Eliciting Heterogeneous Investor Beliefs from Portfolio Holdings and Performance Evaluation

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Abstract

In this paper, I develop a novel approach to elicit heterogeneous investor beliefs about expected returns as well as common investor beliefs about the covariance matrix of the risky assets from a snapshot of cross-section portfolio holdings. Portfolio revealed expectations and the covariance matrix are forward-looking and dynamic, fundamentally different from those estimated from historical return data. As an empirical application, I measure a fund manager's forecasting ability by correlating his revealed beliefs about stock returns with the subsequent realized returns. The results show that, on average, this correlation (termed either semi-belief accuracy index (SBAI) or belief accuracy index (BAI)) is not significantly different from zero, indicating fund managers, on average, may not possess forecasting abilities. However, funds with higher positive correlations outperform funds with lower or negative correlations, indicating some managers have the ability to predict stock returns. Sorting funds into deciles according to this correlation, I find the annualized performance spread between the top and bottom decile is about 4-5%. I also show that new performance measures, SBAI and especially BAI, contain unique information that is not in the existing performance measures.

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1 Introduction

A longstanding tradition in economics is to elicit aspects of decision makers' preferences from their choices (Samuelson 1938). Following this tradition, this paper proposes a novel approach to elicit mean-variance optimizing fund managers' beliefs on expected returns and variance-variance matrix of the risky assets in the economy from their portfolio holdings. The basic intuition is that if a fund manager prefers one portfolio over another, that action reveals his or her belief on the relative risk-return tradeoff of the asset mix in the two portfolios.

To capture the major risk-return tradeoff faced by fund managers, I model an environment where there are k non-redundant assets and investors are mean-variance risk-averse in returns.[1] The variance-covariance matrix of risky assets in this economy is, hence, characterized by a k-eigenfactor structure. To elicit heterogeneous beliefs about expected returns of these k non-redundant assets among investors, I assume that the variance-covariance matrix of asset returns is common knowledge among investors. This assumption is based on the argument that mean returns are a lot harder to estimate than volatilities (Merton 1980) and is not controversial in the portfolio choice literature. For example, the Black-Litterman model (1992), which is well adopted among practitioners, is based on the same assumption. Basak (2005) also employs a similar assumption to study the asset pricing implication of heterogeneous beliefs in a dynamic setting. Under these assumptions, I show that an investor's portfolio indeed exhibits k-fund separation. This result is in clear contrast to that obtained under homogeneous information. For example, a fairly general result in the modern finance theory is that when investors are homogenously informed, their portfolio holdings exhibit two-fund separation (Huang and Litzenberger (1988)).

I further demonstrate that k-fund allocations vary among investors and the heterogeneity of investors' portfolios across the k-fund reveals their respective private information. Intuitively, an investor optimally tilts his portfolio holdings toward assets with better perceived risk-return tradeoffs conditional on his private information. By reverse-engineering the corresponding portfolio optimization problem, I show that heterogeneity in investor portfolio holdings reveal heterogeneous beliefs among investors, and the commonality across portfolio holdings reveal investors' common belief on the variance-covariance matrix. I term this set

[1] The mean-variance analysis of Markowitz (1952) has long been recognized as the cornerstone of modern portfolio theory and is widely used in both academia and industry. Recently, Preuschoff, Quartz, and Bossaerts (2006) also find the support of Markowitz theory in the human brain-scanning evidence.

of beliefs *portfolio (revealed) beliefs* and the inferred variance-covariance matrix *portfolio (revealed) variance-covariance matrix*. Alternatively, the variance-covariance matrix can also be estimated using historical return data. With the estimated covariance matrix as a proxy for the common belief on risk, heterogeneous investor beliefs can be elicited from investor portfolio holdings. I term this set of beliefs *semi-portfolio (revealed) beliefs*. The differences between these two sets of revealed beliefs lie on the use of the covariance matrix – it is elicited from a forward-looking holding matrix in the former and is estimated from historical realized returns in the latter.

This model has several important empirical implications. In this paper, I focus on the following one: If fund managers are able to forecast future stock returns, then their private beliefs about these future stock returns should mimic the subsequent realized returns. To measure such forecasting abilities among investors, I use the correlation between their *ex ante* beliefs as revealed in their portfolio holdings, and the subsequent realized stock returns. I term these correlations the *semi-belief accuracy index* (SBAI) when using semi-portfolio revealed beliefs and the *belief accuracy index* (BAI) when using portfolio revealed beliefs. I regard these correlations as performance metrics since they are measures of managers' abilities to forecasting future stock returns. Specifically, I construct an SBAI and a BAI for each fund by computing the correlation between portfolio revealed beliefs (that is, a manager's private beliefs regarding future stock returns) and the respective subsequent one-month returns for all stocks in this fund portfolio, for each quarter in the sample. I find, on average, the correlation between the portfolio revealed beliefs and the realized returns (i.e., SBAI or BAI) is not significantly different from zero, indicating that mutual fund managers, on average, cannot predict stock returns. This result is not surprising given that Grinblatt and Titman (1989) find that the average fund performance in their sample is close to zero and Spiegel, Mamaysky, and Zhang (2007) find there is little evidence that average funds earn superior returns. I then sort the funds into deciles according to this correlation measure. The Spearman rank correlation coefficient between the rankings of average SBAIs/BAIs of the past one year and the rankings based on the current SBAI/BAIs is around 0.7/0.6. This persistence disappears for SBAI and weakens for BAI after three quarters. This finding indicates that some fund managers possess the ability to forecast future factor returns for short to medium horizons.

If SBAI and BAI measure fund managers' talents, the cross-sectional difference in these measures should contain valuable information that can be used to predict cross-sectional future fund performance. To examine this hypothesis, I sort funds into deciles according to either SBAI or BAI and track decile fund performance over the subsequent three months, and compare this performance over the full sample. I find the performance spread between the sorted top and bottom deciles is 4-5% per annum and statistically significant. This spread remains statistically significant after adjusting for styles and using CAPM, Fama-French, and Carhart models.

To compare SBAI and BAI, I perform double sorts. For each portfolio formation period, I first sort funds into quintiles based on their SBAIs and then sort funds into quintiles based on their BAIs within each SBAI. The 5-1 spread is statistically significant for three SBAI quintiles, indicating that BAI contains unique information that is not in SBAI. I also sort funds into quintiles in reverse, first by BAI and then by the SBAI measure. The 5-1 spread is only statistically significant for one BAI quintile. These results indicate that the portfolio revealed variance-covariance matrix Σ indeed contains some unique information about the future returns relative to Σ estimated using historical return data.

To see whether SBAI and BAI contain information about future fund performance that is not in other holding-based performance measures, I perform additional double-sorts. The closely related portfolio-based measure that is not based on historical return data is the benchmark-free metric proposed in Grinblatt and Titman (1989), henceforth the GT measure. Controlling for the GT measure, the average differences between the top and bottom quintiles of the funds ranked by their SBAIs/BAIs range from 5 to 44 basis points per month or 0.6% to 5.28% per year, with three quintiles statistically significant. These results indicate that SBAI/BAI contain significant information about future fund returns that is not contained in the holding-based GT measure. I also perform double-sorts between the BAI and the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997), and between the BAI and the FundRank measure of Shi, Stoffman, Yuan, and Zhu (2007). The results show that SBAI/BAI has unique information to predict future fund returns. In sum, these findings establish empirical supports for using SBAI/BAI to measure investment performance.

