Optimal Inflation and the Identification of the Phillips Curve*

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Abstract

Several academics and practitioners have pointed out that inflation follows a seemingly exogenous statistical process, unrelated to the output gap, leading some to argue that the Phillips curve has weakened or disappeared. In this paper we explain why this seemingly exogenous process arises, or, in other words, why it is difficult to empirically identify a Phillips curve, a key building block of the policy framework used by central banks. We show why this result need not imply that the Phillips curve does not hold – on the contrary, our conceptual framework is built under the assumption that the Phillips curve always holds. The reason is simple: if monetary policy is set with the goal of minimising welfare losses (measured as the sum of deviations of inflation from its target and output from its potential), subject to a Phillips curve, a central bank will seek to increase inflation when output is below potential. This targeting rule will impart a negative correlation between inflation and the output gap, blurring the identification of the (positively sloped) Phillips curve. We discuss different strategies to circumvent the identification problem and present evidence of a robust Phillips curve in US data.

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1. Introduction

A number of recent papers have pointed out that inflation can be approximated (and forecast) by statistical processes unrelated to the amount of slack in the economy (Atkeson and Ohanian, 2001; Stock and Watson, 2007, 2009; Dotsey, Fujita and Stark, 2018; Cecchetti et al., 2017; Forbes, Kirkham and Theodoridis, 2017). The empirical disconnect between inflation and various measures of slack has been interpreted by some commentators as evidence that the Phillips curve (a positive relation between inflation and the output gap) has weakened or even disappeared (Ball and Mazumder, 2011; IMF, 2013; Hall, 2013; Blanchard, Cerutti and Summers, 2015; Coibion and Gorodnichenko, 2015).[12]. On the face of it, a change in the Phillips Curve relationship could have major implications for monetary policy, so the potential causes of any weakening have been an important topic of discussion for policymakers (Draghi, 2017; Carney, 2017*a*; Powell, 2018).

The Phillips curve is one of the building blocks of the standard macroeconomic models used for forecasting and policy advice in central banks. Its empirical elusiveness could challenge the wisdom of these models and the usefulness of their forecasts. Arguably, it even calls in to question part of the rationale for independent, inflation-targeting central banks. Or does it?

In this paper we use a standard conceptual framework to show why:

- the empirical disconnect between inflation and slack is a result to be expected when monetary policy is set optimally; and
- it is also perfectly consistent with an underlying stable and positively sloped Phillips curve.

More specifically, our framework is built under the assumption that the Phillips curve always holds (an assumption we later corroborate in the data). In other words, in our model, inflation depends positively on the degree of slack in the economy. We also allow for cost-push shocks that can lead to deviations from the curve, but without altering its slope. Monetary policy is set with the goal of minimising welfare losses (measured as the sum of the quadratic deviations of inflation from its target and of output from its potential), subject to the Phillips curve or aggregate supply relationship. In that setting a central bank will seek to increase inflation when output is below its potential. This targeting rule imparts a negative correlation between inflation and the output gap, blurring the identification of the (positively sloped) Phillips curve.[3]

The paper is extended along five dimensions. First, we study differences in the solutions between discretion – our baseline case in which the monetary authority cannot commit to a future path of inflation and the output gap – and the case of commitment, in which the authority credibly commits to a future plan. We show that the main intuition goes through in both cases. The difference lies in

[1]For a selection of the vast media comment on the issue, see articles in the *Financial Times*, *Wall Street Journal* and *The Economist* and opinion piecies by Alan Blinder, Paul Krugman and Lawrence Summers.

[2]The output gap is defined as the deviation of output from its potential; in the original paper of Phillips (1958), the focus was the negative relationship between wage inflation and unemployment

[3]This result follows straightforwardly from the basic New Keynesian model as derived in Clarida, Galí and Gertler (1999), while similar results would obtain in the classic setting of Barro and Gordon (1983).

the implied properties of the statistical process for inflation generated by the optimal policy in each case. In the simple framework studied here, the greater degree of inertia under optimal commitment also offers one potential solution to the identification problem.

A second extension introduces shocks to the targeting rule. These shocks can be interpreted as lags in monetary transmission; as shocks to the monetary policy instrument rule; or, in a multi-region setting, as idiosyncratic demand shocks affecting different regions or countries within a monetary union. We show that the relative variance of these shocks vis-a-vis the cost-push shocks is key for the empirical identification of the Phillips curve using standard regression analysis. This result also rationalises the findings of the vast empirical literature that uses identified monetary policy shocks to estimate the transmission of monetary policy. Effectively, well-identified monetary policy shocks should help in retrieving the Phillips curve.

Third, we study a multi-region (multi-country or multi-sector) setting with a common central bank and discuss conditions under which regional (or sectoral) data can help mitigate the bias from the endogeneity of monetary policy. The discussion, however, also underscores some of the limitations faced by regional analysis.

A fourth extension discusses the estimation of a wage-Phillips curve and compares the identification challenges with those faced in the price-Phillips curve.

The final extension departs from the stylised New Keynesian model of Clarida, Galí and Gertler (1999) and studies the aggregate supply constraint in a large-scale DSGE model of the type designed for forecasting and policy analysis in central banks. In such larger models, the concept of a single, structural relationship between inflation and the output gap is no longer well defined: their reduced-form correlation varies according to which shock hits the economy. Nonetheless, we show that the intuition from the *structural* Phillips curve in the basic model continues to apply to the *reduced-form* Phillips curve in larger-scale DSGE models. In the model of Burgess et al. (2013), designed for policy use at the Bank of England, a positively sloped reduced-form Phillips curve is present when policy is set according to an estimated Taylor rule. But under optimal discretionary policy the slope of the curve changes sign.

We next turn to practical attempts to address the identification issue we raise, focusing on US data. The simultaneity bias arises due to the behaviour of monetary policy in partially accommodating cost-push shocks to the Phillips curve. It is magnified because monetary policy seeks to offset any demand shocks that might otherwise help identify the curve. We discuss three practical solutions that attempt to circumvent these issues by isolating the remaining demand-driven variation in inflation.

First, econometricians can attempt to control for cost-push and other trade-off inducing shocks to aggregate supply, in line with the approach proposed by Gordon (1982). This helps to minimise the remaining cost-push driven variance in the error term, leaving only demand shocks that can correctly identify the Phillips curve. In practice, however, the success of this approach requires successfully controlling for each and every trade-off inducing shock affecting the economy. The ability to do this may be limited in the recent past, where energy price shocks are less dominant than in the 1970s.

Second, if econometricians can find suitable instrumental variables, they can purge their output gap data of any cost-push shocks, leaving only the demand variation needed to consistently estimate the Phillips curve. With highly autocorrelated cost-push shocks (precluding the use of lagged variables as instruments), using measures of monetary policy or other demand shocks may be one set of appropriate external instruments (Barnichon and Mesters, 2019). But if the variance of monetary policy shocks has fallen since the early 1980s and/or the effect of a shock of a given size has reduced, as suggested by Boivin and Giannoni (2006), then these instruments may be too weak to provide a practical solution in the recent data.

We next present evidence on our third solution, using cross-sectional regional variation in unemployment to identify the Phillips curve. Following Fitzgerald and Nicolini (2014) and concurrently with a recent paper by Hooper, Mishkin and Sufi (2019), we use US metropolitan area price and unemployment data to estimate a Phillips curve including metropolitan area fixed effects, to control for time-invariant regional heterogeneity in the natural rate of unemployment, as well as time fixed effects to control for variation over time in monetary policy and the aggregate natural rate. Under our preferred specification, a steeper Phillips curve re-emerges, with a short-run slope at least twice as large as any of our estimates using aggregate data.

The idea that endogenous stabilisation policy can hide structural relationships in the data is an old one, going back at least to Kareken and Solow (1963)'s critique of Milton Friedman's evidence on the effect of money on income. They pointed out that a monetary policy that perfectly stabilised nominal income would completely offset any underlying relationship between income and measures of money. Similarly, Brainard and Tobin (1968) present a model in which the lead-lag correlation between money and income following an exogenous change in fiscal policy depends on the endogenous monetary policy response. Goldfeld and Blinder (1972) study the bias arising from reduced-form OLS estimation of fiscal and monetary policy multipliers when both policies are set endogenously. These identification issues are very well known in the context of monetary policy effects: Cochrane (1994) sets out how they were the primary motivation for the literature on identified monetary policy shocks.

Several authors over the years have also highlighted the general result that under an optimal control policy, the correlation between a policy target and policy instrument should be driven towards zero: including Worswick (1969); Peston (1972); Goodhart (1989) and (in the context of the Phillips curve) Mishkin (2007).[4] This point is perhaps also a specific example of Goodhart's law: 'that any observed statistical relationship will tend to collapse once pressure is placed upon it for control purposes' (Goodhart, 1984).

In a forecasting context, Woodford (1994) shows that if an indicator is a poor predictor of inflation that may just be because monetary policy is already responding to it appropriately. Similarly, Edge

[4]See also a series of blogposts by Nick Rowe (e.g. https://worthwhile.typepad.com/worthwhile_canadian_initi/2010/12/milton-friedmans-thermostat.html), who uses the analogy (credited to Milton Friedman) of the relationship between a room's temperature and its thermostat.

and Gürkaynak (2010) point out that unforecastable inflation is a prediction of DSGE models in which policymakers respond aggressively to stabilise inflation. They suggest that forecasting performance during the Great Moderation is therefore a poor metric of the models' success, since policymakers acted strongly to offset the forecastable component of inflation. Perhaps because measures of slack are one step removed from monetary policy instruments, these issues seem to have been often neglected in discussions of the Phillips curve.

Of course, that the empirical Phillips curve may vary with monetary policy was one of the examples given by Lucas (1976) in his critique. Given their original emphases, both the Lucas critique and Goodhart's law are more often applied to explain *suboptimal* stabilisation policies. Indeed, several authors have explicitly modelled a situation where policymakers set monetary policy based on a misspecified or unidentified Phillips curve (Haldane and Quah, 1999; Primiceri, 2006; Sargent, Williams and Zha, 2006). In these papers, mistakes or imperfect information on the part of policymakers can lead to changes in inflation expectations that cause the reduced-form Phillips curve to disappear.[5]

In contrast, we show how a disappearing reduced-form Phillips curve is also a natural consequence of successful monetary policy. The idea that improvements in monetary policy have flattened the slope of the reduced-form Phillips curve is often ascribed to researchers and policymakers at the Federal Reserve.[6] Most articulations of this view have tended to focus on the role of improved monetary policy in anchoring inflation expectations (e.g. Williams, 2006; Bernanke, 2007; Mishkin, 2007; Bernanke, 2010).[7]

Our point is closely related but distinct: even in a purely static setting in which expectations play no role, the structural relationship between slack and inflation can be masked by the conduct of monetary policy. This effect of monetary policy on the Phillips curve has also been highlighted at various times over the years in the literature and by policymakers. Roberts (2006), Carlstrom, Fuerst and Paustian (2009) and recently Bullard (2018) highlight the role of monetary policy on inflation dynamics in simple New Keynesian models with Taylor rules, while Nason and Smith (2008), Mavroeidis, Plagborg-Møller and Stock (2014) and Krogh (2015) explore Phillips curve identification in detail in similar setups. Haldane and Quah (1999), using a similar model to the one we adopt, show that optimal discretionary policy can flatten or reverse the slope of the reduced-form Phillips curve. Fitzgerald and Nicolini (2014) make the same point using an old Keynesian framework, and like us, use regional data from US metropolitan areas to recover a steeper Phillips curve slope.

Despite these papers, a surprisingly bulky literature has continued searching for a Phillips curve in the data without addressing the key identification challenge. Our first contribution is to frame the issue as simply as possible: as a classical identification problem; and as one that is present in the

[5] Relatedly, others have examined mechanisms through which changes in monetary policy behaviour could change the underlying structural Phillips curve. For example, Ball, Mankiw and Romer (1988) showed how increases in average inflation rates, by changing the frequency with which firms reset prices, could change the deep parameters that determine its slope.

[6] Gordon (2013) terms it the 'Fed view'.

[7] The effect of endogenous monetary policy on inflation expectations also features in some leading explanations of the 'missing disinflation' following the financial crisis, such as Del Negro, Giannoni and Schorfheide (2015).

same standard New Keynesian equations that are taught in graduate economics textbooks. Given that the New Keynesian framework forms the basis for the models used in central banks, it is also a natural platform to respond to criticisms of that framework, and of policymakers for their continued reliance on Phillips curve relationships. A second contribution is to show the extent to which these conclusions generalise to a more complex dynamic, stochastic, general equilibrium (DSGE) quantitative framework and to different measures of inflation and slack, including articulating why one should expect to see stronger wage Phillips curve relationships in the data. Our simple analytical framework also enables us to rationalise findings in various strands of the empirical literature and to critically evaluate some of the practical solutions to the identification problem. This discussion motivates our empirical focus on using regional variation to recover a steeper Phillips curve slope for the United States.

The paper is organised as follows. Section 2 introduces a simple model of optimal policy embedding the Phillips curve and illustrates the 'exogeneity result' or disconnect between equilibrium inflation and output gap under the assumption that the monetary authority cannot commit to a future path of inflation (discretion). Section 3 illustrates the empirical identification problem. Section 4 presents and discusses extensions of the model and notes some conceptual solutions to achieve identification. Section 5 examines the solutions in practice using national and metropolitan area data for the US. Section 6 contains concluding remarks.

2. Optimal inflation in the basic New Keynesian model

This section uses an optimal monetary policy framework to illustrate why, in equilibrium, one should expect inflation to follow a seemingly exogenous process, unrelated (or even negatively related) to measures of slack.

To explain the intuition as starkly as possible, we use the canonical New Keynesian model, as derived in Clarida, Galí and Gertler (1999), Woodford (2003) and elsewhere. Here we closely follow the textbook exposition from Galí (2008). For now, we dispense with the usual IS equation determining aggregate demand. This equation is necessary only to determine how policy is implemented. In the basic model it does not constrain equilibrium outcomes, so we can equivalently consider the policymaker as directly choosing the output gap as their policy instrument. Our model therefore consists of just two equations: a Phillips curve and a description of optimal monetary policy.

The (log-linearised) New Keynesian Phillips curve is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \tag{1}$$

where π_t is the deviation of inflation from its target; x_t is the output gap, measured as the difference between output and its potential level[8] and u_t is a cost-push shock that follows an exogenous AR(1)

[8] In the full model derived in Galí (2008), this is the welfare-relevant gap between output and its efficient level.

process with persistence ρ ($u_t = \rho u_{t-1} + \epsilon_t$, where ϵ_t are i.i.d. and mean zero). We *assume* that the Phillips curve has a strictly positive slope, denoted by $\kappa > 0$.

The Phillips curve is evidently alive and well in the model: it is the *only* equation making up its non-policy block. By construction, we have a positively sloped Phillips curve. Increases in the output gap clearly increase inflation and falls in the output gap reduce it. Nonetheless, once we augment the model with a description of optimal monetary policy, this relationship will not be apparent in the data. Inflation will instead inherit the properties of the exogenous shock process u_t.

