributions, our model provides a mechanism that counteracts the channel highlighted by Barsky et al. (2007) and can thus generate a boom in durables even when their prices are flexible, helping the

---

7 Specifically, when durable goods’ prices are relatively flexible, as appears to be the case in the data, these models predict that following a monetary expansion, non-durable purchases increase, while durable purchases, remarkably, decrease. In the case of fully flexible durable prices, the predicted contraction in the durable goods producing sector is so large that the monetary expansion has almost no effect on total aggregate output. See Klenow and Malin (2011) and references therein for a positive link between the durability of the good and the frequency of price adjustment.
The paper is organized as follows. Section 2 reviews the literature. Section 3 presents and discusses the main empirical facts that motivate key features of our model. Section 4 introduces a simple version of the model and discusses the basic mechanisms at play. Section 5 presents the full model with labor market frictions, which both add realism to the model and increase the persistence of the responses of key macroeconomic aggregates to monetary policy interventions; the section then studies the extent to which the model can quantitatively account for the empirical evidence. Section 6 offers concluding remarks.

2 Relation to the Literature

As emphasized by Woodford (2012) in his influential Jackson Hole symposium paper, in standard modern, general-equilibrium, frictionless asset pricing models, open market purchases of securities by Central Banks have no effect on the real economy. This result, which goes back to Wallace (1981)’s seminal article, is at odds with the widely held view that open market operations (OMO) by Central Banks affect interest rates—and at odds indeed with the very practice of Central Banks. The “irrelevance” or neutrality of OMO is easiest to see in the context of a representative agent model, as explained by Woodford (2012); however, Wallace (1981)’s widely cited result applies to a more general setting with heterogeneous agents. A key premise for Wallace’s irrelevance result, however, is that OMO by the Central Bank are accompanied by fiscal transfers that ensure no change in the income distribution following the monetary policy intervention. In other words, by construction, distributional effects of OMO are muted by fiscal transfers that neutralize distributional changes—and hence preclude any change in individuals’ decisions following the intervention.9

8Suppose the central bank wishes to sell a risky asset (an asset with lower return in a bad state); one would think the private sector would be in principle only willing to buy it at a lower price. However, in the frictionless settings analyzed by Woodford (2012), even if the central bank keeps the risky asset, the risk does not disappear from the economy. The central bank’s earning on its portfolio are lower in the bad state and this means lower earnings distributed to the Treasury (and hence higher taxes to be collected from the private sector in the bad state). So the representative household’s after-tax income is equally exposed to risk, whether or not the household buys the asset. Thus asset prices are unaffected by the open-market operation.

9Wallace (1981) refers to this condition as “unchanged fiscal policy.” An unchanged fiscal policy in that context is one in which there is no change in government consumption and no change in the income or wealth distribution. To implement Wallace’s OMO without the redistributioinal effects, a Central Bank needs to rely on the Treasury to adjust transfers and taxes in a particular way to keep the income distribution unchanged. An alternative way of obtaining this result would be to have a complete set of contingent securities that would undo any change in the income distribution.
In contrast with Wallace (1981), OMO have real effects in our model economy because we allow for redistributional effects. Indeed, the goal of this paper is to study the effects of monetary policy interventions when, realistically, OMO are not accompanied by neutralizing fiscal transfers—nor is there a complete set of state-contingent securities that would ensure an unchanged income distribution following the policy intervention. The motivation is necessarily a practical one. When researchers estimate the causal effects of monetary policy interventions, they do not (cannot) abstract from or control for the distributional effects they cause—and there is no accompanying fiscal policy that undoes them in practice. Hence, to understand the effects of those interventions on activity, researchers need to take into account the potential impact of the redistribution caused by the policy intervention and any interaction with the fiscal policy in place.

To better understand the importance of agents’ life-cycle savings considerations, we also study a limit case of our simple model with an infinitely-lived representative agent. In this limit case, monetary neutrality is obtained, as in Sidrauski (1967). This is because agents suffering a revaluation effect on their financial assets are compensated in equal amounts by current and future transfers from a fiscal authority rebating lump-sum transfers, thus precluding wealth effects and any change in behavior. In the absence of nominal rigidities, real wages and relative prices are thus entirely determined by real factors. Nominal wage income and durable good prices therefore increase in tandem in the presence of inflation, and the increase in nominal wage income exactly offsets the desire to bring forward durable good purchases. This is true even though inflation does reduce the real value of financial wealth.  

Money neutrality in our model obtains under the same conditions in which Ricardian Equivalence holds (Barro 1974). By (realistically) precluding risk sharing of aggregate monetary policy shocks across generations, the model yields money non-neutrality even with flexible prices.  

\[\text{\textsuperscript{10}}\text{Recall the assumption that the government makes lump-sum transfers from seigniorage revenues to agents. Following Weil (1991)’s arguments, based on an endowment economy with helicopter drops, we show that also in an economy with production and durable goods, the reduction in wealth caused by OMO is exactly offset by future increases in government transfers, which renders money neutral. This intuition is perhaps not too easy to see in Weil (1991) because of a key mistake in the derivation of the formula for money holdings, going from equation 3.12 to 3.18, which blurs the interpretation.}\]

\[\text{\textsuperscript{11}}\text{Allowing for fiscal transfers to exactly offset the heterogenous effects of monetary policy across different agents would restore the money neutrality in our model. Realistically, however, monetary policy shocks are not accompanied by targeted fiscal transfers aimed at undoing the monetary effects. Hence, to interpret the data and in particular the empirical evidence on the effect of monetary policy interventions, one cannot assume away the redistributional effects of monetary policy.}\]
The methodological strategy followed in this paper has much in common with the NK literature (e.g., Christiano, Eichenbaum, and Evans, 2005): we construct a dynamic equilibrium model and compare the responses to monetary policy shocks to those obtained from a structural Vector Auto-Regressive (VAR) model. We argue that our model can match most key patterns in the data, and in particular the response of prices and output, without relying on nominal rigidities; the latter could, however, be added to the analysis.

The paper connects with a growing branch of the literature that seeks to study alternative channels for the transmission of monetary policy, which can complement the standard channel based on nominal rigidities.\textsuperscript{12} Examples in this literature are Grossman and Weiss (1983), Rotemberg (1984), and Alvarez and Lippi (2012), who study the role of segmentation in financial markets and the redistributive effects caused by monetary policy.\textsuperscript{13} Lippi, Ragni, and Trachter (2013) provide a general characterization of optimal monetary policy in a setting with heterogeneous agents and incomplete markets. More quantitative analyses can be found in Doepke and Schneider (2006b), Meh, Ríos-Rull, and Terajima (2010), Algan, Allais, Challe and Ragot (2012) and Gottlieb (2012). Like us, they numerically analyze the effects of monetary policy and/or inflation in a flexible price economy with aggregate dynamics and heterogeneous-agents. However, none of these papers models open market operations or consumer durables, both key elements of the transmission mechanism we highlight. More crucially, they do not consider the critical role played by the government as net debtor, which leads to the negative wealth effect in the private sector. The qualitative effects are also different: Doepke and Schneider (2006b) and Meh et al. (2010) generate a contraction in activity following a monetary policy expansion, whereas our model generates a boom in activity driven by the durable good sector. Finally, the heterogeneity in these models typically requires computationally heavy methods, which makes it difficult to incorporate shock processes that are realistic enough for a comparison to responses from a structural VAR.\textsuperscript{14} By contrast, our model is solved quickly using standard linearization methods, allowing for a straightforward comparison to VARs as well as New-Keynesian DSGE models. To achieve

\textsuperscript{12}Part of this literature has been labeled “New Monetarist Economics” See Williamson and Wright (2010) for a survey.

\textsuperscript{13}In our model, there is no financial segmentation: all agents can in principle participate in financial markets, though naturally some may endogenously choose not to hold any positions.

\textsuperscript{14}Doekpe and Schneider (2006) and Meh et al. (2010) model a one-time, unanticipated inflationary episode rather than recurring monetary policy shocks. Algan et al. (2012) and Gottlieb (2012) discretize the monetary shock process, giving rise to relatively stylized dynamics.
this, we follow a simple stochastic ageing structure introduced in Gertler (1999), but differently from Gertler (1999), we work out a computational strategy that allows for standard preferences.\footnote{Gertler’s approach requires the utility function to be in a class of nonexpected utility preferences, excluding for example standard CRRA utility functions, whereas our model is instead compatible with the latter.}

Our paper also relates to recent work by Auclert (2015) and Kaplan, Moll, and Violante (2016). Auclert (2015) focuses on the redistribution of wealth across agents with different marginal propensities to consume and different exposure to interest rate changes. This is of course an important redistribution channel, but distinct from the revaluation effect that we study. Our focus is on the distribution away from the private sector (the negative wealth effect), which is what drives most of our results—the redistribution across households plays a more subdued role in our model, both qualitatively and quantitatively. In terms of aggregate effects from the redistribution, we find that the redistributive effects can have a large impact in the US economy, whereas Auclert (2015) finds relatively small effects for the United States (the effects are larger for countries with a prevalence of adjustable interest rates, like the United Kingdom). Needless to say, the two mechanisms complement each other and should be taken into account when evaluating redistributive effects of monetary policy. Kaplan, Moll, and Violante (2016) study a setting with heterogeneous agents in a NK framework with price rigidities. (See also Werning (2015), Gornemann, Kuester and Nakajima (2012) and Lüticke (2015) for related analyses.)\footnote{Other New Keyensian Models with sticky prices can be found in for example McKay and Reis (2013) and Ravn and Sterk (2013).} Kaplan, Moll, and Violante (2016) find that the standard, direct effect of intertemporal substitution in consumption triggered by changes in interest rates plays a subdued role. What is important for the quantitative response of the economy is the indirect general-equilibrium effect due to the labour demand response, together with the fiscal stance. Our findings are consistent with Kaplan, Moll and Violante (2016) in that i) the government plays a crucial role for the non-neutrality of OMO, a role that is key to generate the impact effect on real activity; and ii) the labour demand response is quantitatively important for the amplification and persistence of the initial impulse. The labour demand response in our model is driven by search-and-matching frictions, while in Kaplan, Moll, and Violante (2016) it stems from price rigidities. We thus seem to arrive to fairly similar conclusions from different modelling assumptions on labour and product markets: Kaplan, Moll, and Violante (2016) assume perfect competition in a frictionless
labour markets and imperfect competition with nominal rigidities in product markets; our paper assumes frictions and imperfect competition in labour markets and perfect competition in product markets (with no nominal rigidities). Arguably, reality sits in between the two stylized extremes.

3 Empirical Evidence

In this Section we first revisit the empirical evidence on the effects of monetary policy shocks on the macroeconomy, highlighting the role of durables and the government debt. We do so by estimating a structural VAR model using Gertler and Karadi (GK, 2015)’s identification strategy. For earlier VAR evidence on the effects of monetary policy on durables, see e.g. Erceg and Levin (2006) and Monacelli (2009).

The model, identification, and results are described in turn. Next, we discuss the empirical evidence on redistributive effects from monetary policy.