One may question whether it is reasonable to extract beliefs assuming that fund managers are mean-variance optimizers given the empirical evidence that fund managers are not fully rational and are exposed to various behavior biases as one may expect. For example, they are likely to engage in non-mean-variance optimizing activities such as window-dressing (Haugen and Lakonishok (1988); Musto (1997; 1999); Carhart, Kaniel, Musto, and Reed

(2002)). They may also have a slightly different objective such as minimizing tracking error or maximizing fund inflow. However, there are three responses to such critiques. First, as long as the mean-variance optimization, the foundation of modern portfolio management, captures the main trade-off faced by a portfolio manager, the results in this paper should hold. Second, one can think that the observable portfolio weights consist of two components. The first and the major component is the outcome of a mean-variance optimization. The second component is the noise introduced by various "irrationalities" or departures from mean-variance optimization. Window dressing, for example, can be thought as contributing to the latter effect. Typically, window dressing refers to the phenomenon where managers reshuffle their portfolios around disclosure dates to include stocks with good immediate *past* performance and could also be used by funds to disguise their positions. Such noises are going to bias downward the estimated forecasting abilities. The estimates of SBAI/BAI, therefore, can be thought as a lower bound of managerial skills. The noises also bias against any findings of the predictability of SBAI/BAIs for future fund returns. Given these considerations, our empirical findings of the predictability of SBAIs/BAIs provide further empirical supports for SBAI/BAI as performance measures. Finally, even though it does not adjust explicitly for the specific index benchmark that funds use if their objective is to minimize tracking errors, the proposed method accounts for the index implicitly by extracting the underlying indexes among index-tracking funds as non-redundant assets and comparing these funds' performance accordingly.

It is also worthwhile to mention that BAI is constructed from a cross-sectional *observation* of fund holdings and subsequent stock returns rather than *estimated* based on a time series of past fund returns or a combination with a time series of historical fund holdings. For example, one can construct a BAI with only one quarterly filing of holding and one subsequent return data point for each fund, without using any further historical time series data points on holdings or returns. This is useful for ranking a large set of newly incepted funds with no performance track record other than one quarterly filing of holdings. In contrast, existing performance metrics would have difficulty in ranking such funds.

Relation to the literature. This paper is closely related to Grinblatt and Titman (1989), Cohen, Coval, and Pastor (2005), and Shi, Stoffman, Zhu, and Yuan (2007). Grinblatt and Titman (1989) also propose a "belief accuracy" measure. They compute the change in portfolio weights between two immediate quarters for a stock in a given fund's portfolio and use the change as a proxy for the manager's belief of the stock's expected return: A large positive (negative) change indicates that the fund manager believes the expected return of this stock is higher (lower). They then correlate these weight changes with realized returns to compute the accuracy of a manager's belief. This logic is very similar to the BAI proposed in this paper. However, unlike the approach proposed in this paper, the information across fund managers' holdings – how similarities and differences may reveal some information about fund managerial skills is not used in Grinblatt and Titman (1989). In this sense, our paper is closely related to Cohen, Coval, and Pastor (2005). Cohen, Coval, and Pastor (2005) propose a performance metric based on the *similarity* of funds' holdings to that of the best performing funds. They start with a performance ranking that is based on the past return performance and then assign a performance ranking for the current period based on the similarity of funds' holdings to that of the best performing funds. By comparison, this paper starts with a cross-section of holdings of the current period to back out the heterogeneous beliefs and then assess the accuracy of these beliefs without using any data of past performance. Therefore, the major differences are that 1) the BAI does not use the information in historical alpha to rank funds, and 2) the BAI explores similarities and differences across fund holdings to extract information on the variance-covariance structures of returns in addition to expected returns. Shi, Stoffman, Zhu, and Yuan (2007) view stocks and funds as a network where the link structure is defined by the portfolio holdings of funds. They apply well-researched ranking algorithms in networks to generate ranking measures of funds (FundRanks) and of stocks (StockRanks). This paper differs by assuming that fund managers are mean-variance optimizers while Shi, Stoffman, Zhu, and Yuan (2007) assume that fund managers engage in some risk-return tradeoffs that are common among themselves – the functional form of this trade-off is not assumed. By assuming fund managers are mean-variance optimizers, the approach suggested in this paper put clear interpretations on the ranking measure while the FundRank measure proposed by Shi, Stoffman, Zhu, and Yuan (2007) captures a fund quality measure that is general but without clear interpretations.[2]

[2]Kacperczyk, Sialm, and Zheng (2005) also motivate their study based on portfolio theory and use only holding data to construct a performance measure. They find funds that have more industry-concentrated portfolios, intuitively, are funds that have superior information, and hence perform better. This paper can be also thought of as a generalized version of Kacperczyk, Sialm, and Zheng (2005) in a mean-variance framework by adding a risk-return trade-off: Funds prefer to concentrate on stocks that they know are likely to perform better but at same time want to diversify away the risk. The characteristic selectivity (CS) metric and the characteristic timing (CT) measure proposed in Daniel, Grinblatt, Titman, and Wermers (1997) can be thought also as related. The major difference is, however, that they rely also on historical return data

Wermers (2000); Ferson and Khang (2002); Kacperczyk, Sialm, and Zheng (forthcoming); and Cremers and Petajisto (2006) also contribute to the performance evaluation literature in investigating the use of the information in the portfolio holdings. However, unlike this paper, they do not explore the information across mean-variance optimizing fund managers' holdings – how similarities and differences in their holdings may reveal some information about fund managers' belief about the risk-return trades of stocks in their portfolio.

Although this paper is the first to elicit beliefs on expected returns and variance-covariance matrix from portfolio holdings, it is not the first attempt to look beyond price or return data to back out return-generating factors. Lo and Wang (2000; 2001) find the turnover satisfies an approximately linear k-factor structure and Goetzmann and Massa (2006) identify factors through a sample of net flows to nearly 1000 U.S. mutual funds over a year and a half period. In general, this paper differs from the existing literature by treating portfolio holdings as solutions to an investment optimization problem. Building on a simple model of portfolio choice, this paper proposes a new way to infer variance-covariance matrix, as well as investor heterogeneous beliefs from portfolio holdings.

Besides improving or adding another dimension to the existing performance metrics, this paper contributes to the finance literature by highlighting a revealed preference approach to extract information on investor expected return and on return generating process from portfolios and potentially improving empirical asset pricing tests. For example, the variance-covariance matrix is inferred in the *observed* portfolio weights in this paper. In the existing literature, the variance-covariance matrix is *estimated* based on historical returns. The information provided in portfolio holdings could potentially improve the existing estimation and hence has implications for tests involving cross-sectional stock returns. There is also an extensive body of empirical asset pricing literature that explores how to estimate the number of factors using price and return data (Connor and Korajczyk (1986; 1988), Bai and Ng (2002; 2006), Ludvigson and Ng (forthcoming), and Jones (2001)). The revealed Σ matrix from portfolio holdings could potentially contributes to this set of tests as well.

The remainder of this paper is organized as follows. In Section 2, I present a partial equilibrium model of portfolio holdings. Section 3 describes the data used in the empirical implementation of the model and outlines the computation of SBAI and BAI. In Section 4,

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and three characteristic factors to measure performances while the BAI proposed in this paper is based on holding only. Wermers, Yao, and Zhao (2007) use portfolio information to predict stock returns, similar to Shi, Stoffman, Zhu, and Yuan (2007).