To show this, we assume that the policymaker sets monetary policy optimally under discretion. Period by period, she minimises the following quadratic loss function

$$L_t = \pi_t^2 + \lambda x_t^2$$

subject to the constraint (1) and taking expectations of future inflation as given.[9] The solution to the minimisation problem is the policymaker's optimal targeting rule

$$\pi_t = -\frac{\lambda}{\kappa} x_t \tag{2}$$

When faced with a positive cost-push shock that creates a trade-off between the inflation and output stabilisation objectives, the policymaker balances them, creating a negative output gap to reduce the degree of above-target inflation. The relative weight placed on each objective depends on the policymaker's preference parameter λ.

The Phillips curve (1) and optimal targeting rule (2) together completely determine the path of inflation in the model. We can solve for equilibrium inflation by using (2) to substitute out for x_t in (1), and by iterating forward to obtain

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda(1 - \beta\rho)} u_t \tag{3}$$

In equilibrium, inflation deviations are at all times perfectly proportional to the exogenous cost-push shock. In other words, with a constant target, equilibrium inflation itself behaves as an exogenous process. In the limit, when the monetary authority does not put any weight on the output gap ($\lambda = 0$), inflation equals the target rate, a point previously made by Haldane and Quah (1999).

This behaviour is entirely consistent with recent empirical work by Cecchetti et al. (2017) and Forbes, Kirkham and Theodoridis (2017) suggesting that inflation data in the US and the UK can be modelled as an exogenous statistical process, unrelated or negatively related to measures of slack.[10] But crucially, the basic theory is also built under the assumption that monetary policy is at all times constrained by a working Phillips curve. There is no discrepancy between the two results. The

[9] Clarida, Galí and Gertler (1999) show how minimising such a loss function is equivalent to maximising the welfare of the representative agent in the model. But it can alternatively be motivated as a simple way to capture the preferences enshrined in the mandates of modern (flexible) inflation targeting central banks: see Carney (2017*b*), for example.

[10] It is also consistent with the observation that in larger DSGE models such as Smets and Wouters (2007), inflation is largely explained by exogenous markup shocks (King and Watson, 2012).

Phillips curve may be the correct structural model of the inflation process, but that does not mean that one should observe it in the empirical relationship between (equilibrium levels of) inflation and the output gap.

The reason is simple. The policymaker in the model is able to set policy to achieve any desired level of the output gap. Successful monetary policy should lean against any undesirable deviations in output from potential, which would otherwise cause inflationary or deflationary pressures. Precisely because monetary policy can be used to offset the effect of such output gaps on inflation, their effect on inflation should not be visible in the data.

Optimal monetary policy does not seek to eliminate all output volatility: from (2) we can see that in response to cost-push shocks, the policymaker will prefer to tolerate output deviations from potential. But such shocks impart a negative correlation between inflation and output, rather than a positive one. Again, the more successful monetary policy is in managing any trade-offs between inflation and output, the more it will blur the underlying positive Phillips curve correlation.

To summarise, we have shown that with an optimizing monetary policy, equilibrium levels of inflation inherit the statistical properties of exogenous cost-push shocks. This does not necessarily tell us that the Phillips curve is not present. In the model, the Phillips curve exists and policymakers are completely aware of its existence. But because they know exactly how the curve operates, they are able to perfectly offset its effects on equilibrium inflation.[11]

3. Phillips curve identification

As may already be apparent from the discussion in Section 2, regression analysis will have difficulty in recovering the Phillips curve. Figure 1 shows data simulated from the model described by (1) and (2), with parameters calibrated as in Galí (2008). Specifically, the slope of the Phillips curve is set at $\kappa = 0.1275$, the policymaker's weight on output deviations relative to quarterly inflation is set as $\lambda = 0.0213$, or around one-third relative to annualised inflation. The discount factor is set to $\beta = 0.99$ and the persistence of the cost-push shock to $\rho = 0.5$.

Of course, there is no Phillips curve visible in the simulated data. As can be seen from the line of best fit, a naive OLS regression of inflation on the output gap,

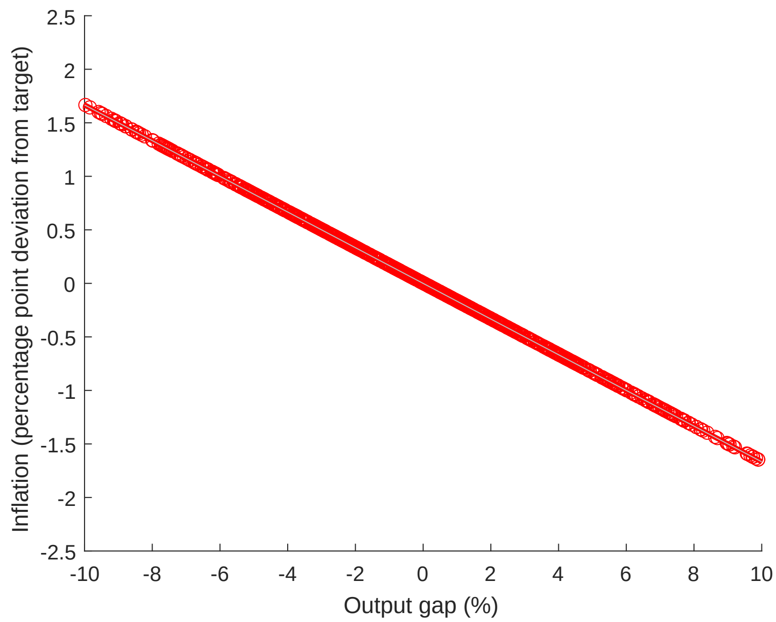
$$\pi_t = \gamma_1 x_t + \varepsilon_t \tag{4}$$

will produce a negative parameter estimate, $\hat{\gamma}_1 = -\frac{1}{6}$, reflecting the targeting rule (2), rather than a consistent estimate of the positive slope of the Phillips curve. Many papers have focused on the difficulty of controlling for inflation expectations in Phillips curve estimation, but the problem here is a more straightfoward one.[12]

[11] Stock and Watson (2009) raise the possibility that, despite its failure to forecast or explain the data, the Phillips curve is still useful for conditional forecasting. They pose the question '...suppose you are told that next quarter the economy would plunge into recession, with the unemployment rate jumping by 2 percentage points. Would you change your inflation forecast?'

[12] See Nason and Smith (2008), Mavroeidis, Plagborg-Møller and Stock (2014) and Krogh (2015) for discussions.

Figure 1: *Inflation/output gap correlation in model-simulated data*



Notes: 1000 periods of data are simulated from the model described by (1) and (2). We draw each ϵ_t from a standard normal distribution.

The identification problem is a simple case of simultaneity bias. The regressor x_t is correlated with the error term ε_t. The naive econometrician does not observe the Phillips curve in the data. Rather, he or she observes equilibrium inflation and output gap outturns: which are the intersection of the Phillips curve (1) and the targeting rule (2). In fact, the case here is an extreme one: the regressor and the error are perfectly negatively correlated.[13]. The issue is completely analagous to the classic case of simultaneity bias: jointly determined supply and demand equations.

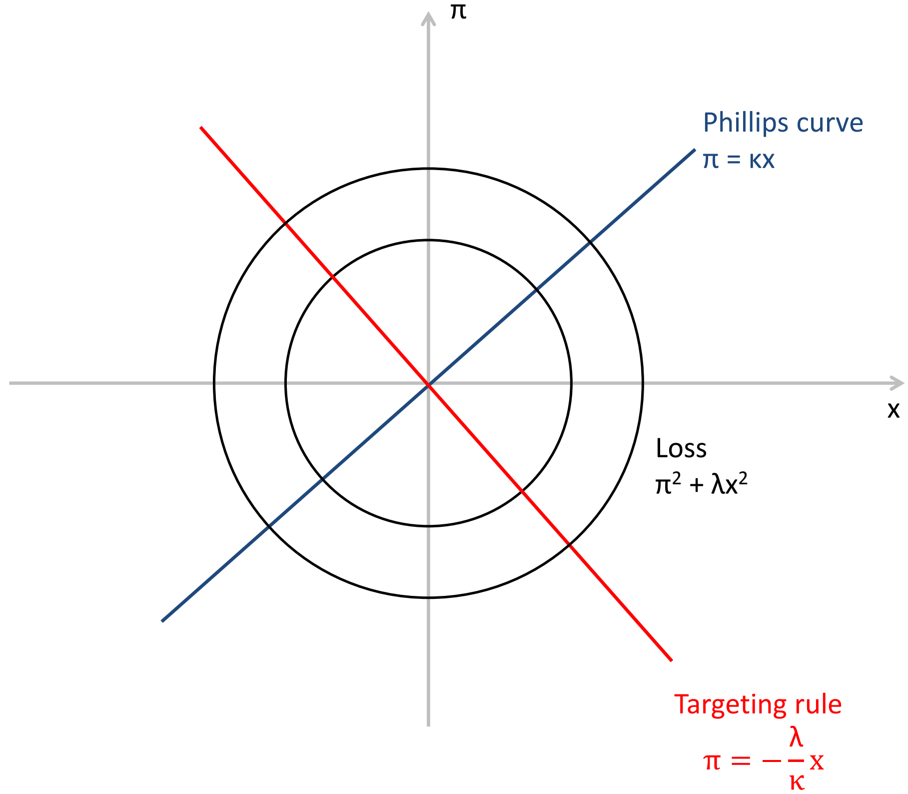
To show the identification challenge, we first plot the two model equations in Figure 2.[14] The Phillips curve (1) is in blue, the optimal targeting rule (2) in red, while the black circles index the policymaker's loss function at different levels of loss. The observed inflation-output gap pairs are the equilbrium where the two lines intersect. With no cost-push shocks to the Philips curve, the first-best outcome of at target inflation and no output gap is feasible, so the lines intersect at the origin.

When the upward sloping Phillips curve is subject to cost-push shocks, the equilibrium shifts to different points along the optimal targeting path, shown in Figure 3. But with monetary policy set optimally, there are no shifts along the Phillips curve: at all times the equilibrium remains on the

[13]Using (3) to substitute out for π_t in (2) gives the equilibirum evolution of the output gap $x_t = -\frac{\kappa}{\kappa^2+\lambda(1-\beta\rho)}u_t$. While the regression error term is equal to $\varepsilon_t = u_t + \beta E_t \pi_{t+1} = (1 + \frac{\rho\lambda}{\kappa^2+\lambda(1-\beta\rho)})u_t$

[14]This graphical illustration of optimal discretionary policy is from Seneca (2018): we are grateful to him for making it available to us. A similar graphical exposition appears in Carlin and Soskice (2005), as well as in papers at least as far back as Kareken and Miller (1976) (with thanks to Marc Giannoni for alerting us to the latter reference).

Figure 2: *Graphical illustration of optimal monetary policy under discretion*



negatively sloped optimal targeting rule line. As a result, the simulated data trace out the optimal targeting rule, not the Phillips curve. The estimated coefficient is $\hat{\gamma}_1 = -\frac{\lambda}{\kappa} = -\frac{1}{6}$.

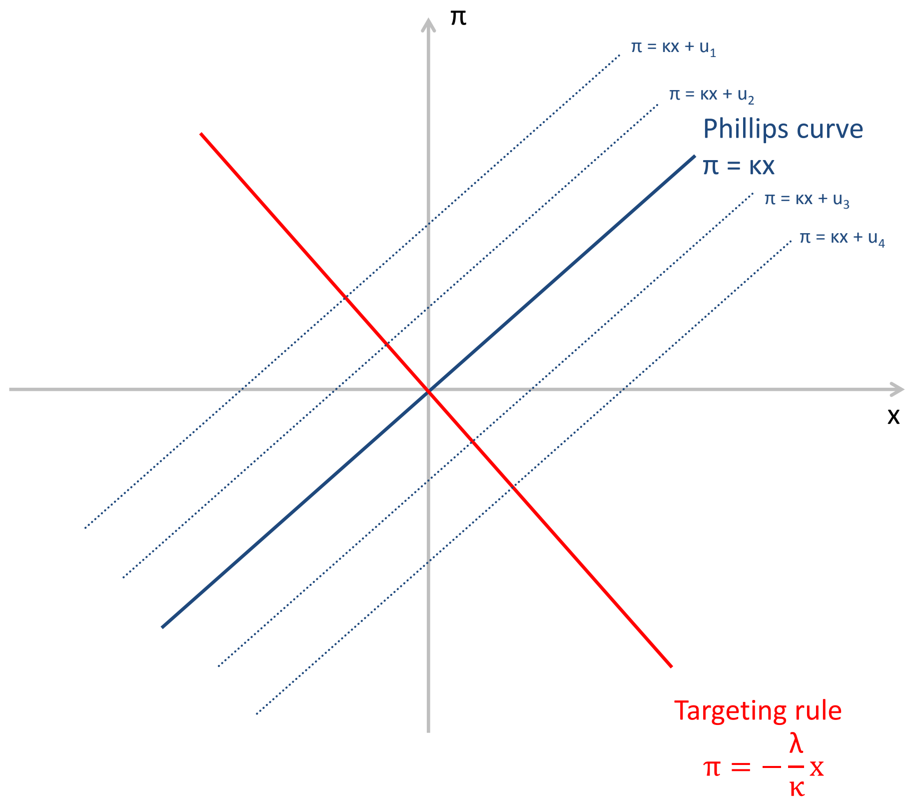
The issue is that the Phillips curve is not identified. Our simple set-up has no exogenous variables shifting monetary policy. Worse, the only shocks are to the equation of interest, so the estimated parameter is almost entirely unrelated to the slope of the Phillips curve.[15] The problem is the same one that arises when trying to identify a supply curve while only observing equilibrium quantities and prices. Without any exogenous demand shifter, there is no way of doing so.

4. Extensions to the basic model and solutions to the estimation challenge

In this section we study a number of extensions to the basic model. For each extension, we discuss whether and how it can help solving the Phillips curve's empirical identification problem. In Subsection 4.1, we discuss the case in which the monetary authority can commit to a path of inflation and output gap. In Subsection 4.2, we allow for shocks to the targeting rule and we discuss how they link to the identified monetary policy shocks in the monetary policy transmission literature. In Subsection 4.3, we study a multi-region setting. In Subsection 4.4 we discuss the mapping into a wage Phillips curve. In Subsection 4.5 we extend our analysis to explore the effect of monetary policy on the Phillips curve in larger DSGE models.

[15] Other than the fact that the slope of the Phillips curve happens to appear in the optimal targeting rule.

Figure 3: *Graphical illustration of optimal discretionary policy in response to cost-push shocks*



4.1. Commitment

First, we show that our main results are unchanged when the monetary policymaker is able to commit to a future plan for inflation and the output gap. In Sections 2 and 3 we assumed that the policymaker was unable to commit. There are a range of practical issues that may make commitment difficult: monetary policy committees often have changes in membership and future policymakers may not feel bound by prior commitments and perhaps relatedly, successful commitment requires that promises are credible, even when they are time inconsistent. Nonetheless, the optimal commitment policy is able to achieve better outcomes in the face of cost-push shocks than optimal policy under discretion, so it is important to know how this affects our results.

