Notes on Shimer (2005)

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Here I briefly discuss Shimer's (2005) puzzle/critique of the search and matching model of the labour market. It is directly relevant for parts (d) and (e) of Problem Set 4 on the labour market (and also the last few slides of the lecture note entitled: The Labour Market II). Here is a link to the paper: http://www.jstor.org.gate2.library.lse.ac.uk/stable/pdfplus/4132669.pdf?cookieSet=1

Shimer's (2005) critique is that the basic search and matching model cannot reconcile the strong procyclicality of θ (and the job finding rate $\theta q(\theta)$) with the weak procyclicality of labour productivity p in US data. In other words, the model delivers an elasticity of θ with respect to p that is much too small.

Recall, in class (part d of PS4) we derived an expression for ε_θ (the elasticity of θ wrt. p) as a function of ε_w and η (the elasticity of the job finding rate) using the Job Creation Condition. We calibrated ε_w and η to US data and our model gave us an expression for ε_θ, which was much too small! This is Shimer's critique: the model is unable to generate enough variation in v, u or θ from changes in labour productivity p to match US business cycle data.

In his 2005 paper, Shimer does not calibrate ε_w to US data, but rather endogenises it by assuming Nash bargaining for wage determination. At the top of pp. 36, Shimer combines the wage equation $[w = z + \beta(p - z + c\theta)]$[2] and the Job Creation condition $\left[p - w - \frac{(r+s)c}{q(\theta)} = 0\right]$[3] to eliminate the wage w, which leaves an implicit function for θ:

$$\frac{r+s}{q(\theta)} + \beta\theta = (1-\beta)\frac{p-z}{c} \tag{1}$$

The second equation on pp. 36 is the expression for the elasticity of θ wrt. net labour productivity $(p - z)$. To derive this, totally differentiate equation (1) wrt. θ and $(p - z)$ to get:

$$\begin{aligned} -\frac{(r+s)\,q'(\theta)}{q(\theta)^2}d\theta + \beta d\theta &= \frac{(1-\beta)}{c}d(p-z) \\ \frac{d\theta}{d(p-z)} &= \frac{1-\beta}{\left[\beta - \frac{(r+s)q'(\theta)}{q(\theta)^2}\right]c} \qquad (2) \\ \frac{d\theta}{d(p-z)}\frac{(p-z)}{\theta} &= \frac{(1-\beta)(p-z)}{\left[\beta - \frac{(r+s)q'(\theta)}{q(\theta)^2}\right]c\theta} \end{aligned}$$

Using equation (1):

$$\begin{aligned} \frac{d\theta}{d(p-z)}\frac{(p-z)}{\theta} &= \frac{\frac{r+s}{\theta q(\theta)} + \beta}{\left[\beta - \frac{(r+s)q'(\theta)}{q(\theta)^2}\right]} \\ &= \frac{r+s+\beta f(\theta)}{\left[\beta - \frac{(r+s)q'(\theta)}{q(\theta)^2}\right]f(\theta)} \\ &= \frac{r+s+\beta f(\theta)}{(r+s)(1-\eta)+\beta f(\theta)} \end{aligned}$$

[1] Any errors are my own.
[2] This is equation (2) of the lecture note entitled: The Labour Market II.
[3] This is equation (1) of the lecture note entitled: The Labour Market II.

where $f(\theta) \equiv \theta q(\theta)$ is the job finding rate and $\eta \equiv \frac{f'(\theta)\theta}{f(\theta)}$ is the elasticity of $f(\theta)$[4]. Shimer remarks that only when $\beta = 0$ (i.e. the worker has no bargaining power) does the elasticity of the $v-u$ ratio with respect to $(p-z)$ rise "appreciably", and even then it is still too small to match the data.

In Section II.D, Shimer calibrates the model to US data and simulates it for labour productivity shocks. In Section II.E (pp. 39) he discusses the results. He concludes that the biggest failure of the model is its inability to match the volatility of vacancies and unemployment. In the simulated model, v and u are less than 10% as volatile as in US data.

Section II.F (pp. 41) looks at the source of the problem: wage determination. With Nash bargaining, wages are flexible to labour productivity and hence "soak up" a large portion of the productivity shock, leaving firms with little incentive to change v. As we discussed in class, a rise in p (a boom) shifts the job creation curve (JC) to the right and the wage curve (WC) up. These shifts have opposite effects on θ: the rightward shift of JC increases θ while the upward shift of WC reduces θ. Although the net result is an increase in θ[5], the increase is small because of the upward shift of WC.