I analyze SBAI and BAI results. In Section 5, I examine whether SBAI and BAI contain valuable information to predict fund performance, especially over and above that contained in the existing performance measures. I conclude in Section 6.

2 Portfolio Allocation under Heterogeneous Beliefs

In this section, I develop a standard portfolio choice model. In this model, investors have heterogeneous beliefs about the excess returns of n risky stocks but possess common knowledge about the variance-covariance matrix of their returns. I solve for optimal portfolio allocation for heterogeneous investors. However, the objective of this model is not to obtain a set of optimal portfolio weights given heterogeneous beliefs, but to see whether observing a set of optimal portfolio weights, one can back out heterogeneous investor beliefs and the variance-covariance matrix of returns. In the model, portfolio optimization by investors results in a portfolio matrix that reveals both the perceived variance-covariance matrix as well as heterogeneous beliefs about future excess returns. In what follows, I first detail the return-generating process for risky and risk-free assets and the information structure among the investors. I then solve the portfolio optimization problem for mean-variance investors and show the variance-covariance matrix as well as investor beliefs can be identified up to a constant through a decomposition of the portfolio matrix. Finally, I define portfolio (revealed) beliefs to construct the belief accuracy index (BAI).

2.1 Assets

To develop the model, I first focus on a standard portfolio allocation problem. In this problem, the available investment opportunities consist of a risk-free asset with a constant return, r_f, and n risky assets where ith asset's *excess return* over the risk-free rate (r_f) is denoted as $\tilde{r}_i$. I assume that $k(< n)$ assets are non-redundant. The n risky assets have the following variance-covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma^2_{1,1} & \dots & \sigma^2_{1,n} \\ \vdots & \ddots & \vdots \\ \sigma^2_{n,1} & \dots & \sigma^2_{n,n} \end{bmatrix}, \tag{1}$$

which is of rank of k.[3] I assume that the spectral decomposition of the Σ matrix has the following form:

$$\Sigma = \begin{bmatrix} \mathbf{b}_1 & \cdots & \mathbf{b}_k \end{bmatrix} \begin{bmatrix} \sigma^2_{f_1} & & \\ & \ddots & \\ & & \sigma^2_{f_k} \end{bmatrix} \begin{bmatrix} \mathbf{b}'_1 \\ \vdots \\ \mathbf{b}'_k \end{bmatrix}, \tag{2}$$

where Σ has k eigenvectors. Intuitively, one can think this economy has k basis assets. Let Σ_f denote the following:

$$\begin{bmatrix} \sigma^2_{f_1} & & \\ & \ddots & \\ & & \sigma^2_{f_k} \end{bmatrix}.$$

Then, the spectral structure of the Σ matrix can be re-written as:

$$\Sigma = \mathbf{b}\Sigma_f\mathbf{b}'. \tag{3}$$

Corresponding to the above spectral structure, I use V to denote a subspace in $\mathbb{R}^n$, which is the span of $\{\mathbf{b}\}_{l=1,\ldots,k}$. Intuitively, one can think that there are k orthogonal basis assets in this economy that spans the risky-asset space.

2.2 Beliefs

I assume that there are m investors in this economy, where $m > k$. These investors possess a common knowledge of Σ, but are heterogeneously informed about the risky assets' excess returns. As mentioned in the introduction, this assumption is motivated by the fact that the second moments can be better estimated than the first moments by using high-frequency historical return data, as shown in the empirical asset pricing literature.

I use μ_{mi} to denote investor m's belief of asset i's expected excess return. Investor m's belief of the n assets' expected excess returns can be written as $\mu_m = [\mu_{m1}, ..., \mu_{mn}]'$, and total m investors' beliefs on n assets can be written as $\mu = [\mu_1, ..., \mu_m]'$, which is $m \times n$ matrix.

To explore the spectral decomposition of the Σ matrix, I decompose μ on the same basis

[3]The models in the Appendix deal with the case where the variance-covariance is of full rank and the case where the variance-covariance is of two components – a part of the risk that investors can form beliefs of expected return on, and a part of risk of which investors do not demand expected return when exposed to. The results remain the same.

by projecting μ to the subspace V spanned by $\mathbf{b}$, namely,

$$\mu = \hat{\mu}\mathbf{b}', \tag{4}$$

where $\hat{\mu}$ is a $m \times k$ matrix and $\hat{\mu} = [\hat{\mu}_1, ..., \hat{\mu}_m]'$. That is, $\hat{\mu}_m$, a $k \times 1$ vector, describes investor m's belief on the k basis assets.

With this characterization, I make one assumption regarding the belief structure. Specifically, I assume that the column vectors of $\hat{\mu}$ are orthonormal. The orthogonality assumption is basically a rationality assumption since the columns of $\hat{\mu}$ reflect beliefs about different orthogonal eigenfactors (or basis assets).[4] This assumption simplifies the derivation for investors' beliefs, as shown later.

2.3 Investor Portfolio Optimization Problem

Let w_{m0} denote the percentage of wealth (or portfolio weight) invested by investor m in the riskless asset and $\mathbf{w}_m = [w_{m1}, \cdots, w_{mn}]'$ denote the vector of portfolio weights in each of the n risky assets by investor m. The portfolio weights satisfy the following equation:

$$w_{m0} + \sum_{i=1}^{n} w_{mi} = 1, \tag{5}$$

where $\mathbf{w} = [\mathbf{w}_1, ..., \mathbf{w}_m]'$ and is a $m \times n$ matrix.

In this economy, investors choose portfolio weights to obtain a standard mean-variance optimization for expected returns. Investor m, conditional on his beliefs, chooses his portfolio weights, $\mathbf{w}_m$, to maximize the following:

$$\max_{\{\mathbf{w}_m\}} \left(\mathbf{w}_m^{'} \mu_m - \frac{1}{2} \gamma \mathbf{w}_m^{'} \Sigma \mathbf{w}_m \right), \tag{6}$$

where γ is assumed to be the same across investors.[5]

[4] In reality, investors may have different degrees of disagreement about the returns for different assets. That is, the length of $\hat{\mu}$ for different eigenfactors (basis assets) could be different. This possibility can be addressed from both theoretical and empirical perspectives. Theoretically, in general equilibrium settings, a larger perceived uncertainty about an asset would result in decreased investor demand. Therefore, it would affect the return distribution of the asset itself. However, the return distribution must be taken as a given in this partial equilibrium model, because investors do not have price impacts. Hence, it is reasonable to assume that any differences in perceived uncertainty (or the length of $\hat{\mu}$) are absorbed in the return distribution. Empirically, one can normalize $\hat{\mu}$ by empirical measures of degrees of perceived uncertainties, such as dispersion of analysis forecasts.

[5] This objective function is commonly used in the literature and can be thought of as a reduced form of the investor portfolio optimization problem. For example, it can be obtained in a standard Merton's

The first-order condition for investor m yields a mean-variance efficient portfolio:

$$\mathbf{w}_m = \frac{1}{\gamma}\Sigma^{\dagger\prime}\mu_m \tag{7}$$

where † denotes Moore-Penrose generalized inverse. Finally, the matrix of optimal portfolio weights by all m investors in this economy can be written as:

$$\mathbf{w} = \frac{1}{\gamma}\mu\Sigma^{\dagger\prime}. \tag{8}$$

2.4 Heterogeneous Beliefs Revealed in Portfolio Holdings

For a given Σ, investor private beliefs can be immediately revealed by $\mathbf{w}$. This result is stated in the following lemma, which is immediate from Equation (8).