It turns out that the same intuition holds, although the precise details slightly differ. Again following Galí (2008), when the policymaker instead minimises the loss function

$$L = E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) \tag{5}$$

subject to the sequence of Phillips curves given by (1) for each period. This gives a pair of optimality conditions

$$\pi_0 = -\frac{\lambda}{\kappa} x_0 \tag{6}$$

$$\pi_t = -\frac{\lambda}{\kappa}(x_t - x_{t-1}) \tag{7}$$

These can be combined to give the targeting rule under commitment

$$p_t = -\frac{\lambda}{\kappa}x_t \tag{8}$$

where p_t is the log deviation of the price level from its level in period -1. Substituting $p_t - p_{t-1}$ for π_t in (1) and substituting out x_t using (8) gives a difference equation in p_t. Galí (2008) shows the solution for this in terms of the previous period's price level and the current period cost-push shock. Iterating backwards and then taking the first difference gives equilibrium inflation

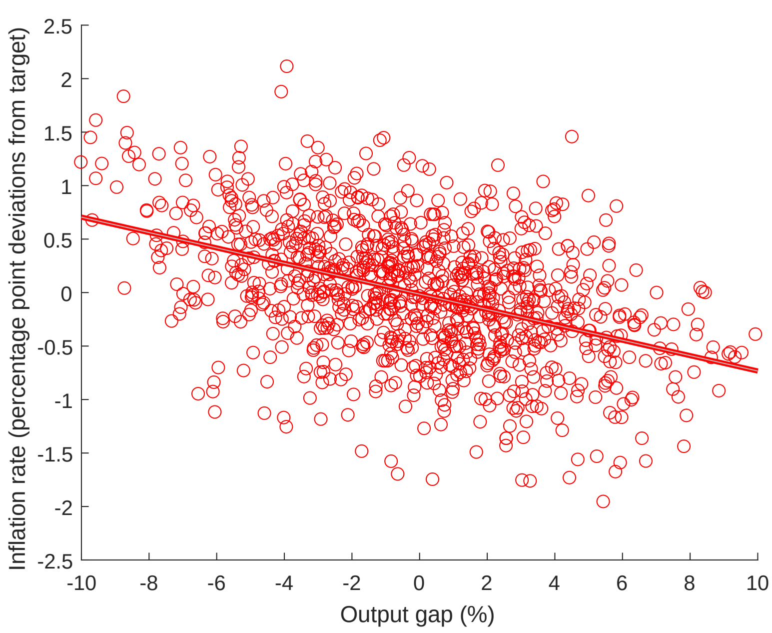
$$\pi_t = \frac{\delta}{1-\delta\beta\rho}(u_t - (1-\delta)\sum_{i=0}^{t-1}\delta^{t-1-i}u_i) \tag{9}$$

where $\delta \equiv \frac{((\lambda(1+\beta)+\kappa^2)-((\lambda(1+\beta)+\kappa^2)^2-4\beta\lambda^2)^{0.5})}{2\lambda\beta}$. Substituting into (7) and iterating backwards gives the equilibrium output gap

$$x_t = \frac{-\delta\kappa}{\lambda(1-\delta\beta\rho)}\sum_{i=0}^{t}\delta^{t-i}u_i \tag{10}$$

Equilibrium inflation under optimal commitment policy depends solely on the cost-push shock

Figure 4: *Inflation/output gap correlation in model-simulated data: optimal commitment*



Notes: 1000 periods of data are simulated from the model described by (1) and (7). We draw each ϵ_t from a standard normal distribution.

process. The equilibrium path is quite different to that under discretion, however. At any point in time inflation displays history dependence, depending on the entire history of cost-push shocks rather than just the one in the current period.

Simple regressions will again fail to uncover the Phillips curve. The only difference is that under commitment, the optimal targeting rule imposes a negative correlation between the output gap and the *price level*. The relationship between inflation and the output gap in the simulated data shown in Figure 4 is noisier, but shows no sign of the Phillips curve embedded in the model. The OLS estimate of γ in (4) gives the coefficient $\hat{\gamma}_1 = -0.085$.

At least in the simple framework here, the history-dependence of optimal commitment policy also suggests a straightforward solution to the identification problem. From (10), the equilibrium output gap will be correlated with its own lagged values. This policy-induced persistence means that the lagged output gap can be used as an instrument for the current output gap. Intuitively, the policymaker chooses to create an output gap even after the cost-push shock has disappeared. They commit to do so in order to achieve better inflation outcomes when the shock originally occurs. The policymaker therefore optimally reintroduces the positive Phillips curve relation that is absent under optimal discretion. As a result, in the simple case here, a suitable choice of instrument will be able to recover the true Phillips curve slope.

4.2. Shocks to the targeting rule

The previous sections have illustrated how successful monetary policy might mask the underlying structural Phillips curve in the data. We now show that the opposite is also true in our model: if monetary policy is set far from optimally, the Phillips curve is likely to reappear.

So far we have assumed policymakers can implement monetary policy by directly choosing their desired observable output gap each period. But alas in practice, policymaking is not quite so simple. In empirical studies we observe lags between changing policy and its impact on the output gap and inflation, which means that in practice central banks are inflation *forecast* targeters (Svensson, 1997; Haldane, 1998). Forecast errors will therefore inject noise into the targeting rule. Potential output is unobservable, so the output gap must be estimated (with error). And the effect of the policy instruments actually available (typically the central bank policy rate and forward guidance on its future path; as well as quantitative easing) on the target variables is also unknown. Errors from any of these sources will insert noise into the desired balance between inflation and output gap deviations. These various shocks to the targeting rule correspond closely to the typical interpretations of identified monetary policy shocks in the empirical literature on this topic (Christiano, Eichenbaum and Evans, 1996, 1999; Romer and Romer, 2004*b*; Faust, Swanson and Wright, 2004; Bernanke, Boivin and Eliasz, 2005; Olivei and Tenreyro, 2007; Gertler and Karadi, 2015; Cloyne and Hürtgen, 2016). That literature is able to identify a positively correlated response of inflation and the output gap to monetary policy shocks, in line with the results below.

Returning to optimal policy under discretion, we model implementation errors by including an

AR(1) shock process e_t in the targeting rule (2) to give

$$\pi_t = -\frac{\lambda}{\kappa}x_t - e_t \tag{11}$$

where $e_t = \rho_e e_{t-1} + \zeta_t$ and ζ_t is zero-mean and i.i.d. with variance σ_e^2.[16] We can show that equilibrium inflation and the output gap now both have an additional term proportional to e_t. Respectively, they are given by $\pi_t = s_1\lambda u_t - s_2\kappa e_t$ and $x_t = -s_1\kappa u_t - s_2(1-\beta\rho_e)e_t$, where $s_1 \equiv \frac{1}{\lambda(1-\beta\rho)+\kappa^2}$ and $s_2 \equiv \frac{\kappa}{\lambda(1-\beta\rho_e)+\kappa^2}$.

With shocks to the targeting rule, neither equation is identified. The equilibrium values of inflation and the output gap both depend on a combination of both shocks. Consequently, if either equation is estimated by OLS, its regressor will be correlated with the regression error term and the resulting parameter estimate inconsistent. In particular, it follows from substituting the equilibrium values of π_t and x_t into the definition of the OLS estimator in the regression (4) that

$$\text{plim}(\hat{\gamma}) = \frac{\text{plim}(\frac{1}{T}\sum_{t=1}^{T} x_t\pi_t)}{\text{plim}(\frac{1}{T}\sum_{t=1}^{T} x_t^2)} = \frac{\frac{-\lambda}{\kappa}\frac{s_1^2(1-\rho_e^2)}{s_2^2(1-\rho^2)}\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2} + (1-\beta\rho_e)\kappa\frac{\sigma_e^2}{\sigma_u^2+\sigma_e^2}}{\frac{s_1^2(1-\rho_e^2)}{s_2^2(1-\rho^2)}\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2} + (1-\beta\rho_e)^2\frac{\sigma_e^2}{\sigma_u^2+\sigma_e^2}} \tag{12}$$

The size of the simultaneity bias to each equation depends on the relative variances of the shocks.[17] Figure 5 plots simulated data for three cases. We set $\rho_e = 0.5$ and set the other parameters as before. First, the red circles show the case where the cost-push shock has a variance 100 times larger than the targeting rule shock. These look almost identical to the case with only a cost-push shock: the circles trace out the targeting rule. Second, the green circles show the case when the shocks have equal variance. The slope is still negative, but flatter. The final case gives the cost-push shock a variance 100 times smaller than the targeting rule shock, and the data trace out a positively sloped line.

Table 1: *OLS regressions of inflation on the output gap in the simulated data*

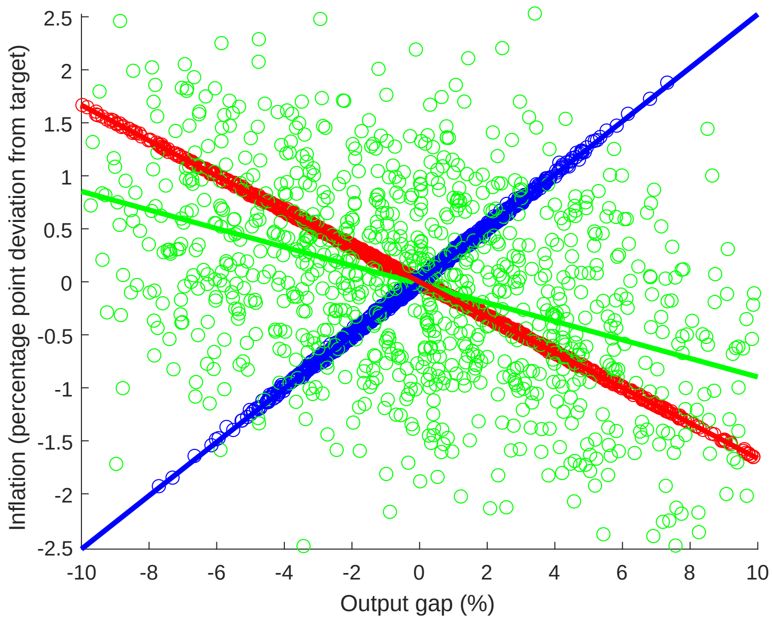
	(1)	(2)	(3)	(4)	(5)	(6)
	$\frac{\sigma_u^2}{\sigma_e^2} = 100$	$\frac{\sigma_u^2}{\sigma_e^2} = 100$	$\frac{\sigma_u^2}{\sigma_e^2} = 1$	$\frac{\sigma_u^2}{\sigma_e^2} = 1$	$\frac{\sigma_u^2}{\sigma_e^2} = 0.01$	$\frac{\sigma_u^2}{\sigma_e^2} = 0.01$
LHS variable	π_t	$\pi_t - E_t\pi_{t+1}$	π_t	$\pi_t - E_t\pi_{t+1}$	π_t	$\pi_t - E_t\pi_{t+1}$
x_t	-0.1667	-0.1805	-0.0873	-0.0792	0.2523	0.1275

Notes: Table shows the OLS regression coefficients of OLS for the shock distributions described in the notes to Figure 5. Specifications (2), (4) and (6) (perfectly) control for inflation expectations by subtracting from the dependent variable the true value of $\beta E_t\pi_{t_1}$. The true slope of the Phillips curve is $\kappa = 0.1275$, while the true slope of the optimal targeting rule is $-\frac{\lambda}{\kappa} = -0.1667$.

[16] Clarida, Galí and Gertler (1999) and Svensson and Woodford (2004) show in the basic New Keynesian model that when there are policy control lags that mean all variables are predetermined in advance, up to an unforecastable shock, the optimal targeting rule will take exactly this form, where e_t is the forecast error. We subtract it from the right hand side of (11) to match the usual convention that a positive monetary policy shock involves a policy tightening.

[17] Carlstrom, Fuerst and Paustian (2009) show a similar equation to illustrate the OLS estimate bias in their framework.

Figure 5: *Inflation/output gap correlation in model-simulated data: optimal discretion with shocks to the targeting rule*



Notes: 1000 periods of data are simulated from the model described by (1) and (11). The green circles show the case when each ϵ_t and ζ_t is drawn from a standard normal distribution. The blue circles show the case when each ϵ_t is drawn from an N(0,10) distribution and the red circles each ζ_t is instead drawn from an N(0,10) distribution.

Looking at the regression coefficients in Table 1, in the first two cases these are both strongly influenced by the endogenous policy response embodied in the optimal targeting rule. It also makes little difference whether or not the econometrician correctly controls for inflation expectations, which also enter the Phillips curve. In the third case however, the regression coefficient turns positive. The estimate is actually upward biased in specification 5, which omits inflation expectations. Once these are controlled for, the bias becomes very small. The regression correctly identifies the slope of the Phillips curve to four decimal places.

The reason the bias disappears is straightforward. When cost-push shocks have a relatively low variance, most of the variation in the simulated data arises from the shocks to the targeting rule. With the Phillips curve stable, these movements in the targeting rule now trace out the Phillips curve, as shown graphically in Figure 6. This suggests that if we can successfully control for the cost-push shocks u_t in (1), then we may be able to limit the bias in estimates of the Phillips curve.

4.3. Regional Phillips curves

Partly to avoid the difficulties associated with identifying the Phillips curve at the national level, a number of authors have estimated Phillips curves at a more disaggregated, regional or sectoral level (Fitzgerald and Nicolini, 2014; Kiley, 2015; Babb and Detmeister, 2017; Leduc and Wilson, 2017; Tuckett, 2018; Vlieghe, 2018; Hooper, Mishkin and Sufi, 2019). In this subsection we show that in an extended version of the basic model, this may also help the econometrician to identify the aggregate

Phillips curve.

The key to identification is that at the regional level, the endogenous response of monetary policy to demand shocks is switched off, ameliorating the simultaneity bias in estimating aggregate Phillips curves. This point was made by Fitzgerald and Nicolini (2014) as motivation for their estimation of Phillips curves at a regional level. The same logic can explain why the Phillips curve may be more evident in countries within a monetary union such as the euro area.[18]

We assume that the aggregate Phillips curve (1) continues to hold, but that aggregate inflation and the aggregate output gap also depend on the weighted average of inflation and the output gap in each of n regions

$$\pi_t = \sum_{i=1}^{n} \alpha_i \pi_t^i \tag{13}$$

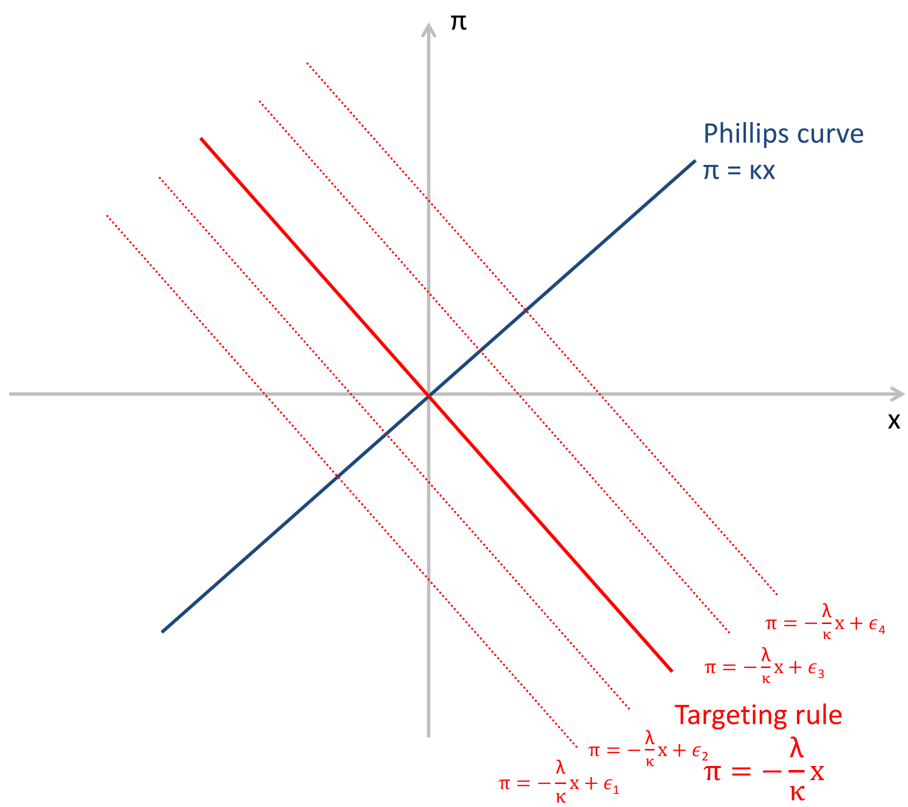
$$x_t = \sum_{i=1}^{n} \alpha_i x_t^i \tag{14}$$

where $\sum_{i=1}^{n} \alpha_i = 1$ and regional inflation is determined by a regional Phillips curve analogous to (1)

$$\pi_t^i = \beta E_t \pi_{t+1}^i + \kappa x_t^i + u_t^i \tag{15}$$

[18]Nakamura and Steinsson (2014) present evidence that endogenous monetary and tax policies reduce national fiscal multipliers relative to local ones.