3.1 Monetary expansions and the responses of durables, prices and public debt

Policy and academic discussions on the economic effects of monetary policy interventions often rely on the relatively high sensitivity of the durables sector to interest rate changes. We corroborate this premise by studying US evidence using a VAR approach. We find that monetary expansions lead to a boom in consumer durables, with little increase in non-durables. This motivates the introduction of consumer durables in our model, as a key variable in the monetary transmission mechanism. Further, we find that monetary expansions trigger a substantial and swift increase in prices and a prolonged decline in real public debt, two important elements of the transmission mechanism which underly a redistribution of wealth.\(^\text{17}\)

The empirical analysis for measuring the effects of monetary policy shocks relies on a general linear dynamic model of the macroeconomy whose structure is given by the following

\(^{17}\text{A typical finding in earlier empirical studies is a puzzling gradual decrease of prices following an expansionary monetary shock. However, many of these studies exploit a recursive identification strategy which restricts prices to respond only with a one-period lag to the shock. Our results highlight the limitations of such restrictions.}\)
system of equations:\textsuperscript{18}

\[ Y_t = \sum_{s=1}^{S} A_s Y_{t-s} + \sum_{s=1}^{S} B_s P_{t-s} + \varepsilon_t^y, \]  
\text{(1)}

\[ P_t = \sum_{s=1}^{S} C_s Y_{t-s} + \sum_{s=1}^{S} D_s P_{t-s} + \varepsilon_t^p. \]  
\text{(2)}

Here, \( Y_t \) is a vector of non-policy variables, \( P_t \) is a policy indicator, \( A_s, B_s, C_s \) and \( D_s \) are coefficient matrices, \( \varepsilon_t^y \) is a vector of reduced-form residuals associated with the non-policy block of the VAR, and \( \varepsilon_t^p \) is the residual of the policy equation. The reduced-form residuals are linear combinations of structural shocks:

\[
\begin{bmatrix}
\varepsilon_t^y \\
\varepsilon_t^p
\end{bmatrix} = \Psi 
\begin{bmatrix}
v_t^y \\
v_t^p
\end{bmatrix},
\]  
\text{(3)}

where \( v_t^p \) is a monetary policy shock, \( v_t^y \) is a vector of other structural shocks, and \( \Psi \) is an unknown matrix which governs the contemporaneous impact of structural shocks on the variables in the VAR.\textsuperscript{19} Together, Equations (1), (2) and (3) state that the variables in the VAR depend on lagged values of \( Y \) and \( P \), as well as on the structural shocks.

The coefficient matrices \( A_s, B_s, C_s \) and \( D_s \) are estimated by Ordinary Least Squares (OLS) and do not depend on how the contemporaneous effects of structural shocks are identified. Following GK, we use monthly data starting in July 1979, when Paul Volcker took office as chairman of the Federal Reserve System, and end the sample in July 2012. Also following GK, we include twelve lags of data and use the one-year rate on government bonds as the policy indicator. The non-policy variables in the system include the seasonally adjusted Consumer Price Index (CPI) in (log) levels, as well as expenditures on durables and non-durables, both seasonally adjusted and deflated with the CPI. Further, we control for the Gilchrist and Zakrajšek (2012) excess bond premium, following GK. Finally, we include total public debt, deflated by the CPI, which is relevant for the monetary transmission mechanism.

\textsuperscript{18}See for example, Olivei and Tenreyro (2007) and the references therein.

\textsuperscript{19}Shocks are assumed to have zero mean and to be uncorrelated among each other and over time. Independence from contemporaneous economic conditions is considered part of the definition of an exogenous policy shock. The standard interpretation of \( v_t^p \) is a combination of various random factors that might affect policy decisions, including data errors and revisions, preferences of participants at the FOMC meetings, politics, etc. (See Bernanke and Mihov 1998).
that we study.\footnote{We have also estimated specifications extended with Industrial Production and the Civilian Unemployment Rate, and obtained very similar results.} This data series has been retrieved manually from the Monthly Statements of Public Debt of the United States, available online via www.treasurydirect.gov.

A standard assumption to identify the effects of a monetary policy shock is that non-policy variables respond to the shock only with a lag, which amounts to the assumption that the right column of $\Psi$ consists of zeros except for its bottom element. While this “recursive identification” assumption is debatable in general, it is especially ill-suited for our purposes, since the redistribution channel that we study relies on a change in prices when a monetary shock hits, which is ruled out by assumption under the recursive identification scheme. We therefore resort to an alternative approach proposed by Gertler and Karadi (2015), which we describe below.

### 3.2 High-frequency identification of monetary policy shocks

Our approach to identifying monetary policy shocks follows GK, who use the methodology of Mertens and Ravn (2013). A key element of the approach is the use of an instrumental variable which is correlated with the monetary policy shock, $v^p_t$, but not with the other macroeconomic shocks, contained in $v^y_t$. The instrument used is the change in the three-month ahead futures rate during a 30 minute window around announcements by the Federal Open Market Committee (FOMC).\footnote{The data series for the instrumental variable is taken from GK, who convert the surprises to a monthly frequency using a weighting procedure which accounts for the precise timing of each FOMC within the month. The instruments are available over the period 1990-2012.\footnote{See Gürkaynak, Sack and Swanson (2005) for an early analysis of the effects of monetary surprises using the high-frequency approach.}}\footnote{See Gürkaynak, Sack and Swanson (2005) for an early analysis of the effects of monetary surprises using the high-frequency approach.}

The instrumental variables estimator is implemented following a simple two-stage procedure. The first stage is to regress the reduced-form policy residual $\varepsilon^p_t$ on the instrument. The fitted value of this regression, denoted $\widehat{\varepsilon}^p_t$, captures variations in the one-year interest rate that are purely due to monetary policy surprises around FOMC meetings. The second stage is to estimate the linear model $\varepsilon^p_t = \psi^p \varepsilon^p_t + \xi_t$, where $\xi_t$ is a vector of i.i.d. residuals and $\psi^p$ is a vector of coefficients, which captures the impact on the non-policy variables of a monetary surprise associated with a unit increase in the policy instrument. Up to a scaling’s factor, the right column of $\Psi$ is thus estimated as $[\widehat{\psi}^p; 1]$, where $\widehat{\psi}^p$ is the OLS estimate of $\psi^p$. Given this vector and the estimates of $A_s$, $B_s$, $C_s$ and $D_s$, Impulse Response Functions
(IRFs) can be computed by iterating on the VAR. We scale the IRFs such that the one-year rate declines by a maximum of 75 basis points.

The estimated IRFs are depicted in Figure 1, together with 95 percent confidence bands. The monetary expansion triggers an increase in inflation. On an annualized basis, the monthly inflation rate increases by more than two percentage points on impact. Thus, our results do not exhibit a “price puzzle”. On the contrary, inflation swiftly increases, even though the increase is short-lived. The inflation response implies that the price level (not plotted) increases persistently, by about 0.5 percent. Further, there is a large, somewhat gradual increase in durables expenditures, up to about 2 percent. By contrast, the increase in non-durables expenditures is much smaller. On impact, non-durables even decline substantially. Furthermore, real public debt shows a large and significant decline. Figure 1 also displays responses from our full quantitative model. We discuss these responses in Section 5. For now, we simply highlight that the model responses fall largely within the empirical confidence bands.

While our identification strategy follows Gertler and Karadi (2015), we include a different set of variables in the VAR. In the Appendix, we directly compare the responses of the one-year interest rate and the CPI level, as implied by our VAR, to those reported in Gertler and Garadi (2015). It turns out that the response of inflation is very similar: the CPI displays a sharp and temporary increase, which will be mimicked by our model.

3.3 Redistributive Effects of Monetary Policy

A main goal of our paper is to study the redistributive effects of monetary policy and their impact on aggregate variables in a quantitative model. A number of recent empirical papers substantiate our motivation. In particular, Doepke, and Schneider (2006a) document significant wealth redistributions in the US economy following (unexpected) inflationary episodes. Their analysis is based on detailed data on assets and liabilities held by different segments of the population, from which they calculate the revaluation effects caused by inflation. The authors find that the main winners from a monetary expansion are the government as well as poor, young households, whereas the losers tend to be richer, middle age and older house-

\footnote{For other VAR approaches that avoid the price puzzle, see e.g. Bernanke, Boivin and Eliazs (2005) and Castelnuovo and Surico (2010).}

\footnote{There is also a decline in the excess bond premium (not plotted), which is in line with the results of GK (given the size and the sign of the shock).}
Figure 1: Responses to an Expansionary Monetary Policy Shock.

Note: The figure shows the model and estimated empirical impulse responses to an expansionary monetary policy shock, together with the 95 percent confidence bands for the empirical responses. The model responses correspond to the full model, as described in Section 5. The empirical responses are scaled to imply a maximum decline in the one-year nominal government bond rate of 75 basis points. The model responses are scaled such that the impact decline in the one-year rate coincides with the empirical point estimate.

...hlds (in their forties or above). Note that households as a whole are net creditors and the government is a net debtor in the US economy. Adam and Zhu (2014) document similar patterns for Euro area countries and Canada, and update the results for the United States. As for the US economy, in most euro-area countries, the household sector is a net creditor and the government is a net debtor.\textsuperscript{25}

Our model embeds these redistributive revaluation effects and brings two additional considerations to the analysis. The first consideration is how these redistributive effects alter the

\textsuperscript{25} Looking at the disaggregated data for households, the age at which households become net creditors differs across countries, with the turning point being around 40-45.
various demographic groups’ incentives to work, consume, and save in different types of assets, the hiring decision of firms, and finally, how these changes affect the macroeconomy. The second consideration is how the Treasury redistributes the higher revenues stemming from an expansionary monetary policy intervention. These higher revenues consist of i) higher value of remittances received from the Central Bank as a result of the interest on bonds earned by the Central Bank; and ii) gains from the revaluation of government debt—assuming the government is a net debtor. The revaluation gains by the government can be large, as Doepke and Schneider (2006a)’s calculations illustrate. The remittances are also considerable, amounting to an average of two percent of total government revenues during our period of analysis, with significant volatility. We assume that these remittances are rebated to the working-age agents, as in practice the taxation burden tends to fall on the working population. However, the framework can be adjusted to allow for different tax-transfer configurations.

An additional empirical paper motivating our analysis is Coibion et al. (2012), who find that unexpected monetary contractions as well as permanent decreases in the inflation target lead to an increase in inequality in earnings, expenditures, and consumption. Their results rely on the CEX survey, and thus exclude top income earners. The authors however argue that their estimates provide lower bounds for the increase in inequality following monetary policy contractions. This is because individuals in the top one-percent of the income distribution receive a third of their income from financial assets—a much larger share than any other segment of the population; hence, the income of the top one-percent likely rises even more than for most other households following a monetary contraction.

Consistent with these findings, in our model, monetary policy expansions cause a redistribution of income from retirees, who rely more heavily on their nominal wealth as a source of finance for consumption, to working agents and future tax payers. The consumption of goods by working agents increases relative to that of retired agents following a monetary expansion. These results are more directly examined by Wong (2015), who finds that total expenditures by the young increase relatively to those of older people following a monetary policy expansion, the latter identified through a recursive VAR assumption. In the Appendix, using a different identification strategy, we study the responses by different demographic groups and furthermore explore the differences in the responses of durables and nondurables by the various groups. We find that indeed working-age households see an increase in expenditures relative to retired households and that this response is almost entirely driven by the purchases of durable goods. These results lend support to the mechanism in our model,
which generates a relative increase in durable consumption by working-age agents vis-à-vis retirees following a monetary expansion.

4 Monetary policy shocks in a simple heterogeneous-agent model

We study the dynamic effects of monetary policy shocks in a general equilibrium model that embeds overlapping generations and a parsimonious life-cycle structure with two stages: working life and retirement. Transitions from working life to retirement and from retirement to death are stochastic but obey fixed probabilities, as in Gertler (1999). Financial markets are incomplete in the sense that there exists no insurance against risks associated with retirement and longevity. As a result, agents accumulate savings during their working lives, which they gradually deplete once retired. These savings can take the form of money, bonds, and durable consumption goods.

The money supply is controlled by a Central Bank, who implements monetary policy using open market operations, that is, by selling or buying bonds. Realistically, we assume that the Central Bank transfers its profits to the Treasury. The Treasury in turn balances its budget by setting lump-sum transfers to households. In this environment we study the dynamic effects of persistent monetary policy shocks. We contrast our benchmark model with an alternative economy in which the Central Bank uses “helicopter drops” of money rather than OMO to implement monetary policy.

We solve the model using a standard numerical method.\textsuperscript{26} This may seem challenging given the presence of heterogeneous households and incomplete markets. In particular, the presence of aggregate fluctuations implies that a time-varying wealth distribution is part of the state of the macroeconomy. To render the model tractable, we introduce a government transfer towards newborn agents which eliminates inequality among working agents. (Wealth inequality among retired agents, as well as between working-age and retired agents, is preserved in our framework.) We show that aggregation then becomes straightforward and only the distribution of wealth between the group of working-age agents and retirees is relevant for aggregate outcomes. At the same time, our setup preserves the most basic life-cycle savings

\textsuperscript{26} Specifically, we use first-order perturbation, exploiting its certainty-equivalence property. See the appendix for details.
pattern: working-age agents save for retirement and retired agents gradually consume their wealth.