Lemma 1 *Once observing a portfolio holding matrix* $\mathbf{w}$*, investor private beliefs on expected return of risky assets for a given* Σ *can be computed as*

$$\mu = \gamma\mathbf{w}\Sigma. \tag{9}$$

Since the variance-covariance matrix Σ can be estimated from historical return data, this leads immediately to the extraction of heterogeneous beliefs from portfolio holdings.

Result 1 *For a given portfolio holding matrix* $\mathbf{w}$ *and a* Σ *estimated from historical returns, investor private beliefs on expected return of risky assets are revealed up to a constant (which is the risk aversion coefficient):*

$$\mu = \gamma\mathbf{w}\Sigma. \tag{10}$$

Note that the degree of risk aversion only affects the allocation across risky and risk-free assets and does not affect the allocation within risky assets. I term these sets of beliefs "*semi-portfolio (revealed) beliefs*" because these beliefs are revealed by corresponding portfolio holdings as well as the variance-covariance matrix estimated using past return data. One potential issue with the semi-portfolio (revealed) beliefs is that one may argue that the estimated Σ used in formulating these semi-portfolio revealed beliefs, is not forward-looking and is subject to various estimation biases. Next, I show that instead of estimating Σ using

(1971) problem where the investment opportunity set is constant (see also Aït-Sahalia, Cacho-Diaz, and Hurd (2007)). Note that assuming all investors have the same risk aversion coefficient γ does not affect the results since difference in γ affects the allocation among risky and risk-free assets but does not affect the allocation among risky assets for a given set of beliefs.

historical return data, one can also elicit the common belief on Σ among investors from $\mathbf{w}$. This Σ matrix, unlike the variance-covariance matrix estimated using return data, is forward-looking.

To see that, let us first denote the spectral decomposition of $\mathbf{w}'\mathbf{w}$ as:

$$\mathbf{w}'\mathbf{w} = \mathbf{u}e\mathbf{u}' = \begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_k \end{bmatrix} \begin{bmatrix} e_1 & & \\ & \ddots & \\ & & e_k \end{bmatrix} \begin{bmatrix} \mathbf{u}_1' \\ \vdots \\ \mathbf{u}_k' \end{bmatrix}. \tag{11}$$

The next proposition shows how $\mathbf{w}$ reveals Σ up to a constant.

Proposition 1 $\Sigma = a\mathbf{u}e^{-\frac{1}{2}}\mathbf{u}'$ *where a is a constant.*

PROOF OF PROPOSITION 1:

Since the column vectors of $\hat{\mu}$ are orthogonal. The following is immediate from Equation (8):

$$\mathbf{w}'\mathbf{w} = \frac{1}{\gamma^2}\Sigma^{\dagger'}\mathbf{b}\hat{\mu}'\hat{\mu}\mathbf{b}'\Sigma^{\dagger} = \frac{1}{\gamma^2}\Sigma^{\dagger'}\mathbf{b}\mathbf{b}'\Sigma^{\dagger}. \tag{12}$$

Note that since $\Sigma = \mathbf{b}\Sigma_f\mathbf{b}'$, Equation (12) can be written as:

$$\begin{aligned} \mathbf{w}'\mathbf{w} &= \frac{1}{\gamma^2}(\mathbf{b}(\Sigma_f\mathbf{b}'))^{\dagger'}\mathbf{b}\mathbf{b}'((\mathbf{b}\Sigma_f)\mathbf{b})^{\dagger} = \frac{1}{\gamma^2}(\Sigma_f\mathbf{b}')^{\dagger}\mathbf{b}^{\dagger'}\mathbf{b}\mathbf{b}'\mathbf{b}'^{\dagger}(\mathbf{b}\Sigma_f)^{\dagger} \\ &= \frac{1}{\gamma^2}(\Sigma_f\mathbf{b}')^{\dagger}(\mathbf{b}\Sigma_f)^{\dagger} = \frac{1}{\gamma^2}\mathbf{b}'^{\dagger}\Sigma_f^{-2}\mathbf{b}^{\dagger} = \frac{1}{\gamma^2}\mathbf{b}\Sigma_f^{-2}\mathbf{b}' \end{aligned} \tag{13}$$

where $1/\gamma^2$ is a constant. The last three equalities are obtained by repeatedly using the facts that 1) $\mathbf{b}^{\dagger} = \mathbf{b}'$ (because $\mathbf{b}$ are orthogonal) and 2) a property of Moore-Penrose generalized inverse: If $rank(A) = rank(B)$, then $(AB)^{\dagger} = B^{\dagger}A^{\dagger}$ (Theorem 5.9 in Schott (2005)). The last equality gives a spectral decomposition of $\mathbf{w}'\mathbf{w}$, and so does Equation (11) Therefore, $\mathbf{b} = a_1 \begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_k \end{bmatrix}$ and $\Sigma_f = a_2 e^{-\frac{1}{2}}$ where a_1 and a_2 are two constants. The rest follows since $\Sigma = \mathbf{b}\Sigma_f\mathbf{b}'$.■

Since $\mathbf{b}$ and Σ are revealed by portfolio matrix, I term them "*portfolio (revealed) betas*" and "*portfolio (revealed) variance-covariance matrix*" respectively. After obtaining $\mathbf{b}$ and Σ, the investor beliefs, μ, can be computed by Equation (9). This result is stated below.

Result 2 *For a given portfolio holding matrix $\mathbf{w}$, investor private beliefs on expected return of risky assets for a given Σ can be computed as*

$$\mu = \gamma\mathbf{w}a\mathbf{u}e^{-\frac{1}{2}}\mathbf{u}'. \tag{14}$$

I term these beliefs "*portfolio (revealed) beliefs*" because these beliefs are revealed by corresponding portfolio holdings only.

It may seem magical that I am able to obtain $\mathbf{b}$, Σ, and μ from a single $m \times n$ matrix $\mathbf{w}$. Equation (8) may provide some intuition. Notice that the investors possess a common knowledge of Σ and hence their portfolio should exhibit k fund separation. The fact that their weights are increasing in μ means they tilt their portfolios toward funds with higher expected returns.

Since *portfolio belief* captures a fund manager's *ex ante* belief about future stock returns, how accurate his beliefs about future stock returns are compared to the realized subsequent returns is an intuitive measure of his forecasting ability. Specifically, the correlation between the realized excess returns and the revealed beliefs about excess returns using all stocks in fund m's portfolio, expressed as $\operatorname{corr}(\mu_m, \tilde{\mathbf{r}})$, captures how closely fund m's manager's expectation of excess returns matches the corresponding subsequent realized returns. I term the correlation between the semi-portfolio (revealed) beliefs, as computed in Result 1, and the realized return the "*semi-belief accuracy index* (SBAI);" the correlation between the portfolio (revealed) beliefs, as computed in Result 2, and the realized return the "*belief accuracy index* (BAI)." In the rest of the paper, I test the empirical relevance of SBAI and BAI.

3 Empirical Analysis

3.1 Data

For the empirical tests, I employ four databases: CRSP stock daily return from January 1981 to December 2005, CRSP stock monthly return, CRSP mutual fund monthly return and the stock holdings of mutual funds from the CDA/Spectrum mutual fund holdings database maintained by Thomson Financial from January 1981 to September 2006. For the mutual fund returns, I use net returns, i.e., returns after fees, expenses, and brokerage commissions but before any front-end or back-end loads. The mutual fund holding database comprises mandatory SEC filings as well as voluntary disclosures by mutual funds. It is typically available quarterly. Wermers (2000) describes this database in more detail.