Figure 6: *Graphical illustration of optimal discretionary policy in response to targeting-rule shocks*



with idiosyncratic cost-push shocks $u_t^i = \rho u_{t-1}^i + \epsilon_t^i$ and ϵ_t^i zero-mean and i.i.d over time, but potentially correlated across regions. We must also specify how idiosyncratic demand shocks and aggregate monetary policy affect the regional output gap with an equation analogous to the IS curve in the basic New Keynesian model, given by

$$x_t^i = E_t x_{t+1}^i - \sigma^{-1}(i_t - E_t \pi_{t+1}^i - r_t^i) \tag{16}$$

where the idiosyncratic demand shocks are given by $r_t^i = \rho_r r_{t-1}^i + e_r^i$ and e_r^i are zero-mean and i.i.d over time, but potentially correlated across regions. The equations can be aggregated together to give the usual aggregate IS relation

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t) \tag{17}$$

We therefore allow inflation and the output gap are determined partly by idiosyncratic shocks to each region, but restrict the monetary policy rate i_t to be the same across all n regions.

We next denote for any regional variable its (log) deviation from the aggregate as $\hat{z}_t^i = z_t^i - \sum_{i=1}^n \alpha_i z_t^i$. We can then subtract (1) from (15) to give a Phillips curve in terms of log deviations from aggregate inflation.

$$\hat{\pi}_t^i = \beta E_t \hat{\pi}_{t+1}^i + \kappa \hat{x}_t^i + \hat{u}_t^i \tag{18}$$

Subtracting (17) from (16) gives an equivalent IS curve

$$\hat{x}_t^i = E_t \hat{x}_{t+1}^i + \sigma^{-1}(E_t \hat{\pi}_{t+1}^i + \hat{r}_t^i) \tag{19}$$

Monetary policy is set (under discretion) by minimising the same aggregate period loss function as in Section 2, subject to the aggregate Phillips curve (1).[19] Policy therefore follows the same targeting rule (2) depending solely on aggregate variables.[20]

The crucial difference to the identification problem at the regional level is that while monetary policy perfectly offsets the aggregate demand shocks, $r_t = \sum_{i=1}^n \alpha_i r_t^i$, it does not respond at all to the idiosyncratic regional deviations from that average, $\hat{r}_t^i$. The regressor in the Phillips curve equation $\hat{x}_t^i$ is now affected by exogenous demand shocks that do not influence the aggregate Phillips curve. As a result, the endogeneity problem is mitigated.

For each region, we can verify that one solution to the model described by (18) and (19) is

$$\hat{\pi}_t^i = c_1(1-\rho)\hat{u}_t^i + c_2 \kappa_i \hat{r}_t^i \tag{20}$$

[19] This differs from the monetary policy that would be welfare-optimal in the model, since welfare would also be lowered by dispersion in prices within a region, even if average inflation was zero. Clarida, Galí and Gertler (2001) show in the context of an open economy model that the welfare-optimal policy would minimise a loss function that included the sum across countries of the squared deviations of inflation, rather than the square of the sum of deviations.

[20] Although to ensure determinacy, the policymaker's instrument rule will need to respond to idiosyncratic variables.

and

$$\hat{x}_t^i = c_1 \rho \sigma^{-1} \hat{u}_t^i + c_2 (1 - \rho_r \beta) \hat{r}_t^i \tag{21}$$

where $c_1 \equiv \frac{1}{(1-\rho)(1-\rho\beta)-\rho\kappa\sigma^{-1}}$ and $c_2 \equiv \frac{\sigma^{-1}}{(1-\rho_r)(1-\rho_r\beta)-\rho_r\kappa\sigma^{-1}}$.[21] Unlike aggregate inflation, which evolves in line with the exogenous shocks to the Phillips curve, regional inflation also depends on idiosyncratic demand shocks. In the simplest case when the shocks are independent and entirely transitory ($\rho = \rho_r = 0$), the equilibrium output gap deviation will be independent of the idiosyncratic cost-push shocks $\hat{u}_t^i$ and a simple regression of $\hat{\pi}_t^i$ on $\hat{x}_t^i$ will give a consistent estimate of κ.

Away from that special case, there remain challenges to identification. First, even if the idiosyncratic cost-push shocks $\hat{u}_t^i$ are uncorrelated with demand (absent any monetary policy response), they will inject additional noise in finite samples. Particularly if there is limited cross-sectional variation in the regional data, this will lead to imprecise estimates of κ. Moreover, in practice the shocks are unlikely to be independent of the forces driving aggregate demand, even absent changes in monetary policy. Many types of regional supply shocks are likely to simultaneously increase regional inflation and reduce regional output below its potential. If such shocks are large, this correlation may still impart a significant negative bias into estimates of κ.

Second, with $\rho > 0$ or $\rho_r > 0$, there will be omitted variable bias unless the econometrician can control for the effect of regional inflation expectations. While possible in principle, reliable data are likely to be less readily available than at the national level. If cross-sectional variation in inflation expectations is important, there is perhaps likely to be more chance of success when estimating at the country level within a single multi-country monetary authority. Alternatively, if that variation is constant over time, it can be controlled for using region fixed effects.

4.4. The wage Phillips curve

While identification of the price Phillips curve is complicated by the endogenous response of optimal monetary policy, the focus of the original Phillips study was the correlation between wage inflation and unemployment in the UK. In this subsection we comment on how optimal monetary policy maps into the original wage Phillips curve relationship between wage inflation and unemployment. Intuitively, one might expect the wage Phillips curve to be less vulnerable to identification issues related to the endogeneity of monetary policy, since wage inflation is one step removed from the price-inflation targeting remit of most central banks.

As well as a different dependent variable (wage inflation rather than price inflation), the typical wage Phillips curve attempts to explain inflation using variation in unemployment or the unemployment gap, rather than the output gap. Using unemployment in the equation is unlikely to solve the identification issues arising from the behaviour of monetary policy for at least two reasons.

[21] While this is one solution, depending on how policy is implemented, there may be a multiplicity of equlibria. It is beyond the scope of this paper to study those, so we assume that the policymaker's instrument rule is able to rule them out. In practice, this will involve responding to deviations of regional inflation or regional output gaps from their equilibrium values, even when those deviations have no impact on aggregate inflation or the aggregate output gap.

First, many central banks' remits explicitly specify unemployment or employment as one of their (secondary or dual) target variables. As such, they will optimally set policy to close any gap between unemployment and its natural rate, unless there is a trade-off between that goal and their inflation targets, in which case they will seek to balance the two goals, as was the case with the output gap in Section 2. Monetary policy will therefore blur the structural relationship between inflation and the unemployment gap in a similar way. Second, even for central banks without an explicit mandate to minimise fluctuations in employment, when there is co-movement between the output gap and the unemployment gap, policy will often implicitly seek to stabilise employment.[22]

There are, however, reasons to think that using wage inflation as the dependent variable might lessen some of the identification problems. Nominal wage rigidities can be incorporated into the basic model in an analogous way to price rigidities, as introduced by Erceg, Henderson and Levin (2000). With both wage and price stickiness, some shocks, such as innovations to firms' desired price-markups, will lead to a wedge between the rate of price inflation and the output gap, but not between the rate of wage inflation and the output gap. Since inflation targeting central banks typically target price inflation, policymakers may respond by adjusting the output gap to achieve their desired trade-off with price inflation. But doing so would lead to variation in wage inflation operating via the wage Phillips curve. Put differently, if some shocks only directly affect the price Phillips curve and not the wage Phillips curve, then the output gap will be correlated with the error term in the former but not the latter, which will be consistently estimated.

The wage Phillips curve may not face quite as severe problems, but there remain limits to how easily it can be identified under optimal monetary policy. First, while there may be some shocks that only affect the price Phillips curve, there are likely to be several more that affect both curves (for a given output gap). Wage mark-up shocks will increase both price and wage inflation relative to the prevailing output gap. Erceg, Henderson and Levin (2000) show that shocks to household consumption or leisure preferences, or to total factor productivity, will conversely move price and wage inflation in opposite directions for a given output gap. Since the inflationary impact of these shocks will lead policymakers to attempt to lean against them via the output gap, this will induce a correlation between the output gap and the shocks affecting the wage Phillips curve (for a given output gap). The direction of the bias will differ according to the shock, but the equation will in general not be identified.

Second, even if price inflation shocks are particularly prevalent, many typical examples of such shocks, such as changes in oil prices, have relatively transitory effects on price inflation. Since monetary policy is typically thought to have its peak effect on inflation with some lag, attempting to offset very transitory shocks may not be possible. As a result, policymakers are perhaps less likely to respond to the very shocks that would otherwise have helped econometricians identify the wage Phillips curve. Conversely, when transitory shocks are affecting price inflation, wage inflation can sometimes give a better signal of underlying price pressures, which may lead policymakers to

[22] Galí (2011) shows how the basic framework can be easily extended to include unemployment in a way that closely resembles the output gap in the basic model.

behave at times as if they were targeting wage inflation.[23]

4.5. Larger DSGE models

In addition to nominal wage rigidities, larger macroeconomic models of the type used for policy analysis in central banks usually have a range of other frictions, additional factors of production and a richer dynamic structure.[24] In this subsection we study how the intuition underlying Phillips curve identification in the basic New Keynesian model translates to the aggregate supply relationship in larger models.

An overriding conceptual issue in larger DSGE models is that there typically is no single, stable Phillips curve relationship between inflation and the output gap. In the basic model the output gap is proportional to firms' real marginal costs, but this is a special case that does not generalise to larger models. The reduced-form Phillips curve correlation therefore varies for different shocks. We illustrate this point in Figure A1 in the online appendix, which shows the inflation-output gap relationship in a large-scale DSGE model *conditional* on each type of shock in the model. We use the COMPASS model, described in Burgess et al. (2013), which was designed for forecasting and policy analysis at the Bank of England. The model is in the tradition of well-known medium-scale DSGE models such as Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007), in which similar findings would emerge, as well as DSGE models used in other central banks. The simulated Phillips curve varies markedly depending on the shock. Conditional on demand-type shocks, such as to government spending or world demand, there is a positive relationship between inflation and the output gap. Conditional on cost-push type shocks to wage or price markups, the correlation turns negative.

Even when we restrict our attention to those shocks we typically think of as demand, there are different reduced-form Phillips curves for different shocks: the investment adjustment cost shock has a slope over twice as steep as a government spending shock, for example. These different reduced-form slopes arise for several reasons. First, the shocks do not all have the same impact on the output gap relative to real marginal costs and inflation. Second, they each have different dynamic effects (some shock processes are estimated to be more persistent than others, for example), which influences the contemporaneous Phillips curve correlations. And related to both points, the simulations incorporate an endogenous monetary policy response via the model's Taylor rule. While the Taylor rule is not sufficient to hide the positive Phillips curve relationships completely, it will be exerting some influence, the scale of which will depend on the specific shock.[25]

Given these conceptual difficulties, how should we think of the Phillips curve in larger DSGE

[23]In addition, the welfare optimal policy in models with sticky wages typically involves placing a positive weight on avoiding wage inflation (Erceg, Henderson and Levin, 2000). But we are not aware of any central banks who officially target wage inflation in practice.

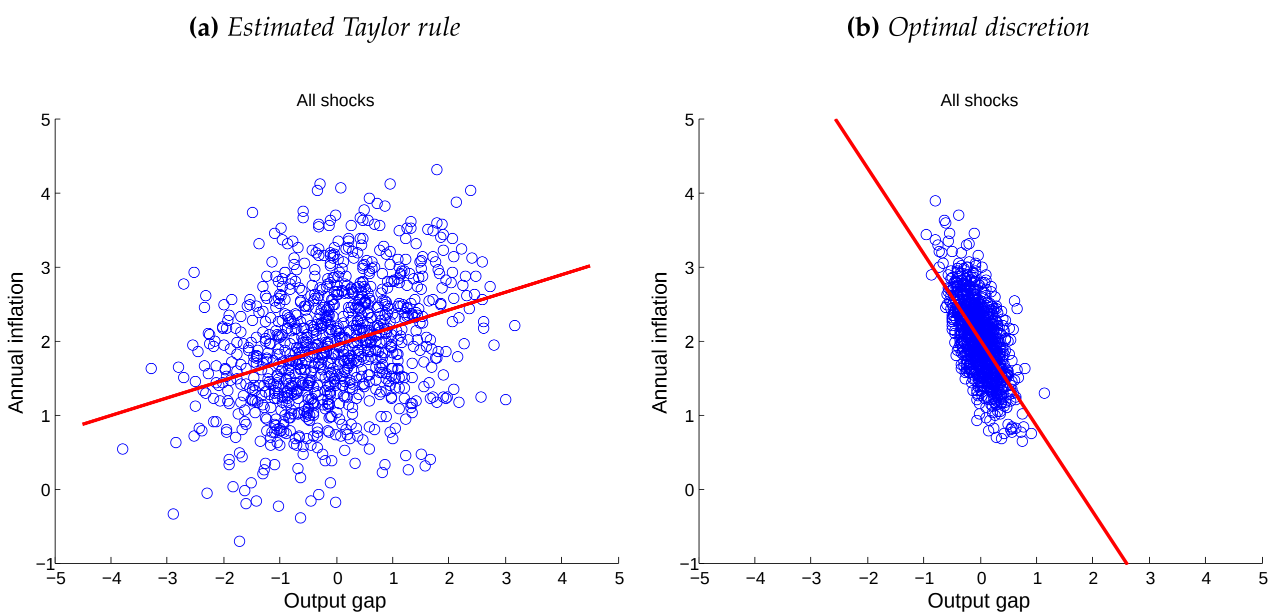
[24]See for example Edge, Kiley and Laforte (2010); Burgess et al. (2013); Brubakk and Sveen (2009); Adolfson et al. (2013) for descriptions of models used respectively at the Federal Reserve Board, the Bank of England, Norges Bank and the Riksbank.

