Bond Supply and Excess Bond Returns

Robin Greenwood
Harvard Business School and NBER
rgreenwood@hbs.edu

Dimitri Vayanos
LSE, CEPR and NBER
d.vayanos@lse.ac.uk

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Abstract

We examine empirically how the maturity structure of government debt affects bond yields and excess returns. We organize our investigation around a model of preferred habitat, in which supply shocks impact an arbitrage-free term structure. Consistent with the model, we find that the relative supply of long-term bonds is positively related to bond yield spreads and excess returns, with the effects being larger for longer-maturity bonds. Moreover, supply predicts positively excess returns even after controlling for term-structure slope, and supply and slope become stronger predictors of returns following periods when arbitrageurs lose money.

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1 Introduction

How does the maturity structure of government debt affect interest rates? If, for example, the government raises the supply of long- relative to short-term bonds, would this raise the spread between long and short rates? According to standard representative-agent models, there should be no effect because of Ricardian equivalence (Barro (1974)). Intuitively, the consumption of the representative agent, and hence interest rates, depend on government spending but not on how spending is financed.

The irrelevance result is at odds with a preferred-habitat view, held by policy makers and emphasized in early term-structure theories (e.g., Culbertson (1957), Modigliani and Sutch (1966)). According to that view, there are investor clienteles with preferences for specific maturities, and the interest rate for a given maturity is influenced by demand and supply shocks local to that maturity. For example, an increase in the relative supply of long-term bonds should lower their prices relative to short-term bonds, thus raising the spread between long and short rates.

Determining empirically how the maturity structure of government debt affects interest rates is important for informing the theory of the term structure, especially given the conflicting predictions. An empirical investigation of supply effects is also relevant from a policy viewpoint. For example, during the 2007-2009 financial crisis, central banks around the world conducted unprecedented open-market purchases of long-term bonds, greatly shortening the average maturity of government debt. Drawing on preferred-habitat intuition, the central banks hoped that their purchases, also known as quantitative easing, would lower long rates and stimulate private investment.[1] In light of these policy actions, it is surprising how little empirical evidence there is correlating the maturity structure of government debt to interest rates. In this paper, we provide such evidence.

We use time-series data to examine empirically how the maturity structure of government debt affects bond yields and excess returns in the U.S. We organize our investigation around a set of predictions from a formal model of preferred habitat (Vayanos and Vila (2009)), which we extend to allow for time-series variation in maturity structure. The model assumes that investor clienteles with preferences for specific maturities trade with arbitrageurs. In the absence of arbitrageurs, the markets for different maturities would be segmented, and interest rates would be determined solely

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[1] The largest purchases were conducted by the U.S. Federal Reserve. In its December 2008 minutes, the Federal Open Market Committee announced its intention to buy $500 billion of agency mortgage-backed securities and $100 billion of agency debt. The March 2009 minutes announced further purchases of $300 billion of long-term Treasury bonds, $750 billion of agency mortgage-backed securities, and $100 billion of agency debt. The objective behind these purchases is stated in the December 2008 minutes as follows: "... participants discussed the merits of purchasing large quantities of longer-term securities such as agency debt, agency mortgage-backed securities, and Treasury securities. The available evidence indicated that such purchases would reduce yields on those instruments, and lower yields on those securities would tend to reduce borrowing costs for a range of private borrowers ..." Large purchases with similar objectives were conducted by the Bank of England, the European Central Bank, the Swiss National Bank, the Bank of Japan and the Bank of Canada.

by local demand and supply. For example, an increase in the supply of 10-year zero-coupon bonds would lower their price, raising the 10-year interest rate, but would not affect the rates for years 9 and 11. Arbitrageurs integrate maturity markets, but because they are risk averse, demand and supply still affect the term structure. A basic prediction of the model is that an increase in the relative supply of long-term bonds should lower their prices, raising their yields and excess returns over short-term bonds. The model generates further predictions, which broaden and sharpen our empirical investigation of supply effects. These predictions concern how changes in the relative supply of long-term bonds affect the entire cross section of bonds, and how the effects vary over time depending on arbitrageur risk aversion.

We construct measures of the maturity structure of promised payments on U.S. government bonds starting in 1952. For every bond, CRSP maintains a record of the coupon structure and face value outstanding at the end of each month. This allows for precise estimates of the maturity structure of government debt. We relate these estimates to yield spreads and subsequent excess bond returns. Consistent with the model, we find that the relative supply of long-term bonds is positively related to their yield spreads and subsequent excess returns over short-term bonds. The effects are economically significant: a one standard deviation increase in the fraction of government debt due in ten years or longer is associated with a 38bps (hundredths of one percent) increase in the yield spread between the 20- and the one-year bond, and a 218bps increase in the predicted one-year return of the 20-year bond in excess of the one-year bond.[2] The R-squared of the latter regression is 5.7%, but rises to 22.5% and 37.4%, respectively, when predicting three- and five-year returns. Thus, the effects of supply are not only economically significant, but also account for a sizeable fraction of the variation in long-horizon returns.

One surprising prediction of the model is that holding the total supply of debt constant, an increase in the relative supply of long-term bonds can lower the prices of all bonds—including short-term ones, whose supply decreases. The intuition is that changes in supply affect the term structure by altering the market prices of the risk factors, which are common to all bonds because of the absence of arbitrage. Arbitrageurs accommodate an increase in the relative supply of long-term bonds by increasing their holdings of these bonds and reducing their holdings of short-term bonds. Since long-term bonds are more sensitive to changes in the short rate, arbitrageurs become more exposed to short-rate risk and require higher compensation for bearing that risk. Therefore, the market price of short-rate risk increases, and this raises risk premia and yields on all bonds. The data support this prediction.

Three additional predictions of the model that are supported in the data are as follows. First,

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[2] Evaluating statistical significance is complicated by the fact that yields and expected returns contain slow-moving components that can be unrelated to the slow-moving debt maturity structure. We compute t-statistics to account for this issue. Statistical significance is relatively low for the yield spread results, but higher for the return results.

the effects of supply on yields and excess returns are increasing with maturity since longer-maturity bonds are more sensitive to changes in the market price of short-rate risk. Second, supply predicts positively excess bond returns even after controlling for the slope of the term structure, which is a well-known return predictor (e.g., Fama and Bliss 1987, Campbell and Shiller 1991). Indeed, an increase in supply raises both term-structure slope and bond risk premia—but slope is a noisy measure of premia because it also depends on expected short rates. Third, supply has stronger effects on excess returns than on yields. Indeed, the effect of a supply shock on a bond's yield is equal to the shock's average effect on the bond's instantaneous expected return over the bond's life. Moreover, the effect on instantaneous expected return decreases as the bond approaches maturity both because the supply shock is mean-reverting (in the model as well as in the data) and because supply effects are weaker for short-term bonds.

One concern with our analysis is that the maturity structure of government debt is an endogenous variable chosen by the government. In particular, maturity structure could be driven by macroeconomic variables, which could also affect bond risk premia regardless of any causal effect of supply. We find, however, that supply predicts excess bond returns even after controlling for a host of macroeconomic variables used in the literature. Moreover, variation in maturity structure is driven to a significant extent by regulation and political constraints, which are not closely tied to the macroeconomy. For example, a regulatory ceiling on bonds' coupon rates induced the Treasury to issue only notes and bills between 1965-1973, and this shortened the average maturity of government debt. Debt maturity started increasing only in 1976, when Congress raised the maturity of notes to ten years.[3] Finally, the interpretation of our findings as indicating a causal effect of supply is strengthened from the additional predictions that we test. In particular, the model characterizes how risk premia should vary with measures of arbitrageur risk aversion, and the data support these predictions. A complete macroeconomic story would need to address this together with our other findings.

Arbitrageur risk aversion characterizes two related effects in the model. When risk aversion is high, arbitrageurs are less able to accommodate supply shocks, and therefore supply has stronger effects on yields and excess bond returns. Moreover, the slope of the term structure, which in the model predicts positively excess bond returns, becomes a stronger predictor because arbitrageurs are less able to exploit, and hence reduce, the predictability. To measure arbitrageur risk aversion, we note that when long-term bonds are in large supply, they yield more than short-term bonds, and arbitrageurs short short-term bonds to invest in long-term bonds. Therefore, arbitrageurs lose money when an upward-sloping term structure is followed by under-performance of long- relative to short-term bonds. A similar argument implies that arbitrageurs lose money when

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[3] Garbade (2007) provides a comprehensive account of the evolution of debt maturity since the 1950s. We provide a short description in Section 3.1.

a downward-sloping term structure is followed by over-performance of long- relative to short-term bonds. Proxying for risk aversion through the product of term-structure slope times long-term bonds' subsequent excess returns, we find that supply and term-structure slope are stronger predictors of excess returns when risk aversion is high. These findings provide suggestive evidence for theories in which arbitrageur capital influences asset returns.[4]

A number of papers measure supply effects by analyzing the behavior of bond yields around specific policy events. Such events include Operation Twist, a program undertaken by the U.S. Treasury and Federal Reserve during 1962-1964 with the objective to shorten the average maturity of government debt (e.g., Modigliani and Sutch (1966), Ross (1966), Wallace (1967), Van Horne and Bowers (1968) and Holland (1969)), and the 2000-2002 buybacks by the U.S. Treasury, undertaken with a similar objective (e.g., Garbade and Rutherford (2007) and Greenwood and Vayanos (2010)). An advantage of such event studies is that because the exact dates of policy events are known, it is easier to map changes in supply to changes in yields and so establish causality. For example, within three weeks of the buyback announcement in 2000, the spread between the 20- and the five-year interest rate had decreased by 65bps, a clear indication of an anticipated supply effect.[5] On the other hand, event studies typically rely on a small number of data points and thus do not have much power at long horizons. Our time-series approach is more suitable for measuring longer-term effects of supply on yields and excess returns.

Simon (1991,1994), Duffee (1996) and Fleming (2002) document supply effects in the cross section of Treasury Bills by correlating the supply of individual bills with the idiosyncratic component of their yields.[6] Krishnamurthy (2002) finds that when a new "on-the-run" bond is issued in large quantity, it trades at a low price premium relative to an old "off-the-run" bond with comparable characteristics. We focus instead on effects at a more aggregate scale. Reinhart and Sack (2000) and Dai and Philippon (2005) find that government deficits raise the spread between long- and short-term interest rates. The latter paper also shows that this effect is partly through an increase in the risk premia of long-term bonds. These findings are consistent with ours because the debt-to-gdp ratio is positively correlated with the relative supply of long-term bonds. Kuttner (2006) finds that shifts in Federal Reserve holdings of government debt towards longer maturities lower the risk premia of two- to five-year bonds. We test instead for a level effect (i.e., does the fraction of

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[4] See Gromb and Vayanos (2010) for a survey.

[5] Other events indicating strong demand or supply effects include the 2001 discontinuation of the 30-year U.S. Treasury bond, the 2004 pension reform in the U.K., and the 2008-2009 quantitative-easing operations of the U.S. Federal Reserve. Upon the announcement that the 30-year bond would be discontinued, the bond's price soared from 102.56 to 107.88, a yield drop of 34 bps. For evidence on the latter two events, see Greenwood and Vayanos (2010).

[6] See also Park and Reinganum (1986) and Ogden (1987), who argue that bills maturing at the end of calendar months are more expensive than other bills because of cash-management demand by corporations. Price pressure at higher frequencies is documented in Fleming and Rosenberg (2007), who find that Treasury dealers are compensated by high excess returns when holding large inventories of newly issued Treasury securities.

long-term debt rather than its first difference predict excess returns) because this is what our model predicts, and we do so for the aggregate amount issued by the Treasury.[7] Longstaff (2004) finds that U.S. Treasury bonds trade at a high price premium relative to bonds issued by Refcorp, a U.S. government agency, during those months of the 2000-2002 buybacks when the Treasury made large purchases. Krishnamurthy and Vissing-Jorgensen (2010) find that when government bonds are is small supply, i.e., high debt-to-gdp ratio, they trade at a high price premium relative to AAA-rated corporate bonds. These findings suggest that there exists an investor clientele valuing government bonds significantly above close substitutes such as AAA-rated corporate or agency bonds.[8]

The rest of this paper is organized as follows. Section 2 develops the theoretical framework and derives the empirical hypotheses. Section 3 describes the data and our measures of maturity structure. Section 4 presents the results and Section 5 concludes.

2 Theoretical Predictions

A theoretical framework helps organize our empirical investigation of supply effects on the term structure. The theory builds on the preferred-habitat model of Vayanos and Vila (2009), in which investors with strong preferences for specific maturities trade with arbitrageurs. In the absence of arbitrageurs, the markets for different maturities would be segmented because each maturity has its own clientele of investors. Arbitrageurs integrate markets, rendering the term structure arbitrage-free. However, because arbitrageurs are risk averse, investor demand has an effect. We extend this analysis by considering explicitly the role of supply and maturity structure of government debt. We determine how these variables affect bond prices and risk premia, and derive the model's predictions.

2.1 Model

The model is set in continuous time. The term structure at time t consists of a continuum of zero-coupon bonds with maturities in the interval $(0, T]$ and face value one. We denote by $P_t^{(\tau)}$ the price of the bond with maturity τ at time t, and by $y_t^{(\tau)}$ the bond's yield (i.e., the spot rate for

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[7] Because we organize our empirical investigation around a theoretical model, we can also test for a more comprehensive set of predictions, e.g., how supply effects manifest themselves in the cross section of bonds, how they compare for yields and excess returns, and how they vary over time depending on arbitrageur risk aversion.

[8] Other papers documenting price pressure in the government bond market include Fernald, Mosser and Keane (1994) and Kambhu and Mosser (2001), who describe incidents where interest-rate hedging by mortgage or options traders affected the term structure, and Baker and Wurgler (2009), who find that government bonds comove more with large stocks with high earnings during "flights to quality," reflecting correlated demand for both types of securities.

maturity τ). The yield is related to the price through

$$y_t^{(\tau)} = -\frac{\log P_t^{(\tau)}}{\tau}. \tag{1}$$

The short rate r_t is the limit of $y_t^{(\tau)}$ when τ goes to zero. We take r_t as exogenous and assume that it follows the Ornstein-Uhlenbeck process

$$dr_t = \kappa_r(\overline{r} - r_t)dt + \sigma_r dB_t, \tag{2}$$

where $(\overline{r}, \kappa_r, \sigma_r)$ are constants and B_t is a Brownian motion. The short rate could be determined by the central bank and the macroeconomic environment, but we do not model these mechanisms. We focus instead on how exogenous movements in r_t influence the bond prices $P_t^{(\tau)}$ that are endogenously determined in equilibrium.