For this study, I focus on domestic all-equity funds. To construct the sample for this analysis, I first merge the CRSP mutual fund database with the CDA/Spectrum holdings using the MFLINKS programs provided by Wermers (2000).

Finally, I include only funds with equity holdings greater than $5 million, with at least 10 stocks and with stocks that can be matched with the CRSP stock return database.

The final sample for the mutual fund holdings includes the period 1981-2006 (103 quarters in total). However, note that monthly stock and fund return data only cover periods until September 2006 and daily stock return data are available until December 2005.

3.2 Eliciting Beliefs and Variance-Covariance Matrix

For each quarter in the sample, I construct a portfolio weight matrix $\mathbf{w}$, with the row indexing the funds and the column indexing the stocks. Since there are 103 quarters in the sample, there are 103 $\mathbf{w}$ matrices.

Results 1 and 2 show that, to compute portfolio revealed beliefs, besides $\mathbf{w}$, Σ is also needed. Result 1 indicates to use Σ estimated from historical return data and correlate the resulting beliefs with the realized returns to form SBAI; while Result 2 depends on extracting Σ from $\mathbf{w}$ and correlating the elicited beliefs with the realized returns to form BAI.

To construct SBAI, I experiment with two estimators for Σ using non-overlapping three-month weekly stock return data before each quarter-end. Since the sample size is smaller relative to the dimension of the variance-covariance matrix, I adopt the shrinkage approach as highlighted in Schäfer and Strimmer (2007). The advantage of this shrinkage approach is that the optimal shrinkage intensity is calculated analytically. I also try to get around the dimensionality problem by estimating the covariance matrix of a smaller number of assets such as Fama-French three factors and estimating the corresponding factor loadings using weekly data based on Scholes-William approach. However, SBAI computed using the latter approach has performed poorly and therefore the results are not reported. This finding indicates that the covariances among Fama-French factors do not capture the covariance among stocks well.

To construct BAI, I need to deal with an empirical issue – in reality some assets may be redundant and the variance-covariance matrix may not be full rank. That is, the number of non-redundant assets, k is unknown. Hence, one needs to first determine k, the rank of Σ or equivalently the number of eigenfactors in Σ. Since the Σ matrix is revealed from *observed* data and is not estimated as in the existing literature, a straightforward application of Bayesian information criterion is sufficient to obtain k. In fact, Bai and Ng (2002) have shown that the eigenvalue scree diagram analysis yields a similar result to Bayesian information criterion.[6] A scree diagram analysis shows that top 6-7 factors explain most of variation in the weight matrix. Further, a principal component analysis shows that, on average, 10 to 20 factors explain about 30 to 40% of the variation in the weight matrix. Beyond 20 factors, each factor explains less than 1% variation of the matrix. Given these findings, I conduct analyses based on k of 10, but perform robustness checks using k up to 20 and find the results are robust for $k > 10$.

After computing semi-portfolio revealed beliefs according to Equation 10 and portfolio revealed beliefs according to Equation 14, for every stock in the portfolio matrix $\mathbf{w}$ of each manager at the quarter when he or she makes the portfolio decision, I calculate semi-belief accuracy index (SBI) and belief accuracy index (BAI) for each fund manager by correlating revealed beliefs with immediate future (e.g., one-month ahead) excess returns $\tilde{\mathbf{r}}$.[7] Therefore, SBAI and BAI for a fund manager presented in this paper measures how closely his or her beliefs about excess returns for stocks in his or her portfolio mimic the one-month ahead realized returns for these stocks.

4 Can Mutual Fund Managers Predict Returns?

Since active portfolio management, based on the idea that managers can predict returns, is so widespread, it is worthwhile to examine whether fund managers can, on average, forecast stock returns. The results in Table 1 show that, for an average mutual fund manager, the correlation between private beliefs and one-month ahead returns (SBAI or BAI) is close to zero and statistically insignificant. This indicates that fund managers, on average, cannot forecast factor returns. This result is not surprising given that Grinblatt and Titman (1989) also find the average performance based on their benchmark-free GT measure is close to zero.

However, after sorting funds into deciles according to SBAI and BAI, I find that fund

[6] There is an extensive body of empirical asset pricing literature that explores how to determine the number of factors using price/return data (Connor and Korajczyk (1986; 1988), Bai and Ng (2002; 2006), Ludvigson and Ng (forthcoming), and Jones (2001)).

[7] Since the fund manager's information should last at least one month, the most obvious testing period is the subsequent one-month period. However, the results also hold if correlating beliefs with the returns in the subsequent two-month or three-month period. For expositional clarity, only one-month ahead results are presented.

managers in the top half deciles are able to predict future returns to some extent. For example, the beliefs of those with the top decile funds sorted according to BAI, on average, have a positive and statistically significant correlation of about 0.0402, while the beliefs of those with the bottom decile funds have a negative and statistically significant correlation of about -0.0377. This difference between the bottom and the top deciles is about 0.0778 and is statistically significant at the one-percent level. Similar results hold when sorting funds according to SBAI.

**** Insert Table 1 about here ***

The results in Table 1 also show that the size of funds is not statistically different across the top and bottom SBAI and BAI deciles. The average fund size in the SBAI (BAI) sample is around $417.1913 ($424.67) million, with the average size in the top decile at about $429.356 ($469.525) million and that in the bottom decile at around $389.0151 ($394.533) million. This indicates that the size of the funds is unlikely to be related to the ability to forecast future returns.

I also compute the Spearman rank correlation between the ranking based on a fund's average SBAI or BAI during the previous one year and the current SBAI or BAI, respectively, during the sample period. The ranking based on BAI is more persistent than that based on SBAI. For example, the Spearman rank correlation between the ranking based on a fund's average BAI during the previous one year and the one-quarter ahead BAI is 0.64226. This correlation is 0.39191 for two-quarter ahead BAI, 0.19905 for three-quarter ahead. All Spearman correlation correlation coefficients are statistically significant at the one-percent level. This correlation turns negative and insignificant for the four-quarter ahead BAI and back to positive and statistically significant for the five-quarter ahead BAI. By comparison, the Spearman rank correlation for rankings based on SBAI remains positive and statistically significant for one to three-quarter ahead windows but turns negative and statistically significant for four to five-quarter ahead windows. These results are presented in Table 2. This indicates a persistence of forecasting ability.

**** Insert Table 2 about here ***

5 Predicting Mutual Fund Returns Using SBAI and BAI

I have shown that although, on average, managers cannot forecast factors returns, managers of some funds can do so. Given that the SBAI and BAI are persistent, one should expect funds with higher SBAIs or BAIs to outperform funds with lower SBAIs or BAIs, respectively. Next, I test whether SBAI and BAI for each manager indeed contain valuable information that can be used to predict future fund performance.

5.1 Trading Strategy Using SBAI and BAI

To examine the performance of a trading strategy based on a manager's SBAI/BAI, I first sort all funds in the sample into deciles according to their most recent SBAI/BAI. The decile portfolios are formed by weighting all the funds in the decile either equally or according to their size. To evaluate the performance of the decile portfolios, I compute the returns of these decile portfolios for the subsequent three-month period. Figure 1 outlines an example of the timeline for this trading strategy based on SBAI/BAI. It includes the strategy formation period (including belief extraction and belief accuracy index formation) and the subsequent (three-month) strategy testing period. Since holdings are not disclosed immediately after the effective holding date, correlating the subsequent stocks introduces a one-month lag in computing SBAI/BAI.[8] Also note that the return observations for the testing periods in this trading strategy do not overlap.