Another advantage of our model is that it straightforwardly nests a model with an infinitely-lived representative agent. One can show analytically that monetary policy shocks do not affect real activity under the representative agent assumption, provided that money and consumption enter the utility function separably. This is because, by construction, redistributive effects are absent in an economy without heterogeneity.

We consider two versions of the model. The simple version does not incorporate any form of product or labor market friction. It highlights the source of the transmission mechanism due exclusively to the redistributive effect of the intervention. The simplest version has very limited persistence when compared to the empirical responses (a result that is also true in a simple NK framework). To add persistence and amplification, we incorporate search and matching frictions in the labour market. This is done in Section 5, where we quantitatively study a more realistic model to gauge the extent to which the proposed mechanism can quantitatively account for the VAR evidence.

4.1 Agents and demographics

We model a closed economy which consists of a continuum of households, a continuum of perfectly competitive firms and a government, which is comprised of a Treasury and a Central Bank. In every period a measure of new working agents is born. Working-age agents retire and turn into retirees with a time-invariant probability \( \rho_R \in [0, 1) \) in each period. Upon retirement, agents face a time-invariant death probability \( \rho_x \in (0, 1] \) in each period, including the initial period of retirement. The population size and distribution over the age groups remains constant over time and the total population size is normalized to one. The fraction of working-age agents in the economy, denoted \( \nu \), can be solved for by exploiting the implication that the number of agents retiring equals the number of deaths in the population.

\[ This \text{ result by itself is not surprising, as (super)neutrality results for representative agent models with productive durables, have been known since the seminal work of Sidrauski (1967) and Fischer (1979). Sidrauski (1967) shows that when money enters the utility function separably, the rate of inflation does not affect real outcomes in the steady state. Fischer (1979) shows that under logarithmic utility this is also true along transition paths. Under alternative utility functions this is generally not true, but in quantitative exercises deviations from neutrality are often found to be quantitatively small, see for example Danthine, Donaldson and Smith (1987). In our benchmark model we will assume logarithmic utility and thus focus on a different source of non-neutrality. \]
\[ \rho_R \nu = \rho_x (1 - \nu + \rho_R \nu). \]

(4)

The life-cycle status of an agent is denoted by a superscript \( s \in \{ N, W, R \} \), with \( N \) denoting a newborn agent ready to work, \( W \) a pre-existing working agent, and \( R \) a retiree.

Households derive utility from non-durables, denoted \( c \in \mathbb{R}^+ \), a stock of durables, \( d \in \mathbb{R}^+ \), and real money balances, denoted \( m \in \mathbb{R}^+ \). They can also invest in nominal bonds, the real value of which we label \( b \in \mathbb{R} \). Bonds pay a net nominal interest rate \( r \in \mathbb{R}^+ \).

Working-age agents, including the newborns, supply labor to firms in a competitive labor market whereas retirees are no longer productive. Durables depreciate at a rate \( \delta \in (0, 1) \) per period and are produced using the same technology as non-durables. Because of the latter, durables and non-durables have the same market price. All agents take laws of motion of prices, interest rates, government transfers, and idiosyncratic life-cycle shocks as given. We describe the decision problems of the agents in turn.

### 4.2 Retired agents

Agents maximize expected lifetime utility subject to their budgets, taking the law of motion of the aggregate state, denoted by \( \Gamma \), as given. Letting primes denote next period’s variables, we can express the decision problem for retired agents (\( s = R \)) recursively and in real terms as:

\[
V^R(a, \Gamma) = \max_{c, d, m, b} U(c, d, m) + \beta (1 - \rho_x) \mathbb{E} V^R(a', \Gamma')
\]

s.t.
\[
\begin{align*}
& c + d + m + b = a + \tau_R, \\
& a' \equiv (1 - \delta) d + \frac{m}{1 + \pi'} + \frac{(1 + r) b}{1 + \pi'}, \\
& c, d, m \geq 0,
\end{align*}
\]

(5)

where \( V^R(a, \Gamma) \) is the value function of a retiree which depends on the aggregate state and the real value of wealth, denoted by \( a \), \( \mathbb{E} \) is the expectation operator conditional on information available in the current period, \( \beta \in (0, 1) \) is the agent’s subjective discount factor, and \( \pi \in \mathbb{R} \) is the net rate of inflation. \( U(c, d, m) \) is a utility function and we assume that \( U_j(c, d, m) > 0 \), \( U_{jj}(c, d, m) < 0 \) and \( \lim_{j \to 0} U_j(c, d, m) = \infty \) for \( j = c, d, m \). Finally, \( \tau^s \in \mathbb{R} \) is a transfer from
the government to an agent with age status \( s \), so \( \tau^R \) is the transfer to any retired agent.

The budget constraint implies that retirees have no source of income other than the interest stemming from previously accumulated wealth. Implicit in the recursive formulation of the agent’s decision problem is a transversality condition

\[
\lim_{t \to \infty} \mathbb{E}_t \beta^t (1 - \rho_x)^t U_{c,t} x_t = 0,
\]

where \( x = d, m, b \) and where \( U_{c,t} \) denotes the marginal utility of non-durable consumption. Finally, we assume that agents derive no utility from bequests and that the wealth of the deceased agents is equally distributed among the currently working-age agents.

### 4.3 Working agents

Working-age agents supply labor in exchange for a real wage \( w \in \mathbb{R}^+ \) per hour worked. The optimization problem for newborn agents (\( s = N \)) and pre-existing working-age agents (\( s = W \)) can be written as:

\[
V^s(a, \Gamma) = \max_{c, d, m, b, h} U(c, d, m) - \zeta \frac{h^{1+\kappa}}{1+\kappa} + \beta (1 - \rho_R) \mathbb{E} V^W(a', \Gamma') + \beta \rho_R (1 - \rho_x) \mathbb{E} V^R(a', \Gamma')
\]

\[s = N, W\]

\[s.t.\]

\[
c + d + m + b = a + wh + \tau^{bg} + \tau^s,
\]

\[
a' \equiv (1 - \delta) d + \frac{m}{1+\pi'} + \frac{(1 + r) b}{1+\pi'},
\]

\[c, d, m \geq 0,
\]

where working-age agents too obey transversality conditions. The term \( \zeta \frac{h^{1+\kappa}}{1+\kappa} \) captures the disutility obtained from hours worked, denoted \( h \), with \( \zeta > 0 \) being a scaling’s parameter and \( \kappa > 0 \) being the Frisch elasticity of labor supply. Bequests from deceased agents are denoted \( \tau^{bg} \); as before, \( \tau^s \) is a lump-sum transfer from the government. When making their optimal decisions, working agents take into account that in the next period they may be retired, which occurs with probability \( \rho_R (1 - \rho_x) \), or be deceased which happens with probability \( \rho_R \rho_x \). We thus allow the possibility that upon retirement, agents may be immediately hit by a death shock.
4.4 Firms

Goods are produced by a continuum of perfectly competitive and identical goods firms. These firms operate a linear production technology:

\[ y_t = h_t. \]  \hspace{1cm} (7)

Profit maximization implies that \( w_t = 1 \), that is, the real wage equals one.

4.5 Central bank

Although we do not model any frictions within the government, we make a conceptual distinction between a Central Bank conducting monetary policy and a Treasury conducting fiscal policy. We make this distinction for clarity and in order to relate the model to real-world practice.

The Central Bank controls the nominal money supply, \( M_t \in \mathbb{R}^+ \), by conducting open market operations. In particular, the Central Bank can sell or buy government bonds. We denote the nominal value of the bonds held by the Central Bank by \( B_t^{\text{CB}} \in \mathbb{R} \). The use of open market operations implies that in every given period the change in bonds held by the Central Bank equals the change in money in circulation, that is,

\[ B_t^{\text{CB}} - B_{t-1}^{\text{CB}} = M_t - M_{t-1}. \]  \hspace{1cm} (8)

The Central Bank transfers its accounting profit—typically called seigniorage—to the Treasury.\(^{28}\) The real value of the seigniorage transfer, labeled \( \tau_t^{\text{CB}} \in \mathbb{R} \), is given by:

\[ \tau_t^{\text{CB}} = \frac{\tau_{t-1}^{\text{CB}} b_{t-1}^{\text{CB}}}{1 + \pi_t}. \]  \hspace{1cm} (9)

The above description is in line with how Central Banks conduct monetary policy, as well as with the typical arrangement between a Central Bank and the Treasury. By contrast, many models of monetary policy assume monetary policy is implemented using “helicopter drops,” that is, expansions of the money supply that are not accompanied by a purchase of assets but instead by a fiscal transfer equal to the change in the money supply. Modern monetary

---

\(^{28}\)We abstract from operational costs incurred by the central bank.
models are often silent on how monetary policy is implemented and directly specify an interest rate rule. In our framework, however, the specific instruments used to implement monetary policy are critical, since the associated monetary-fiscal arrangements pin down redistributive effects and hence the impact of changes in monetary policy on the real economy.

When we implement the model quantitatively, we simulate exogenous shocks to monetary policy. We do so by specifying a stochastic process that affects the growth rate of the money supply $M_t$. The change in $M_t$ is implemented through open market operations.

### 4.6 Treasury

The Treasury conducts fiscal policy. For simplicity, we abstract from government purchases of goods and assume that the Treasury follows a balanced budget policy. The government has an initial level of bonds $B_{t-1}^G$ which gives rise to interest income (or expenditure if the government has debt) on top of the seigniorage transfer from the Central Bank. To balance its budget, the government makes lump-sum transfers to the households, which can be either positive or negative. The government’s budget policy satisfies:

$$\nu \rho_R \tau_t^N + \nu (1-\rho_R) \tau_t^W + (1-\nu) \tau_t^R = \frac{r_t b_{t-1}^G}{1+\pi_t} + \tau_t^{CB}. \quad (10)$$

Here, the left-hand size denotes the total transfer. In particular, $\nu \rho_R \tau_t^N$ is the total transfer to the newborns, $\nu (1-\rho_R) \tau_t^W$ is the transfer to pre-existing working agents and $b_t^G$ is the real value of government bonds. The right-hand side denotes total government income.

For tractability we also assume that the government provides newborn agents with an initial transfer that equalizes their wealth levels with the average after-tax wealth among pre-existing agents, that is,

$$\tau_t^N = a_t^W + \tau_t^W, \quad (11)$$

where $a_t^W \equiv \int_{i:s=W} a_{i,t}di$ is the average wealth among pre-existing working agents (before transfers). Since before-tax wealth is the only source of heterogeneity among working agents, all working agents make the same decisions and what arises is a representative agent. This implication makes the model tractable. Note that although we eliminate heterogeneity among working agents by assumption, the framework preserves the heterogeneity between working and retired agents, as well as the heterogeneity among retired agents.

Finally, we assume that only productive agents are affected by transfers or taxes, that is,
we set $\tau_t^R = 0$. This assumption is motivated by the observation that the majority of the tax burden falls on people in their working life, due to the progressivity of tax systems. Note, however, that the framework is highly flexible and can be used to analyze more complex fiscal settings. For the sake of expositional clarity and brevity, we focus on this baseline setting; solutions for alternative fiscal settings are available at request from the authors.\(^{29}\)

### 4.7 Market clearing and equilibrium

Aggregate non-durables and durables are given by:

\[
\begin{align*}
c_t &= \nu c_t^W + (1 - \nu) c_t^R, \\
d_t &= \nu d_t^W + (1 - \nu) d_t^R,
\end{align*}
\]

where superscripts $W$ and $R$ denote the averages among working and retired agents, defined analogously to the definition of $a_t^W$.\(^{30}\) Clearing in the markets for goods, money and bonds requires:

\[
\begin{align*}
c_t + d_t &= \nu h_t^W + (1 - \delta) d_{t-1}, \\
m_t &= \nu m_t^W + (1 - \nu) m_t^R, \\
0 &= b_t^G + b_t^{CB} + \nu b_t^W + (1 - \nu) b_t^R.
\end{align*}
\]

Finally, the size of the bequest received per working-age agent is given by:

\[
\tau_t^{bq} = \frac{\rho_x^R \alpha_t^R + \rho_x \rho_x^W a_t^W}{\nu}.
\]

We are now ready to define a recursive competitive equilibrium:

**Definition.** A recursive competitive equilibrium is defined by policy rules for non-durable consumption, $c^*(a, \Gamma)$, durable consumption, $d^*(a, \Gamma)$, money holdings, $m^*(a, \Gamma)$.