[25]Estimated Taylor rules often find large coefficients on interest rate smoothing, which will limit the amount the policymaker in the model chooses to offset large movements in contemporaneous inflation.

models? One interpretation, consistent with the Phillips curve's inception as an empirical regularity in the UK data, is that is simply the average reduced-form relationship, conditional on a demand shock having occurred. The slope of such an object would clearly change over time if some types of shock became more or less frequent. It would also be vulnerable to the Lucas critique. But if policymakers judged that such changes were relatively slow-moving, they may still find such an empirical Phillips curve a useful input into their decisions.

Under that interpretation, the logic we have outlined for the basic model continues to complicate estimation of empirical Phillips curves in larger models. Figure 7a shows another DSGE simulation using Burgess et al. (2013), this time for all shocks in the model. Despite the presence of supply shocks and an endogenous monetary policy response, a positively sloped Phillips curve emerges.

Figure 7: *Inflation/output gap correlation in simulated data from a large-scale DSGE model.*



Notes: 1000 periods of data are simulated from the model in Burgess et al. (2013) using the MAPS toolkit described in the same paper. Each period a set of unanticipated shocks are drawn independently from a standard normal distribution. The red lines show the lines of best fit from an OLS regression of the simulated annual inflation data on the (contemporaneous) flexible price output gap. The first panel shows the results using the estimated Taylor rule in the model. The second panel replaces the Taylor rule with the optimal discretionary monetary policy, where the policymaker minimises, period by period, an ad hoc loss function containing the discounted sum of squared deviations of annual inflation from target (with a weight of 1) and the output gap (with a weight of 0.25). The solution is calculated using the algorithm of Dennis (2007).

Figure 7b runs an otherwise identical simulation with the model's Taylor rule replaced by the optimal monetary policy under discretion. As in the examples from the basic model, the positively sloped Phillips curve disappears and its estimated sign turns negative. This is true irrespective of the shock.[26]

Even in larger models, we would argue one can still interpret the Phillips curve as a structural equation. Although they need not feature a simple structural relationship between inflation and the output gap, larger New Keynesian models will contain some kind of equivalent aggregate supply constraint. Typically this will contain measures of real marginal costs rather than the output gap.[27] It is also likely to have a richer dynamic structure. Given that structure and wider variety of shocks, if one is able to estimate the full structural model and there is enough variation in the data, then it may be possible to recover any structural aggregate supply relationship. But precisely because we do not know the true model of the economy, such an approach may be less robust to misspecification than the empirical Phillips curve described above.

Moreover, as long as the structural aggregate supply relationship can be specified as a relationship between inflation and some measure of slack, then the identification issues we raise in the simple model may still apply. In Burgess et al. (2013), the Phillips curve for consumer price inflation is a function of past and future inflation; the marginal cost of final output production; and a markup shock. Figure A3 in the online appendix shows simulated data from the model under a specification of optimal discretionary policy where the policymaker targets inflation and (instead of the output gap) the marginal cost of final output production. Just as with the effect of demand shocks on the output gap in the basic model, the policymaker is able to perfectly offset the effect of all shocks on the marginal cost. In equilibrium, the only shock that has any effect on the policymaker's chosen target variables is the markup shock, which creates a trade-off between them.

These findings from a larger model designed for practical policy use in central banks suggest another source of variation to identify the structural Phillips curve or aggregate supply relationship. If the measure of slack targeted by the policymaker is different to the one that directly influences inflation, then the policymaker will not seek to offset all variation in the inflation-relevant measure. In the example above, if the policymaker seeks to minimise fluctuations in the output gap this will not always minimise movements in real marginal costs, since the relationship between the two measures of slack will vary according to the mix of shocks. The reasoning is analogous to the discussion of the wage Phillips curve in the previous section. The policymaker's actions will only blur the structural Phillips curve in equilibrium to the extent her policy targets are correlated with the measures of inflation and slack in the aggregate supply relationship.

[26] Figure A2 in the online appendix shows the correlation under discretion conditional on each shock. In this more complex setting, the reduced-form slope does not represent any single optimal targeting rule. But the same intuition continues to hold: monetary policy will seek to minimise any variation in the output gap that would cause inflation to move in the same direction. Conversely, following a markup (or cost-push) shock, monetary policy will aim to reduce the output gap at times when inflation is above target.

[27] In the model simulated above, there is a more stable positive relationship across different shocks between inflation and the relevant measure of real marginal costs than with the output gap.

5. Solutions to the estimation challenge in practice

In this section we examine Phillips curve identification in practice using US data. The previous subsection suggested at least three ways econometricians could recover the structural Phillips curve:

1. Supply shocks: if we can control for these well enough, we should be able to recover the Phillips curve.

2. Instrumental variables: with good instruments for the output gap, uncorrelated with cost-push shocks, then the structural Phillips curve can be recovered.

3. Regional data: monetary policy does not offset regional demand shocks, while time fixed effects can control for aggregate supply shocks.

In summary, the identification challenge arises from the presence of cost-push shocks to the Phillips curve and the partial accommodation of these by monetary policymakers. The size of the simultaneity bias is magnified because monetary policy seeks to offset any demand shocks that, in practice, might otherwise help identify the curve.

Each solution attempts to circumvent these issues by isolating the remaining demand-driven variation in inflation. The first two solutions use aggregate time-series data and the third turns to the regional cross-section. While a large number of papers have estimated Phillips curves without addressing the identification issue we raise here, many others over the years have followed one or more of these approaches, either implicitly or explicitly. Our discussion provides a framework which ties together these different solutions.

The econometric solutions to simultaneity in economics are well known. And econometricians will no doubt continue to come up with other innovative ways to successfully identify Phillips curves.[28] But there are reasons to think that using aggregate data, the task is likely to become ever more difficult. Boivin and Giannoni (2006) showed that both the variance and the effect of monetary policy shocks had become smaller in the period since the early 1980s, while similar arguments have recently been made by Ramey (2016). Both suggest that in economies such as the US, with established policy frameworks, policy is now largely conducted systematically. This limits the remaining exogenous variation in aggregate demand needed to recover the Phillips curve.

An alternative avenue, therefore, is to turn to cross-sectional data. As in Fitzgerald and Nicolini (2014), we next show that using regional data on inflation and unemployment by metropolitan area, a steeper Phillips curve re-emerges.

5.1. The empirical Phillips curve in the aggregate data

For our empirical exploration, we turn our attention to the US, where Phillips' UK findings were translated by Samuelson and Solow (1960). Our inflation data are the (seasonally adjusted) quarterly

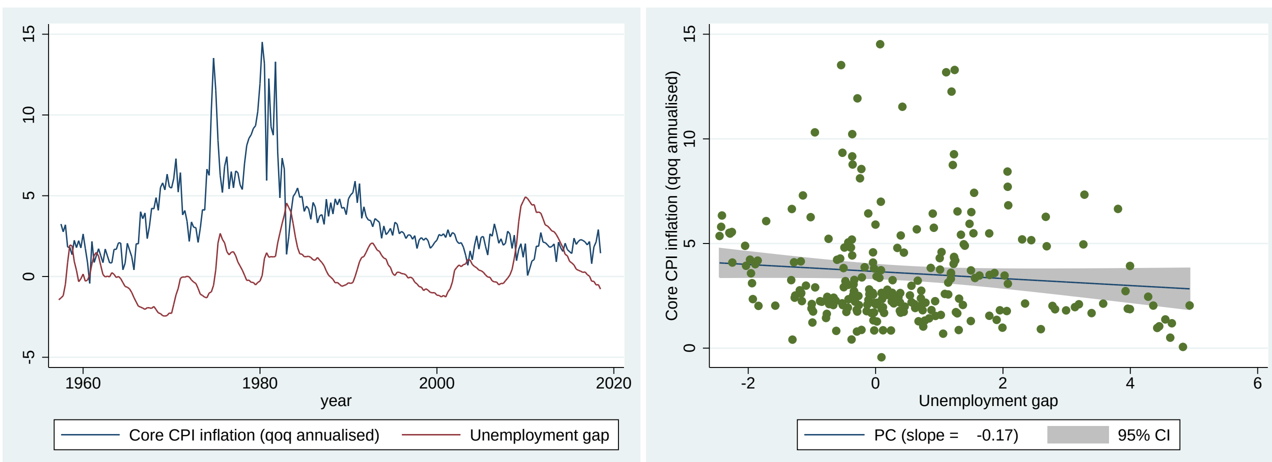
[28] See Barnichon and Mesters (2019), Galí and Gambetti (Forthcoming) and Jordà and Nechio (Forthcoming) for some recent examples, discussed further below.

annualised log change in *core* CPI inflation. While PCE inflation has been the FOMC's preferred measure since 2000, for most of our sample monetary policy focused on CPI inflation.[29] It also allows us to more readily compare with the US regional price data, which is a CPI measure. Using core inflation rather than headline is a straightforward mechanical way of stripping out a subset of the cost-push shocks affecting headline inflation, in line with our first solution above.

Again for comparability with the regional data, we use the (seasonally adjusted) quarterly unemployment gap as our proxy for slack, measured as the civilian unemployment rate less the CBO estimate of the long-term natural rate of unemployment. Using the unemployment gap, we would therefore expect to see a negative structural relationship with inflation. Figure 8 plots the

Figure 8: *US core CPI inflation and the unemployment gap: 1957 Q1 to 2018 Q2*

(a) *Time series* **(b)** *Scatter plot*



Notes: Figures show plots of quarterly annualised core CPI inflation against the CBO estimate of the unemployment gap. Phillips curve slope and the confidence interval around it is estimated using OLS.

two time series, alongside a simple scatter plot of the data over our sample period of 1957-2018. The reduced-form Phillips curve slope is flat and not significantly different from zero. But as is clear from the time series and has been well-documented elsewhere, the full sample masks a great deal of time variation in the relationship.

Figure 9 shows how the correlation has varied over time. We split the time periods according to Fed Chair over our sample period.[30] We split Paul Volcker's chairmanship into two periods, given the very different inflation and output dynamics at the start and end of his tenure.[31]

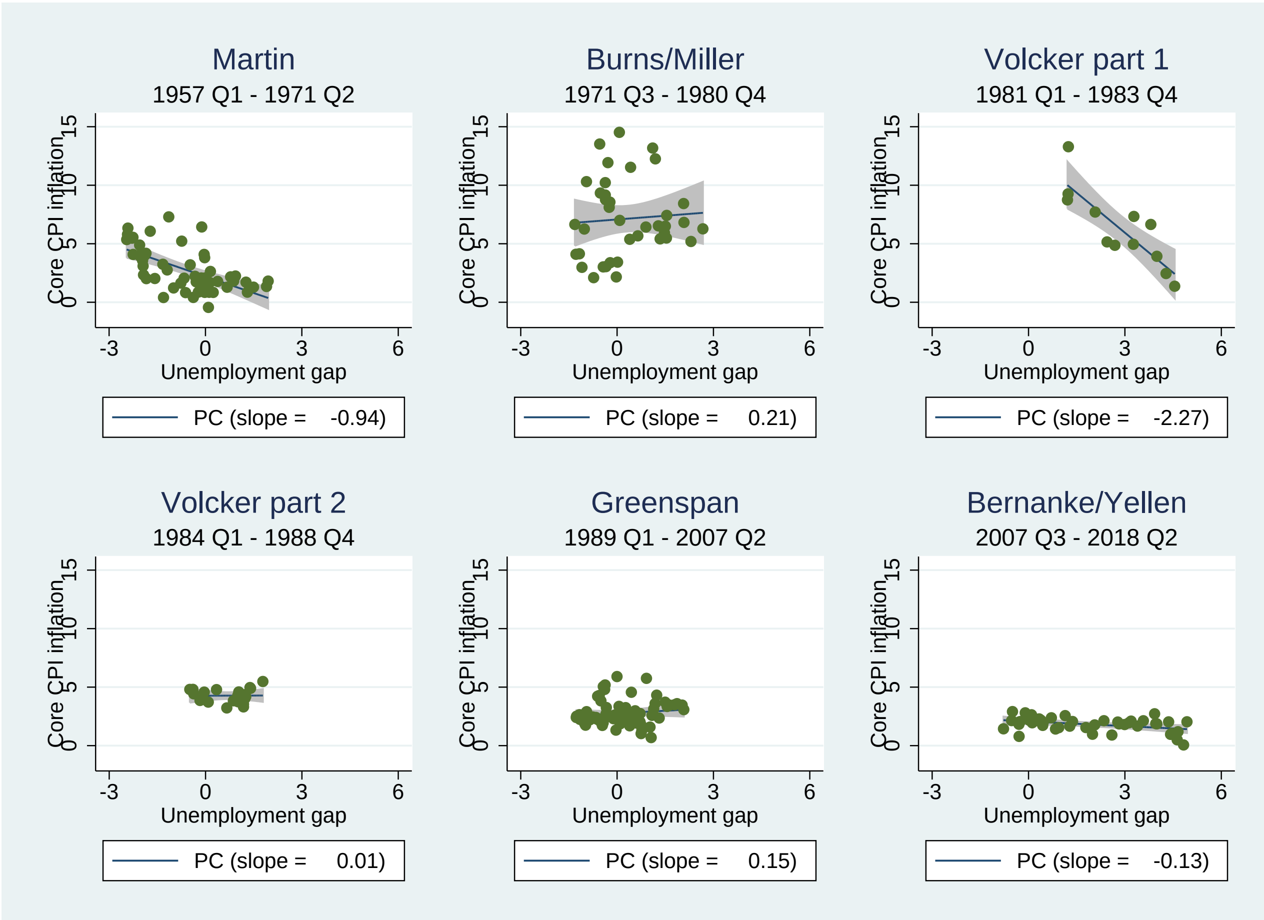
The data can be explained with the traditional narrative of the US Phillips curve over the second

[29] Board of Governors of the Federal Reserve System (2000).

[30] We also lag the tenure dates by six quarters to reflect the lags between monetary policy actions and their effect on real activity and inflation. Christiano, Eichenbaum and Evans (2005) and Boivin and Giannoni (2006) both find that monetary policy has its peak impact on output after around four quarters, and on quarterly inflation after eight quarters.

[31] We split the sample at the end of 1983 in line with convention in dating the Volcker disinflation (Goodfriend and King, 2005).

Figure 9: *Phillips correlation by Fed Chair*



Notes: Figure shows scatter plots of quarterly annualised core CPI inflation against the CBO estimate of the unemployment gap, split by time period. We lag the tenure dates of each Fed chair by six quarters as a way of reflecting the lags between monetary policy actions and their effect on real activity and inflation. Phillips curve slopes and confidence intervals are estimated using OLS.

half of the 20th century, as discussed in histories by King (2008) and Gordon (2011). In the latter years of William McChesney Martin's 23 year term, with the Phillips curve viewed as an exploitable long-run trade off, overly accommodative fiscal and monetary policies led to unemployment falling steadily below today's estimate of its natural rate (Romer and Romer, 2004*a*). Inflation rose at the same time, resulting in a downward sloping Phillips curve visible in the data (driven by rises in x_t in (1)).