For SBAIs, since the weekly return is available from January 1981 to December 2005, I can estimate variance-covariance matrices for 100 quarters during this period and hence there are in total 100 strategy-formation dates. The first portfolio formation date is April 1981 and the last portfolio formation date is December 2005. There are 100 strategy formation dates from April 1981 (inclusive) to December 2005 (inclusive). For each of these 100 strategy-formation dates, there are three month-observations of returns for each decile portfolio. In total, there are 300 non-overlapping monthly return observations for each decile portfolio in the sample.

For BAIs, since the holding data is available quarterly from January 1981 to September 2006, I can form in total 102 strategy-formation periods. The first portfolio formation date

[8] The results are robust to lags varying from one month to three months.

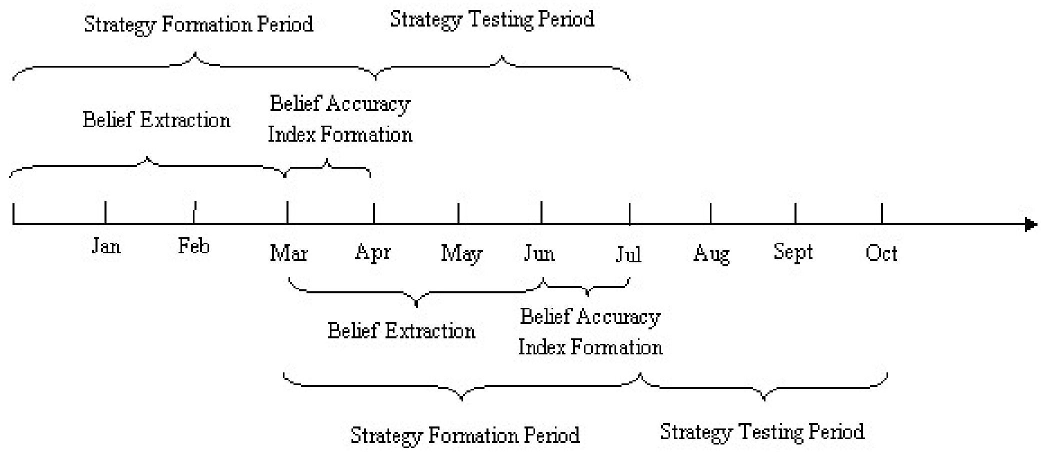

Figure 1: **Timeline.** This figure presents an example of the timeline for forming a trading strategy and testing this strategy.

is April 1981 and the last portfolio formation date is July 2006. There are 102 strategy formation dates from April 1981 (inclusive) to July 2006 (inclusive). For each of these 102 strategy-formation dates, there are three month-observations of returns for each decile portfolio. For the last quarter portfolio formation (July 2006), there are only two month-observations (August 2006 and September 2006) of returns for each decile portfolio. In total, there are 305 non-overlapping monthly return observations for each decile portfolio in the sample.

The risk- and style-adjusted net returns for each value- and equal-weighted decile portfolio using SBAI are reported in Tables 3 and 4, respectively. The third column in Tables 3 and 4 reports the average returns for funds in each decile. The next column reports the excess returns (that is, returns over the risk-free rate).[9] The next three columns report the intercepts from a time-series regression based on the one-factor CAPM model, the three-factor model of Fama and French (1993), and the four-factor model of Carhart (1997), respectively.

The risk- and style-adjusted net returns for each value- and equal-weighted decile portfolio using BAI (based on ten eigenfactors) are similarly reported in Tables 5 and 6, respectively.

*** Insert Table 5 about here***

[9]The t-statistics in this column are the corresponding Sharpe ratios.

*** Insert Table 6 about here***

The results in these tables indicate that funds with the best forecasting skills in the immediate past quarter (decile ten) significantly outperform funds with the worst forecasting skills (decile one) in the subsequent three-month testing period. In other words, investing in equal-weighted (value-weighted) decile-ten SBAI funds generates an additional 37 (35) basis points per month, or about 4.44% (4.2%) per year compared to investing in decile-one SBAI funds. Similarly, investing in the equal-weighted (value-weighted) decile-ten funds would generate an additional 34 (42) basis points per month, or about 4.08% (5.16%) per year compared to investing in decile-one funds. The relationship between the past BAI and the future fund performance is highly monotonic for the equal-weighted trading strategy and slightly less so for the value-weighted trading strategy. These results are not influenced by variations in the risk or style factors, as reported in the next four columns of Tables 3 to 6.

Tables 7 and 8show characteristics of equal-weighted decile SBAI and BAI funds, respectively. The results show that all decile funds have a statistically significant positive CAPM beta, invest statistically more in small stocks, hold more value stocks (but statistically significant only for decile-5 BAI funds), and have a negative (statistically insignificant) exposure to the momentum factor (except decile-9 and decile-10 SBAI funds).

Tables 7 and 8 also show that the top performing SBAI and BAI funds invest less in high market-beta, small or momentum stocks; and the top performing BAI funds invest more in value stocks, although these results are not statistically significant. The top performing BAI funds, as shown in Table 7 , however, has a significantly lower beta than bottom performing BAI funds and the statistical significance of this difference is at the ten-percent level.

Next to see whether SBAI and BAI have different information in predicting future fund returns, or equivalently, whether Σ estimated from historical returns and portfolio revealed Σ elicit portfolio beliefs differently, I perform a series of conditional sorts. For each portfolio formation period, I first sort funds into quintiles based on their SBAIs. I then sort funds into quintiles based on their BAIs within each SBAI. Finally, I examine the subsequent three-month returns for the resulting 25 (equal-weighted) portfolios, which are reported in Panel A of Table 9. Controlling for SBAI, the average differences between the top and bottom quintiles of the funds ranked by their BAIs range from 18 to 36 basis points per month or 2.16% to 4.32% per year. Two of these difference are strongly statistically significant at the one-percent level and one at five-percent level. This indicates that the BAI measure contains much more additional information about future fund performance that is not included in the

SBAI index.

I also examine how much information about future fund returns is contained in the SBAI index but not captured in the BAI measure. To do so, I sort funds into quintiles in reverse, first by BAI and then by the SBAI measure. These results are reported in Panel B of Table 9. Controlling for BAI, the 5-1 spreads produced by the SBAI measure are statistically significant only for the 4th BAI quintile (at five-percent level). This indicates that most information contained in the SBAI index about better-performing and worse-performing funds is already contained in the BAI index. These results indicate that the portfolio revealed variance-covariance matrix Σ indeed contains some unique information about the future returns.

For robustness check, the performance of a trading strategy based on twenty eigenfactors is reported in Tables 10 and 11. The results are similar to (and in many cases slightly stronger than) those based on ten eigenfactors. These results are reasonable since principal component analysis indicates that most variations in the portfolio are captured by 10 factors.

*** Insert Table 10 about here***

*** Insert Table 11 about here***

5.2 Do SBAI and BAI Contain Valuable Information in Predicting Future Fund Performance?

One way to assess the usefulness of SBAI and BAI in capturing true managerial skills is to conduct a simulation analysis. However, if the simulation environment follows the theoretical model in Section 2, SBAI and BAI measures, by construction, dominate all existing performance metrics. Therefore, such an assessment must come from the empirical relevance of SBAI and BAI measures.

One may also question whether SBAI and BAI measures contain more information than the existing holding-based performance measures. The comparable one is the benchmark-free GT measure in Grinblatt and Titman (1989), which does not use any past return information.