---

\(^{29}\)We have solved a version of our model in which instead taxes are proportional to wealth levels, and obtained results similar to the ones obtained from our benchmark model. An alternative, behavioural assumption, suggested by David Laibson, would be to realistically assume that agents are not aware of these future transfers. We do not follow this avenue here, but we highlight that this would cause even bigger perceived wealth effects and intensify the aggregate responses we document.

\(^{30}\)Due to the transfer to newborns $c_t^W = c_t^N$, $d_t^W = d_t^N$, $b_t^W = b_t^N$ and $m_t^W = m_t^N$. 

---
As well as laws of motion for inflation, the nominal interest rate, and the real wage, such that households optimize their expected life-time utility subject to their constraints and the law of motion for the aggregate state, the Treasury and Central Bank follow their specified policies, and the markets for bonds, money, goods, and labor clear in every period. The aggregate state $\g$ includes the value of the monetary policy shock, the distribution of wealth among agents, as well as the initial holdings of assets by households, the Treasury, and the Central Bank.

4.8 Absence of wealth effects in a representative-agent model

A special case of our simple model is obtained when we set the death probability to one, that is, $\rho_x = 1$. In this case, agents immediately die upon retirement and retired agents are effectively removed from the model. Given the absence of heterogeneity among working agents, the model becomes observationally equivalent to one with an infinitely-lived representative household with a subjective discount factor equal to $\tilde{\beta} = \beta (1 - \rho R)$. Without heterogeneity among households, shocks to monetary policy do not create net wealth effects and do not impact on real economic activity. This subsection explains why this is the case, using arguments that closely follow Sidrauski (1967), Barro (1978), and Weil (1991).

Consider the simple model with $\rho_x = 1$.\textsuperscript{31} We assume that non-durables, durables and real money balances enter the utility function separably. In particular, we assume the following logarithmic preferences: $U(c, d, m) = \ln c + \eta \ln d + \mu \ln m_t$, where $\eta, \mu > 0$ are preference parameters. This special case is useful to understand the role of household heterogeneity in the transmission of monetary policy, as several analytical results can be derived. The first result is:

**Result 1.** Monetary policy is neutral with respect to real activity in the representative agent model.

The arguments for the monetary neutrality follow Sidrauski (1967). The representative agents’ first-order conditions for durables and labor supply, and the aggregate resource con-

\textsuperscript{31}We focus on the baseline model for simplicity. It is straightforward to show that the same results are obtained in a representative agent version of the model with search and matching frictions.
straint are, respectively:

\[ U_{c,t} = U_{d,t} + \beta (1 - \delta) E_t U_{c,t+1}, \quad (18) \]

\[ U_{c,t} = h_t^e, \quad (19) \]

\[ c_t + d_t = h_t + (1 - \delta) d_{t-1}, \quad (20) \]

where \( U_{c,t} = \frac{1}{c_t}, U_{d,t} = \frac{n}{d_t} \) and for \( t = 0, 1, \ldots \) Given an initial level of durables and given that the utility function is separable in its arguments, these three equations pin down the equilibrium solution paths for \( c_t, d_t, \) and \( h_t \) in any period \( t \) without any reference to variables related to monetary policy. Given this solution it is straightforward to pin down output and the real interest rate as well.

Next, we consider the wealth effects of monetary policy shocks and derive the following key result:

**Result 2.** Changes in monetary policy do not create net wealth effects in the representative agent model.

To arrive at this result, consider the government’s consolidated (expected) present value budget constraint. The Appendix demonstrates that this constraint can be written as:

\[ E_t \sum_{s=t}^{\infty} D_s \frac{r_s}{1+r_s} m_s = \frac{m_{t-1} - (1+r_{t-1}) (b_{t-1}^G + b_{t-1}^{CB})}{1+\pi_t} + E_t \sum_{s=t}^{\infty} D_s \tau_s^G, \quad (21) \]

where \( D_s \equiv \prod_{k=t}^{s-1} \frac{1+\pi_{k+1}}{1+\pi_k} \) is the agent’s valuation of one unit of nominal wealth received in period \( s > t, D_t \equiv 1, \) and \( \tau_t^G \equiv \nu \rho_R \tau_t^N + \nu (1 - \rho_R) \tau_t^W \) is the total transfer to the household sector in period \( t. \) The left-hand side of Equation (21) represents the expected present value of government income, in real terms. Here, \( \frac{r_t}{1+r_t} m_t \) is the opportunity cost that households pay for holding money. This cost to the households represents a source of income to the government, which enjoys an interest-free liability. The left-hand side of the equation represents the present value of this income to the government. The right hand side captures the present value of government liabilities. The first term represents the real value of the outstanding stock of money and government debt, whereas the second term is the present value of transfers to households, another liability to the government.

Importantly, both components of government liabilities are a source of wealth to the
household. Equation (21) makes clear that a monetary shock can affect household wealth via two channels. First, it can trigger a change in current inflation, \( \pi_t \), affecting the real value of wealth held in nominal assets by the households. Second, monetary policy shocks can affect the present value of transfers to households, via Central Bank remittances to the Treasury.

The Appendix demonstrates that the left hand side of Equation (21) does not respond to monetary policy shocks. In particular, from Result 1 it follows that both \( D_s \) and \( r_s m_s \), with \( s \geq 1 \), remain constant. It follows that the right-hand side of the equation remains constant as well. Thus, monetary policy shocks have no net wealth effects on households: any downward (upward) revaluation of nominal wealth due to a change in the price level is exactly offset by an increase (decline) in the present value of transfers. This insight is closely related to the seminal work of Barro (1978) and was spelled out by Weil (1991) in the context of a monetary model.

4.9 The dynamic effects of open market operations

We now analyze the effects of open market operations in our simple model using numerical simulations. Before doing so, we specify the details of household preferences and the monetary policy rule, as well as parameter values.

4.9.1 Functional forms and parameter values

We assume that the utility function is a CES basket of non-durables, durables and money, nested in a CRRA function:

\[
U(c_{i,t}, d_{i,t}, m_{i,t}) = \frac{x_{i,t}^{1-\sigma} - 1}{1 - \sigma},
\]

\[
x_{i,t} \equiv \left[ c_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \eta d_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \mu m_{i,t}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{1}{\epsilon-1}},
\]

(22)

where \( \epsilon, \sigma, \eta, \mu > 0 \). Here, \( \epsilon \) is the elasticity of substitution between non-durables, durables and money, \( \sigma \) is the coefficient of relative risk aversion, and \( \eta \) and \( \mu \) are parameters giving utility weights to durables and money, respectively. Computation of the dynamic equilibrium path seems complicated due to the high dimensionality of the aggregate state \( \Gamma_t \). In the Appendix we show that solving the model using a standard first-order perturbation (lin-
earization) method is nonetheless straightforward under the above preference specification.\textsuperscript{32}

The Central Bank is assumed to set the money supply according to the following process:

\[
\frac{M_t}{M_{t-1}} = 1 + z_t,
\]

where \( z_t \) is an exogenous shock process to the rate of nominal money growth, assumed to be of the following form:

\[
\begin{align*}
\dot{z}_t &= \xi \left( \bar{m} - m_{t-1} \right) + \varepsilon_t, \quad \xi \in (0, 1),
\end{align*}
\]

where \( \varepsilon_t \) is an i.i.d. shock innovation and \( \bar{m} \) is the steady-state value of real money balances. A positive shock increases the money supply on impact. The above feedback rule implies that this increase is gradually reversed in subsequent periods when \( \xi \in (0, 1) \).\textsuperscript{33}

The model period is set to one quarter and parameter values are presented in Table 1, in the column labeled “simple”. The subjective discount factor, \( \beta \), is set to 0.9745 which implies an annual real interest rate of 4 percent in the deterministic steady state. The steady state real interest rate is lower than the subjective discount rate, \( 1/\beta - 1 \), due to the retirement savings motive arising in the presence of incomplete insurance markets. The durable preference parameter \( \eta \) is chosen to target a steady-state consumption spending ratio of 20 percent on durables. To set the money preference parameter, we target a quarterly money velocity, defined as \( \frac{y}{m} \), of 1.8. The intratemporal elasticity of substitution between non-durables, durables and money, \( \epsilon \), is set equal to one, as is the coefficient of relative risk aversion, \( \sigma \). These two parameter settings imply that money and consumption enter the utility function additively in logs. Hence, our benchmark results are not driven by non-separability of money and consumption in the utility function. In the simple model, we set the Frisch elasticity of labor supply \( \kappa \) equal to one following many macro studies. (We shut down the labour supply response in the extension.) The parameter scaling the disutility of labor, \( \zeta \), is set so as to normalize aggregate quarterly output to one.

Life-cycle transition parameters are set to imply a life expectancy of 60 years, with an expected 40 years of working life and expected 20 years of retirement. Accordingly, we set

\textsuperscript{32}In particular, we exploit the properties of first-order perturbation and show that the implied certainty equivalence with respect to the aggregate state allows us to express the decision rules of retired agents as linear functions of their wealth levels. This in turn implies that aggregation is straightforward and that only the distribution of wealth between between retired and working agents is relevant for aggregate outcomes.

\textsuperscript{33}In equilibrium, both real an nominal money balances increase following the shock. Also, the rule implies that the net rate of inflation is zero in the steady state.
\( \rho_R = 0.0063 \) and \( \rho_x = 0.0125 \) which imply \( \nu = 0.6677 \). The depreciation rate of durables, \( \delta \), is set to 0.04 following Baxter (1996). The initial level of government debt is set to eighty percent of annual output. For simplicity we assume that the Central Bank starts off without any bond holdings or debt. The parameter \( \xi \), which governs the persistence of the shock process, is set to 0.1. Section 5 further discusses this parameter.

<table>
<thead>
<tr>
<th>simple</th>
<th>full</th>
<th>description</th>
<th>motivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.9745</td>
<td>subjective discount factor</td>
<td>4% s.s. annual interest rate</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.31</td>
<td>durables preference param.</td>
<td>20% s.s. spending on durables (NIPA)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.0069</td>
<td>money preference param.</td>
<td>1.8 s.s. M2 velocity ( \left( \frac{m}{y} \right) ) (FRB/NIPA)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>1.0</td>
<td>coef. rel. risk aversion</td>
<td>convention literature</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>1.0</td>
<td>intratemp. elast. of subst.</td>
<td>convention literature</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>1.0</td>
<td>inv. elasticity labour supply</td>
<td>convention literature</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>0.5781</td>
<td>disutility of labor</td>
<td>normalize agg. output to one</td>
</tr>
<tr>
<td>( \rho_R )</td>
<td>0.0063</td>
<td>retirement probability</td>
<td>avg duration working life 40 years</td>
</tr>
<tr>
<td>( \rho_x )</td>
<td>0.0125</td>
<td>death probability</td>
<td>avg duration retirement 20 years</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.04</td>
<td>depreciation rate durables</td>
<td>Baxter (1996)</td>
</tr>
<tr>
<td>( \varphi_0^G )</td>
<td>-3.2</td>
<td>initial bonds Treasury</td>
<td>government debt 80% of annual output</td>
</tr>
<tr>
<td>( \varphi_0^{CB} )</td>
<td>0.0</td>
<td>initial bondsCentral Bank</td>
<td>no initial central bank debt/bonds</td>
</tr>
<tr>
<td>( \xi )</td>
<td>0.1</td>
<td>coefficient monetary rule</td>
<td>persistence nominal interest rate</td>
</tr>
<tr>
<td>( \lambda_0 )</td>
<td>-</td>
<td>variable hiring cost</td>
<td>s.s. hiring cost 0.5% of output</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>-</td>
<td>fixed hiring cost</td>
<td>( \lambda_1/\lambda_0 = 20 ) (Pissarides (2009))</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-</td>
<td>matching function elasticity</td>
<td>convention search literature</td>
</tr>
<tr>
<td>( w )</td>
<td>-</td>
<td>real wage</td>
<td>5% s.s. unemployment rate</td>
</tr>
<tr>
<td>( \bar{\gamma} )</td>
<td>-</td>
<td>scaling matching function</td>
<td>vacancy filling probability 0.74</td>
</tr>
</tbody>
</table>

### 4.9.2 Responses to a monetary policy shock under OMO

Figure 2 presents the responses to an expansionary monetary policy shock in the simple model. We first study the responses under the (realistic) premise that monetary policy is implemented using open market operations. These responses are indicated by the blue solid...
lines. The magnitude of the shock is scaled to imply a reduction in the nominal interest rate of about 75 basis points on impact. For now, we focus on the qualitative effects of the shock. In the next section, we use the full model to evaluate the quantitative effects in light of the empirical evidence.