During Arthur Burns's tenure in the 1970s, a combination of factors increased both inflation and increased unemployment, leading to a disappearance of any discernible Phillips curve correlation. Those factors were a series of large cost shocks (increases in u_t in (1)) brought about by oil supply disruption[32], and the Federal Reserve's inability, unwillingness[33] or miscalculations[34] in trying to

[32] Gordon (1977); Blinder (1982).
[33] DeLong (1997).
[34] Orphanides (2002).

lean against them (falls in e_t in (11)) and their impact on inflation expectations[35] (increases in $E_t\pi_{t+1}$ in (1)).

The beginning of Paul Volcker's tenure saw a re-emergence of a steep negative Phillips curve slope, as tighter monetary policy induced rises in unemployment and a sustained fall in inflation (driven by falls in σ_e^2 or ρ_e in (11), or equivalently a fall in λ and a related fall in $E_t\pi_{t+1}$ in (1): Clarida, Galí and Gertler (2000)).

For the subsequent two decades under Paul Volcker and then Alan Greenspan, the Phillips correlation all but disappeared. The causes of the Great Moderation are often divided into those relating to good policy, good luck (in the form of lower shock variance, particularly of supply shocks), and changes in the structure of the economy (Stock and Watson, 2002).

Despite the Great Moderation coming to an end with the 2008 financial crisis and a large rise in unemployment, the Phillips curve correlation that reappeared under the tenures of Ben Bernanke and Janet Yellen has been at best weak. The lack of a large deflation following the crisis has sparked a burgeoning literature attempting to explain the 'missing disinflation' by appealing to one or more of: a flatter structural Phillips curve slope; better anchored inflation expectations *or* increases in inflation expectations; the inflationary effects of financial frictions; or weaker potential supply growth (see Coibion and Gorodnichenko, 2015, for a discussion).

The reduced-form evidence in Figure 9 has led many commentators to conclude that the Phillips curve has flattened over time. It is also consistent with estimates using more sophisticated techniques. In an influential contribution, Ball and Mazumder (2011) estimate a time-varying Phillips curve using median inflation as a measure of core inflation. They report that the Phillips curve steepened from -0.23 in 1960-72 to -0.69 in 1973-84 and then flattened to -0.14 in 1985-2010. Blanchard, Cerutti and Summers (2015) and Blanchard (2016), extending the non-linear Kalman filter estimates of IMF (2013), find that the Phillips curve slope fell from around -0.7 in the 1970s to around -0.2 from the 1990s onwards.

Over the period since 1990 (spanning the Great Moderation, then the financial crisis and its aftermath) a flat Phillips curve is common across a range of typical empirical specifications. Table 2 presents simple OLS estimates using data on quarterly annualised core CPI inflation and the unemployment gap/rate, over a sample from 1990-2018. The first column shows a simple bivariate regression of inflation on the CBO measure of the unemployment gap. The second estimates a typical New Keynesian Phillips curve by replacing the constant term with a survey-based measure of forward-looking inflation expectations from the SPF.[36] The third estimates an acelerationist-style Phillips curve (Phelps, 1967; Friedman, 1968) by using (three) lags of inflation as a proxy for inflation expectations. The fourth, fifth and sixth columns nest both models in a hybrid Phillips curve (Galí and Gertler, 1999), which feature both forward-looking expectations and lags of inflation (either as an alternative proxy for inflation expectations or as an additional source of inflation dynamics).

[35] Barro and Gordon (1983); Chari, Christiano and Eichenbaum (1998).

[36] We use five to ten year ahead inflation expectations, as suggested by Bernanke (2007) and Yellen (2015) as having a stronger empirical fit with the data. See Coibion, Gorodnichenko and Kamdar (2018) for an extensive review of the use of survey expectations in the Philllips curve.

Table 2: *OLS Phillips curve regressions using aggregate US data: 1990-2018*

Phillips curve:	(1) Bivariate	(2) New Keynesian	(3) Acceler-ationist	(4) Hybrid $(U_t - U_t^*)$	(5) Hybrid (U_t)	(6) Hybrid $B(L)(U_t - U_t^*)$
Unemployment rate					-0.081**	
					[0.038]	
Unemployment gap	-0.204***	-0.170***	-0.010	-0.078**		0.503*
	[0.074]	[0.048]	[0.042]	[0.037]		[0.272]
First lag						-1.008**
						[0.458]
Second lag						0.291
						[0.437]
Third lag						0.152
						[0.237]
Sum						-0.062*
						[0.037]
Constant	2.583***				-0.054	
	[0.179]				[0.284]	
Inflation expectations		0.943***		0.388***	0.641***	0.384***
		[0.037]		[0.105]	[0.152]	[0.103]
Core CPI inflation						
First lag			0.404***	0.252**	0.223**	0.278***
			[0.091]	[0.103]	[0.096]	[0.097]
Second lag			0.475***	0.343***	0.312***	0.331***
			[0.083]	[0.098]	[0.095]	[0.107]
Third lag			0.092	-0.013	-0.050	-0.029
			[0.089]	[0.083]	[0.091]	[0.079]
Observations	118	118	118	118	118	118
R-squared	0.100	0.950	0.957	0.963	0.745	0.965

Newey-West standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Notes: The first five columns in the Table show the estimated OLS coefficients and standard errors for regressions nested by the hybrid Phillips curve $\pi_t = \alpha + \gamma_1(U_t - U_t^*) + \gamma_2 E_t\pi_{t+1} + \Sigma_{i=1}^{3}\gamma_{2+i}\pi_{t-i} + \varepsilon_t$. Specification (1) constrains $\gamma_2 = 0$, $\gamma_3 = 0$, $\gamma_4 = 0$, and $\gamma_5 = 0$. Specification (2) constrains $\alpha = 0$, $\gamma_3 = 0$, $\gamma_4 = 0$, $\gamma_5 = 0$. Specification (3) constrains $\alpha = 0$ and $\gamma_2 = 0$. Specification (4) constrains $\alpha = 0$ while specification (5) omits U_t^* and uses U_t as the measure of activity. Specification (6) constrains $\alpha = 0$ while also including three lags of $(U_t - U_t^*)$. $B(L)$ represents a third order lag polynomial. Data are quarterly seasonally adjusted measures from 1990 Q1 to 2018 Q2.

The three hybrid curves feature different specifications for unemployment: they use either the unemployment rate; or else the unemployment gap, with or without additional lags.

Across the different specifications, the steepest Phillips curves slopes are only -0.20 (for the bivariate regression) and -0.17 (augmenting with survey-based inflation expectations). These are in line with the flattened Phillips curve slope found by Blanchard, Cerutti and Summers (2015). In all of the specifications featuring lags of inflation (either to proxy for inflation expectations or as an additional source of dynamics), the slope is flatter still and not always significant. The sum of the coefficients on the forward and backward looking inflation terms is close to 1 in each of the estimates (ranging from 0.9-1.1), in line with natural rate theories of unemployment, which predict stable long-run inflation if and only if $U = U^*$.

In all, the results from these "naive" Phillips curves estimates would suggest that the relationship still exists, but that the slope is relatively flat. Since policymakers also pay close attention to similar estimates, then the identification issue we highlight has the potential to provide misleading inferences for monetary policy. A flatter Phillips curve implies a higher "sacrifice ratio" associated with bringing inflation back to target, which could lead policymakers to place greater weight than optimal on avoiding volatility in output and employment relative to inflation (Blanchard, Cerutti and Summers, 2015). At worst, weaker evidence of a clear link between real activity and inflation could be interpreted as a sign that there is no short-run policy trade-off between the two goals, leading policymakers to abandon the natural rate hypothesis (Taylor, 1998; Cogley and Sargent, 2001). Given its importance for policy, we next discuss the different approaches to identifying the Phillips curve using aggregate data.

5.2. Identification using aggregate data

In the extensive literature estimating Phillips curves, a number of papers have adopted approaches similar to those we suggest, implicitly or explicitly addressing the identification difficulties we highlight here.[37] Encouragingly, even in the period since the first draft of this paper was circulated, several others have proposed new identification strategies to mitigate simultaneity bias in Phillips curve estimation. In this subsection we discuss the findings from some of those contributions and categorise them according to our conceptual framework.

5.2.1 Controlling for supply shocks

In principle, if econometricians can perfectly control for the effect of any cost-push or other trade-off inducing shocks, then any remaining variation in the output gap and inflation must be due to movements in aggregate demand. As in our estimates above, the many papers that estimate Phillips curves using core inflation are already implicitly controlling for cost-push shocks to some degree, by stripping out their direct effects on the price data.[38] Others include the change in the oil price as a regressor (e.g. Roberts, 1995).

The idea of controlling for supply shocks was even present in the original Phillips (1958) article, which describes periods during which cost-push effects led to deviations from the fitted curve. More recently it has been associated with the "triangle model" of Gordon (1982), originally developed to account for the shift in inflation dynamics in the 1970s.[39] As described in Gordon (2013), the model

[37]See Mavroeidis, Plagborg-Møller and Stock (2014) for a comprehensive summary.

[38]See Hasenzagl et al. (2019) for evidence on the different channels through which cost-push shocks to energy prices impact inflation.

[39]And subsequently refined in a series of papers, most recently in Gordon (2013).

includes several variables to control for changes in aggregate supply: food and energy price inflation; relative import price inflation; changes in trend labour productivity; and dummies reflecting the start and end of the Nixon price controls in the 1970s.[40]

Despite including these variables to control for supply shocks, Gordon (2013) still finds a flattening in the Phillips curve slope coefficient on the long-term unemployment gap: from -0.50 to -0.31 when he extends his sample from 1962-96 to 1962-2013.[41] The smaller absolute coefficient could be due to a flattening in the structural Phillips curve slope, but could also be due to increasing difficulties with the practical implementation of the approach in the recent data. The solution is arguably more suited to helping identify the Phillips curve in a period such as the 1970s, when were large, easily identifiable cost-push shocks *and* a higher variance of monetary policy shocks than more recently.[42]

A related idea is that of Coibion and Gorodnichenko (2015), who argue that the supply shock imparted by higher oil prices also pushed up inflation between 2009 and 2011 by increasing firms' inflation expectations, which they proxy using household expectations.[43] Following Roberts (1995), they use the Michigan Survey of Consumers, and find a stable Phillips curve slope of between -0.2 and -0.3 (using the unemployment gap) in both the 1981-2007 and 1981-2013 periods.

In both cases, the large number of supply variables in Gordon's model point towards a more general practical difficulty, which is that there are many trade-off inducing shocks that need to be controlled for, and which of these are most important may vary over time. As an example, the explanations in the DSGE models of Christiano, Eichenbaum and Trabandt (2015) and Gilchrist et al. (2017) for the lack of disinflation during the financial crisis rely on financial frictions that simultaneously increased inflation and decreased real activity. That suggests one may also need to add a measure of financial frictions as an additional explanatory variable.

In some senses, the many papers that estimate the slope of a Phillips curve as part of a fully specified New Keynesian DSGE model are also adopting a variant of this approach. Schorfheide (2008) shows how full-information maximum likelihood (FIML) estimation of a simple New Keynesian model corrects for the simultaneity bias that markup shocks introduce into the slope of the Phillips curve. But he also reports evidence from the literature on how sensitive such estimates are to model specification, with estimates of the coefficient on the output gap varying from 0 to 4.

5.2.2 Instrumental variable estimation

An alternative solution is to use instrumental variable methods. The econometrician must find a valid instrument that correlates with the demand variation in the output gap, and is uncorrelated

[40] The model also includes a large number of lags of inflation (up to 6 years) to capture additional dynamic factors affecting inflation.

[41] Gordon instead emphasises the smaller flattening in the point estimate when using the short-term unemployment rate as the relevant concept of slack, although this measure correlates less closely with estimates of the overall output gap than the total unemployment rate – largely due to the large negative output gap during the financial crisis.

[42] The standard deviation of the Romer and Romer (2004*b*) monetary policy shock series is 2.5 times smaller in the period from 1990 onwards.

[43] See also Hasenzagl et al. (2019).

with the cost-push shock. The fitted value from a first-stage regression will then purge the output gap measure of the endogenous response of monetary policy to the cost-push shock, meaning it can be used to recover the true Phillips curve slope.

Instrumental variable methods, and especially IV-GMM estimation, have been common in much of the literature estimating New Keynesian Phillips curves, including influential papers by Galí and Gertler (1999) and Galí, Gertler and López-Salido (2001). These papers use only lagged variables as instruments. Although these should be orthogonal to the current period cost-push innovation, the exclusion restriction will not generally be satisfied if the cost-push shocks exhibit autocorrelation. As discussed in Mavroeidis, Plagborg-Møller and Stock (2014) and more recently in Barnichon and Mesters (2019), the shocks will in this case still be correlated with the lagged variables. The instruments used must be of a greater lag length than the lag order of the cost-push shocks, but with highly autocorrelated cost-push shocks, such instruments are likely to have low relevance.

Alternatively, separately identified demand shocks can be used as a set of *external* instruments, as recently proposed by Barnichon and Mesters (2019). To satisfy the exclusion restriction, the candidate instruments should be uncorrelated with the cost-push shocks in (1). Monetary policy shocks, which are not usually thought to affect supply, are a natural candidate.

Essentially, this strategy applies the findings from the large literature on identifying monetary policy shocks to recover the Phillips curve.[44] Given the major focus of that literature has been to try to remove the systematic response of monetary policy to economic developments, it should be able to successfully distill the Phillips curve relationship.

Recent work by Barnichon and Mesters (2019) follows exactly this approach. Using the Romer and Romer (2004*b*) narrative measure of monetary policy shocks as instruments for the output gap, they find a much steeper Phillips curve slope than under OLS.

The approach faces the same challenges as outlined by Ramey (2016) for the monetary policy shock literature. She argues that in the period since 1990, monetary policy has been set more systematically, and as a result, there is only a limited amount of true exogenous variation in the data, leading to weak instrument issues.

Identification of monetary policy shocks using high-frequency data may offer one solution (Kuttner, 2001; Faust, Swanson and Wright, 2004; Gertler and Karadi, 2015; Nakamura and Steinsson, 2018). The short-time windows over which these shocks are identified help remove any traces of endogenous monetary policy (Nakamura and Steinsson, 2018), which might otherwise be amplified if the shocks were weak instruments for the output gap. Barnichon and Mesters (2019) use the high frequency identified shocks of Gertler and Karadi (2015) for the post 1990 period and find evidence of a flatter Phillips curve slope than in the earlier period.

Other demand shocks, such as fiscal shocks to government spending or taxes, could in principle also be used as external instruments. But for them to successfully capture sufficient variation in the output or unemployment gap, the shocks must not be offset by any endogenous monetary policy

[44] For example, Christiano, Eichenbaum and Evans (1996, 1999); Romer and Romer (2004*b*); Uhlig (2005); Bernanke, Boivin and Eliasz (2005); Olivei and Tenreyro (2007); Cloyne and Hürtgen (2016).

response. In the basic model presented in Section 2, fiscal shocks do not help identify the Phillips curve, since they are completely offset by optimal monetary policy. Relative to monetary policy shocks, a second drawback is that some fiscal changes are more likely to affect aggregate supply, and so they may not satisfy the exclusion restriction.

Both drawbacks are evident in the large-scale DSGE model simulations we show in the online appendix. Figure A2 shows that under the loss minimising monetary policy, there is little remaining variation in inflation and the output gap following government spending shocks. And as the shock affects supply and therefore induces a small trade-off between these two policy goals, the variation that does remain results in a negative correlation between the two variables. These simulations also highlight that the Phillips curve may vary for different types of demand shock. If so, then the curve conditional on a monetary policy shock is arguably the more relevant one for monetary policymakers, since it relates directly to their policy instrument.