To implement this test, for a given quarter, I compute the GT measure by correlating the changes in a fund's holdings with the realized excess returns in the following month.

To see whether SBAI and BAI contain any additional information that is not in the GT measure, I perform a series of conditional sorts. For each portfolio formation period, I first sort funds into quintiles based on their SBAIs. I then sort funds into quintiles based on their GTs within each SBAI. Finally, I examine the subsequent three-month returns for the resulting 25 (equal-weighted) portfolios, which are reported in Panel A of Table 12. Controlling for the GT measure, the average differences between the top and bottom quintiles of the funds ranked by their SBAIs range from 5 to 34 basis points per month or 0.6% to 4.98% per year. Three of these differences are strongly statistically significant at or above the five-percent level. This indicates that the SBAI measure contains much more additional information about future fund performance that is not included in the GT index.

I also examine similarly how much information about future fund returns is contained in the BAI index but not captured in the GT measure. To do so, I sort funds into quintiles, first by GT and then by BAI. These results are reported in Panel B of Table 12. Controlling for GT, the 5-1 spreads produced by BAI are also statistically significant for three BAI quintiles (one at ten-percent and two are at five-percent level). This indicates that the BAI measure contains much more additional information about future fund performance that is not included in the GT index.

**** Insert Table 12 about here ***

I also compare SBAI/BAI with the FundRank measure of Shi, Stoffman, Yuan, and Zhu (2007), another holding-based performance measures. Since Shi, Stoffman, Yuan, and Zhu (2007) have shown the FundRank measure is similar to the measure in Cohen, Coval, and Pastor (2005), this doublesort is an indirect comparison between SBAI/BAI and Cohen, Coval, and Pastor (2005). To do so, I perform a series of conditional sorts. For each portfolio formation period, I first sort funds into quintiles based on their SBAIs. I then sort funds into quintiles based on their FundRank within each SBAI. Finally, I examine the subsequent three-month returns for the resulting 25 (equal-weighted) portfolios, which are reported in Panel A of Table 13. Controlling for the FundRank measure, the average differences between the top and bottom quintiles of the funds ranked by their SBAIs range from 20 to 36 basis points per month or 2.4% to 4.32% per year. Two of these differences are strongly statistically significant at the five-percent level. This indicates that the SBAI measure contains much more additional information about future fund performance that is not included in the FundRank index.

Similarly, I also examine how much information about future fund returns is contained in the BAI index but not captured in the FundRank measure. To do so, I sort funds into quintiles, first by GT and then by BAI. These results are reported in Panel B of Table 13. Controlling for FundRank, the 5-1 spreads produced by BAI are also statistically significant for two BAI quintiles at five-percent level, range from 19 to 41 basis points per month or 2.28% to 4.92% per year. This also indicates that the BAI measure contains much more additional information about future fund performance that is not included in the FundRank index.

I also compare BAI with the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997); the 5-1 spreads produced by the BAI measure are statistically significant for one quintile. These results, presented in Table 14, show that SBAI and BAI have some unique information in predicting next three-month stock returns.

6 Conclusion

This paper proposes new fund performance measures based on how closely fund managers' beliefs regarding future stock returns match realized returns. These measures are constructed by correlating beliefs about future returns revealed from historical holdings of funds with subsequent realized returns. To infer the revealed beliefs, this paper assumes that managers are rational mean-variance optimizers. The key idea is that managers tilt their portfolios toward stocks with better risk-return tradeoffs, according to their private beliefs. Hence, observing their holdings, one can determine whether fund managers' beliefs on future returns are accurate. The evidence in this paper suggests that although, on average, fund managers may not be able to predict future returns, some fund managers do possess forecasting abilities. Therefore, mutual fund investors could benefit significantly from investing in funds selected through the proposed SBAI and BAI measure. Further, the SBAI and BAI measure contains the information that is not in the existing performance measures. Theoretically, these performance metrics contribute to the existing performance measures by exploring information contained in the cross-sectional similarities and differences of fund holdings with the guidance of a mean-variance preference optimizing framework.

In addition to the contribution to the performance evaluation literature, the paper contributes to the general finance literature by pointing out a new angle from which to study asset pricing questions. That is, instead of estimating investor expectation regarding risk and risks from historical returns, one can extract their expectations regarding risks and returns in portfolio holdings using a revealed preference approach. For example, the variance-covariance matrix estimated in the *observed* portfolio weights is dynamic and quarter-to-quarter estimates are different. In the existing literature, it is challenging to estimate a dynamic variance-covariance matrix based on historical returns. The information provided in portfolio holdings could potentially improve the existing estimation and hence has important implications for tests involving cross-sectional stock returns.

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Table 1: Summary Statistics for Decile Funds Formed Based on SBAI and BAI

Table 1 reports the means and the standard errors for SBAI and BAI and the corresponding total net asset (TNA) for funds in each SBAI or BAI decile. The SBAI (BAI) is the correlation between a fund manager's beliefs, computed using Result 1 (Result 2), at each quarter-end and the following month realized excess returns for all stocks in the fund manager's portfolio. We compute means of the SBAI (BAI) and the corresponding TNA for funds in each decile for each portfolio formation date and then average these means across 103 portfolio formation dates over the period of January 1981 to September 2006 for BAI and across 100 portfolio formation dates over the period of January 1981 to January 2006 for SBAI.

	SBAI			BAI		
Decile	N	Mean	Mean TNA	N	Mean	Mean TNA
1	100	-0.028	389.0151	103	-0.0377	395.4141
		(0.0012)	(34.1083)		(0.0029)	(34.8588)
2	100	-0.017	391.7293	103	-0.0258	412.9296
		(0.0011)	(31.6804)		(0.0029)	(35.5099)
3	100	-0.011	392.2132	103	-0.0179	389.0201
		(0.0010)	(31.8428)		(0.0030)	(32.0562)
4	100	-0.0063	401.9889	103	-0.0106	426.7532
		(0.0010)	(34.1151)		(0.0029)	(33.6422)
5	100	-0.0021	421.2617	103	-0.0036	390.1184
		(0.0010)	(36.8365)		(0.0028)	(26.2115)
6	100	0.0019	464.8567	103	0.0033	457.2082
		(0.0010)	(38.8423)		(0.0028)	(35.9811)
7	100	0.006	443.5446	103	0.0106	445.107
		(0.0009)	(37.2056)		(0.0029)	(29.8550)
8	100	0.0107	407.1994	103	0.0183	424.4548
		(0.0009)	(29.9629)		(0.0031)	(33.7275)
9	100	0.0166	430.7482	103	0.0271	436.7588
		(0.0010)	(35.4613)		(0.0032)	(33.1155)
10	100	0.0278	429.356	103	0.0402	469.7214
		(0.0011)	(36.6736)		(0.0031)	(41.8781)
Average		-0.0001	417.1913		0.0004	424.7485
		(0.0009)	(10.9741)		(0.0027)	(10.7075)
Top Decile -		0.0557	40.3409		0.0779	74.3073
Bottom Decile		(0.0015)**	(48.6280)		(0.0033)**	(47.0552)

Table 2: Persistence of SBAI and BAI

Table 2 reports the Spearman rank correlation for SBAI and BAI. The Spearman rank correlation is computed by correlating the ranking based on the fund's current SBAI (BAI) with the ranking based on the fund's average lagged SBAI (BAI) during the previous one to four quarters over the period of January 1981 to September 2006 for SBAI (over the period of January 1981 to January 2006 for SBAI).