Figure 2: Responses to an Expansionary Monetary Policy Shock in the Simple Model.

Note: The figure shows the simple model’s impulse responses to an annualized 75 basis point decline in the quarterly nominal interest rate, when policy is implemented, correspondingly, through OMO or helicopter drops. The model responses correspond to the simple model without search and matching frictions. Horizontal axes denote quarters after the shock.

Following the monetary expansion, the inflation rate increases on impact, as the price level jumps up.\textsuperscript{34} In the periods after the initial shock, the nominal interest rate and the price level gradually revert back to their initial levels, which happens as a result of the reversion

\textsuperscript{34}The intuition for the price increase is standard. As the central bank buys government bonds, it increases the amount of money in circulation. Since agents’ utility is concave in real money holdings, they are induced to substitute some of the extra cash for consumption goods. The increased demand for goods in turn drives up prices, which dampens the demand increase as it reduces the real value of money holdings.
in the monetary policy rule. During this period, inflation is slightly negative and the price level gradually reverts back to its initial level before the shock.

The monetary expansion increases aggregate output on impact. The responses of durables and non-durables make clear that this increase in output is entirely driven by an increase in expenditures on durables. Non-durables decline on impact, although the magnitude of the response is much smaller than the response of durables. Finally, there is a decline in the real value of public debt (i.e. debt issued by the Treasury), which mirrors the response of the price level and which reflects a financial gain for the government at the expense of the public due to a revaluation of its debt.\(^\text{35}\)

Figure 3 plots several variables that provide insight into the impact of monetary policy shocks, as well as into their endogenous propagation over time. Consider again the model version in which monetary policy is implemented using open market operations (indicated by blue solid lines). The real interest rate, plotted in the upper left panel, declines, reflecting an increased desire to save. The top right panel plots the transfer to the working households as a fraction of output, which on impact increases by about 0.8 percent, after which it gradually reverts back to the steady state. Thus, the government gradually remits its financial gains from the monetary expansion back to the households.

The middle two panels show the responses of consumption by working agents, whereas the bottom panels show the consumption responses of working agents vis-à-vis retired agents. Relative to the retirees, consumption of durables and non-durables by working agents increases. All households face a reduction in their real wealth due to the increase in prices, but the retirees are not compensated by an increase in transfers; hence, they lose relative to working agents.\(^\text{36}\) In absolute terms, consumption of durables by working agents increases as well. The response of non-durables expenditures by working agents is negative on impact.

To understand the effects of monetary policy on real activity more deeply, note that the increase in prices creates a negative wealth effect to the households as it reduces the real value of their money and bond holdings. These losses are only partly compensated for by an increase in (expected) government transfers. Thus, the policy shock reduces the households’ permanent income levels. Furthermore, households become less well insured

\(^{35}\)A second financial gain for the government stems from a downward revaluation of the outstanding stock of money, which is a liability to the government alongside debt.

\(^{36}\)Additionally, for retired agents wealth is the only source of income, whereas working agents also receive wage income, which in real terms is not directly affected by inflation. This is another reason why working agents are less vulnerable to inflation.
Figure 3: Responses to an Expansionary Monetary Policy Shock in the Simple Model.

Note: The figure shows the simple model’s impulse responses to an annualized 75 basis point decline in the quarterly nominal interest rate, when policy is implemented, correspondingly, through OMO or helicopter drops. The model responses correspond to the simple model without search and matching frictions. Horizontal axes denote quarters after the shock.

against idiosyncratic shocks after a decline in the value of their assets. These effects induce the households to consume less and enjoy less leisure, that is, to work more, in order to re-build their savings. However, as the aggregate resource constraint in equation (14) makes clear, in equilibrium it is not possible for the household sector as a whole to reduce all consumption expenditures and work more, since the additional labour effort generates more output. Thus, while the household sector desires to save a larger fraction of the real income that it generates through production, it is not possible to increase its aggregate holdings of bonds since the economy is closed and the government’s financial position is determined by its policies. However, it is possible for households to save more by accumulating more durables, which are partly consumption goods and partly assets. This implies a substitution
from non-durables expenditures towards durables expenditures. Thus, the negative wealth effect triggered by a monetary expansion induces households to work more and save more for retirement, which leads to an expansion in output and a substitution of consumption towards durables.

4.10 Helicopter drops

We now contrast the effects of open market operations to the effects of shocks in a version of the simple model in which monetary policy is implemented using “helicopter drops” of money. By a helicopter drop, we mean an expansion in the money supply that is not accompanied by an increase in Central Bank bond holdings, but rather by an outright transfer to the Treasury.\(^{37}\) It then follows that the total transfer from the Treasury to the households is given by its interest earnings on bond holdings (which can be negative) plus the change in the money supply. In real terms, the transfer to the households becomes:

\[
m_t - \frac{m_{t-1}}{1 + \pi_t} + \frac{r_{t-1}}{1 + \pi_t} \frac{b_{t-1}^G}{1 + \pi_t} = \nu \rho_R \tau_t^N + \nu (1 - \rho_R) \tau_t^W, \tag{25}
\]

We assume again that helicopter drops are gradually reversed after the initial shock, following the same feedback rule as used in the economy with market operations.\(^{38}\)

The red dashed lines in Figures 3 and 4 plot the responses for the economy with helicopter drops. Note first that the response of the nominal interest rate is virtually the same as it is in the case of OMO. The figures show that although response of prices to the helicopter drop is comparable to the one in our economy with OMO, the effects on real economic outcomes are very different. In particular, with helicopter drops, output and durable expenditures decline following an expansion of the money supply, whereas in the decline in the real interest rate is much more muted than under OMO. Thus, the transmission of monetary policy depends importantly on the operating procedures of the Central Bank and the associated monetary-fiscal arrangements.

The response of government transfers, plotted in the lower right panel, reveals why the effects of a monetary expansion are so different when helicopter drops are used. Upon impact, there is a large one-time positive transfer to working households, whereas transfers in later

\(^{37}\)Consequently, \(b_t^{CB}\) remains zero at all times.

\(^{38}\)For comparability, we do not re-scale the magnitude of the shock relative to the benchmark model.
periods are negligible. Thus, a helicopter drop creates mostly a redistribution between current generations, favoring currently working agents, who receive the government transfer, at the expense of the retirees. Future generations are largely unaffected. As a result of their wealth gains, working agents increase consumption of both types of goods and reduce their labor supply, the latter creating a drop in output. By contrast, in the economy with OMO the transfers are spread out over time. As a result, future generations gain at the expense of the current generations (both working and retired agents), who face net losses of wealth. These losses induce working agents to increase labor supply, which generates an increase in output. As a result, the transmission mechanism is essentially reversed when helicopter drops are used.

5 Full model and quantitative exploration

Before we compare the model’s predictions directly to the data, we add two more ingredients. First, we introduce search and matching frictions in the labor market. Second, we enrich the model’s description of fiscal policy.

5.1 Adding search and matching frictions

In the simple model described above, fluctuations in aggregate output due to monetary policy shocks arise from labour-supply effects. To appreciate this point, recall that labour is the only input in production and note that the working households’ first-order condition for labour can be written as:

\[ w_t \lambda_t = \zeta h_t^\kappa, \]

where \( \lambda_t \) is the Lagrange multiplier on the working households’ budget constraint, which measures the marginal utility of wealth. After a negative shock to wealth, \( \lambda_t \) increases, which pushes up aggregate labour supply and therefore aggregate output. Vice versa, any increase in aggregate output following a monetary expansion derives from an increase in labour supply. Various empirical studies indicate that reductions in wealth can depress labour supply, see e.g. Imbens, Rubin and Sacerdote (2001) and Holtz-Eakin, Joulfaian and

\[ 39 \text{Recall that } w_t = 1, \text{ so any increase in } h_t \text{ must be accompanied by an increase in } \lambda_t. \]
Rosen (1993). However, at high frequency and for small shocks, the labour supply response may not be strong.

We verify robustness of our transmission mechanism in an environment in which the labour supply channel is suppressed completely. The new assumptions we introduce are arguably more realistic and in line with the macro-labour literature. Specifically, we introduce search and matching frictions in the labour market. Workers inelastically supply labour if they have a job and firms hire workers by posting costly vacancies. Operational firms make positive profits and hence firm equity is a valuable asset, which is a form of savings to households alongside money, bonds, and consumer durables.

We introduce matching frictions following the approach of Diamond, Mortensen, and Pissarides, see e.g. Pissarides (1990). Working-age agents can be either unemployed or matched with a firm.\footnote{We set $\zeta = 0$ in this model version, i.e. there is no disutility from work. We do not model unemployment benefits.} A separation between a worker and a firm takes place if the worker retires at the end of the period. If the worker does not retire, the match dissolves with an exogenous probability $\rho_S$. The overall separation rate, denoted $\tilde{\rho}_S$, is therefore given by $\tilde{\rho}_S = \rho_R + (1 - \rho_R) \rho_S$. Newborn agents enter the workforce as unemployed. It follows that the number of job searchers in the economy, which we denote $s_t$, is given by $s_t = \rho_R \nu + (1 - \rho_R) \rho_S n_{t-1}$. Hiring takes place at the beginning of the period, after aggregate and individual shocks have realized, but before production takes place. The evolution of the employment rate among working-age agents, denoted $n_t$, is given by:

$$n_t = (1 - \tilde{\rho}_S) n_{t-1} + g_t,$$

where $g_t$ denotes the number of new hires in period $t$. We assume that there is full income sharing among working-age agents, following Merz (1995) and many others. Hence, we preserve our setup without heterogeneity among working-age agents.