There are three types of agents: the government, preferred-habitat investors, and arbitrageurs. The government determines the supply of bonds through its issuance policy. We restrict attention to policies where the supply of bonds with maturity τ can vary over time only in response to changes in the corresponding yield $y_t^{(\tau)}$.[9]

Preferred-habitat investors have preferences for bonds of specific maturities. Examples are pension funds, whose typical preferences are for maturities longer than fifteen years, life-insurance companies, with preferences for maturities around fifteen years, and asset managers and banks' treasury departments, with preferences for maturities shorter than ten years. For simplicity, we assume that preferences take an extreme form whereby each investor demands only a specific maturity. Such preferences can arise when investors consume only once and are infinitely risk averse.[10] Precluding investors from substituting across maturities simplifies the analysis without affecting the main intuitions. Indeed, if investors could tilt their portfolios towards maturities with attractive yields, this would simply add to the arbitrageurs' activity. Therefore, the analysis would be qualitatively similar to assuming less risk-averse arbitrageurs. The set of investors demanding maturity τ constitutes the clientele for the bond with the same maturity. We assume that the demand of that clientele can vary over time only in response to changes in the corresponding yield $y_t^{(\tau)}$.

Taken together, the government and investors generate a net supply (government supply minus

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[9] For example, the government could maintain the face value for each maturity constant over time.

[10] See Appendix B of Vayanos and Vila (2009) for more details. Besides simplifying the model, our assumed preferences have been documented in real-world bond markets, where they pose a problem for government buybacks (e.g., Informed Budgeteer (2001), Greenspan (2001)).

investor demand) that we express in terms of time-t market value and denote by $s_t^{(\tau)}$ for maturity τ. The net supply $s_t^{(\tau)}$ can depend only on $y_t^{(\tau)}$, and a natural assumption is that it is a decreasing function. This is because of two effects operating in the same direction: an increase in $y_t^{(\tau)}$ reduces gross supply because it reduces the present value of bonds issued by the government, and raises investor demand if investors can substitute between bonds and other asset classes (e.g., real estate). For analytical simplicity, we assume that $s_t^{(\tau)}$ depends linearly in $y_t^{(\tau)}$, i.e.,

$$s_t^{(\tau)} = \alpha(\tau)\tau \left[\beta_t^{(\tau)} - y_t^{(\tau)}\right]. \tag{3}$$

We impose no restrictions on the function $\alpha(\tau)$ except that it is positive.

In the absence of arbitrageurs, the yield for maturity τ would be $y_t^{(\tau)} = \beta_t^{(\tau)}$, the value that renders the net supply $s_t^{(\tau)}$ in (3) equal to zero. We allow $\beta_t^{(\tau)}$ to depend both on maturity τ and time t, and assume that it takes the form

$$\beta_t^{(\tau)} = \overline{\beta} + \sum_{k=1}^{K} \theta_k(\tau)\beta_{k,t}, \tag{4}$$

where $\overline{\beta}$ is a constant, $\{\beta_{k,t}\}_{k=1,..,K}$ are risk factors characterizing demand and supply, and $\{\theta_k(\tau)\}_{k=1,..,K}$ are functions characterizing how each factor would impact the cross-section of maturities in the absence of arbitrageurs. For example, when $\theta_k(\tau)$ is independent of τ, a change in $\beta_{k,t}$ would impact all maturities equally and cause a parallel shift in the term structure. When instead $\theta_k(\tau)$ is single-peaked around a specific maturity, a change in $\beta_{k,t}$ would impact that maturity the most, and can be interpreted as a local demand or supply shock. We assume that the factor $\beta_{k,t}$ follows the Ornstein-Uhlenbeck process

$$d\beta_{k,t} = -\kappa_{\beta,k}\beta_{k,t}dt + \sigma_{\beta,k}dB_{\beta,k,t}, \tag{5}$$

where $(\kappa_{\beta,k}, \sigma_{\beta,k})$ are positive constants and $B_{\beta,k,t}$ is a Brownian motion independent of $B_{r,t}$ and $B_{\beta,k',t}$ for $k' \neq k$. We impose no restrictions on the functions $\{\theta_k(\tau)\}_{k=1,..,K}$.

Arbitrageurs can invest in all maturities, and choose a portfolio to trade off instantaneous mean and variance. They solve the optimization problem

$$\max_{\{x_t^{(\tau)}\}_{\tau\in(0,T]}} \left[E_t(dW_t) - \frac{a}{2}Var_t(dW_t)\right], \tag{6}$$

where W_t denotes the arbitrageurs' time-t wealth, $x_t^{(\tau)}$ denotes their dollar investment in the bond with maturity τ, and a is a risk-aversion coefficient. The assumption that preferences are over instantaneous mean and variance is for analytical convenience, and the assumption that the risk-aversion coefficient a is constant simplifies the analysis by suppressing wealth effects. We appeal informally to wealth effects, however, when deriving some of our empirical hypotheses.

2.2 Equilibrium Term Structure

In the absence of arbitrageurs, the yield for maturity τ would be $y_t^{(\tau)} = \beta_t^{(\tau)}$. Since $\beta_t^{(\tau)}$ is a general function of τ, the term structure could have an arbitrary shape. It would also be disconnected from the short rate r_t since $\beta_t^{(\tau)}$ is independent of r_t. Such a term structure exhibits extreme segmentation: each maturity constitutes a separate market, with the yield being determined by the demand of the corresponding clientele and the supply of bonds with that maturity. Of course, segmentation does not occur in equilibrium because of arbitrageurs. Arbitrageurs integrate maturity markets, rendering the term structure arbitrage-free.

One consequence of arbitrageur activity is to bridge the disconnect between the short rate and bond yields. Suppose, for example, that the short rate increases, becoming attractive relative to investing in bonds. Investors do not take advantage of this opportunity because they prefer the safety of the bond that matures at the time when they need to consume. But arbitrageurs do take advantage by shorting bonds and investing at the short rate. Through this carry "roll-up" trade, bond prices decrease, thus responding to the high short rate. Conversely, following a negative shock to the short rate, arbitrageurs engage in a carry "roll down" trade, borrowing short and buying bonds. In both cases, the carry trades of arbitrageurs provide the mechanism through which bond yields move to reflect changes in current and expected future short rates.

In addition to bringing bond yields in line with the short rate, arbitrageurs bring yields in line with each other, smoothing local demand and supply pressures. Suppose, for example, that supply increases at a particular maturity segment. In the absence of arbitrageurs, yields in that segment would increase but other segments would remain unaffected. Arbitrageurs exploit the difference in yields, buying that segment and shorting other segments to hedge their risk exposure. This brings yields in line with each other, spreading the local effect of supply over the entire term structure.

We conjecture that equilibrium bond yields are affine functions of the risk factors, i.e., the short rate r_t and the demand and supply factors $\{\beta_{k,t}\}_{k=1,..,K}$. Equilibrium bond prices thus are

$$P_t^{(\tau)} = e^{-\left[A_r(\tau)r_t + \sum_{k=1}^{K} A_{\beta,k}(\tau)\beta_{k,t} + C(\tau)\right]} \tag{7}$$

for $K+2$ functions $A_r(\tau), \{A_{\beta,k}(\tau)\}_{k=1,..K}, C(\tau)$ that depend on maturity τ. The function $A_r(\tau)$ characterizes the sensitivity of bond returns to the short rate r_t, and the functions $\{A_{\beta,k}(\tau)\}_{k=1,..K}$ characterize the sensitivity to the demand and supply factors $\{\beta_{k,t}\}_{k=1,..,K}$. This can be seen from Ito's lemma, which implies that a bond's instantaneous return is

$$\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} = \mu_t^{(\tau)} dt - A_r(\tau)\sigma_r dB_{r,t} - \sum_{k=1}^{K} A_{\beta,k}(\tau)\sigma_{\beta,k} dB_{\beta,k,t}, \tag{8}$$

where $\mu_t^{(\tau)}$ denotes the bond's expected return.

Absence of arbitrage requires that a bond's expected return in excess of the short rate is a linear function of the bond's sensitivities to the risk factors:

$$\mu_t^{(\tau)} - r_t = A_r(\tau)\lambda_{r,t} + \sum_{k=1}^{K} A_{\beta,k}(\tau)\lambda_{\beta,k,t}. \tag{9}$$

The coefficients $\lambda_{r,t}$ and $\{\lambda_{\beta,k,t}\}_{k=1,..,K}$, which are the same for all bonds, are the market prices of risk corresponding to the factors r_t and $\{\beta_{k,t}\}_{k=1,..,K}$, respectively. Eq. (9) holds in any arbitrage-free model with $K+1$ factors. Therefore, it also holds in our model because arbitrageurs render the term structure arbitrage-free. The contribution of our model is to link the market prices of risk (on which absence of arbitrage imposes no restrictions) to the supply and maturity structure of government debt: it is through their effect on the market prices of risk that supply and maturity structure affect bond yields and returns.

Vayanos and Vila (2009) show that the functions $A_r(\tau), \{A_{\beta,k}(\tau)\}_{k=1,..K}, C(\tau)$ are linear combinations of exponentials, and are given by generalized Vasicek (1977) formulas. Computing these functions amounts to solving a system of $(K+1)^2$ equations. The system yields simple closed-form solutions in the case where arbitrageur risk aversion is low, i.e., a is close to zero. We mainly focus on that case because it conveys useful intuitions and suffices to derive most of our empirical hypotheses.

Proposition 1. *Suppose that the risk aversion a of arbitrageurs is low. Then, a unit increase in the factor $\beta_{k,t}$ lowers a bond's return by*

$$A_{\beta,k}(\tau) \approx \frac{a\sigma_r^2 \int_0^T \alpha(\tau)\tau\theta_k(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r} d\tau}{\kappa_r - \kappa_{\beta,k}} \left(\frac{1-e^{-\kappa_{\beta,k}\tau}}{\kappa_{\beta,k}} - \frac{1-e^{-\kappa_r\tau}}{\kappa_r}\right). \tag{10}$$

Proposition 1 determines how a demand or a supply shock affects the term structure. We next draw the implications of this proposition when the shock is a change in the maturity structure of debt.

2.3 Effects of Debt Maturity

Without loss of generality, we identify changes in debt maturity structure with the factor $\beta_{1,t}$. We assume that an increase in $\beta_{1,t}$ corresponds to an increase in average maturity, achieved through government issuance of long-term bonds and buyback of short-term bonds. Denoting by S_1 the set of long maturities issued by the government and by S_2 the set of short maturities bought back, the function $\theta_1(\tau)$ is positive for $\tau \in S_1$, negative for $\tau \in S_2$, and zero otherwise. Following a unit increase in $\beta_{k,t}$, the government issues $\int_{S_1} \alpha(\tau)\tau\theta_1(\tau)d\tau$ dollars worth of long-term bonds and buys back $-\int_{S_2} \alpha(\tau)\tau\theta_1(\tau)d\tau$ dollars worth of short-term bonds. We assume that $\int_0^T \alpha(\tau)\tau\theta_1(\tau)d\tau = 0$, thus focusing on a pure change in debt maturity structure without any change in the total market value of debt (and in our empirical analysis we can control for the overall size of the government bond market). The effects we derive below would become stronger if $\int_0^T \alpha(\tau)\tau\theta_1(\tau)d\tau > 0$, i.e., the increase in average maturity is accompanied by an increase in the total value of debt. Propositions 2-6 examine how the increase in average maturity affects bond yields and risk premia. These propositions generate our testable hypotheses.

Proposition 2 (**Debt Maturity and Yields**). *Suppose that the risk aversion a of arbitrageurs is low. Then, an increase in average maturity (i.e., an increase in $\beta_{1,t}$) raises the yields of all bonds. Moreover, yields of intermediate- or long-term bonds are the most affected.*

That an increase in average maturity raises the yields of all bonds is surprising: why do short-term bonds experience an increase in yields, thus becoming cheaper, even though their supply decreases? The intuition is that demand and supply effects operate through changes in the market prices of risk. Since market prices of risk are common to all bonds because of the absence of arbitrage, demand and supply shocks local to one segment of the term structure have global effects. In particular, an increase in the supply of long-term bonds raises the yields of short-term bonds. This effect can dominate the decrease in short yields caused by a decrease in the supply of short-term bonds.

The reason why short yields are more influenced by the supply of long-term bonds than of short-term bonds has to do with the risk aversion of arbitrageurs. When arbitrageurs have low risk aversion, they only require small price changes to accommodate shocks to demand and supply. Therefore, the short rate is the dominant driver of yields, and demand and supply effects operate through changes in the market price of short-rate risk: if a demand or a supply shock raises the market price of short-rate risk, it raises the yields of all bonds, and vice-versa.

An increase in average maturity raises the market price of short-rate risk because it raises the duration of arbitrageurs' portfolio. Indeed, arbitrageurs accommodate the shock by increasing their holdings of long-term bonds and reducing those of short-term bonds. Even though the two transactions have the same market value, arbitrageurs' overall exposure to the short rate increases because long-term bonds have higher duration than short-term bonds. Therefore, arbitrageurs require higher compensation to bear short-rate risk.[11]

We next examine how the increase in average maturity affects bond risk premia, defined as bonds' instantaneous expected returns in excess of the short rate. Since the increase in average maturity raises the market price of short-rate risk, it raises the risk premia of all bonds.

Proposition 3 (Debt Maturity and Risk Premia). *Suppose that the risk aversion a of arbitrageurs is low. Then, an increase in average maturity (i.e., an increase in $\beta_{1,t}$) raises the risk premia of all bonds. Moreover, risk premia of long-term bonds are the most affected.*

The increase in risk premia is largest for long-term bonds because they are the most sensitive to the short rate.[12] The increase in yields is not necessarily largest for long-term bonds, however: Proposition 2 shows that yields of intermediate-term bonds can increase the most. Intuitively, the effect of a demand or a supply shock on a bond's yield is equal to the average effect on the bond's risk premium over the bond's life. This average effect can be largest for intermediate-term bonds if the shock mean-reverts quickly. Regardless of mean-reversion, however, demand and supply shocks have small effects on the yields and risk premia of short-term bonds. Intuitively, short-term bonds are close substitutes to investing in the short rate, and arbitrageurs can tie their yields closely to current and expected future short rates. Similar to Greenwood (2005) and Garleanu, Pedersen and Poteshman (2009), the pressure that demand and supply shocks exert on prices depends on the degree of substitutability of the assets whose supplies are being changed.