Related to these ideas, a recent paper by Galí and Gambetti (Forthcoming) estimates Phillips curves *conditional* on identified demand shocks in a VAR. They find that while endogeneity issues do lead to downward bias in estimates of the US wage Phillips curve, there has also been a structural flattening over time.

5.3. Identification using regional data

Given some of the practical difficulties using aggregate data in the presence of systematic monetary policy, an alternative solution is to exploit cross-sectional variation. An interesting recent approach in this vein is Jordà and Nechio (Forthcoming), who take advantage of the fact that economies with fixed exchange rates are unable to implement independent monetary policies.

To show the possibility of using regional data to identify the aggregate US Phillips curve, we use a panel of city-level price inflation and unemployment data, as in Fitzgerald and Nicolini (2014). Hooper, Mishkin and Sufi (2019) also make use of US city-level (and state-level data) in their detailed study of the US wage and price Phillips curves. Our city-level dataset, containing price data, is an extended and updated version of the one used by Kiley (2015) and Babb and Detmeister (2017).

5.3.1 Data description

We use data from 28 US metropolitan areas published by the BLS.[45] Together these areas account for over one-third of the US population (Babb and Detmeister, 2017). There is significant size heterogeneity across the sample - weighted by average labour force, the largest three areas (New York, Los Angeles and Chicago) account for 31% of the total, while the smallest thirteen areas account for less than 2% each. Since six cities in our sample were discontinued after 2017, we opt to exclude the observations from 2018 H1 onwards.[46] Our full sample runs from the first half (H1) of

[45] We list the full set of areas we use in the online appendix. The earlier conference draft of this paper used a smaller sample of only 23 areas. Moving to the full set yields almost identical results.

[46] We use the terms city and metropolitan area interchangeably.

1990 to the second half (H2) of 2017,[47] with some gaps for metropolitan areas where the data were only published in the later part of the sample.[48]

The inflation series is the annualised log change in the semiannual CPI excluding food and energy. For the majority of metropolitan areas, data are also available at a higher frequency, but in order to maximise our cross-sectional sample, we opt to convert these to semiannual data.[49] The city-level CPI data are not seasonally adjusted by the BLS.

For unemployment, we take the BLS's metropolitan statistical area measures of unemployed as a percentage of the share of civilian labour force.[50] The BLS publish both seasonally adjusted and unadjusted labour force data at the metro area level – we use the unadjusted series, consistent with the CPI data. We take the average of the unemployment *rate* to convert the monthly published data to semiannual averages.

We also run specifications using survey-based measures of 12 month inflation expectations from the University of Michigan Consumer Survey. The Michigan Survey includes data published for four broad geographical regions: the North East, North Central, South, and West. We assign each metropolitan area to its appropriate region (or the region containing most of the metropolitan area's population, for metro areas that span more than one region).

5.3.2 Regional data results

To motivate our regional empirical specification, first note that we only have data on the unemployment rate at the regional level, rather than the unemployment gap to proxy for the output gap. If the regional Phillips curves are of a form similar to (15), transformed to include the regional unemployment gap $(U_t^i - {U^*}_t^i)$:

$$\pi_t^i = \beta E_t \pi_{t+1}^i - \kappa(U_t^i - {U^*}_t^i) + u_t^i \tag{22}$$

If, as is likely, the regional equilibrium unemployment rate, ${U^*}_t^i$ is positively correlated with the actual unemployment rate, then in a pooled OLS regression such as

$$\pi_{it} = \alpha + \gamma_1 E_t \pi_{it+1} + \gamma_2 U_{it} + \varepsilon_{it}, \tag{23}$$

[47]Metropolitan area unemployment is published from 1990. In the conference draft of this paper we also used CPI price level data only from 1990 onwards. Here we make use of the longer published backrun of the CPI to construct a inflation (and lagged inflation) rates for 1990.

[48]CPI data for Tampa are published only from 1997 H2; Phoenix from 2002 H1. Our results are robust to excluding both cities.

[49]Where the semiannual CPI figure is published by the BLS, we use that. Where only monthly data are published, we take the semiannual average. Where the published data are published only in certain months, we follow BLS methodology and estimate the missing months via interpolation, before taking the semiannual average (see also Fitzgerald and Nicolini, 2014).

[50]The local unemployment data use the CBSA delineations of metropolitan areas, which the CPI data have also used since 2018, having previously used sightly different MSA definitions. We match the unemployment data to the currently used definition, since the BLS treats this as continuous with the old one for CPI. For the subset of cities where CPI data were only ever published under the old definition, we sum unempoyment and the labour force data for the matching CBSA metropolitan and micropolitan areas.

the omitted variable will bias the estimated coefficient $\hat{\gamma}_2$ towards zero. To partially address this we run specifications including metropolitan area fixed effects (α_i):

$$\pi_{it} = \alpha_i + \gamma_1 E_t \pi_{it+1} + \gamma_2 U_{it} + \varepsilon_{it}, \tag{24}$$

which control for time-invariant regional differences in U^* (as well as time-invariant inflation expectations), although not for time-variation in those regional differences.

Second, as long the regional unemployment rate is correlated with the aggregate unemployment rate, and regional inflation is affected by aggregate cost-push shocks, then the slope estimate will still be biased by the endogenous response of monetary policy to aggregate cost-push shocks. To avoid this, note that our theoretical Phillips curve in terms of regional deviations from the aggregate, (18), can be rearranged to give:

$$\begin{aligned} \pi_t^i &= \pi_t + \beta E_t(\pi_{t+1}^i - \pi_{t+1}) + \kappa(x_t^i - x_t) + \hat{u}_t^i \\ &= \beta E_t \pi_{t+1}^i + \kappa x_t^i + (\pi_t - \beta E_t \pi_{t+1} - \kappa x_t) + \hat{u}_t^i \end{aligned} \tag{25}$$

where x_t^i are uncorrelated with $\hat{u}_t^i$ but are correlated with the aggregate cost-push shock $u_t = \pi_t - \beta E_t \pi_{t+1} - \kappa x_t$. We can therefore remove any monetary-policy induced correlation between the regressor and the error term by also including time fixed effects (δ_t):

$$\pi_{it} = \alpha_i + \gamma_1 E_t \pi_{it+1} + \gamma_2 U_{it} + \delta_t + \varepsilon_{it} \tag{26}$$

which will also control for any time-varying changes in the aggregate equilibrium unemployment rate.

To compare across the different specifications, we estimate each of equations (23), (24) and (26). As additional controls we include seasonal dummies and, given the data are semiannual, just a single lag of inflation. For completeness we also show results including time fixed effects but not including metropolitan area fixed effects. The results are shown in Table 3. All four estimates of the Phillips curve slope are statistically significant and with the correct sign. In the first column, the pooled OLS estimate of -0.15 suggests a flat Phillips curve. It is slightly larger than the estimates with lagged dependent variables using aggregate data in Table 2, but no steeper than the estimates without lagged inflation.[51]

Figures 10a and 10b illustrate the slope coefficient. In Figure 10a, the scatter plots core inflation against unemployment. Both variables are shown as the residuals following a regression on the other controls in the first column of Table 3, such that the line of best fit shows the estimated Phillips curve slope. Figure 10b shows averages of the same data, where the unemployment and inflation data are averaged across 100 equal sized bins according to the unemployment rate.

In the second column we include area fixed effects and the point estimate of the slope is slightly

[51]Note that the estimated coefficient on inflation expecatations is not robust to changes in the sample. Estimating pooled OLS on a sample beginnning in 1991 instead of 1990 reduces the point estimate from 0.60 to 0.36.

Table 3: *US Metro area Phillips curve: 1990-2017*

Regression	(1) Pooled OLS	(2) Metro area FE only	(3) Year FE only	(4) Year and Metro area FE
Unemployment rate	-0.150***	-0.162***	-0.272***	-0.379***
	[0.016]	[0.019]	[0.036]	[0.052]
Inflation expectations	0.598***	0.589***	0.259*	0.225
	[0.058]	[0.059]	[0.147]	[0.141]
Core CPI inflation				
First lag	0.362***	0.371***	0.122***	0.105***
	[0.035]	[0.036]	[0.035]	[0.034]
Observations	1,525	1,525	1,525	1,525
R-squared	0.321	0.350	0.450	0.487
Metro area FE	No	Yes	No	Yes
Year FE	No	No	Yes	Yes
Seasonal dummies	Yes	Yes	Yes	Yes

Robust standard errors (clustered by metro area) in brackets
*** p<0.01, ** p<0.05, * p<0.1

Notes: The table shows coefficients and standard errors estimated from four regional Phillips curve specifications. Core CPI inflation is the dependent variable in each case. Specification (1) estimates equation 23 (plus controls) by pooled OLS. Specification (2) estimates equation 24 (plus controls) using group (area) fixed effects. Specification (3) is identical to (1) apart from the inclusion of a set of year dummy variables. Specification (4) is identical to (2) apart from the inclusion of a set of year dummy variables. The additional controls are one lag of core CPI inflation and a seasonal dummy variable for each metropolitan area that takes the value of 1 in H2 and 0 in H1. All specifications contain a constant. Data are semiannual non-seasonally adjusted measures from 1990 H1 to 2017 H2.

larger, although not significantly so.

In the third column we include year fixed effects but not area fixed effects, purging the data of any aggregate-level variation over time, including changes in monetary policy and in the natural rate of unemployment. The estimated Phillips curve slope steepens to -0.27, as shown in Figures 11a and 11b.

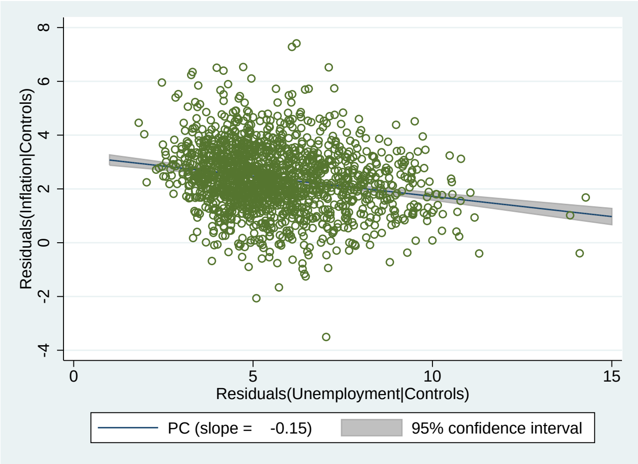
In the fourth column, metro area fixed effects are also included, controlling for any time-invariant unobserved factors such as different average levels of U^* across regions. The resulting Phillips curve is -0.38, 2.5 times larger than the pooled OLS estimate.[52] The residuals and slopes including both sets of fixed effects are shown in Figures 12a and 12b; as well as in 13, which plots the estimated Phillips curve by metropolitan area, with different intercept terms for each city.

These results provide evidence of a steeper US Phillips curve at the regional level. They are

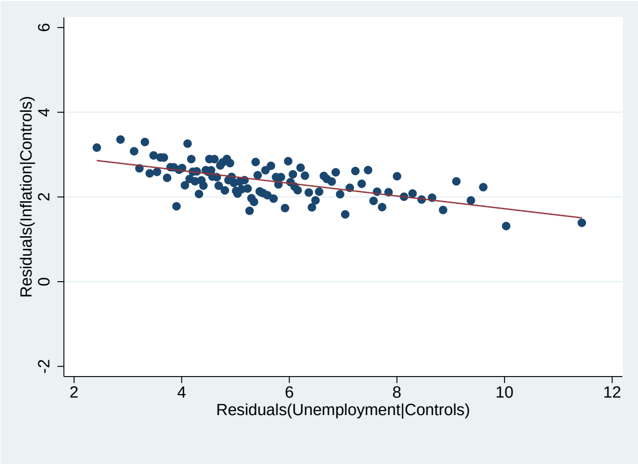
[52]Since the pooled OLS results have a higher coefficient on lagged inflation, then taken literally, the estimates suggest that the medium run Phillips curve slopes are more similar across specifications, a point made by our discussant Matthew Rognlie. But we are inclined to focus more on the instantaneous slope coefficient, since the coefficient on lagged inflation is likely to be picking up inflation persistence unrelated to changes in unemployment. Moreover, the Phillips curve slope coefficients we report are relatively robust to including different dynamic specifications (or no dynamics), or to estimating using annual or biannual data.

Figure 10: *Pooled OLS: metropolitan area core CPI inflation versus unemployment (both regressed on controls)*

(a) *Raw residuals*



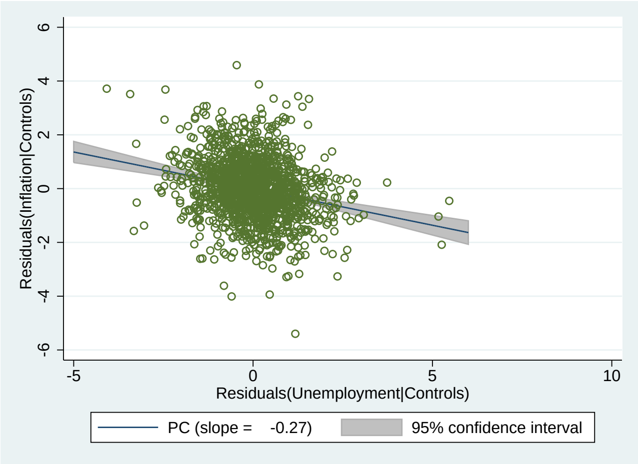
(b) *Residuals grouped into bins*



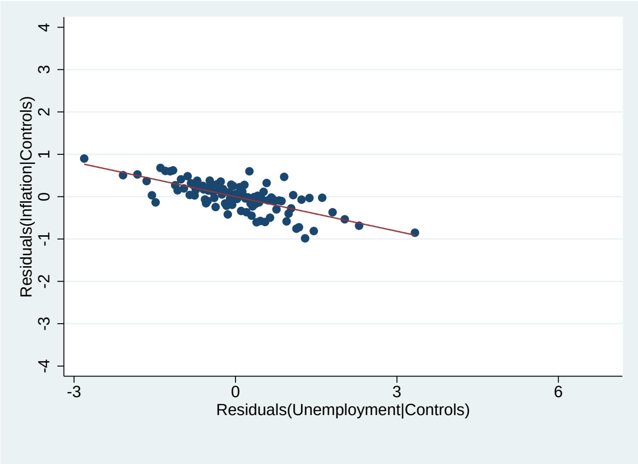
Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (1) in table 3. Panel 10a plots the residuals from a regression of core CPI inflation on all regressors other than the unemployment rate, against the residuals from a regression of the unemployment rate on all other regressors. Panel 10b shows averages of the same data, where the unemployment and inflation data are averaged across 100 equal sized bins according to the unemployment rate.