Firms are either matched with a worker or are inactive. The equity value of an active firm is given by:

$$V_t = \theta - w_t + (1 - \tilde{\rho}_S) \mathbb{E}_t \Lambda_{t,t+1} V_{t+1},$$

(26)

where $w_t$ is the real wage, $\theta$ is worker productivity, and $\Lambda_{t,t+1}$ is the stochastic discount factor of the owner of the firms. Inactive firms may search on the labor market for a worker after
posting a vacancy, which comes at a flow cost $\chi_0$ per period. If the firm is successful in finding a worker, the firm pays a fixed cost $\chi_1$ to hire the worker. The latter cost represents all hiring costs that are not proportional to the duration of the vacancy, such as training costs, see Pissarides (2009).\footnote{As emphasized by Pissarides (2009), the presence of fixed component in vacancy creation helps to alleviate the well-known problem that search and matching models tend generate much smaller unemployment fluctuations than those observed in the data. Similarly, in our model, the fixed component helps to align the model response in output with the one observed in the VAR.} Creating an inactive firm is costless which gives rise to the following free-entry condition:

$$\frac{\chi_0}{\lambda_t} + \chi_1 \leq V_t,$$

where $\lambda_t \in [0, 1]$ is the probability of filling a vacancy. The free-entry condition states that the total (expected) cost of activating a firm cannot exceed the equity value. We calibrate the model such that the condition holds with equality at all times. Given a number of vacancies and a number of searchers, the total number of new matches follows from an aggregate matching function given by $g_t = \overline{g}s_t^\alpha v_t^{1-\alpha}$, where $v_t$ is the aggregate number of vacancies, $\overline{g}$ is a scaling’s parameter and $\alpha$ is the elasticity of the number of new matches with respect to the number of searchers. The probability of filling vacancy is given by $\lambda_t = \frac{\omega}{w_t}$. We assume the real wage is fixed, i.e. $w_t = w < \theta$. Further, we assume that firms use the working-age agents’ stochastic discount factor.\footnote{Thus, the firms’ discount factor is given by $\Lambda_t^{\,t+1} = \beta (1 - \rho_R) \frac{U_{t+1}^{\mathcal{N}}}{U_{t+1}^{\mathcal{X}}} + \beta \rho_R (1 - \rho_x) \frac{U_{t+1}^{\mathcal{W}R}}{U_{t+1}^{\mathcal{W}N}}$. This assumption simplifies the analysis but is not very restrictive since it can be shown that the stochastic discount factor of all households is the same to a first-order approximation.} \footnote{Consistent with this assumption we assume that agents sell off all firm their equity upon retirement. The budget constraint of a working-age household becomes: $c_t + d_t + m_t + b_t + V_t (x_t - (1 - \tilde{\rho}_S) x_{t-1}) = a_t + (\theta - w_t) x_t + w_{t+1} + \tau^b q + \tau^a$, where $x_t$ is the amount of firm equity held by the household. The aggregate supply of firm equity is equal to $n_t$.}

### 5.2 Fiscal policy rule

The second modification we make relative to the simple model is the introduction of a more general fiscal policy rule. The motivation for this is essentially empirical. Recall that in the simple model, the government follows a balanced budget policy and transfers any income to the households, period-by-period. This implies that, following a monetary expansion, real public debt declines as the price level increases. Subsequently, however, debt reverts back to the mean, as the price level recovers. In the VAR, however, we observe that real public debt further declines in the two years after the shock. (See Figure 1.)
Given that public debt plays a key role in the transmission mechanism, we devise a fiscal rule which mimics the behavior of real public debt in the VAR. We achieve this by allowing the government to transfer its income to the household with some delay. Realistically, such delays can arise from the fact that it takes time for a government to adjust tax rates.

Specifically, we generalize fiscal policy to imply a period-by-period Treasury Budget constraint of the following form:

\[
\nu \rho_R r_t^N + \nu (1 - \rho_R) r_t^W = \sum_{i=0}^{\infty} \gamma_i \left( \frac{r_{t-i-1}^G}{1 + \pi_{t-i}} + \tau_{t-i}^C \right)
\]

The above equation simply states that the total transfer to the households (the left-hand side) equals a weighted combination of government income in the past. We impose a long-run balanced budget by imposing that \(\sum_{i=0}^{\infty} \gamma_i = 1\), that is, all government income will be transferred to households at some point in time. When we set \(\gamma_0 = 1\) and \(\gamma_i = 0\) for any lag \(i > 0\), we obtain the fiscal rule of the simple model. In the full model, we set \(\gamma_0 = -1.15\), \(\gamma_1 = -1\), \(\gamma_7 = 2\), and \(\gamma_{12} = 1.15\). Below we will show that, with these parameter values, the model can mimic the debt response in the VAR reasonably well.

5.3 Calibration

The calibration of the full model targets the same steady-state values for the interest rate, the durables spending ratio, and money velocity as the simple model. Accordingly, \(\beta\), \(\eta\), and \(\mu\) are set to, respectively, 0.9770, 0.31 and 0.0048. The labour utility parameters \(\kappa\) and \(\zeta\) are irrelevant in the search and matching version. Instead, five parameter pertaining to the labour market frictions are calibrated: \(\alpha\), \(\chi_0\), \(\chi_1\), \(w\) and \(\zeta\). The matching function elasticity, \(\alpha\), is set to 0.5, a conventional value in the search and matching literature. The other parameters are set to hit four steady-state targets. The first target is a steady-state unemployment rate of 5 percent. Second, we target the average hiring cost to be 0.5 percent of the quarterly output generated by a worker. Third, we target the ratio of the vacancy cost to the fixed cost of hiring, \(\chi_1/\chi_0\), equal to 20, which is at the upper end of the range considered by Pissarides (2009). Finally, set we set \(\zeta\) to 0.7, which delivers a vacancy filling probability of 0.74, in line with Den Haan, Ramey and Watson (2000). The parameter \(\theta\) is normalized to 1.05, in order to imply an aggregate output level of roughly one in the steady state. Finally, the persistence parameter, \(\xi\), is set to 0.1, in order to obtain a degree of
persistence in the nominal interest rate similar to the VAR.\textsuperscript{44} We further modify the fiscal policy rule as described above. All other parameter values are the same as in the simple model.

### 5.4 Model vis-à-vis empirical evidence

We now compare the model’s predictions to the VAR. The blue lines in Figure 1 plot the impulse responses to an expansionary monetary policy shock in the full model. Recall that the black lines in Figure 1 are the point estimates obtained from the VAR. Two aspects of the model’s parametrization are chosen to directly match the VAR by construction. First, as it is standard, the size of the shock is chosen to match the decline of the one-year nominal interest rate, plotted in the top left panel. Second, and as discussed above, the parameters of the fiscal policy rule are chosen to match the dynamics of real public debt, plotted in the bottom right panel.

The remaining four panels inform on the model’s quantitative performance vis-à-vis the empirical VAR estimates. The top right panel shows that the inflation dynamics predicted by the model is similar to the VAR, although the initial spike in the model is somewhat larger than the VAR’s point estimate. The middle left panel shows that the model can account for much of the increase in durables expenditures. Like the VAR, the model predicts a hump-shaped increase in durables expenditures. Compared to the predictions of the simple model, displayed in Figure 2, the full model predicts a much more persistent increase in durables expenditures, due to the introduction of search and matching frictions. The responses of non-durables consumption and total consumption expenditures in the model are also in line with the VAR evidence: although the model responses are quantitatively smaller than the point estimates, they fall comfortably within the 95-percent confidence bands. We conclude that the model can quantitatively mimic, to a large extent, the empirical responses obtained from the estimated VAR.\textsuperscript{45}

Finally, let us elaborate on how the presence of search and matching frictions affects the impact of monetary policy shocks on the real economy. In the full model, the labour supply channel is absent and aggregate output is determined by firms’ hiring decisions. In this

\textsuperscript{44}The response of the interest rate in the VAR features some oscillations, which we do not try to match.

\textsuperscript{45}Both Figure 1 and Figure 2 are consistent with the VAR evidence provided by Uhlig (2005) who follows an agnostic identification approach and emphasizes that his empirical evidence is consistent with either an increase, a decrease, or no change in output following a monetary shock.
economy, the household sector can increase real savings not only through consumer durables, but also via investment in firm equity. An increased desire to save among households pushes up the market value of the firms, which encourages vacancy posting and boosts employment.\footnote{From Equation (26) it can be seen that an increase in the discount factor, $\lambda_{t,t+1}$, leads to an increase in the firm value, $V_t$. The free-entry condition dictates that an increase in $V_t$ must be offset by a decline in $\lambda_t$, the rate at which vacancies are filled. From the matching function it follows that hiring increases.} Thus, in this version of the model aggregate output increases because of an increase in labor demand rather than in labor supply. Furthermore, aggregate output dynamics are governed by the employment rate, which is a slow-moving state variable that adds to the degree of endogenous persistence in the model.

6 Concluding remarks

We study the redistributive and aggregate effects of monetary policy in an economy in which the government is a large net debtor. An expansionary open market operation causes a downward revaluation of public debt and a negative wealth effect for the private sector as a whole, as households’ revaluation losses are not fully compensated by fiscal rebates. Households respond to the fall in wealth by increasing their saving rate, which pushes down the real interest rate. Lower interest rates generate a substitution towards durable goods, causing a boom in the durable good sector. In the simple model, aggregate hours worked increase due to a labour supply effect. With search and matching frictions, aggregate hours increase as firms post more vacancies. In all, the expansionary OMO causes an increase in output driven by the durable good sector. This response, together with the redistributive effects embedded in the model are consistent with the empirical evidence on the effects of monetary interventions in the US economy. In this respect, our paper provides new evidence that that following an unexpected monetary policy expansion, the real value of public debt falls and the price level increases. (The results indicate a swift and significant response of the aggregate price level, without the so called “price puzzle” obtained using other identification strategies.)

Our model thus offers a setting consistent with i) the way in which Central Banks affects the policy rate; ii) empirical estimates on how such changes affects the macroeconomy and more specifically, the durable good sector and the real value of public debt; and iii) empirical evidence on the distributional effects of monetary policy. Our results address the challenge
posed by Barsky, House and Kimball (2007), who pointed out to a counterfactual prediction of the standard New Keynesian representative-agent model with durable goods. The mechanism emphasized in our model can thus be used to complement the workhorse New Keynesian model in monetary policy analyses.

The model also shows that implementation matters: specifically, expansionary OMO can have sharply different effects from helicopter drops. We stress that in economies with a largely indebted government sector, monetary policy can have significant fiscal repercussions and it is hence important to take them into account to fully understand the effect of monetary interventions. In other words, fiscal policy, even if passive, can play a critical role on how monetary policy affects the macroeconomy. Understanding how the government redistributes its losses or windfalls through spending, investment, and taxes is important and we plan to study this second round of redistributions in future work.

References


Appendix

In this Appendix we present additional evidence supplementing the empirical results, provide full derivation of the model equations and study extensions of the model that allow for search and matching frictions in the labour market as well as wage rigidity.

We replicate the VAR of Gertler and Karadi (2015) and compare their results to ours. The figure below plots the responses for the 1-year rate and the CPI level in our VAR specification (black line, shaded areas are confidence bands), as well as those from Gertler and Karadi’s specification (red line).\footnote{In the main text, we plot the annualized monthly inflation rate rather than the price level.} For the sake of comparison, we scale the shock such that impact response of the 1 year rate coincides in the two specifications. While in our specification the CPI level declines somewhat more strongly in the initial periods following the shock, the overall responses are very similar.

![Response Comparison Graph]

A2. The response of different demographic groups

Wong (2014) explores the response of expenditures to monetary policy shocks by different demographic groups. In this Section, we replicate her results and decompose expenditures in durables and nondurables to check the soundness of our model. We find that nondurable expenditures respond very little to monetary policy expansions, as in the aggregate results—and consistent with our model. This is true for all demographic groups. Durable good expenditures are the key variable responding to monetary interventions. Consistent with our model, we find that the increase in expenditures during a monetary expansion is almost entirely driven by the response of working-age people; hence as in the model we present, the relative response of durables by working-age vis-à-vis retired agents, increases significantly.
following an expansion. In what follows, we describe the data and approach.

A2.1. Data

The longitudinal data is based on the microdata of the consumer expenditure survey obtained from the ICPSR at the University of Michigan for years 1980-2007. Each household is surveyed for 4 subsequent quarters, where they report monthly expenditures at a disaggregated level. Information about the household demographics and finances are also available. Following Aguiar and Hurst (2012), only households that respond for all quarters (that is, with at least 12 months of data) are kept. Moreover, we keep only urban households, as rural households were not surveyed in the first covered years. This leaves a total of about 80,000 households.

Our measure of durables includes residential investments and other long-term expenditures such as vehicle purchases (new cars, parts) or recreational equipment. Nondurables include services as well as food, alcohol, tobacco, clothing, fossils consumption, and other miscellaneous categories.

The identified monetary policy shocks were obtained through the methodology of Romer and Romer (2004), extended up to 2007.

A2.2. Method

We run the following regression:

\[
\Delta \ln y_{it} = \sum_{s=0}^{20} \beta_s mps_{t-s} + \text{Dummies} + e_{it}\]

Where the left-hand-side is the log-change in consumption for household \(i\) at time \(t\), \(mps_{t-s}\) is the monetary policy shock at \(t\) with \(s\) lags, and dummies include household and cohort (the year of birth) fixed-effects, as well as family size, the only demographic variable whose coefficient is significant. As we work in log-changes, we compute the cumulative IRF, that is, the cumulative sum of the beta coefficients. For the 95 percent confidence interval band, we follow Romer & Romer (2004)’s Monte Carlo approach in that we draw 10,000 coefficients from a multivariate normal distribution with mean vector and variance-covariance matrix.