The effect of average maturity on risk premia, derived in Proposition 3, is larger than the effect on yields, derived in Proposition 2. This follows from the property that the effect of a demand or

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[11] The effects of a change in average maturity can be seen formally from Proposition 1. According to (1) and (7), a unit increase in the factor $\beta_{1,t}$ raises the yield for maturity τ by $\frac{A_{\beta,1}(\tau)}{\tau}$. The integral $\int_0^T \alpha(\tau)\tau\theta_1(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r}d\tau$ in Proposition 1 can be interpreted as the duration of the bond portfolio acquired by arbitrageurs following the increase in $\beta_{1,t}$: the change in the supply of the bond with maturity τ is $\alpha(\tau)\tau\theta_1(\tau)$, and the sensitivity $A_r(\tau)$ of that bond's return to the short rate is approximately $\frac{1-e^{-\kappa_r\tau}}{\kappa_r}$ when arbitrageur risk aversion is low. Since long-term bonds have higher duration than short-term bonds ($\frac{1-e^{-\kappa_r\tau}}{\kappa_r}$ is increasing in τ), and the bond portfolio acquired by arbitrageurs has zero market value ($\int_0^T \alpha(\tau)\tau\theta_1(\tau)d\tau = 0$), the portfolio's duration is positive. Therefore, $A_{\beta,1}(\tau) > 0$, meaning that an increase in average maturity raises the yields of all bonds.

[12] This can be seen formally from (9): an increase in $\lambda_{r,t}$ affects long premia the most because $A_r(\tau) \approx \frac{1-e^{-\kappa_r\tau}}{\kappa_r}$ is increasing in τ.

a supply shock on a bond's yield is equal to the average effect on the bond's risk premium over the bond's life. The effect on the bond's average premium is smaller than the effect on the current premium for two reasons. Since the shock mean-reverts, its effect on the premia of all bonds dies down over time. And even if the absence of mean reversion, the shock's effect on the premium of any given bond decreases over time as the bond's time to maturity decreases.

Proposition 4 (**Risk Premia vs. Yields**). *Suppose that the risk aversion a of arbitrageurs is low. Then, the effect of an increase in average maturity on risk premia exceeds that on yields.*

Propositions 2 and 3 imply that changes in average maturity induce a positive relationship between bond risk premia and the slope of the term structure, defined as a bond's yield minus the short rate. For example, when average maturity increases, premia increase, and so does the slope because yields increase. Thus, the model can help explain the positive premia-slope relationship found by Fama and Bliss (1987). This, however, raises the question whether the effects of supply on premia are fully captured by slope. Proposition 5 shows that the answer to this question is no: supply predicts positively excess bond returns even after controlling for slope. The intuition is that slope is driven not only by demand and supply, but also by the short rate. And while demand and supply affect slope only through their effect on premia, the effect of the short rate is only partly through premia. For example, a decrease in the short rate increases the slope even if premia remain unchanged: simply because the current short rate moves below expected future short rates. Therefore, an increase in slope is accompanied by a larger increase in premia when caused by demand and supply than when caused by the short rate.

Proposition 5 (**Debt Maturity vs. Other Return Predictors**). *Suppose that the risk aversion a of arbitrageurs is low. Then, average maturity predicts positively excess bond returns both in a univariate regression, and when controlling for the slope of the term structure.*

How do the results of Propositions 2-5 extend to higher values of arbitrageur risk aversion? When risk aversion is high, the effects of demand and supply shocks local to one segment of the term structure tend to become more local. The extent to which this happens depends on the number of risk factors. In the extreme case where there is a large number of factors, arbitrageurs cannot hedge all sources of risk when intermediating across maturities. Therefore, if they are highly risk averse, they refrain from intermediation, and demand and supply effects are fully local. The polar opposite case is when there is only one factor, which has to be the short rate.[13] Demand and supply shocks are then one-off, and a previous version of this paper shows that they have global effects, as in Propositions 2-5. Adding a small number of demand and supply factors yields intermediate results.[14] We do not take a stance on the number of factors, and test for local effects

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[13] If the short rate is constant, the term structure is flat and constant.

[14] See Vayanos and Vila (2009).

in our empirical analysis.

We finally examine the effects of variation in arbitrageur risk aversion, within the set of low values assumed in Propositions 2-5. Not surprisingly, when risk aversion increases, demand and supply effects become stronger. Moreover, the slope of the term structure becomes a stronger predictor of excess bond returns, which are zero on average when arbitrageurs are risk-neutral.

Proposition 6. *Suppose that the risk aversion a of arbitrageurs is low. When a increases*

- *An increase in average maturity has larger effects on bond yields and risk premia.*
- *The regression coefficient of excess bond returns on the slope of the term structure increases.*

Proposition 6 is a comparative-statics result because the parameter a is constant in the model. Stepping outside of the model, however, we can interpret the proposition as concerning the effects of time-variation in a. Indeed, if a is decreasing in arbitrageurs' wealth, then it increases in periods when arbitrageurs lose money. Identifying such periods requires a measure of arbitrageurs' returns. Possible measures are the returns of hedge funds or the profit-loss positions of proprietary-trading desks. But the model suggests an even more direct measure (in the sense of requiring only term-structure data), derived from arbitrageurs' trading strategies. For example, at times when the term structure is upward sloping, the model predicts that arbitrageurs are engaged in the carry roll-down trade. Therefore, arbitrageurs' wealth decreases when that trade loses money. Conversely, when the term structure is downward sloping, arbitrageurs are engaged in the carry roll-up trade, and their wealth decreases when that trade loses money.

The empirical hypotheses derived from the model can be summarized as follows:

Hypothesis 1. *The spread between the yield of a τ-year bond and a one-year bond is increasing in the average maturity of government debt.*

Hypothesis 2. *The expected return of a τ-year bond in excess of a one-year bond is increasing in the average maturity of government debt. This positive relationship appears both in a univariate regression, and when controlling for the slope of the term structure.*

Hypothesis 3. *The effect of average maturity on expected excess returns is stronger for larger τ. The effect on yields is stronger for larger τ, or is hump-shaped.*

Hypothesis 4. *The effect of average maturity on expected excess returns is stronger than on yields.*

Hypothesis 5. *Arbitrageurs lose money when the term structure slopes up and long-term bonds subsequently underperform short-term bonds, or when the term structure slopes down and long-term bonds subsequently outperform short-term bonds. Following such events:*

- *The regression coefficient of excess bond returns on average maturity increases.*
- *The regression coefficient of excess bond returns on the slope of the term structure increases.*

3 Data

3.1 Debt Maturity

We collect data on every U.S. government bond that was issued between 1940 and 2006 from the CRSP historical bond database. CRSP collects data on bond characteristics (issue date, coupon rate, maturity, callability features) as well as providing monthly observations of face value outstanding. We break the stream of each bond's cash flows into a series of principal and coupon payments.[15] At date t, the future payments are the sum of principal and coupon payments from all bonds, bills, and notes that were issued at t or before and have not yet retired, all scaled by the face value outstanding at t. For example, consider the 7-year bond issued in February 1969 (CRSP ID 19760215.206250) with a coupon payment of 6.25%, and suppose that t denotes the last day of March 1972. On this day, investors who held the bond were expecting eight more coupon payments of \$3.125 per \$100 of face value, starting August 1972 and ending February 1976 (the maturity of the bond), with the full principal to be repaid in February 1976. CRSP reports a total face value of \$882 million outstanding as of March 1972. Thus, as of this date, there are eight coupon payments of \$27.56 million and the principal payment of \$882 million.

The face value of a bond can change throughout its traded life. This comes from the Treasury issuing additional shares, or occasionally repurchasing all or some of the outstanding value of the bond in an open market operation, such as in the 2000-2002 buyback program. Considering the example above, if the Treasury were to repurchase the entire face value of the bond during April 1972, then at the end of April we would register a remaining payment stream of zero. Despite generally complete data from CRSP, there are some reporting gaps in face values. Where these occur, we fill in missing values with the face value outstanding at the end of the previous month.

In early sample years, face values are reported only occasionally. By the early 1950s, face values are reported for over 95% of Treasury instruments, and so we start our analysis there. This coincides with the beginning of the Fama-Bliss bond data, which starts in June 1952. Thus, the majority of our tests rely on debt maturity measured between 1952 and 2006, with excess returns extending beyond. Constraining our estimation to the 1964-2003 period studied in Cochrane and Piazzesi (2005) strengthens our results.

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[15] Our calculations are similar to those in Doepke and Schneider (2006), who use the CRSP bond data to determine the maturity structure of holdings of U.S. government bonds by domestic households.

For a large fraction of securities, CRSP reports both the entire face value and the face value held by the public. In principle, we would prefer to have the latter measure, as this appropriately nets out Federal Reserve holdings and interagency holdings. However, we find that the face value held by the public is reported only sporadically for some bonds, and tends to be missing for bills until quite recently. We thus use the entire face value, although we have experimented with adjustments for Federal Reserve holdings, which we discuss in the robustness section.[16]

Total principal payments due τ years from date t are given by

$$PR_t^{(\tau)} = \sum_i PR_{it}^{(\tau)},$$

where i subscripts the individual bonds. To verify that we have captured all bonds outstanding, we collect data from the back issues of the Bureau of Public Debt and match reported totals to total principal payments at various points in time. Total coupon payments due τ years from date t are given by

$$C_t^{(\tau)} = \sum_i C_{it}^{(\tau)},$$

where coupons are one half the annual coupon rate times the face value. Total payments due τ years from date t are the sum of principal and coupons

$$D_t^{(\tau)} = PR_t^{(\tau)} + C_t^{(\tau)}.$$

Figure 1 shows expected future payments at a single point in time, which we take to be June 1975. The figure marks principal and coupon payments separately. Figure 1 is fairly typical in our time series in that coupons constitute a fairly small fraction of total payments on a face value basis, and less on a present value basis. Coupons constitute a larger fraction of total payments as maturity lengthens, because bonds and notes pay coupons while bills do not.

We measure debt maturity by the long-term debt share, defined as the fraction of payments

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[16] Simple measures of the maturity of Federal Reserve holdings correlate strongly with our debt maturity variables, suggesting that the Federal Reserve does not take an active approach to managing the maturity of its debt portfolio. Federal Reserve holdings, measured in dollar terms, increase steadily throughout the sample period, during periods of considerable fluctuation in the debt-to-gdp ratio. Thus, when debt-to-gdp is low, the Federal Reserve holds a disproportionately large share of government debt.

due in ten years or longer as of total payments due in all future years:

$$m_t^{(10+)} = \frac{\sum_{10 \le \tau \le 30} D_t^{(\tau)}}{\sum_{0 \le \tau \le 30} D_t^{(\tau)}}.$$

An alternative measure of debt maturity is the dollar-weighted average maturity

$$M_t = \frac{\sum_{0 \le \tau \le 30} D_t^{(\tau)} \tau}{\sum_{0 \le \tau \le 30} D_t^{(\tau)}},$$

which weighs the times of future payments by the fraction that these payments represent as of total payments. The dollar-weighted average maturity resembles Macaulay duration, except that weights are computed on a face rather than a present value basis. In our empirical analysis we use a measure closer in spirit to Macaulay duration, discounting all payments by the historical average $\overline{r}$ of the short rate.[17] This measure, to which we refer as the duration, is

$$Duration_t = \frac{\sum_{0 \le \tau \le 30} \frac{D_t^{(\tau)}}{(1+\overline{r})^\tau} \tau}{\sum_{0 \le \tau \le 30} \frac{D_t^{(\tau)}}{(1+\overline{r})^\tau}}.$$

The long-term debt share, the dollar-weighted average maturity and the duration are highly correlated, with the lowest correlation being 93%. Figure 2 plots these measures and Table I reports summary statistics. Debt maturity shortened in the 1950s, from the mid-1960s to the mid-1970s, and after 2000. It lengthened in the early 1960s, and from the mid-1970s to the late 1980s, while it remained approximately flat during the 1990s. Drivers of these movements included regulatory constraints, changes in the size of government debt, changes in Treasury attitudes about the benefits of long-term debt, and the desire to keep debt maturity within a target level.

The drop in debt maturity from the mid-1960s to the mid-1970s, and the subsequent rise, were partly driven by the 4.5% regulatory ceiling on bonds' coupon rates. Because of the ceiling, the Treasury did not issue bonds between 1965-1973, leading to a decline in debt maturity. Congress gradually loosened the ceiling, first by extending the maturity of notes (to which the ceiling did not apply) from five to seven years in 1967, and then by allowing limited issuance of bonds not meeting the ceiling in 1971. Debt maturity, however, started increasing only in 1976, when Congress raised the maturity of notes to ten years. The ceiling was eliminated in 1988.

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[17] We do not discount by the term structure at date t because this would create an endogeneity problem: since interest rates in the model depend on the maturity structure of government debt, duration would be endogenous.

An additional driver of the rise in debt maturity during the late 1970s and the 1980s was the expansion of government debt: the Treasury issued at long maturities to reduce the risk of having to refinance large amounts of short-term debt at high rates. Debt maturity also rose slightly in the late 1990s, but for the opposite reason, namely, because the government was running budget surpluses. Indeed, because maturing debt issues were not being reissued, and most of them were short-term, debt maturity was rising mechanically. It was because of a concern with the rising debt maturity that the Treasury engaged in the 2000-2002 buybacks. The buybacks, together with the discontinuation of the 30-year bond in 2001, were behind the decrease in average maturity after 2000.[18]

Table I confirms the visual impression from Figure 1 that most debt is short-term: on average, 18% of face value comes after ten years, and the dollar-weighted average maturity is 4.67 years. Table I also confirms that debt maturity varies significant over time: the standard deviation of the long-term debt share is 5%, and of dollar-weighted average maturity is 0.84 years. This implies economically significant shifts that arbitrageurs must accommodate. For example, approximately five trillion dollars of government securities were held in December 2007. A one standard deviation increase in the long-term debt share would mean that $250 billion of short-term debt is replaced by the same amount of long-term debt. These amounts are sizeable compared with the capital devoted to term-structure arbitrage. For example, Hedge Fund Research reports that total assets in macro and fixed-income-arbitrage hedge funds were less than $150 billion in 2005.