Figure 11: *Year fixed effects only: metropolitan area core CPI inflation versus unemployment (both regressed on controls)*

(a) *Raw residuals*



(b) *Residuals grouped into bins*

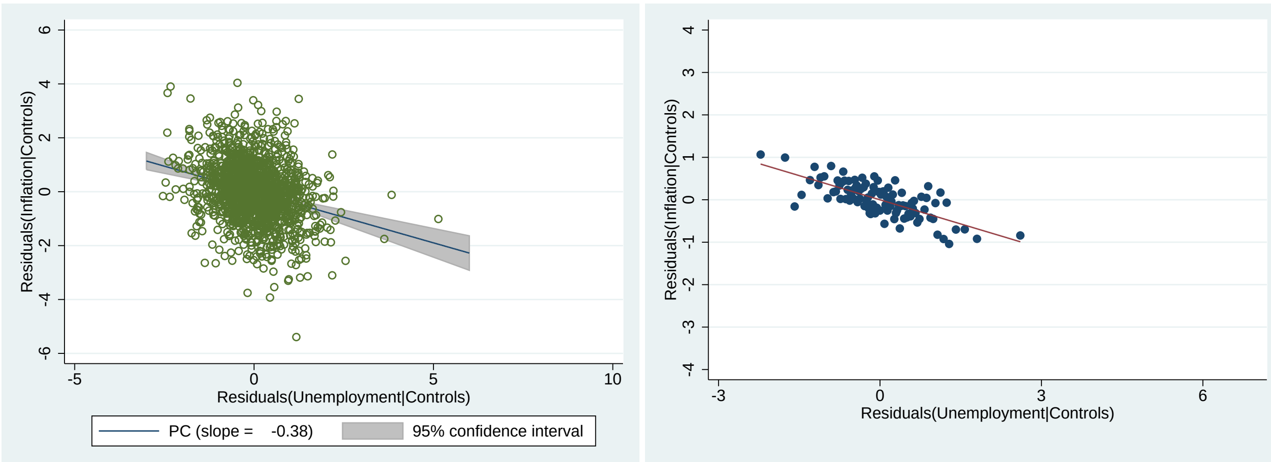


Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (3) in table 3. See the notes to Figures 10a and 10b for details.

consistent with the idea that because monetary policy endogenously offsets changes in aggregate demand, and leans against cost-push shocks, identification is blurred at the aggregate level.

Figure 12: *Year and metro area fixed effects: metropolitan area core CPI inflation versus unemployment (both regressed on controls)*

(a) *Raw residuals* **(b)** *Residuals grouped into bins*



Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (4) in table 3. See the notes to Figures 10a and 10b for details.

5.3.3 Robustness

As discussed in Section 4.3, including time fixed effects in our regional Phillips curve estimates removes the bias from aggregate supply shocks and the endogenous monetary policy response to them. But regional inflation may still be affected by idiosyncratic regional cost-push or supply shocks. Although aggregate monetary policy should not respond to regional deviations in inflation, the shocks themselves may still be positively correlated with regional unemployment. If so, our estimates will still be biased against finding a steep negative slope. If regional supply shocks are important, our estimate of -0.38 should be interpreted as a lower bound (in absolute terms), with the true Phillips curve slope steeper still.

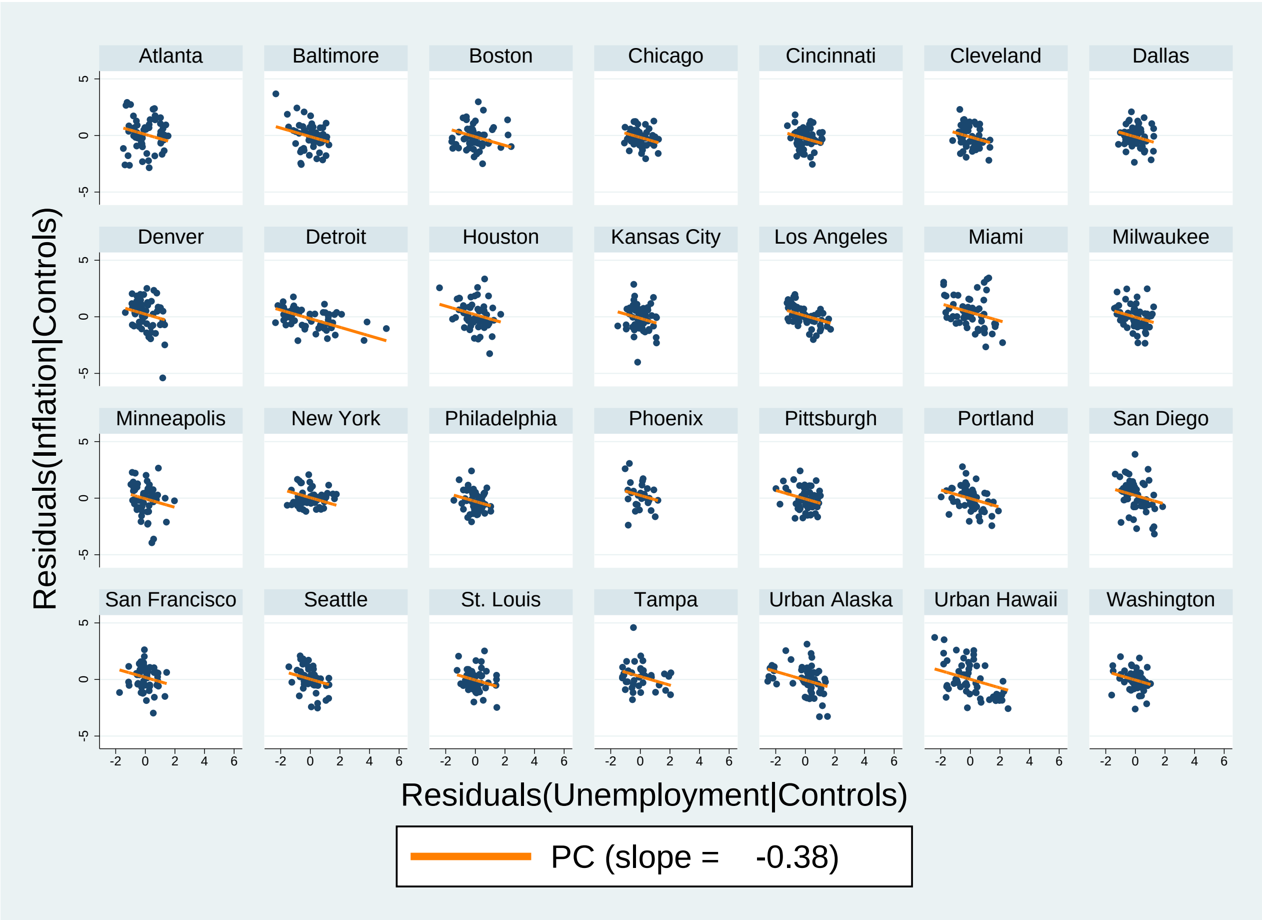
To examine the robustness of our results, we next explore two strategies that may help mitigate simultaneity bias from regional supply shocks. Each is analogous to one of our suggested solutions using aggregate data.

First, one option is to use a regional demand instrument to purge the unemployment data of regional supply shocks. We do this using a Bartik (1991) type instrument for regional government spending. In doing so we adapt the methodologies in Nekarda and Ramey (2011), who use a Bartik instrument to examine the effect of government spending at the industry-level and Nakamura and Steinsson (2014) who compute the effect of military spending on different US states and regions.

Bartik-type instruments are formed by interacting a time-invariant, region specific 'exposure' variable, which we denote B_i, and a national (or industry) growth rate or shock.[53] In our setting, we construct a Bartik exposure variable, B_i, that aims to capture which cities are likely to be more

[53] See Jaeger, Ruist and Stuhler (2018), Goldsmith-Pinkham, Sorkin and Swift (2018) and Borusyak, Hull and Jaravel (2018) for recent critical discussions of the use of these instruments.

Figure 13: *Year and metro area fixed effects: metropolitan area core CPI inflation versus unemployment by metro area (both regressed on controls)*



Notes: The figures are a graphical illustration of the Phillips curve slope estimated in specification (4) in table 3. For each metropolitan area, the figure plots the residuals from a fixed effects regression of core CPI inflation on all regressors other than the unemployment rate, with a different area fixed effect plotted for each city, against the residuals from a fixed-effects regression of the unemployment rate on all other regressors.

affected by changes in national government spending. To do so, we take the inner product of each industry j share of nominal shipments to government (in 1992), θ_j, from the dataset of manufacturing industries constructed by Nekarda and Ramey (2011); and the city's share of employment in that industry (in 1993), $\frac{E_{ij}}{E_i}$, from the Census Bureau's County Business Patterns:

$$B_i \equiv \sum_j \theta_j \frac{E_{ij}}{E_i} \tag{27}$$

We combine these data sources at the two-digit SIC level, which gives us 20 distinct industries.[54] We then interact our exposure variable with a measure of the growth rate of real aggregate federal government consumption, or federal government defense consumption, taken from the BEA's NIPA tables. The intuition underlying the instrument is that increases in national government spending should increase demand more in cities where employment is skewed towards industries that are more heavily involved in producing shipments to government, particularly defense orientated industries.

Table 4 shows the results of the instrumental variable estimation. For convenience, the first column repeats the OLS results with year and metro area fixed effects. The remaining columns show results with different variants of the instrument. The second column interacts the exposure variable with the 3 year log change in (real) federal government defense consumption; the third column uses the 3 year log change in total federal government consumption. The fourth column uses 6 leads and lags of the semiannual change in total federal government consumption. The final column uses a variant of the instrument used by Nakamura and Steinsson (2014), and interacts the 3 year log change in defense consumption with a metro-area fixed effect rather than the Bartik exposure variable.

Examining the results, they raise questions about the usefulness of our government demand instrument at the city level, in contrast to the findings of Nakamura and Steinsson (2014) at the state level. The second-stage results give broadly similar point estimates of the Phillips curve slope, albeit with much higher standard errors. But the first-stage results consistently suggest that increases in national government spending lead to significant *increases* in unemployment in areas with higher exposure relative to areas with lower exposure. If the instrument was successfully capturing variation in aggregate demand, we would expect these coefficients to be negative. It therefore seems highly unlikely that the instrument is successfully purging the data of any regional supply shocks.[55]

Second, we already control for regional cost-push shocks to some extent by excluding from our CPI measure some of the products which are most likely to be impacted them: food and energy. If some areas are more exposed to increases in food and energy inflation, then headline regional inflation will be subject to greater regional cost-push shocks in those areas. But if such shocks are important we would expect them to exert a smaller direct influence on core CPI inflation, leading to a smaller negative bias.

In Table A1 in the online appendix, we compare our baseline results to regional Phillips curves

[54] The County Business Patterns publish employment data at the MSA level from 1993, but only at the two digit level of aggregation. We have also experimented with aggregating the underlying county data, which is published at the four-digit SIC code level. This has the drawback that for a large fraction of the industry-county pairs, the employment data is published only as a range. A smaller fraction of industry-MSA pairs are also published only as a range. Where this is the case, we take the midpoints of the range.

[55] Instead, the instrument appears to be combining the fact that national government has been countercyclical over our sample, with the fact that those cities with higher values of B_i also seem to be more cyclical. As evidence of the latter fact, a regression of the regional unemployment rate on our exposure variable interacted with the simple average of metro area unemployment rates also leads to a significant positive coefficient. This is in contrast to the finding reported by Nakamura and Steinsson (2014) when carrying out a similar test at the state level using their instrument.

Table 4: *US Metro area Phillips curve, instrumental variables estimates: 1990-2017*

	(1)	(2)	(3)	(4)	(5)
Regression	OLS	2SLS	2SLS	2SLS	2SLS
Instrument		$\Delta_{3y} lnG_t^D$ x B_i	$\Delta_{3y} lnG_t$ x B_i	$C(L)\Delta lnG_t$ x B_i	$\Delta_{3y} lnG_t^D$ x α_i
Unemployment rate	-0.379***	-0.454**	-0.392*	-0.252	-0.508***
	[0.052]	[0.209]	[0.207]	[0.158]	[0.105]
Inflation expectations	0.225	0.219	0.224	0.201	0.215
	[0.141]	[0.141]	[0.139]	[0.138]	[0.139]
Core CPI inflation					
First lag	0.105***	0.089	0.103*	0.119**	0.077**
	[0.034]	[0.057]	[0.056]	[0.049]	[0.034]
Observations	1,525	1,525	1,525	1,413	1,525
R-squared	0.487	0.485	0.487	0.486	0.482
			2SLS first stage estimates Dependent variable: unemployment rate		
Gov. spending instrument		9.237***	11.027***		
		[2.104]	[2.434]		
Sum of leads/lags				5.642	
				[4.446]	
R-squared		0.828	0.828	0.836	0.849
Instrument(s) F-stat		83.5	84.8	12.0	10.4
Cluster robust F-stat		19.3	20.5	11.5	n/a

Robust standard errors (clustered by metro area) in brackets
*** p<0.01, ** p<0.05, * p<0.1

Notes: Specification for all regressions is the same as Specification (4) in Table 3. B_i is as defined in the text. G_t is the semiannual level of real federal government consumption. G_t is the semiannual level of real federal government defense consumption. $C(L)$ is the sum of a 6th order lag and 6th order lead polynomial. α_i is a metro area fixed effect. All regressions include year and metro area fixed effects, and a set of seasonal dummies for each metro area. Regression (4) is estimated over a sample from 1990 H1 to 2015 H2. All other regressions are estimated over a sample from 1990 H1 to 2017 H2. All instruments are standardized to have a unit variance.

estimated using alternative subsets of the CPI basket, and find that the slopes are broadly similar across different measures. This provides some reassurance that regional supply shocks are not exerting a significant bias on our results.

6. Conclusion

We use standard analytical framework to explain why inflation follows a seemingly exogenous statistical process, or, in other words, why the Phillips curve cannot be easily identified with macroeconomic data. In the framework, a monetary authority minimizes welfare losses, measured as deviations of inflation and output from their targets, subject to a Phillips curve. This leads the authority to follow an optimal targeting rule in which it seeks to increase inflation when the output gap decreases. This imparts a negative relation between inflation and the output gap that blurs the identification of the positively sloped Phillips curve. In equilibrium, inflation inherits the statistical properties of any cost-push shocks affecting the Phillips curves (e.g., energy price shocks, exchange rate changes, and so on).

We show that shocks to the targeting rule are key for the identification of the Phillips curve. These targeting shocks can take the form of monetary policy shocks in a Taylor rule or, in a multi-region setting or a multi-country monetary union, idiosyncratic demand shocks affecting the various regions or countries in different ways. In a univariate regression analysis, if the relative variance of these shocks is sufficiently high, vis-a-vis the remaining variance of the cost-push shocks that cannot be controlled for, the slope of the Phillips curve can be identified. Similarly, identification of monetary policy or other demand shocks allows the positive relationship between inflation and output gap to be distilled.

We have also shown how the simple framework here can jointly rationalise several empirical findings on the Phillips curve. First, it should be weaker in periods when there are large cost shocks – such as the 1970s – *and* when monetary policy is relatively successful in achieving its targets – as in the inflation targeting era. Second, wage Phillips curves should be more evident in the data that price Phillips curves. And third, the Phillips curve relationship should appear stronger in disaggregated panel data than in aggregate data.

To summarise, the paper explains the identification problem posited by the estimation of Phillips curves; rationalises findings in the empirical literature and discusses practical solutions to the identification problem, showing evidence of a steeper Phillips curve in US regional data. In doing so, the paper hopes to address a recent wave of work questioning the existence of a link between inflation and slack, a key building block of the prevalent monetary policy framework.

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