---

48The CEX data is available up to 2012, but the analysis here is restricted to take place before 2007.
from the OLS regression. For each of these draws, the cumulative IRF is computed, and the 2.5 and 97.5 quantiles are kept to produce the bands of the confidence interval. Then, this regression is run for the entire cohort, and for different age groups.

**A2.3. Results**

The responses of household expenditures to expansionary monetary policy shocks suggest that the increase in consumption is triggered mainly by young households (25-34), as is shown in Figure A4. More specifically, the increase is preliminary due to the durable good response, as Figure A5 indicates.

Figure A4. Response of Total Expenditures by Age group

![Figure A4](attachment:image.png)

Note: The plots show the response of total expenditures to an identified monetary policy shock using Romer&Romer dates. Shaded areas show 90 percent confidence bands.
Figure A5. Response of Durable and Non-durable Good Expenditures by Age group

Note: The plots show the response of durable and non-durable good expenditures to an identified monetary policy shock using Romer&Romer dates. Shaded areas show 90 percent confidence bands.
A3. Model derivations

This Section derives the present-value budget constraint of the government and provides details on the model and the solution strategy.

A3.1 The government’s budget constraint

The consolidated government budget constraint in real terms can be written as:

$$b^g_t + b^{cb}_t - m_t = \frac{1 + r_{t-1}}{1 + \pi_t} \left( b^g_{t-1} + b^{cb}_{t-1} \right) - \frac{m_{t-1}}{1 + \pi_t} - \tau^g_t$$

where \( \tau^g \equiv \nu \rho_R \tau^N_t + \nu (1 - \rho_R) \tau^W_t + (1 - \nu) \tau^R_t \) is the total transfer to the households. We now derive the present-value government budget constraint, see also Ireland (2005). Define:

$$\omega_{t+1} \equiv \frac{1 + r_t}{1 + \pi_{t+1}} \left( b^g_t + b^{cb}_t \right) - \frac{m_t}{1 + \pi_{t+1}}$$

and use this definition to express the period-\( t \) budget constraint as:

$$\omega_{t+1} = \frac{1 + r_t}{1 + \pi_{t+1}} \left( \frac{1 + r_{t-1}}{1 + \pi_t} \left( b^g_{t-1} + b^{cb}_{t-1} \right) - \frac{m_{t-1}}{1 + \pi_t} - \tau^g_t + \frac{r_t}{1 + r_t} m_t \right),$$

Also, define \( D_s \) as in the main text note that \( \frac{1 + r_s}{1 + \pi_{s+1}} D_{s+1} = D_s \). Consider budget constraint for period \( s \) and multiply both sides by \( D_{s+1} \):

$$D_{s+1} \omega_{s+1} = D_s \left( \omega_s - \tau^g_s + \frac{r_s}{1 + r_s} m_s \right).$$

Sum all constraints from period \( t \) to infinity:

$$\sum_{s=t}^{\infty} D_{s+1} \omega_{s+1} = \sum_{s=t}^{\infty} D_s \left( \omega_s - \tau^g_s + \frac{r_s}{1 + r_s} m_s \right),$$

where we impose the limit condition \( \sum_{s=\infty} D_s \omega_s = 0 \). Finally, rearrange to obtain:

$$\sum_{s=t}^{\infty} D_s \left( \frac{r_s}{1 + r_s} m_s - \tau^g_s \right) = \frac{m_{t-1} - (1 + r_{t-1}) \left( b^g_{t-1} + b^{cb}_{t-1} \right)}{1 + \pi_t}.$$
Furthermore, in the representative agent version of the model we can express the household’s first-order condition for money and bonds, respectively, as:

\[ U_{c,t} = U_{m,t} + \beta E_t \frac{1}{1 + \pi_{t+1}} U_{c,t+1} \]

\[ U_{c,t} = \beta E_t \frac{1 + r_t}{1 + \pi_{t+1}} U_{c,t+1} \]

which can be combined as:

\[ U_{c,t} = \frac{1 + r_t}{r_t} U_{m,t} \]

Under the logarithmic preferences assumed in Section 3.1.8 this equation becomes \( \mu c_t = \frac{r_t}{1 + r_t} m_t \). Given that non-durable consumption is not affected by monetary policy in the representative agent version, it follows that \( \frac{r_t}{1 + r_t} m_t \) is not affected either.

### A3.2 Solving the model

The model is solved using first-order perturbation (linearization). This part of the Appendix describes the first-order conditions for the optimization problems of the individuals and discusses aggregation of the individuals’ choices.

**Retired agents and aggregation.** Although the model features a representative working-age agent, there is wealth heterogeneity among retired agents. Typically, dynamic models with a large number of heterogeneous agents are challenging to solve. For our model, however, it turns out that the decision rules of the retirees are linear in wealth, which implies that aggregation is straightforward. Hence we can solve for aggregates without reference to the distribution of wealth among retired agents. Wealth heterogeneity between working-age and retired agents, however, is a key factor driving aggregate dynamics.

We exploit that the use of first-order perturbation implies certainty equivalence (see Schmitt-Grohé and Uribe (2004)). As a consequence, first-order approximations to the equilibrium laws of motion of the model coincide with those obtained for a version without aggregate uncertainty.\(^{49}\) In what follows, we therefore omit expectations operators.\(^{50}\)

The first-order conditions for the choices of durables, money and bonds by a retired

---

\(^{49}\)Both versions preserve \emph{idiosyncratic} uncertainty.

\(^{50}\)Alternatively, one could first linearize the model equations and then perform the steps described below.
household $i$ can be written, respectively, as:

$$U_{c,i,t} = U_{d,i,t} + \beta (1 - \rho_x) (1 - \delta) U_{c,i,t+1},$$

$$U_{c,i,t} = U_{m,i,t} + \frac{\beta (1 - \rho_x)}{1 + \pi_{t+1}} U_{c,i,t+1},$$

$$U_{c,i,t} = \frac{\beta (1 - \rho_x) (1 + r_t)}{1 + \pi_{t+1}} U_{c,i,t+1}.$$

Now introduce four auxiliary variables $\gamma_{c,i,t} \equiv \frac{c_{i,t}}{a_{i,t}}$, $\gamma_{d,i,t} \equiv \frac{d_{i,t}}{a_{i,t}}$, $\gamma_{m,i,t} \equiv \frac{m_{i,t}}{a_{i,t}}$ and $\gamma_{b,i,t} \equiv \frac{b_{i,t}}{a_{i,t}}$. The crucial step is to show that there are four restrictions that pin down $\gamma_{c,i,t}$, $\gamma_{d,i,t}$, $\gamma_{m,i,t}$ and $\gamma_{b,i,t}$ as functions of only aggregate variables. To find these coefficients, first combine the first-order conditions to obtain:

$$U_{c,i,t} = U_{d,i,t} + (1 - \delta) (1 + \pi_{t+1}) (U_{c,i,t} - U_{m,i,t})$$

$$U_{c,i,t} = (1 + r_t) (U_{c,i,t} - U_{m,i,t})$$

Under the assumed nested CES preferences we obtain:

$$U_{c,i,t} = x_{i,t}^{\frac{\gamma_{c,i,t}}{\gamma_{c,i,t}}} \left[ \frac{c_{i,t}^{\gamma_{c,i,t}}}{c_{i,t}} + \eta d_{i,t}^{\gamma_{d,i,t}} + \mu m_{i,t}^{\gamma_{m,i,t}} \right] \left[ \frac{c_{i,t}^{\gamma_{c,i,t}}}{c_{i,t}} \right]$$

$$U_{d,i,t} = x_{i,t}^{\frac{\gamma_{d,i,t}}{\gamma_{d,i,t}}} \eta d_{i,t}^{\gamma_{d,i,t}}$$

$$U_{m,i,t} = x_{i,t}^{\frac{\gamma_{m,i,t}}{\gamma_{m,i,t}}} \mu m_{i,t}^{\gamma_{m,i,t}}.$$

The combined first-order conditions become:

$$\gamma_{c,i,t} = \eta \gamma_{d,i,t} + (1 - \delta) (1 + \pi_{t+1}) \left( \gamma_{c,i,t} - \mu \gamma_{m,i,t} \right)$$  \hspace{1cm} (27)

$$\gamma_{c,i,t} = (1 + r_t) \left( \gamma_{c,i,t} - \mu \gamma_{m,i,t} \right)$$  \hspace{1cm} (28)

To get the third restriction, consider the Euler equation for bonds, which can be written as:

$$\left( \frac{x_{i,t}}{x_{i,t+1}} \right)^{\frac{-\gamma_{b,i,t}}{\gamma_{b,i,t}}} \left( \frac{c_{i,t}}{c_{i,t+1}} \right)^{\frac{\gamma_{b,i,t}}{\gamma_{b,i,t}}} = \frac{\beta (1 - \rho_x) (1 + r_t)}{1 + \pi_{t+1}}$$  \hspace{1cm} (29)
and use the fact that \( a_{i,t+1} = \left( (1 - \delta) \gamma_{d,i,t} + \gamma_{m,i,t} \frac{1}{1 + \pi_t} + \frac{(1 + r_t)\gamma_{b,i,t}}{1 + \pi_t} \right) a_{i,t} \) to write:

\[
\frac{c_{i,t}}{c_{i,t+1}} = \frac{\gamma_{c,i,t}}{\gamma_{c,i,t+1} \left( (1 - \delta) \gamma_{d,i,t} + \gamma_{m,i,t} \frac{1}{1 + \pi_t} + \frac{(1 + r_t)\gamma_{b,i,t}}{1 + \pi_t} \right)}
\]

\[
\frac{x_{i,t}}{x_{i,t+1}} = \left[ \left( \frac{\frac{c_{i,t}}{c_{i,t+1}} + \frac{\gamma_{c,i,t}}{\gamma_{c,i,t+1}} + \frac{\gamma_{d,i,t}}{\gamma_{d,i+1}} + \frac{\gamma_{m,i,t}}{\gamma_{m,i,t+1}} + \frac{\gamma_{b,i,t}}{\gamma_{b,i,t+1}}}{\gamma_{c,i,t+1}} \right) \right]^{-\frac{\gamma_{c,i,t+1}}{\gamma_{c,i,t}} - 1}
\]

The budget constraint gives the fourth restriction since it can be written as:

\[
\gamma_{c,i,t} a_{i,t} + \gamma_{d,i,t} a_{i,t} + \gamma_{m,i,t} a_{i,t} + \gamma_{b,i,t} a_{i,t} = a_{i,t}
\]

or:

\[
\gamma_{c,i,t} + \gamma_{d,i,t} + \gamma_{m,i,t} + \gamma_{b,i,t} = 1
\]

Equations (27)-(30) pin down \( \gamma_{c,i,t}, \gamma_{d,i,t}, \gamma_{m,i,t} \) and \( \gamma_{b,i,t} \) as functions of only aggregate variables, as we have substituted out individual wealth from all the equations. Hence we can omit individual \( i \)-subscripts for these variables. Given the average wealth level among retired agents, \( a^{R, t} \), we can now compute averages for retired agents’ decision variables as \( c^{R, t} = \gamma_{c,t} a^{R, t} \), \( d^{R, t} = \gamma_{d,t} a^{R, t} \), \( m^{R, t} = \gamma_{m,t} a^{R, t} \) and \( b^{R, t} = \gamma_{b,t} a^{R, t} \). Note that these objects do not depend on the distribution of wealth among retired agents. Finally, we can express aggregate wealth owned by retired agents as:

\[
a^{R, t} = (1 - \rho_x) \left( (1 - \delta) d^{R, t-1} + \frac{m^{R, t-1} + (1 + r_{t-1})b^{R, t-1}}{1 + \pi_t} \right)
\]

\[
+ \rho_R (1 - \rho_x) \frac{\nu}{1 - \nu} \left( (1 - \delta) d^{W, t-1} + \frac{m^{W, t-1} + (1 + r_{t-1})b^{W, t-1}}{1 + \pi_t} \right).
\]