3.2 Bond Prices and Returns

We use the Fama-Bliss discount bond database to compute yields, returns, and forward rates for two-, three-, four- and five-year bonds. Beyond five years, yields are not available for most maturities, making it difficult to compute returns and forward rates. However, Ibbotson Associates provides yields and returns on a bond with an approximate maturity of 20 years, and we use this to obtain a long-term yield and return. Ibbotson Associates also provides yields and returns on an intermediate-term bond with an approximate maturity of 10 years.

Yields and returns are computed in logs. Yield spreads and excess returns are constructed relative to the one-year bond. We denote by $r_{t+1}^{(\tau)}$ the return of the τ-year bond during the year following date t, and by

$$rx_{t+1}^{(\tau)} \equiv r_{t+1}^{(\tau)} - y_t^{(1)}$$

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[18] Our description of the evolution of debt maturity draws on Garbade (2007) and Garbade and Rutherford (2007). Garbade (2007) attributes the drop in debt maturity in the 1950s and the rise in the early 1960s to changes in Treasury attitudes about the benefits of long-term debt.

the bond's excess return during the same period. We denote by

$$rx_{t+k,k}^{(\tau)} \equiv \sum_{k'=1}^{k} rx_{t+k'}^{(\tau)}$$

the bond's excess return during the k years following date t.

4 Results

4.1 Debt Maturity and Bond Risk Premia

Table II reports results from regressing yield spreads and excess returns on our measures of debt maturity. For brevity, we report results for the long-term debt share and duration measures; results for dollar-weighted average maturity are similar. The yield spread regression is

$$y_t^{(\tau)} - y_t^{(1)} = a + bX_t + u_t,$$

where X_t is the measure of debt maturity, observed monthly. The results are in Rows (1)-(5) of Table II. Because yield spreads depend on persistent variables other that debt maturity (e.g., expected short rates), the regression residuals are serially correlated and t-statistics must be adjusted accordingly. The t-statistics in Table II are computed using Newey and West (1987) standard errors allowing for 36 months of lags. This amounts to estimating non-parametrically the correlation structure between residuals up to 36 months apart, but assuming no correlation for longer lags. An alternative method to adjust for serial correlation is to estimate a parametric process for the residuals. Under an AR(1) process, t-statistics are lower than in Table II: the median t-statistic across Rows (1)-(5) decreases from 1.82 in Table II to 1.34. The discrepancy might be because the serial correlation extends to lags longer than 36 months—although increasing the number of lags in the Newey and West procedure does not seem to affect the t-statistics in Table II.

While the yield spread results are weakly statistically significant, they go in the direction predicted by the model. Consistent with Hypothesis 1, debt maturity is positively related to yield spreads. Consistent with Hypothesis 3, the positive relationship strengthens for long-term bonds. For example, the coefficient of the long-term debt share rises from 0.016 for the two-year bond to 0.076 for the 20-year bond. Thus, a one-standard deviation increase in the long-term debt share is associated with a $0.076 \times 5\% = 38$bps increase in the yield spread between the 20- and the one-year bond. Using the duration measure, the effect is 41bps.

We next turn to the results on excess returns. Panel A of Figure 3 plots the five-year excess return of the 20-year bond over years $t+1$ to $t+5$ together with the long-term debt share in year t. The figure shows a strong positive correlation. Periods where debt maturity is short, such as the late 1960s and the 1970s, are associated with low returns, while periods in which debt maturity is longer, such as the late 1980s, the 1990s, and the early 2000s, are associated with high returns. The horizontal axis on the figure ends in 2003 because the five-year excess return includes holding period returns between 2004 and 2008. Panel B of Figure 3 plots the one-year excess return of the 20-year bond over year $t+1$ together with the long-term debt share in year t. The figure also shows a positive correlation, albeit more noisy than in Panel A.

Rows (6)-(10) of Table II report the results of the regression

$$rx_{t+1}^{(\tau)} = a + bX_t + u_{t+1},$$

of one-year excess returns on the measure X_t of debt maturity, observed monthly. As in the case of yields, t-statistics must be adjusted for serial correlation in the regression residuals. In the case of returns, serial correlation arises from two sources. First, persistent variables other than debt maturity could affect expected returns (e.g., clientele demand or macroeconomic variables). Second, because returns are measured over one year but are sampled monthly, measurement periods overlap. The overlap problem does not arise when returns are sampled annually. We report the results of the corresponding regression for the 20-year bond and returns sampled in December of each year in Row (13).

As in the case of yields, t-statistics are computed using Newey and West standard errors and allowing for 36 months of lags. An alternative set of t-statistics can be computed by estimating parametric processes for the residuals. A plausible process in the case of annual sampling is an ARMA(1,1): such a process would arise if the component of expected return that is driven by variables other than debt maturity is AR(1), and the realized return is expected return plus *i.i.d.* noise.[19] Under an ARMA(1,1) process, the t-statistic in Row (13) increases from 2.41 to 5.72. This might be because of the negative serial correlation of residuals at long lags. Adjustments derived under other processes and for monthly sampled returns yield also high t-statistics.[20]

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[19] We are grateful to John Cochrane for suggesting the ARMA(1,1) adjustment, as well as the AR(1) adjustment for yield spreads. Duffee (2007) also emphasizes the importance of accounting for slow-moving components of expected returns when computing t-statistics in return-forecasting regressions.

[20] For example, the t-statistic under an AR(1) process and annually sampled returns is 2.42. In the case of monthly sampling, we compute t-statistics under an ARMA(1,q) process, with q ranging from one to 12. The lowest t-statistic for the 20-year bond is 1.75 and the median is 3.43. One issue with these estimations, however, is that the moving average coefficients tend to be unstable depending on the number of lags. In the case of monthly sampling, we also compute t-statistics using Hansen and Hodrick (1980) standard errors, which account explicitly for the overlapping windows in measurement periods. Hansen and Hodrick t-statistics are generally comparable to Newey and West. For example, for the 20-year bond, the t-statistic in Row (10) changes from 2.56 to 2.21, and that in Row (11) changes from 3.27 to 2.98.

As an additional robustness check, we use the bootstrap approach suggested by Bekaert and Hodrick (2001) and Bekaert, Hodrick and Marshall (2001). We compute bootstrapped *p*-values by comparing the Newey-West *t*-statistic to the distribution of bootstrapped *t*-statistics. To preserve the time-series dependence of the original data, we create pseudo time series using the stationary block bootstrap of Politis and Romano (1994). This yields a *p*-value of 0.026 for the 20-year bond and one-year excess returns (Row (10)), and 0.022 for the same bond and three-year excess returns (Row (11)).[21]

Besides being statistically significant, the excess return results go in the direction predicted by the model. Consistent with Hypothesis 2, debt maturity is positively related to subsequent excess returns. Consistent with Hypothesis 3, the positive relationship strengthens for long-term bonds. For example, the coefficient of the long-term debt share rises from 0.085 for the two-year bond to 0.436 for the 20-year bond. These coefficients are economically significant. For example, a one-standard deviation increase in the long-term debt share predicts a $0.436\times5\%$=218bps increase in the return of the 20-year bond in excess of the one-year bond. Using the duration measure, the effect is 208bps. The corresponding effects for the two-year bond are 43bps and 35bps, respectively.

Consistent with Hypothesis 4, debt maturity has larger effects on excess returns than on yields: the regression coefficients in Rows (6)-(10) are larger than in Rows (1)-(5). Comparing the two sets of coefficients reveals the extent to which shocks to debt maturity have transitory or permanent effects on excess returns. Indeed, recall that in the model, the effect of a supply shock on a bond's yield is equal to the shock's average effect on the bond's instantaneous expected return over the bond's life. Moreover, the effect on instantaneous expected return decreases as the bond approaches maturity both because the supply shock is mean-reverting and because supply effects are weaker for short-term bonds. In the absence of mean-reversion, the effect on instantaneous expected return would decrease with the time τ to maturity according to the function $A_r(\tau) \approx \frac{1-e^{-\kappa_r \tau}}{\kappa_r}$, which is concave in τ.[22] This means that the effect on the current instantaneous excess return would not exceed twice that on the current yield. Since, however, the regression coefficients for excess returns exceed those for yields by a multiple of five, mean-reversion appears to be important. This means that shocks to debt maturity have transitory effects on excess returns, and arbitrageurs realize the highest returns in the early years following the shocks.

Rows (11)-(12) and (14)-(15) of Table II concern the excess returns of the 20-year bond over longer horizons, of three and five years. As the degree of return overlap increases, we adjust standard

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[21] An additional concern related to statistical significance is that the OLS coefficients may be biased if innovations in the forecasting variable, i.e., debt maturity, are correlated with innovations in returns (Mankiw and Shapiro (1986), Stambaugh (1986)). This bias is relatively minor in our data. For example, in the case of annual sampling, the coefficient 0.435 in Row (13) is corrected to 0.474.

[22] This follows from (9) because a supply shock affects the market price of short-rate risk $\lambda_{r,t}$.

errors accordingly, i.e., we compute Newey and West standard errors allowing for $12k + 24$ months of lags, where k is the number of years. Increasing the number of lags does not seem to affect the t-statistics. However, as can be seen in Table II, debt maturity becomes a stronger predictor of excess returns when the forecast horizon increases. For example, the R-squared in the case of the long-term debt share increases from 5.7% for the one-year return to 22.5% for the three-year return, to 37.4% for the five-year return. The improvement in regression fit confirms the visual impressions from Panel A of Figure 3, which plots five-year excess returns together with the long-term debt share.

The regression coefficients in Rows (11)-(13) increase approximately linearly as the forecast horizon increases. For example, the 1.525 coefficient from regressing the three-year excess return on the long-term debt share is approximately three times the 0.436 coefficient for the one-year return. Likewise, the 2.651 coefficient for the five-year return is approximately five times the coefficient for the one-year return. Thus, while shocks to debt maturity have transitory effects on expected returns, these effects appear to last for several years before dying out.

We next turn to multivariate tests. Table III reports results from regressing excess returns on our measures of debt maturity and the slope of the term structure. The first five columns report the results of the regression

$$rx_{t+1}^{(\tau)} = a + bX_t + c(y_t^{(\tau)} - y_t^{(1)}) + u_{t+1},$$

of one-year excess returns on the measure X_t of debt maturity and the τ-year yield spread, which we use as measure of term-structure slope (Fama and Bliss (1987), Campbell and Shiller (1991)).[23] Consistent with Hypothesis 2, debt maturity is positively related to subsequent excess returns even after controlling for slope, although the effect is not statistically significant in some of the regressions. The coefficient of slope is positive, which is consistent with the model since slope could capture time-series variation in unobserved demand by clienteles. Since slope and debt maturity are correlated, the coefficient of debt maturity is smaller in the bivariate than in the univariate regression. For example, in the case of the 20-year bond, the coefficient of the long-term debt share drops from 0.436 in the univariate regression to 0.278 in the bivariate regression. Controlling for slope also increases predictive power: the R-squared in the same regressions rises from 5.7% to 13.6%.

The last two columns of Table III concern the excess returns of the 20-year bond over three- and five-year horizons. At these longer horizons, controlling for term-structure slope makes essentially no difference, and the coefficients of debt maturity are similar across the univariate and bivariate

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[23] Using the forward-rate spread as a measure of term-structure slope yields similar results.

regressions. For example, the coefficient from regressing the five-year excess return on the long-term debt share is 2.651 in the univariate and 2.694 in the bivariate regression.

4.2 Robustness Checks

4.2.1 Debt Maturity and the Macroeconomy

One concern with our analysis is that the maturity structure of government debt is an endogenous variable chosen by the government. One possible source of endogeneity is that the government chooses maturity structure to cater to clientele demands. For example, an increase in the demand for long-term bonds could push long rates down and induce the government to increase issuance at the long end.[24] Catering considerations would bias our analysis, but towards finding smaller effects: if supply is chosen to offset demand, it would appear to have smaller effects than it truly does.

A more serious endogeneity concern is that maturity structure might be driven by macroeconomic variables, which could also affect bond risk premia regardless of any causal effect of supply. Macroeconomic variables that have been shown to help predict bond risk premia include output growth, inflation and the T-bill rate.[25] We consider these variables, together with the output gap, inflation risk, T-bill risk and stock market risk, where the risk variables are measured by the standard deviation of monthly inflation over the past year, monthly T-bill rate over the past year, and daily stock market returns over the past month, respectively. We include each of the variables together with debt maturity in a bivariate regression. We also perform a regression with debt maturity and all eight of the Ludvigson and Ng (2010) macroeconomic composite factors derived from principal components. Table IV shows that the coefficient of debt maturity remains positive and statistically significant in our baseline predictive regressions when controlling for these macroeconomic variables.

A macroeconomic variable that is highly correlated with debt maturity is the debt-to-gdp ratio: its correlation with the long-term debt share is 66% and with duration is 69%. A common explanation for the high correlation is that as governments increase the size of their debt, they issue at long maturities to reduce the risk of having to refinance large amounts of short-term debt

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[24] Guibaud, Nosbusch and Vayanos (2007) study issuance policy in the presence of incomplete markets and investor clienteles. They show that a welfare-maximizing government tailors the maturity structure to the clientele mix, e.g., issuing more long-term bonds if the fraction of long-horizon investors increases. A supply response can also be generated by the private sector. Koijen, Van Hemert, and Van Nieuwerburgh (2009) show that households are more likely to take fixed-rate mortgages (effectively issuing long-term bonds) when long-term bonds are expensive relative to short-term bonds.

[25] See, for example, Ferson and Harvey (1991), Baker, Greenwood and Wurgler (2003), and Ludvigson and Ng (2009).

at high rates.[26] In untabulated regressions we find that the debt-to-gdp ratio predicts positively excess bond returns, but its predictive power disappears when controlling for debt maturity. Note that the predictive power of the debt-to-gdp ratio is consistent with supply exerting price pressure. Namely, when debt supply is high, bonds must earn high expected returns so that arbitrageurs are induced to hold large positions.