A natural question to ask then is that, given wage determination is at the heart of the problem, is Nash bargaining correct? As Shimer puts it, "there is no single compelling theory of wage determination" when there are gains to be split[6] The search and matching literature assumes Nash bargaining. Empirically, the elasticity of w wrt. p (ε_w) is close to 1 for new hires. In other words, wages for new matches are very flexible, which is what Nash bargaining gives. So a solution to the puzzle must look elsewhere.

Something which can reconcile the model with the data is if the firm's profit margin $(p-w)$ is small (see lecture slide 15 of The Labour Market II). Again, this is an empirical question. Alternatively, one could enrich the canonical search and matching model of the labour market by adding a hiring cost K (see Problem Set 4). We showed in parts (d) and (e) of Problem Set 4 that the addition of a hiring cost has two effects which help to reconcile the volatility of θ in the model with that observed in US data. The first was seen in part (d); mathematically K subtracts from the denominator of the expression for $\frac{\partial \theta}{\partial p}\frac{p}{\theta}$. Intuitively, the inclusion of K reduces equilibrium θ (as shown in part c), which means that in a boom (i.e. when p increases) firms have an incentive to increase θ by more than before because there exist diminishing returns to θ in the matching function. The second effect (discussed in part e) was that the inclusion of K reduced the firm's profit margin $(p-w)$ which - as mentioned above - improves the fit of the model.

1 Appendix on comparative statics

Comparative statics traces the response of the endogenous variables after a change in an exogenous variable. Here we are interested in the variation of θ, w over the business cycle. In the search and matching model, the business cycle is captured by changes in p, the productivity of the worker.[7] There are at least two ways of deriving $\frac{\partial \theta}{\partial p}$ and $\frac{\partial w}{\partial p}$. Crucially, both methods account for the joint determination of θ, w by the intersection of the WC and JC curves.

Method 1 The first method is by substitution as shown above. Simply combine the JC and WC equations to eliminate w. The result, equation (1) above, is an implicit function for θ. If we know the matching function then we know $q(\theta)$ and we can get an explicit function for θ. In this case we just take the partial derivative of θ with respect to p to get $\frac{d\theta}{dp}$. However, in many cases we do not know the explicit form of the matching function, so we have an implicit function for θ. In this case, total differentiate w.r.t. θ and p and rearrange to get $\frac{d\theta}{dp}$, the result is equation (2) above.

[4]Note, by definition $q(\theta) \equiv \frac{f(\theta)}{\theta}$, and $q'(\theta) = \frac{f'(\theta)\theta - f(\theta)}{\theta^2}$. Hence, $-\frac{q'(\theta)}{q(\theta)^2} f(\theta) = \frac{[f(\theta)]^2 - f'(\theta)\theta f(\theta)}{[\theta q(\theta)]^2} = 1 - \frac{f'(\theta)\theta}{f(\theta)} = 1 - \eta$

[5]The condition for this result is $\beta < 1$, which by assumption is true.

[6]Recall, because we assume $p > z$, there is a surplus from a job match which has to be split between worker and firm.

[7]Shimer (2005) also considers changes in s, the exogenous job destruction rate.

Method 2 The second method starts by taking the partial derivative of w with respect to p of both the JC and WC equations.[8] The result is two equations which are linear in the two unknowns $\frac{\partial \theta}{\partial p}$ and $\frac{\partial w}{\partial p}$. From the wage curve $w = z + \beta \left(p - z + c\theta\right)$ we get:

$$\frac{dw}{dp} = \beta \left(1 + c\frac{d\theta}{dp}\right)$$

From the job creation condition $w = p - \frac{(r+s)c}{q(\theta)}$ we get:

$$\frac{dw}{dp} = 1 + \frac{\left(r+s\right) c q'\left(\theta\right)}{q\left(\theta\right)^2}\frac{d\theta}{dp}$$

We simply solve these two equations simultaneously to get the two unknowns $\frac{\partial \theta}{\partial p}$ and $\frac{\partial w}{\partial p}$. In particular, $\frac{d\theta}{dp}$ is given by:

$$\frac{d\theta}{dp} = \frac{1-\beta}{\left[\beta - \frac{(r+s)q'(\theta)}{q(\theta)^2}\right] c}$$

which is equation (2). This expression is positive because $q'\left(\theta\right) < 0$ and by assumption $\beta \in (0,1)$.

[8] To do this, you must first write JC and WC as explicit functions for w.