**Working agents.** As discussed in the main text there is effectively a representative working-age agent. Its first-order conditions for the choices of labour, durables, money and bonds can
be written, respectively, as:

\[
U_{c,t}^W = \zeta h_t^c \\
U_{c,t}^W = U_{d,t}^W + \beta (1 - \rho_R) (1 - \delta) U_{c,t+1}^W + \beta \rho_R (1 - \rho_x) (1 - \delta) U_{c,t+1}^W \\
U_{c,t}^W = U_{m,t}^W + \beta \left( \frac{1 - \rho_R}{1 + \pi_{t+1}} \right) U_{c,t+1}^W + \beta \rho_R (1 - \rho_x) \frac{U_{WR}^W}{1 + \pi_{t+1}} \\
\frac{U_{c,t}^W}{(1 + r_t)} = \beta \frac{1 - \rho_R}{1 + \pi_{t+1}} U_{c,t+1}^W + \beta \rho_R (1 - \rho_x) \frac{U_{WR}^W}{1 + \pi_{t+1}}.
\]

Here, \(U_{c,t}^W\) and \(U_{c,t}^{WR}\) are the marginal utility of non-durables of working and newly retired agents, respectively, which satisfy:

\[
U_{c,t}^W = (x_t^W)^{\alpha_{c,t+1} \frac{1}{\epsilon}} (c_t^W)^{\frac{1}{\epsilon}} \\
U_{d,t}^W = (x_t^W)^{\frac{-\alpha_{c,t+1}}{\epsilon}} \eta (d_t^W)^{\frac{-1}{\epsilon}} \\
U_{m,t}^W = (x_t^W)^{\frac{-\alpha_{c,t+1}}{\epsilon}} \mu (m_t^W)^{\frac{-1}{\epsilon}} \\
U_{c,t}^{WR} = (x_t^{WR})^{\frac{-\alpha_{c,t+1}}{\epsilon}} (c_t^{WR})^{\frac{-1}{\epsilon}}
\]

where \(x_t^{WR} = \left[ (c_t^{WR})^{\frac{-1}{\epsilon}} + \eta (d_t^{WR})^{\frac{-1}{\epsilon}} + \mu (m_t^{WR})^{\frac{-1}{\epsilon}} \right]^{\frac{1}{-\epsilon}}\). Note that for the newly retired agents it holds that \(c_t^{WR} = \gamma_{c,t} d_t^W\). Finally, the wealth of a working agent can be expressed as:

\[
a_t^W = (1 - \rho_R + \rho_R \rho_x) \left( (1 - \delta) d_t^W + \frac{m_t^W + (1 + r_{t-1}) b_t^W}{1 + \pi_t} \right) \\
+ \frac{1 - \nu}{\nu} \rho_x \left( (1 - \delta) d_t^R + \frac{m_t^R + (1 + r_{t-1}) b_t^R}{1 + \pi_t} \right).
\]
The full system. We are now ready to collect the equations and summarize the entire model. Retired agents:

\[
\begin{align*}
\gamma_{c,t} &= \eta \gamma_{d,t} + (1 - \delta) (1 + \pi_{t+1}) \left( \gamma_{c,t} - \mu \gamma_{m,t} \right) \\
\gamma_{c,t} &= (1 + r_t) \left( \gamma_{c,t} - \mu \gamma_{m,t} \right) \\
\beta (1 - \rho_x) (1 + r_t) &= \left( \Phi_t \right)^{-\sigma_{c+1} \varepsilon_c} \left( \frac{\gamma_{c,t+1}}{\gamma_{c,t+1} + \eta \gamma_{d,t+1} + \mu \gamma_{m,t+1}} \right)^{\frac{1}{\varepsilon_c}} \\
\Phi_t &= \left( \frac{\gamma_{c,t} + \eta \gamma_{d,t} + \mu \gamma_{m,t}}{\gamma_{c,t+1} + \eta \gamma_{d,t+1} + \mu \gamma_{m,t+1}} \right)^{\frac{1}{\varepsilon_c}} \\
\phi_t &= \gamma_{c,t} \alpha_t^R \\
\phi_t &= \gamma_{d,t} \alpha_t^R \\
\phi_t &= \gamma_{m,t} \alpha_t^R \\
\phi_t &= \gamma_{b,t} \alpha_t^R \\
\phi_t &= (1 - \rho_x) \left( 1 - \delta \right) \phi_{t-1} + \frac{m_t^R + (1 + r_{t-1}) b_{t-1}^R}{1 + \pi_t} \\
&\quad + \rho_R (1 - \rho_x) \frac{\nu}{1 - \nu} \left[ (1 - \delta) d_{t-1}^W + \frac{m_{t-1}^W + (1 + r_{t-1}) b_{t-1}^W}{1 + \pi_t} \right] \\
\phi_t &= \phi_t^R + m_t^R + b_t^R
\end{align*}
\]
Working agents:

\[
(x_t^W)^{-\frac{\sigma+1}{\epsilon}} (c_t^W)^{\frac{1}{\epsilon}} = \zeta h_t^c \tag{42}
\]

\[
(c_t^W)^{\frac{1}{\epsilon}} = \eta (d_t^W)^{\frac{1}{\epsilon}} + \beta (1 - \rho_R) (1 - \delta) \left( \frac{x_{t+1}^W}{x_t^W} \right)^{-\frac{\sigma+1}{\epsilon}} (c_{t+1}^W)^{\frac{1}{\epsilon}} + \beta \rho_R (1 - \rho_x) (1 - \delta) \left( \frac{x_{t+1}^{WR}}{x_t^W} \right)^{-\frac{\sigma+1}{\epsilon}} (c_{t+1}^W)^{\frac{1}{\epsilon}}, \tag{43}
\]

\[
(c_t^W)^{\frac{1}{\epsilon}} = \mu (m_t^W)^{\frac{1}{\epsilon}} + \beta \left( \frac{1 - \rho_R}{1 + \pi_{t+1}} \right) \left( \frac{x_{t+1}^W}{x_t^W} \right)^{-\frac{\sigma+1}{\epsilon}} (c_{t+1}^W)^{\frac{1}{\epsilon}} + \beta \rho_R (1 - \rho_x) \left( \frac{x_{t+1}^{WR}}{x_t^W} \right)^{-\frac{\sigma+1}{\epsilon}} (c_{t+1}^W)^{\frac{1}{\epsilon}}, \tag{44}
\]

\[
(c_t^W)^{\frac{1}{\epsilon}} = \frac{\beta (1 - \rho_R) (1 + rt)}{1 + \pi_{t+1}} \left( \frac{x_{t+1}^W}{x_t^W} \right)^{-\frac{\sigma+1}{\epsilon}} (c_{t+1}^W)^{\frac{1}{\epsilon}} + \beta \rho_R (1 - \rho_x) \frac{(1 + rt)}{1 + \pi_{t+1}} \left( \frac{x_{t+1}^{WR}}{x_t^W} \right)^{-\frac{\sigma+1}{\epsilon}} (c_{t+1}^W)^{\frac{1}{\epsilon}}. \tag{45}
\]

\[
a_t^W = (1 - \rho_R + \rho_R \rho_x) \left( (1 - \delta) d_{t-1}^W + \frac{n_{t-1}^W + (1 + r_{t-1}) b_{t-1}^W}{1 + \pi_t} \right) + \frac{1 - \nu}{\nu} \rho_x \left( (1 - \delta) d_{t-1}^R + \frac{n_{t-1}^R + (1 + r_{t-1}) b_{t-1}^R}{1 + \pi_t} \right) \tag{46}
\]

\[
c_t^W + d_t^W + m_t^W + b_t^W = a_t^W + h_t^W + \tau_t^s \tag{47}
\]

\[
c_t^{WR} = \gamma_c a_t^W \tag{48}
\]

\[
x_t^{WR} = \left[ (\gamma_c a_t^W)^{\frac{1}{\epsilon_1}} + \eta (\gamma d_t^W)^{\frac{1}{\epsilon_1}} + \mu (\gamma m_t^W)^{\frac{1}{\epsilon_1}} \right]^{\frac{1}{\epsilon_1}} \tag{49}
\]

\[
x_t^W = \left[ (c_t^W) + \eta (d_t^W)^{\frac{1}{\epsilon_1}} + \mu (m_t^W)^{\frac{1}{\epsilon_1}} \right]^{\frac{1}{\epsilon_1}} \tag{50}
\]

Government policy:

\[
r_{t-1} \left( \frac{b_{t-1}^G + b_{t-1}^{CB}}{1 + \pi_t} \right) = \nu (1 - \rho_R) \tau_t^s \tag{51}
\]

\[
\frac{m_t}{m_{t-1}} (1 + \pi_t) = 1 + z_t \tag{52}
\]

\[
z_t = \xi (m_t - m_{t-1}) + \varepsilon_t \tag{53}
\]
Market clearing:

\[
\begin{align*}
    c_t + d_t &= \nu h^W_t + (1 - \delta) d_{t-1} \\
    c_t &= \nu c_t^W + (1 - \nu) c_t^R \\
    d_t &= \nu d_t^W + (1 - \nu) d_t^R \\
    m_t &= \nu m_t^W + (1 - \nu) m_t^R \\
    0 &= b_t^G + b_t^{CB} + \nu b_t^W + (1 - \nu) b_t^R
\end{align*}
\]

These are 28 equations in 28 variables, being \(c_t, c_t^R, c_t^{WR}, c_t^W, d_t, d_t^R, d_t^W, m_t, m_t^R, m_t^W, b_t^R, b_t^W, b_t^G, b_t^{CB}, x_t^W, x_t^{WR}, \Phi_t, \gamma_{c,t}, \gamma_{d,t}, \gamma_{m,t}, \gamma_{b,t}, h^W_t, r_t, \pi_t, \tau_t, z_t, a_t^R, \) and \(a_t^{WR}\). We leave out the government’s budget constraint, which is redundant by Walras’ law.

**Special cases** We present two simplifying special cases of the model.

**Special case 1** \((\epsilon = 1)\). When the utility elasticity \(\epsilon\) equals one, the utility function becomes a Cobb-Douglas basket nested in a CRRA function:

\[
U(c_{i,t}, d_{i,t}, m_{i,t}) = \left(\frac{c_{i,t} d_{i,t}^\sigma m_{i,t}^{\mu}}{1 - \sigma}\right)^{1-\sigma} - 1
\]

and the marginal utilities become \(U_{c_{i,t}} = \frac{x_{i,t}^{1-\sigma}}{c_{i,t}}\) \(U_{d_{i,t}} = \eta \frac{x_{i,t}^{1-\sigma}}{d_{i,t}}\) and \(U_{m_{i,t}} = \mu \frac{x_{i,t}^{1-\sigma}}{m_{i,t}}\). In the system to be solved, we correspondingly set:

\[
\begin{align*}
    x_t^W &= (c_t^W) (d_t^W)^\eta (m_t^W)^\mu \\
    x_t^{WR} &= (c_t^{WR}) (d_t^{WR})^\eta (m_t^{WR})^\mu \\
    \Phi_t &= \left(\frac{\gamma_{c,t}}{\gamma_{c,t+1}}\right) \left(\frac{\gamma_{d,t}}{\gamma_{d,t+1}}\right)^\eta \left(\frac{\gamma_{m,t}}{\gamma_{m,t+1}}\right)^\mu \left(1 - \delta\right) \frac{\gamma_{d,t}}{1 + \pi_{t+1}} + \frac{\gamma_{m,t}}{1 + \pi_{t+1}} + (1 + r_t) \frac{\gamma_{b,t}}{1 + \pi_{t+1}}
\end{align*}
\]

**Special case 2** \((\sigma = \epsilon = 1)\). When both the risk aversion coefficient \(\sigma\) and the utility elasticity \(\epsilon\) are unity, the utility function further simplifies to:

\[
U(c_{i,t}, d_{i,t}, m_{i,t}) = \ln c_{i,t} + \eta \ln d_{i,t} + \mu \ln m_{i,t}
\]

52
and the marginal utilities become $U_{c,i,t} = \frac{1}{c_{i,t}}$, $U_{d,i,t} = \frac{d}{d_{i,t}}$ and $U_{m,i,t} = \frac{m}{m_{i,t}}$. We can therefore set $(x_t^W)^{-\frac{\sigma+1}{\epsilon}} = (x_t^{WR})^{-\frac{\sigma+1}{\epsilon}} = (\Phi_t)^{-\frac{\sigma+1}{\epsilon}} = 1$.

**Additional References**