In summary, it appears difficult to attribute the predictive power of debt maturity to a macroeconomic mechanism. This is not entirely surprising: macroeconomic risk premia mainly vary at business-cycle frequency, but debt maturity captures a lower-frequency component of expected bond returns. For example, while debt maturity is positively correlated with the debt-to-gdp ratio (a stock), it is statistically uncorrelated with current or future deficits (flows), which mainly vary at business-cycle frequency. A related point is that debt maturity does not appear to have a simple relationship with most macroeconomic variables. Indeed, as our discussion in Section 3.1 illustrates, non-macroeconomic considerations such as regulation and political constraints, changes in Treasury attitudes about the benefits of long-term debt, and the desire to keep debt maturity within a target level, account for a significant portion of the historical variation in debt maturity.

4.2.2 Other Checks

We subject our baseline predictive regressions to a series of additional robustness checks, reported in Table V. For purposes of comparison, Row (1) reports the baseline results. Rows (2)-(4) report results from subsamples. Results in the period 1964-2003, studied by Cochrane and Piazzesi (2005), are stronger. Results in the first half of our sample (1952-1977) are not significant; as can be seen in Figure 3, significance for the entire sample is driven by the second half and by variation in debt maturity between the first and second half.

Rows (5)-(9) show that our results remain significant after the following controls and adjustments. Row (5) controls for the tent-shaped forecaster of Cochrane and Piazzesi (2005). Row (6) controls for a time trend. Row (7) controls for future changes in debt maturity. Row (8) uses a different measure of the long-term debt share that nets out Federal Reserve holdings. While bond-level data on Federal Reserve holdings are not available, data on aggregate Federal Reserve holdings on U.S. Treasuries are available from Banking and Monetary Statistics between 1941 and 1970, and from issues of the Federal Reserve Bulletin after 1970. Row (9) makes a different adjustment for Federal Reserve holdings by simply computing the long-term debt share associated to these holdings, and including it as a control variable.

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[26] The following quote by Treasury Secretary Summers, reported in Greenwood, Hanson and Stein (2010), is consistent with this explanation: "I think the right theory is that one tries to [borrow] short to save money but not [so much as] to be imprudent with respect to rollover risk. Hence there is certain tolerance for [short-term] debt but marginal debt once [total] debt goes up has to be more long-term."

We finally test for local effects of supply shocks. In the presence of local effects, an increase in the supply of long- relative to intermediate-term bonds should raise the risk premia of the former relative to the latter. Since the long- and intermediate-term bonds in our data have approximate maturities of 20 and 10 years, respectively, we measure the supply of long- relative to intermediate-term bonds by

$$m_t^{long-int} = \frac{\sum_{15\leq\tau\leq 25} D_t^{(\tau)}}{\sum_{5\leq\tau<15} D_t^{(\tau)}},$$

i.e., the fraction of payments due between fifteen and twenty-five years as of payments due between five and fifteen years. We regress the return of the long-term bond in excess of the intermediate-bond on this relative supply measure, controlling for our measures of debt maturity. When, for example, debt maturity is measured by duration, this yields

$$rx_{t+1}^{(20)} - rx_{t+1}^{(10)} = \underset{(\text{-}2.60)}{-0.082} + \underset{(3.50)}{0.065} m_t^{long-int} + \underset{(1.68)}{0.012} Duration_t + u_{t+1}$$

for the one-year return, and

$$rx_{t+3,3}^{(20)} - rx_{t+3,3}^{(10)} = \underset{(\text{-}2.96)}{-0.252} + \underset{(2.18)}{0.110} m_t^{long-int} + \underset{(2.32)}{0.047} Duration_t + u_{t+3}$$

for the three-year return. These results suggest the existence of local effects: holding duration of debt constant, an increase in the supply of long- relative to intermediate-term bonds appears to raise the risk premia of the former relative to the latter.

4.3 Arbitrageur Wealth and Bond Returns

If the arbitrageurs in the model were to become more risk averse, debt maturity and yield spreads would become stronger predictors of excess returns. This section explores time-series implications of this idea under the assumption that risk aversion increases following losses. The model delivers sharp predictions as to when arbitrageurs are likely to realize losses. Indeed, arbitrageurs buy long-term bonds when these are in large supply or when the short rate is low, and in both cases the term structure slopes up. Therefore, arbitrageurs lose money when an upward-sloping term structure is followed by under-performance of long- relative to short-term bonds. A similar argument implies that arbitrageurs lose money when a downward-sloping term structure is followed by over-performance of long- relative to short-term bonds. Thus, the change in arbitrageur wealth over year t can be proxied by

$$\Delta W_t^{Arb} = (y_{t-1}^{(\tau)} - y_{t-1}^{(1)}) \cdot rx_t^{(\tau)}, \tag{11}$$

the product of the yield spread at the end of year $t-1$ times excess bond returns over year t. We assume that both yield spread and excess returns concern the 20-year bond, i.e., set $\tau = 20$.[27]

Table VI reports results from regressing the excess returns of the 20-year bond on the 20-year yield spread, debt maturity as measured by the long-term debt share, lagged changes in arbitrageur wealth, and interactions of lagged changes in wealth with the 20-year yield spread and debt maturity:[28]

$$rx_{t+k,k}^{(20)} = a + b(y_t^{(20)} - y_t^{(1)}) + cm_t^{(10+)} + d\Delta W_t^{Arb} + e\Delta W_t^{Arb}(y_t^{(20)} - y_t^{(1)}) + f\Delta W_t^{Arb} m_t^{(10+)} + u_{t+k}.$$

Specification (1) uses the yield spread, lagged changes in wealth, and their interaction. According to Hypothesis 5, the interaction term should have a negative coefficient: the spread predicts returns positively, and more so when wealth decreases. Specification (1) confirms this prediction. Specification (2) shows that the interaction term becomes more statistically significant when changes in wealth are dropped. The stronger result of Specification (2) arises because of a multi-collinearity problem with Specification (1): changes in wealth are highly correlated with the interaction term because the yield spread is persistent. The -720 coefficient of the interaction term in Specification (2) is economically significant: a one standard deviation drop in arbitrageur wealth increases the coefficient on the yield spread by 1.01, which is approximately one third of the value of this coefficient when the change in arbitrageur wealth is zero.

Specifications (3)-(4) are the counterparts of (1)-(2) with debt maturity replacing the yield spread and the yield spread kept as a control. According to Hypothesis 5, the interaction term should have a negative coefficient: debt maturity predicts returns positively, and more so when

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[27] In untabulated regressions, we also consider the proxies

$$\Delta W_t^{Arb} = sign(y_{t-1}^{(\tau)} - y_{t-1}^{(1)}) \cdot rx_t^{(\tau)} \tag{12}$$

and

$$\Delta W_t^{Arb} = m_{t-1}^{(10+)} \cdot rx_t^{(\tau)}. \tag{13}$$

Proxy (12) is similar to (11), except that we use the sign of the yield spread rather than the spread itself. The results are similar as for (11). Proxy (13) is based on the idea that arbitrageurs hold more long-term bonds when these are in large relative supply. We expect (11) and (12) to be more accurate than (13) because they capture trades that arbitrageurs are making in response to general changes in term-structure slope, whether these arise because of changes in supply, short rates, or investor demand. Regression results are indeed stronger for (11) and (12).

[28] Note that the change ΔW_t^{Arb} in arbitrageur wealth over year t does not use information revealed during the k subsequent years, over which the excess return $rx_{t+k,k}^{(20)}$ is computed.

wealth decreases. Specification (3) shows that the coefficient is indeed negative. Specification (4) shows that the interaction term becomes more statistically significant when changes in wealth are dropped.

Besides eliminating the multi-collinearity, Specifications (2) and (4) have an economic rationale. Indeed, if debt maturity is zero, arbitrageurs play no role, and their risk aversion is immaterial. Arbitrageurs also play no role if the term structure is flat. Therefore, changes in arbitrageur wealth can matter only because of their interaction with debt maturity or the yield spread, consistent with Specifications (2) and (4). The results of these specifications thus support our Hypothesis 5.

Table VI assumes that arbitrageur risk-aversion is influenced by trading performance over a one-year horizon. The relevant horizon might be different, however, and is influenced by the speed at which fresh capital can enter the arbitrage industry. Determining this horizon can shed light into the speed of capital flows, and may be useful for calibrated models of limited arbitrage (e.g., He and Krishnamurthy (2009)). We investigate this issue in Table VII, where we proxy arbitrageur wealth by

$$\Delta W_{k,t}^{Arb} = \sum_{j=1}^{k} (y_{t-j}^{(\tau)} - y_{t-j}^{(1)}) \cdot rx_{t-j+1}^{(\tau)},$$

i.e., the sum of wealth changes over the past k years, where the change in wealth over year t is measured as in the baseline case (Eq. (11)). For example, the two-year change in wealth is the product of the yield spread at the end of year $t-2$ times the excess return of long-term bonds over year $t-1$, plus the product of the yield spread at the end of year $t-1$ times the excess return of long-term bonds over year t. We report results for horizons of six months, one, two, three and five years, and for Specifications (1) and (3).

Table VII shows that the interaction term in Specification (1) achieves its highest economic and statistical significance at the two-year horizon. Economic significance is also highest at the two-year horizon for the interaction term in Specification (3). The relevant horizon thus seems to be around two years. This corresponds approximately to the horizon over which investor flows respond to past returns. For example, Coval and Stafford (2007) show that mutual fund flows respond to returns over the past two years.

5 Conclusion

The supply and maturity structure of government debt play no role in standard term-structure theories. Yet, their effects on bond yields and risk premia are the subject of policy debates, ranging from issuance by treasury departments to monetary policy and quantitative easing by central banks. Given the importance of these debates, it is surprising how little empirical evidence there is correlating maturity structure to interest rates. This paper is an attempt to fill that gap.

We study the time-series relationship between government debt maturity and bond yields and excess returns. We draw on significant variation in the average maturity of government debt between the 1950s and the late 2000s: average maturity fell from over 70 months in 1955 to 39 months in 1976, and then more than doubled through the 1980s, before falling again in the 2000s. Then, in early 2009, the Federal Reserve engineered a massive shift in the net supply of long-term bonds available to the public. As we show, these shifts in average maturity have been associated with changes in bond risk premia.

We organize our empirical investigation around a set of predictions from a formal model of preferred habitat, in which supply shocks impact an arbitrage-free term structure. Our results are consistent with the model's predictions. We find that the relative supply of long-term bonds is positively related to their yield spreads and subsequent excess returns over short-term bonds. Moreover, relative supply appears to impact the yields and excess returns of all bonds in the same direction, to impact long maturities the most, to impact excess returns more than yields, and to predict excess returns even after controlling for the slope of the term structure. Finally, following periods when arbitrageurs lose money, both supply and the slope of the term structure are stronger predictors of excess returns.

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Figure 1
Principal and coupon payments

The maturity structure of marketable government debt as of June 1975, at which point the Treasury reported \$315.7 billion of securities outstanding, including \$128.6 billion of Bills, \$150.3 billion of Notes, and \$36.8 billion of Bonds. Including principal and coupon payments, total payments are \$376 billion. The grey bars denote total principal payments, the solid bars denote total coupon payments, all sorted by month of maturity. For maturities of five years and longer, payments are aggregated at the yearly level.

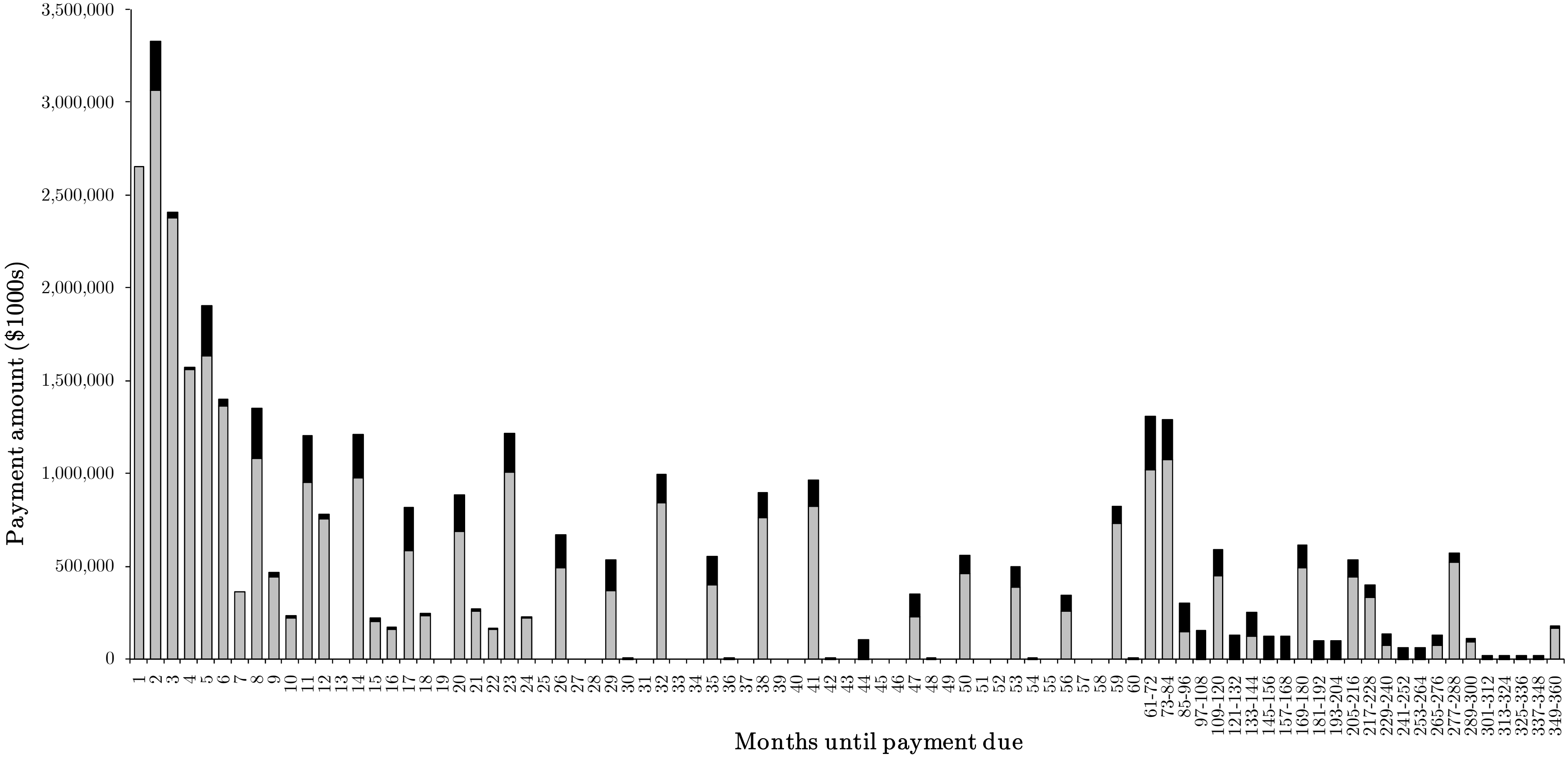


Figure 2
The maturity structure of government debt 1952-2006

Maturity is the dollar-weighted average maturity of all outstanding principal and coupon payments. Long-term debt share is the fraction of principal and coupon payments due in ten years or longer. Duration is the Macaulay duration computed using the historical average nominal Treasury-bill yield.

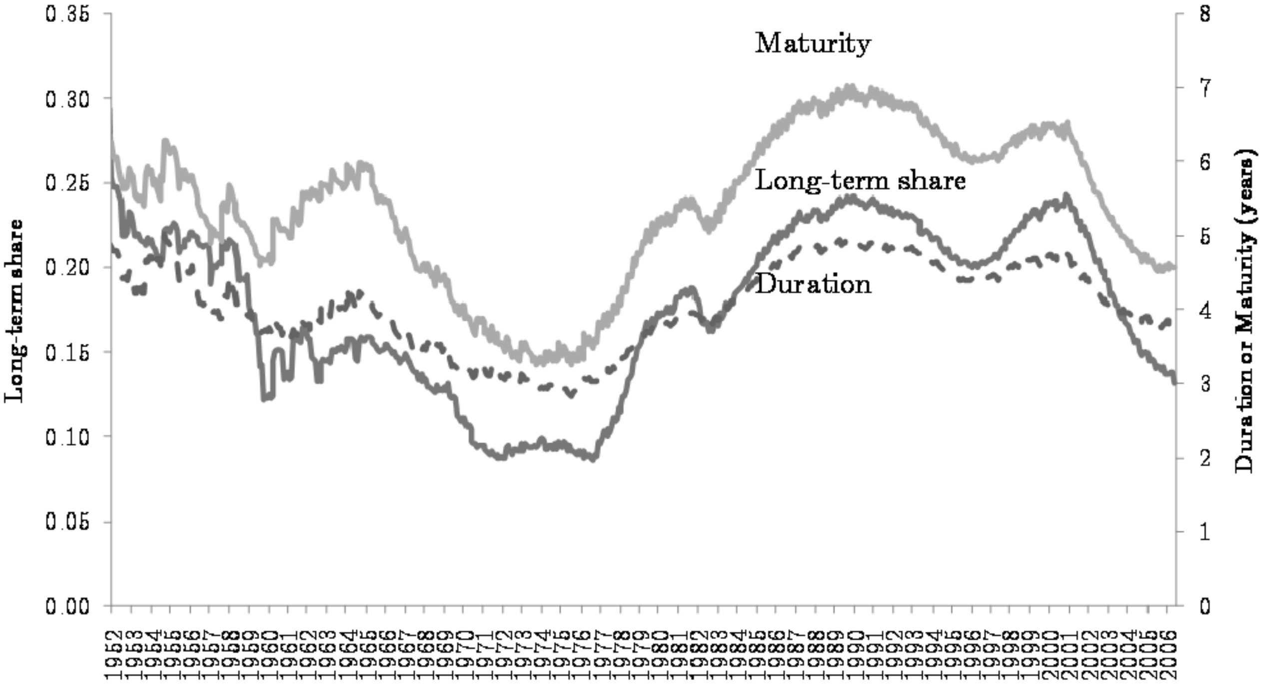
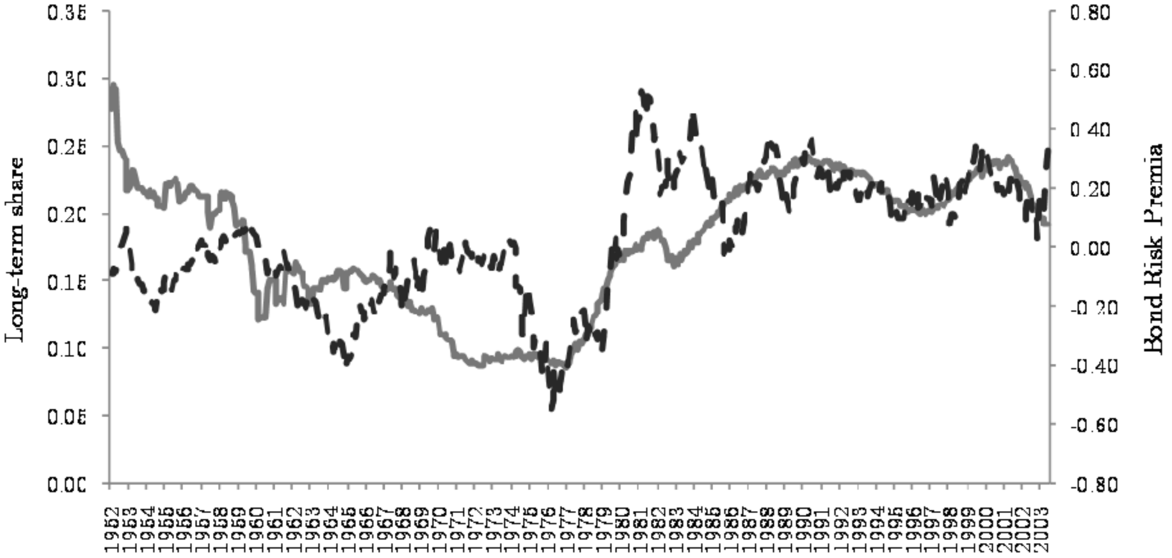


Figure 3
Long-term bond risk premia and government debt maturity

Time series plot of realized bond risk premia (dashed) and government debt maturity (solid). In each panel, the solid line denotes the long-term debt share. In Panel A, the dashed line shows excess returns of the 20-year bond over the subsequent five years. In Panel B, the dashed line shows excess returns of the 20-year bond over the next year.

Panel A. Long-term debt share (solid) and 5-year realized bond risk premia (dashed)



Panel B. Long-term debt share (solid) and 1-year realized bond risk premia (dashed)

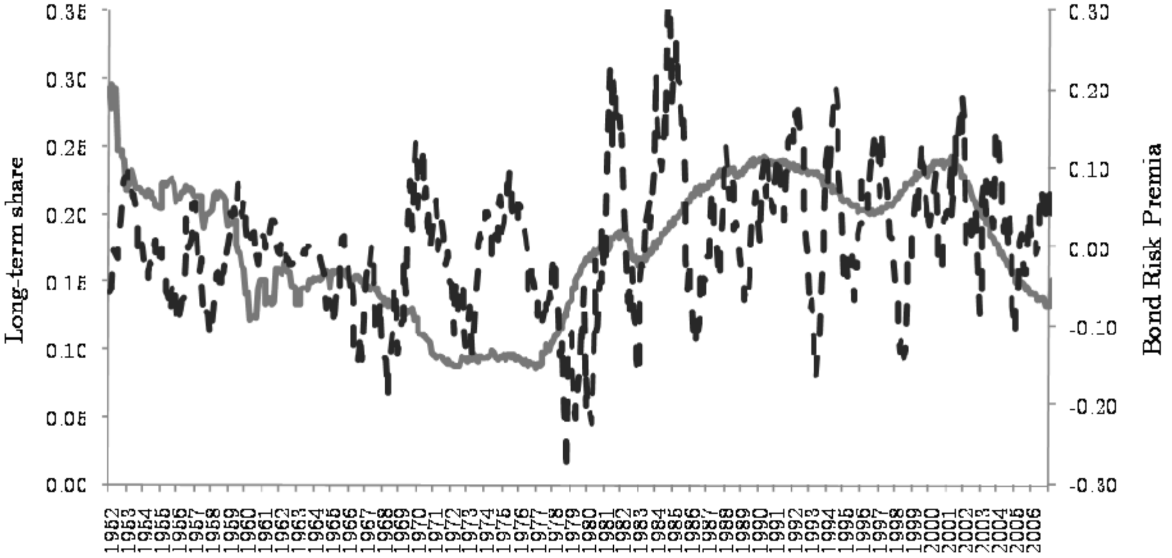


Table I
Summary Statistics

Panel A summarizes variables which describe the quantity and maturity structure of government debt. Maturity is the dollar-weighted average maturity of all outstanding principal and coupon payments. Long-term debt share $m^{(10+)}$ is the fraction of principal and coupon payments due in ten years or longer. Duration is the Macaulay duration computed using the historical average nominal Treasury bill yield. Panel B summarizes other variables and controls. $y^{(\tau)}$-$y^{(1)}$ is the log yield spread between the τ-year bond and the one-year note. Output gap is the residual from a Hodrick-Prescott filter of log GDP. Output growth is the difference between log real GDP in the most recent quarter t and log real GDP at t-4. Inflation risk is the standard deviation of monthly inflation over the past twelve months. Treasury bill risk is the standard deviation of the monthly short-term bill yield over the past twelve months. Stock market risk is the standard deviation of daily CRSP value-weighted stock returns in that month. F_1-F_8 denote the eight macro-factors in Ludvigson and Ng (2010). Change in arbitrageur wealth ΔW^{Arb} is the product of the one-year lagged 20-year yield spread times the excess return of the 20-year bond. The Cochrane and Piazzesi factor is the linear combination of forward rates in Cochrane and Piazzesi (2005).

	Mean	Median	SD	Min	Max
		Panel A: Government Debt Maturity			
Number of Bonds Outstanding	149.48	156	65.17	51.00	251.00
Debt/GDP	37.51	36.58	9.45	22.34	60.77
Maturity (years)	4.67	4.80	0.84	2.77	6.31
$m^{(10+)}$ (long-term debt share)	0.18	0.18	0.05	0.09	0.30
Duration (years)	4.06	4.03	0.58	2.84	5.02
		Panel B: Other variables and controls (%)			
$y^{(2)}$-$y^{(1)}$	0.20	0.21	0.35	-1.07	1.01
$y^{(3)}$-$y^{(1)}$	0.37	0.37	0.55	-1.86	1.76
$y^{(4)}$-$y^{(1)}$	0.50	0.49	0.69	-2.60	2.24
$y^{(5)}$-$y^{(1)}$	0.58	0.57	0.79	-2.65	2.54
$y^{(20)}$-$y^{(1)}$	0.83	0.82	1.24	-3.20	4.05
Output gap	0.02	0.03	1.58	-4.76	3.83
Output growth	3.25	3.33	2.39	-3.09	9.12
Inflation	3.78	3.14	2.78	-0.87	13.78
Inflation risk	0.22	0.20	0.09	0.08	0.56
Tbill yield	5.08	4.78	2.98	0.36	17.42
Tbill risk	0.75	0.61	0.58	0.11	3.47
Stock market risk	0.73	0.64	0.40	0.18	4.93
F_1 (1964+)	0.00	-0.15	1.00	-2.31	5.04
F_2 (1964+)	0.00	0.13	1.00	-4.68	2.68
F_3 (1964+)	0.00	0.01	1.00	-5.32	5.10
F_4 (1964+)	0.00	0.07	1.00	-4.79	4.67
F_5 (1964+)	0.00	0.00	1.00	-3.30	4.63
F_6 (1964+)	0.00	0.03	1.00	-3.47	8.27
F_7 (1964+)	0.00	-0.03	1.00	-12.17	6.67
F_8 (1964+)	0.00	-0.05	1.00	-2.99	4.01
ΔW^{Arb}	0.04	0.01	0.14	-0.42	0.65
Cochrane and Piazzesi factor	3.14	3.05	2.42	-5.28	14.57

Table II

Term structure slope, bond returns, and government debt maturity

Univariate time-series regressions of the form:

$$y_t^{(\tau)} - y_t^{(1)} = a + bX_t + u_t$$

$$rx_{t+k,k}^{(\tau)} = a + bX_t + u_{t+k}$$

The dependent variable is the yield spread, and the one-year, three-year, or five-year excess return of the τ-year bond. The independent variable X is the long-term debt share $m^{(10+)}$ or the duration. *t*-statistics, reported in parentheses, follow Newey-West (1987) and allow for three years of lags in the case of yield spreads and one-year returns, five years in the case of three-year returns, and seven years in the case of five-year returns.

		X=Long-term share $m^{(10+)}$			X=Duration		
		b	(t)	R^2	b	(t)	R^2
	Term structure slope:						
(1)	$y^{(2)}$-$y^{(1)}$	0.016	(1.663)	0.048	0.001	(1.758)	0.059
(2)	$y^{(3)}$-$y^{(1)}$	0.026	(1.717)	0.051	0.002	(1.842)	0.064
(3)	$y^{(4)}$-$y^{(1)}$	0.035	(1.788)	0.058	0.003	(1.916)	0.072
(4)	$y^{(5)}$-$y^{(1)}$	0.040	(1.796)	0.059	0.004	(1.897)	0.072
(5)	$y^{(20)}$-$y^{(1)}$	0.076	(2.246)	0.087	0.007	(2.354)	0.107
	Excess returns sampled monthly:						
(6)	1-year excess return 2-year bond	0.085	(2.088)	0.053	0.006	(1.754)	0.043
(7)	1-year excess return 3-year bond	0.144	(2.053)	0.045	0.011	(1.805)	0.041
(8)	1-year excess return 4-year bond	0.193	(2.077)	0.042	0.016	(1.916)	0.041
(9)	1-year excess return 5-year bond	0.223	(2.024)	0.036	0.019	(1.922)	0.037
(10)	1-year excess return 20-year bond	0.436	(2.564)	0.057	0.036	(2.368)	0.059
(11)	3-year excess return 20-year bond	1.525	(3.269)	0.225	0.114	(3.133)	0.189
(12)	5-year excess return 20-year bond	2.651	(4.074)	0.374	0.195	(4.141)	0.304
	Excess returns sampled annually:						
(13)	1-year excess return 20-year bond	0.435	(2.414)	0.053	0.036	(2.319)	0.056
(14)	3-year excess return 20-year bond	1.523	(2.982)	0.217	0.115	(3.104)	0.191
(15)	5-year excess return 20-year bond	2.697	(4.021)	0.374	0.195	(4.167)	0.301

Table III

Bond returns and government debt maturity: Multivariate tests

Bivariate time-series regressions of the form:

$$rx^{(\tau)}_{t+k,k} = a + bX_t + c(y^{(\tau)}_t - y^{(1)}_t) + u_{t+k}$$

The dependent variable is the one-year, three-year, or five-year excess return of the τ-year bond. The independent variable X is the long-term debt share $m^{(10+)}$ or the duration. The τ-year yield spread is included as a control. *t*-statistics, reported in parentheses, follow Newey-West (1987) and allow for three years of lags in the case of one-year returns, five years in the case of three-year returns, and seven years in the case of five-year returns.

	Excess 1-yr return on the:										Excess 3-yr return		Excess 5-yr return	
	2-yr bond		3-yr bond		4-yr bond		5-yr bond		20-yr bond		20-yr bond		20-yr bond	
							Panel A: Monthly Sampling of Excess Returns							
$m^{(10+)}$	0.062		0.101		0.123		0.144		0.278		1.378		2.694	
	(1.667)		(1.653)		(1.585)		(1.573)		(2.062)		(3.087)		(3.843)	
Duration		0.004		0.010		0.009		0.011		0.022		0.101		0.200
		(1.283)		(1.749)		(1.392)		(1.477)		(1.850)		(2.778)		(3.713)
$y^{(\tau)}$-$y^{(1)}$	1.435	1.451	1.692	0.235	2.027	2.021	1.975	1.962	2.072	2.049	1.950	1.942	-0.583	-0.745
	(3.717)	(3.689)	(3.641)	(1.000)	(3.685)	(3.722)	(3.293)	(3.313)	(3.298)	(3.316)	(2.299)	(1.887)	(-0.314)	(-0.311)
R^2	0.127	0.119	0.121	0.048	0.131	0.128	0.108	0.107	0.136	0.134	0.248	0.211	0.375	0.305
							Panel B: Annual Sampling of Excess Returns							
$m^{(10+)}$	0.059		0.102		0.127		0.138		0.287		1.343		2.638	
	(1.478)		(1.514)		(1.447)		(1.376)		(1.970)		(2.729)		(3.500)	
Duration		0.004		0.007		0.010		0.012		0.023		0.099		0.190
		(1.187)		(1.321)		(1.417)		(1.479)		(1.834)		(2.580)		(3.334)
$y^{(\tau)}$-$y^{(1)}$	1.506	1.505	1.941	1.944	2.016	2.006	2.068	2.048	2.116	2.087	2.728	2.644	0.875	0.766
	(2.743)	(2.751)	(2.947)	(2.949)	(2.671)	(2.621)	(2.434)	(2.389)	(3.168)	(3.138)	(2.302)	(2.068)	(0.486)	(0.360)
R^2	0.123	0.115	0.144	0.140	0.129	0.128	0.112	0.113	0.136	0.136	0.265	0.235	0.376	0.303

Table IV

Bond returns and government debt maturity: Macroeconomic controls

Bivariate time-series regressions of the form:

$$rx^{(\tau)}_{t+k,k} = a + bX_t + cZ_t + u_{t+k}$$

The dependent variable is the one-year or three-year excess return of the 20-year bond. The independent variable X is the long-term debt share $m^{(10+)}$. Macroeconomic control variables Z include the output gap, inflation, inflation risk, Treasury bill yield, Treasury bill yield risk, stock market risk, and the eight macro factors from Ludvigson and Ng (2010). In the case of the eight macro factors, we report only the coefficient on the first one. These controls are summarized in Table 1. t-statistics, reported in parentheses, follow Newey-West (1987) and allow for three years of lags in the case of one-year returns and five years in the case of three-year returns.

	Excess 1-yr return								Excess 3-yr return							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
$m^{(10+)}$	0.453	0.451	0.393	0.443	0.437	0.448	0.422	0.289	1.558	1.498	1.684	1.530	1.588	1.589	1.410	1.618
	(2.944)	(2.537)	(2.179)	(2.528)	(2.649)	(2.603)	(2.436)	(1.950)	(3.289)	(3.434)	(4.555)	(3.285)	(3.870)	(3.739)	(3.257)	(5.413)
Output gap	-0.882								-2.075							
	(-1.294)								(-1.763)							
Output growth		0.009								-0.797						
		(0.024)								(-0.944)						
Inflation			-0.255								0.696					
			(-0.451)								(0.698)					
Inflation risk				-3.784								35.878				
				(-0.395)								(2.438)				
Tbill yield					0.087								1.230			
					(0.206)								(1.562)			
Tbill risk						0.154								5.991		
						(0.088)								(2.149)		
Stock market risk							0.789								7.616	
							(0.597)								(2.224)	
F_1								0.010								0.029
								(1.727)								(1.822)
R^2	0.082	0.060	0.064	0.060	0.057	0.059	0.057	0.242	0.265	0.239	0.237	0.26	0.276	0.273	0.258	0.392

Table V
Robustness and subsamples

Robustness checks and extensions of return-forecasting regressions:

$$rx_{t+k,k}^{(\tau)} = a + bX_t + u_{t+k}$$

where the dependent variable is the one-year or three-year excess return of the 20-year bond and the independent variable X is the long-term debt share $m^{(10+)}$. *t*-statistics, reported in parentheses, follow Newey-West (1987) and allow for three years of lags in the case of one-year returns and five years in the case of three-year returns. The robustness tests relative to the baseline case (1) include: (2) the 1963-2004 sample period used in Cochrane and Piazzesi; (3) the first half of the sample only; (4) the second half of the sample only; (5) controlling for the Cochrane and Piazzesi tent-shaped factor; (6) controlling for a time trend; (7) controlling for future changes in debt maturity; (8) adjusting our maturity measure for debt holdings by the Federal Reserve, (9) including Federal Reserve holdings separately as a control.

		Excess 1-yr return			Excess 3-yr return		
		b	(t)	R^2	b	(t)	R^2
(1)	Baseline case	0.436	(2.564)	0.057	1.525	(3.269)	0.225
(2)	1963-2004	0.567	(3.064)	0.089	1.878	(4.288)	0.316
(3)	1952-1977	-0.054	(-0.294)	0.001	0.118	(0.409)	0.002
(4)	1978-2006	0.897	(2.238)	0.085	2.435	(1.815)	0.254
(5)	Cochrane and Piazzesi	0.353	(2.564)	0.266	1.435	(3.429)	0.350
(6)	Time trend	0.327	(2.040)	0.094	1.102	(2.518)	0.336
(7)	Future changes in debt maturity	0.409	(2.398)	0.064	1.450	(3.400)	0.239
(8)	Fed holdings adjustment	0.444	(2.378)	0.054	1.521	(2.981)	0.201
(9)	Alternate Fed holdings adjustment	0.407	(2.006)	0.058	1.297	(2.018)	0.242

Table VI

Bond returns, government debt maturity, and arbitrageur wealth

The dependent variable is the one-year or three-year excess return of the 20-year bond. The independent variables include the 20-year yield spread, debt maturity, lagged changes in arbitrageur wealth, and interactions of lagged changes in arbitrageur wealth with the 20-year yield spread and with debt maturity. Debt maturity is measured by the long-term debt share $m^{(10+)}$.

$$rx^{(20)}_{t+k,k} = a + b(y^{(20)}_t - y^{(1)}_t) + cm^{(10+)}_t + d\Delta W^{Arb}_t + e\Delta W^{Arb}_t(y^{(20)}_t - y^{(1)}_t) + f\Delta W^{Arb}_t m^{(10+)}_t + u_{t+k}$$

Arbitrageurs make money when the term structure is upward (downward) sloping and subsequent excess bond returns are high (low), thus changes in arbitrageur wealth are measured by the 20-year yield spread times subsequent the subsequent excess return of the 20-year bond. *t*-statistics follow Newey and West (1987), and allow for three years of lags in the case of one-year returns and five years in the case of three-year returns.

	Excess 1-yr return				Excess 3-yr return			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$y^{(20)} - y^{(1)}$	3.191	3.336	2.564	2.520	4.648	4.462	2.302	2.198
	(3.678)	(3.786)	(3.772)	(3.647)	(3.552)	(3.537)	(2.605)	(2.437)
$m^{(10+)}$			0.363	0.328			1.501	1.418
			(2.335)	(2.206)			(2.975)	(3.095)
ΔW^{Arb}	-8.314		32.13		10.344		72.394	
	(-1.671)		(1.549)		(0.859)		(1.367)	
$\Delta W^{Arb}(y^{(20)} - y^{(1)})$	-430.937	-719.691			-1,114.76	-755.277		
	(-2.165)	(-3.124)			(-2.613)	(-2.574)		
$\Delta W^{Arb} m^{(10+)}$			-239.291	-80.987			-408.449	-52.016
			(-2.223)	(-3.138)			(-1.653)	(-1.291)
R^2	0.182	0.173	0.212	0.206	0.103	0.099	0.265	0.254

Table VII

The effects of arbitrageur wealth measured over different horizons

The dependent variable is the one-year excess return of the 20-year bond. The independent variables include the 20-year yield spread, debt maturity, lagged changes in arbitrageur wealth, and interactions of lagged changes in arbitrageur wealth with the 20-year yield spread and with debt maturity. Debt maturity is measured by the long-term debt share $m^{(10+)}$.

$$rx^{(20)}_{t+1} = a + b(y^{(20)}_t - y^{(1)}_t) + cm^{(10+)}_t + d\Delta W^{Arb}_{k,t} + e\Delta W^{Arb}_{k,t}(y^{(20)}_t - y^{(1)}_t) + f\Delta W^{Arb}_{k,t} m^{(10+)}_t + u_{t+1}$$

Arbitrageurs make money when the term structure is upward (downward) sloping and subsequent excess bond returns are high (low). Changes in arbitrageur wealth are measured over lookback periods of 6 months and 1, 2, 3, and 5 years. For lookback periods of 2 years or longer, changes in arbitrageur wealth are measured by the product of the 20-year yield spread times the subsequent one-year excess return of the 20-year bond, summed over years. *t*-statistics follow Newey and West (1987), allowing for three years of lags.

	Return Forecasting Horizon: Excess 1-yr return									
Lookback period:	6 months		1 year		2 years		3 years		5 years	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$y^{(20)} - y^{(1)}$	2.817	2.251	3.191	2.564	3.358	2.275	2.376	2.102	2.334	2.025
	(3.922)	(3.569)	(3.678)	(3.772)	(4.666)	(3.027)	(3.038)	(2.956)	(3.352)	(3.605)
$m^{(10+)}$		0.327		0.363		0.373		0.323		0.347
		(2.239)		(2.335)		(2.400)		(1.912)		(1.908)
ΔW^{Arb}	1.675	43.355	-8.314	32.13	5.781	50.594	6.238	24.641	-1.214	5.363
	(0.310)	(1.484)	(-1.671)	(1.549)	(1.051)	(2.009)	(1.493)	(1.077)	(-0.368)	(0.299)
$\Delta W^{Arb}(y^{(20)} - y^{(1)})$	-516.261		-430.937		-791.900		-547.055		494.396	
	(-2.893)		(-2.165)		(-3.325)		(-1.976)		(2.206)	
$\Delta W^{Arb} m^{(10+)}$		-250.026		-239.291		-283.425		-126.644		-28.098
		(-1.653)		(-2.223)		(-2.218)		(-1.028)		(-0.323)
R^2	0.13	0.151	0.182	0.212	3.358	2.275	0.155	0.154	0.164	0.148

A Proofs

Proof of Proposition 1: From Proposition 10 of Vayanos and Vila (2009), the functions $A_r(\tau)$, $\{A_{\beta,k}(\tau)\}_{k=1,..K}$ are given by

$$A_r(\tau) = \frac{1-e^{-\nu_1\tau}}{\nu_1} + \sum_{l=1}^{K}\gamma_{r,l}\left(\frac{1-e^{-\nu_{l+1}\tau}}{\nu_{l+1}} - \frac{1-e^{-\nu_1\tau}}{\nu_1}\right), \tag{14}$$

$$A_{\beta,k}(\tau) = \sum_{l=1}^{K}\gamma_{\beta,k,l}\left(\frac{1-e^{-\nu_{l+1}\tau}}{\nu_{l+1}} - \frac{1-e^{-\nu_1\tau}}{\nu_1}\right), \tag{15}$$

where the $(K+1)^2$ scalars $\{\nu_l\}_{l=1,..,K+1}, \{\gamma_{r,l}\}_{l=1,..,K}, \{\gamma_{\beta,k,l}\}_{k,l=1,..,K}$ solve the system of

$$\left(1-\sum_{l=1}^{K}\gamma_{r,l}\right)(\nu_1-\kappa_r+M_{1,1}) - \sum_{l,l'=1}^{K}\gamma_{\beta,l',l}M_{1,l'+1} = 0, \tag{16}$$

$$\gamma_{r,l}\left(\nu_{l+1}-\kappa_r+M_{1,1}\right) + \sum_{l'=1}^{K}\gamma_{\beta,l',l}M_{1,l'+1} = 0, \tag{17}$$

$$\left(\sum_{l=1}^{K}\gamma_{\beta,k,l}\right)(\nu_1-\kappa_{\beta,k}) - \left(1-\sum_{l=1}^{K}\gamma_{r,l}\right)M_{k+1,1} + \sum_{l,l'=1}^{K}\gamma_{\beta,l',l}M_{k+1,l'+1} = 0, \tag{18}$$

$$-\gamma_{\beta,k,l}(\nu_{l+1}-\kappa_{\beta,k}) - \gamma_{r,l}M_{k+1,1} - \sum_{l'=1}^{K}\gamma_{\beta,l',l}M_{k+1,l'+1} = 0, \tag{19}$$

and

$$M_{1,1} \equiv -a\sigma_r^2\int_0^T \alpha(\tau)A_r(\tau)^2 d\tau, \tag{20}$$

$$M_{1,l+1} \equiv -a\sigma_\beta^2\int_0^T \alpha(\tau)A_r(\tau)A_{\beta,l}(\tau)d\tau, \tag{21}$$

$$M_{k+1,1} \equiv a\sigma_r^2\int_0^T \alpha(\tau)\left[\tau\theta_k(\tau)-A_{\beta,k}(\tau)\right]A_r(\tau)d\tau, \tag{22}$$

$$M_{k+1,l+1} \equiv a\sigma_\beta^2\int_0^T \alpha(\tau)\left[\tau\theta_k(\tau)-A_{\beta,k}(\tau)\right]A_{\beta,l}(\tau)d\tau, \tag{23}$$

To determine the asymptotic behavior of $\{\nu_l\}_{l=1,..,K+1}, \{\gamma_{r,l}\}_{l=1,..,K}, \{\gamma_{\beta,k,l}\}_{k,l=1,..,K}$ when $a \approx 0$, we use the same argument as in Proposition 5 of Vayanos and Vila (2009). We "guess" that $\gamma_{r,l}$ is of order a^3, and $\gamma_{\beta,k,l}$ is of order a for $k=l$ and of order a^3 for $k \neq l$. We accordingly set

$\gamma_{r,l} = a^3 c_{r,l}$, $\gamma_{\beta,k,k} = ac_{\beta,k,k}$, and $\gamma_{\beta,k,l} = a^3 c_{\beta,k,l}$ for $k \neq l$, in (16)-(19). We also divide (16)-(19) by the appropriate power of a so that each of the resulting equations has terms of order one. For $a = 0$, these equations have a non-zero solution $\{\nu_l\}_{l=1,..,K+1}$, $\{c_{r,l}\}_{l=1,..,K}$, $\{c_{\beta,k,l}\}_{k,l=1,..,K}$, and this validates our guess. The solution for $a = 0$ satisfies $\nu_1 = \kappa_r$, $\nu_{k+1} = \kappa_{\beta,k}$ for $k \geq 1$, and

$$c_{\beta,k,k} = \frac{\sigma_r^2 \int_0^T \alpha(\tau)\tau\theta_k(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r}d\tau}{\kappa_r - \kappa_{\beta,k}}.$$

Combining with (15), we find (10). ∎

Proof of Proposition 2: When $a \approx 0$, an increase in average maturity raises yields by

$$\frac{\partial y_t^{(\tau)}}{\partial \beta_{1,t}} = \frac{A_{\beta,1}(\tau)}{\tau} \approx \frac{a\sigma_r^2 \int_0^T \alpha(\tau)\tau\theta_1(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r}d\tau}{\kappa_r - \kappa_{\beta,1}} \left(\frac{1-e^{-\kappa_{\beta,1}\tau}}{\kappa_{\beta,1}\tau} - \frac{1-e^{-\kappa_r\tau}}{\kappa_r\tau}\right), \tag{24}$$

where the first step follows from (1) and (7), and the second from (10). Since $\theta_1(\tau) > 0$ for $\tau \in S_1$, $\theta_1(\tau) < 0$ for $\tau \in S_2$, S_1 has larger elements than S_2, $\int_0^T \alpha(\tau)\tau\theta_1(\tau) = 0$, and $\frac{1-e^{-\kappa_r\tau}}{\kappa_r}$ is increasing in τ,

$$\int_0^T \alpha(\tau)\tau\theta_1(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r}d\tau > 0. \tag{25}$$

Since, in addition, the function

$$f(\tau) \equiv \frac{1}{\kappa_r - \kappa_{\beta,1}}\left(\frac{1-e^{-\kappa_{\beta,1}\tau}}{\kappa_{\beta,1}} - \frac{1-e^{-\kappa_r\tau}}{\kappa_r}\right)$$

is positive for all τ, (24) implies that an increase in average maturity raises all yields. The relative strength of the effect across maturities depends on the monotonicity of the function $\frac{f(\tau)}{\tau}$. This function is equal to zero for $\tau = 0$ and $\tau = \infty$. Its derivative has the same sign as the function $g(\tau) \equiv \tau f'(\tau) - f(\tau)$, and the derivative of $g(\tau)$ has the same sign as

$$f''(\tau) = \frac{\kappa_r e^{-\kappa_r\tau} - \kappa_{\beta,k}e^{-\kappa_{\beta,k}\tau}}{\kappa_r - \kappa_{\beta,k}}.$$

The function $f''(\tau)$ is positive for $\tau = 0$, negative for large τ, and switches sign once. Therefore, $g(\tau)$ is increasing and then decreasing. Since $g(0) = 0$ and $g(\infty) < 0$, $g(\tau)$ is positive and then negative. Therefore, $\frac{f(\tau)}{\tau}$ is increasing and then decreasing. If the maximum of $\frac{f(\tau)}{\tau}$ is achieved for a value $\tau^* > T$, then an increase in average maturity impacts yields of long-term bonds the most. If instead $\tau^* < T$, then yields of intermediate-term bonds are impacted the most. ∎

Proof of Proposition 3: The effect of average maturity on bond risk premia is

$$\begin{aligned}\frac{\partial\left[\mu_t^{(\tau)} - r_t\right]}{\partial \beta_{1,t}} &= \frac{\partial\left[A_r(\tau)\lambda_{r,t} + \sum_{l=1}^K A_{\beta,l}(\tau)\lambda_{\beta,l,t}\right]}{\partial\beta_{1,t}} \\ &= A_r(\tau)M_{2,1} + \sum_{l=1}^K A_{\beta,l}(\tau)M_{2,l+1}, \qquad (26)\end{aligned}$$

where the first step follows from (9) and the second by the same argument as in Propositions 4 and 10 of Vayanos and Vila (2009). (Eq. (26) coincides with the right-hand side of (A.71) in Vayanos and Vila, written for $k = 1$.) Using the asymptotic behavior of $A_r(\tau)$, $\{A_{\beta,k}(\tau)\}_{k=1,..K}$ when $a \approx 0$, we find

$$\begin{aligned}A_r(\tau)M_{2,1} + \sum_{l=1}^K A_{\beta,l}(\tau)M_{2,l+1} &\approx A_r(\tau)M_{2,1} \\ &\approx a\sigma_r^2 \frac{1-e^{-\kappa_r\tau}}{\kappa_r}\int_0^T \alpha(\tau)\tau\theta_1(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r}d\tau, \qquad (27)\end{aligned}$$

Eqs. (25)-(27) imply that an increase in average maturity raises the risk premia of all bonds, with long-term bonds being affected the most. ∎

Proof of Proposition 4: Propositions 2 and 3 imply that the effect of average maturity on bond risk premia exceeds that on yields if

$$\frac{1-e^{-\kappa_r\tau}}{\kappa_r} > \frac{1}{\kappa_r - \kappa_{\beta,1}}\left(\frac{1-e^{-\kappa_{\beta,1}\tau}}{\kappa_{\beta,1}\tau} - \frac{1-e^{-\kappa_r\tau}}{\kappa_r\tau}\right). \qquad (28)$$

Since

$$\begin{aligned}\frac{1}{\kappa_r - \kappa_{\beta,1}}\left(\frac{1-e^{-\kappa_{\beta,1}\tau}}{\kappa_{\beta,1}\tau} - \frac{1-e^{-\kappa_r\tau}}{\kappa_r\tau}\right) &= \frac{1}{\tau}\int_0^\tau \frac{1-e^{-\kappa_r(\tau-\tau')}}{\kappa_r}e^{-\kappa_{\beta,1}\tau'}d\tau' \\ &< \frac{1}{\tau}\int_0^\tau \frac{1-e^{-\kappa_r(\tau-\tau')}}{\kappa_r}d\tau' \\ &< \frac{1-e^{-\kappa_r\tau}}{\kappa_r},\end{aligned}$$

where the second step follows from $\kappa_{\beta,1} > 0$ and the third because $\frac{1-e^{-\kappa_r \tau}}{\kappa_r}$ is increasing in τ, (28) holds. ∎

Proof of Proposition 5: That average maturity predicts positively excess bond returns in a univariate regression follows from Proposition 3 and because the risk factors r_t and $\{\beta_{k,t}\}_{k=1,..,K}$ are independent of each other. The coefficient γ_1 of average maturity in the bivariate regression

$$\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt = \alpha dt + \gamma_1 \beta_{1,t} dt + \gamma_2 \left[y_t^{(\tau_1)} - r_t \right] dt + d\epsilon_t$$

that controls for slope as measured by $y_t^{(\tau_1)} - r_t$ is

$$\gamma_1 = \frac{\frac{1}{dt}\text{Cov}\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt, \beta_{1,t}\right] \text{Var}\left[y_t^{(\tau_1)} - r_t\right] - \frac{1}{dt}\text{Cov}\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt, y_t^{(\tau_1)} - r_t\right] \text{Cov}\left[\beta_{1,t}, y_t^{(\tau_1)} - r_t\right]}{\text{Var}(\beta_{1,t})\text{Var}\left[y_t^{(\tau_1)} - r_t\right] - \text{Cov}\left[\beta_{1,t}, y_t^{(\tau_1)} - r_t\right]^2}. \tag{29}$$

The denominator in (29) is positive because of the Cauchy-Schwarz inequality. To show that the numerator is also positive, we compute the sensitivity of premia and slope to the risk factors. Same arguments as in Propositions 2 and 3 imply that when $a \approx 0$, the sensitivity to the demand and supply factors is

$$\Delta_{y,\beta_k,\tau} \equiv \frac{\partial \left[y_t^{(\tau)} - r_t \right]}{\partial \beta_{k,t}} \approx \frac{a\sigma_r^2 \int_0^T \alpha(\tau)\tau\theta_k(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r} d\tau}{\kappa_r - \kappa_{\beta,k}} \left(\frac{1-e^{-\kappa_{\beta,k}\tau}}{\kappa_{\beta,k}\tau} - \frac{1-e^{-\kappa_r\tau}}{\kappa_r\tau} \right), \tag{30}$$

$$\Delta_{\mu,\beta_k,\tau} \equiv \frac{\partial \left[\mu_t^{(\tau)} - r_t \right]}{\partial \beta_{k,t}} \approx a\sigma_r^2 \frac{1-e^{-\kappa_r\tau}}{\kappa_r} \int_0^T \alpha(\tau)\tau\theta_k(\tau)\frac{1-e^{-\kappa_r\tau}}{\kappa_r} d\tau. \tag{31}$$

Eqs. (1) and (7) imply that the sensitivity of slope to the short rate is

$$\Delta_{y,r,\tau} \equiv \frac{\partial \left[y_t^{(\tau)} - r_t \right]}{\partial r_t} = \frac{A_r(\tau)}{\tau} - 1 \approx \frac{1-e^{-\kappa_r\tau}}{\kappa_r\tau} - 1. \tag{32}$$

The sensitivity of premia to the short rate is

$$\begin{aligned}
\Delta_{\mu,r,\tau} \equiv \frac{\partial\left[\mu_t^{(\tau)} - r_t\right]}{\partial r_t} &= \frac{\partial\left[A_r(\tau)\lambda_{r,t} + \sum_{l=1}^{K} A_{\beta,l}(\tau)\lambda_{\beta,l,t}\right]}{\partial r_t} \\
&= A_r(\tau)M_{1,1} + \sum_{l=1}^{K} A_{\beta,l}(\tau)M_{1,l+1} \\
&\approx A_r(\tau)M_{1,1} \\
&\approx -a\sigma_r^2 \frac{1-e^{-\kappa_r\tau}}{\kappa_r}\int_0^T \alpha(\tau)\left(\frac{1-e^{-\kappa_r\tau}}{\kappa_r}\right)^2 d\tau, \qquad (33)
\end{aligned}$$

where the first step follows from (9), the second by the same argument as in Propositions 4 and 10 of Vayanos and Vila (2009), and the third and fourth from the asymptotic behavior of $A_r(\tau)$, $\{A_{\beta,k}(\tau)\}_{k=1,..K}$ when $a \approx 0$.

Since the risk factors r_t and $\{\beta_{k,t}\}_{k=1,..,K}$ are independent of each other,

$$\frac{1}{dt}\mathrm{Cov}\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt, \beta_{1,t}\right] = \mathrm{Cov}\left[\mu_t^{(\tau)} - r_t, \beta_{1,t}\right] = \Delta_{\mu,\beta_1,\tau}\mathrm{Var}(\beta_{1,t}), \qquad (34)$$

$$\mathrm{Var}\left[y_t^{(\tau_1)} - r_t\right] = \Delta_{y,r,\tau_1}^2 \mathrm{Var}(r_t) + \sum_{k=1}^{K} \Delta_{y,\beta_k,\tau_1}^2 \mathrm{Var}(\beta_{k,t}), \qquad (35)$$

$$\frac{1}{dt}\mathrm{Cov}\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt, y_t^{(\tau_1)} - r_t\right] = \mathrm{Cov}\left[\mu_t^{(\tau)} - r_t, y_t^{(\tau_1)} - r_t\right]$$

$$= \Delta_{\mu,r,\tau}\Delta_{y,r,\tau_1}\mathrm{Var}(r_t) + \sum_{k=1}^{K} \Delta_{\mu,\beta_k,\tau}\Delta_{y,\beta_k,\tau_1}\mathrm{Var}(\beta_{k,t}), \qquad (36)$$

$$\mathrm{Cov}\left[\beta_{1,t}, y_t^{(\tau_1)} - r_t\right] = \Delta_{y,\beta_1,\tau_1}\mathrm{Var}(\beta_{k,t}). \qquad (37)$$

Eqs. (30)-(33) imply that when $a \approx 0$, (34), (36) and (37) are of order a, while (35) is of order one. Since, in addition, (34) and (35) are positive, (29) implies that $\gamma_1 > 0$. ∎

Proof of Proposition 6: When $a \approx 0$, (24) implies that $\frac{\partial y_t^{(\tau)}}{\partial \beta_{1,t}}$ is approximately a linear function of a with a positive coefficient. Eqs. (26) and (27) imply the same for $\frac{\partial\left[\mu_t^{(\tau)} - r_t\right]}{\partial \beta_{1,t}}$. Therefore, the positive effect of average maturity on bond yields and risk premia becomes larger when a increases.

The regression coefficient γ of excess bond returns on term-structure slope as measured by

$y_t^{(\tau_1)} - r_t$ is

$$\gamma = \frac{\frac{1}{dt}\mathrm{Cov}\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} - r_t dt, y_t^{(\tau_1)} - r_t\right]}{\mathrm{Var}\left[y_t^{(\tau_1)} - r_t\right]}. \tag{38}$$

Eqs. (30)-(33), (35) and (36) imply that when $a \approx 0$, the numerator in (38) is positive and of order a, and the denominator in (38) is positive and of order one. Therefore, γ increases when a increases. ∎