Spearman Rank Correlation Coefficient		
	SBAI	BAI
1-Quarter Lead	0.70371 **	0.64226 **
2-Quarter Lead	0.44372 **	0.39191 **
3-Quarter Lead	0.18164 **	0.19905 **
4-Quarter Lead	-0.05294**	-0.00153
5-Quarter Lead	-0.04465**	0.05987 **

Table 3: Equal-Weighted Portfolio Returns Based on SBAI

At the end of each quarter, fund managers' beliefs regarding excess stock returns are elicited according to Result 1. For each fund, these beliefs are then correlated with the following month's excess returns to form its semi-belief accuracy index (SBAI). Funds are next sorted into decile portfolios according to their SBAIs. Table 3 reports the average monthly return, return over risk-free rate (excess return), and CAPM alpha, Fama-French alpha (1993), and Carhart alpha (1997) with corresponding Newey-West standard errors for the equal-weighted decile portfolios in the subsequent three-month testing period. The table also reports the performance spread between the top and bottom deciles. The portfolio formation period is from January 1981 to January 2006.

Decile	N	Average Return	Excess Return	CAPM Alpha	Fama-French Alpha	Carhart Alpha
1	300	0.0079	0.0034	-0.0029	-0.0029	-0.0027
		(0.0028)**	(0.0028)	(0.0009)**	(0.0010)**	(0.0008)**
2	300	0.0086	0.0041	-0.0020	-0.0021	-0.0018
		(0.0026)**	(0.0026)	(0.0007)**	(0.0007)**	(0.0006)**
3	300	0.0090	0.0044	-0.0016	-0.0016	-0.0016
		(0.0025)**	(0.0025)†	(0.0006)**	(0.0005)**	(0.0006)**
4	300	0.0095	0.0049	-0.0009	-0.0011	-0.0011
		(0.0024)**	(0.0025)*	(0.0005)†	(0.0004)**	(0.0004)**
5	300	0.0092	0.0047	-0.0012	-0.0013	-0.0012
		(0.0024)**	(0.0024)†	(0.0005)**	(0.0004)**	(0.0004)**
6	300	0.0099	0.0053	-0.0006	-0.0008	-0.0007
		(0.0025)**	(0.0025)*	(0.0005)	(0.0004)†	(0.0005)
7	300	0.0101	0.0056	-0.0003	-0.0006	-0.0006
		(0.0024)**	(0.0025)*	(0.0005)	(0.0004)	(0.0005)
8	300	0.0097	0.0051	-0.0008	-0.0011	-0.0009
		(0.0025)**	(0.0025)*	(0.0005)	(0.0004)**	(0.0005)†
9	300	0.0108	0.0062	0.0003	0.0001	-0.0001
		(0.0026)**	(0.0026)*	(0.0007)	(0.0005)	(0.0006)
10	300	0.0116	0.0071	0.0010	0.0011	0.0006
		(0.0028)**	(0.0028)*	(0.0011)	(0.0011)	(0.0009)
Top 10% -	300	0.0037	0.0037	0.0039	0.0040	0.0033
Bottom 10%		(0.0015)*	(0.0015)*	(0.0015)*	(0.0018)*	(0.0014)*
Top 20% -	300	0.0029	0.0029	0.0031	0.0031	0.0025
Bottom 20%		(0.0011)**	(0.0011)**	(0.0011)**	(0.0014)*	(0.0011)*
Top 30% -	300	0.0022	0.0022	0.0023	0.0023	0.0019
Bottom 30%		(0.0009)*	(0.0009)*	(0.0009)*	(0.0010)*	(0.0009)*
Top 40% -	300	0.0018	0.0018	0.0019	0.0018	0.0016
Bottom 40%		(0.0007)*	(0.0007)*	(0.0007)*	(0.0008)*	(0.0007)*

Table 4: Value-Weighted Portfolio Returns Based on SBAI

At the end of each quarter, fund managers' beliefs regarding excess stock returns are elicited according to Result 1. For each fund, these beliefs are then correlated with the following month's excess returns to form its belief accuracy index (SBAI). Funds are next sorted into decile portfolios according to their SBAIs. Table 4 reports the average monthly return, return over risk-free rate (excess return), and CAPM alpha, Fama-French alpha (1993), and Carhart alpha (1997) with corresponding standard errors for the value-weighted decile portfolios in the subsequent three-month testing period. The table also reports the performance spread between the top and bottom deciles. The portfolio formation period is from January 1981 to January 2006.

Decile	N	Average Return	Excess Return	CAPM Alpha	Fama-French Alpha	Carhart Alpha
1	300	0.0078	0.0033	-0.0030	-0.0028	-0.0025
		(0.0028)**	(0.0028)	(0.0009)**	(0.0010)**	(0.0009)**
2	300	0.0084	0.0038	-0.0023	-0.0018	-0.0016
		(0.0026)**	(0.0027)	(0.0008)**	(0.0008)*	(0.0008)*
3	300	0.0085	0.0040	-0.0021	-0.0017	-0.0017
		(0.0026)**	(0.0026)	(0.0007)**	(0.0008)*	(0.0007)*
4	300	0.0096	0.0051	-0.0007	-0.0007	-0.0007
		(0.0024)**	(0.0024)*	(0.0005)	(0.0006)	(0.0006)
5	300	0.0089	0.0044	-0.0017	-0.0015	-0.0016
		(0.0025)**	(0.0026)†	(0.0004)**	(0.0004)**	(0.0005)**
6	300	0.0094	0.0049	-0.0009	-0.0009	-0.0011
		(0.0024)**	(0.0025)*	(0.0005)	(0.0005)†	(0.0006)†
7	300	0.0098	0.0052	-0.0007	-0.0004	-0.0007
		(0.0025)**	(0.0025)*	(0.0005)	(0.0005)	(0.0005)
8	300	0.0100	0.0055	-0.0006	-0.0004	-0.0000
		(0.0026)**	(0.0026)*	(0.0006)	(0.0005)	(0.0006)
9	300	0.0101	0.0056	-0.0005	-0.0000	-0.0003
		(0.0026)**	(0.0026)*	(0.0007)	(0.0006)	(0.0006)
10	300	0.0113	0.0068	0.0005	0.0008	0.0003
		(0.0029)**	(0.0029)*	(0.0011)	(0.0011)	(0.0010)
Top 10% -	300	0.0035	0.0035	0.0035	0.0037	0.0028
Bottom 10%		(0.0016)*	(0.0016)*	(0.0017)*	(0.0020)†	(0.0016)†
Top 20% -	300	0.0026	0.0026	0.0026	0.0027	0.0021
Bottom 20%		(0.0012)*	(0.0012)*	(0.0012)*	(0.0014)†	(0.0012)†
Top 30% -	300	0.0022	0.0022	0.0022	0.0022	0.0019
Bottom 30%		(0.0010)*	(0.0010)*	(0.0011)*	(0.0012)†	(0.0010)†
Top 40% -	300	0.0017	0.0017	0.0017	0.0017	0.0014
Bottom 40%		(0.0008)*	(0.0008)*	(0.0009)†	(0.0010)†	(0.0009)†

Table 5: Equal-Weighted Portfolio Returns Based on BAI

At the end of each quarter, fund managers' beliefs regarding excess stock returns are elicited according to Result 2. For each fund, these beliefs are then correlated with the following month's excess returns to form its belief accuracy index (BAI). Funds are next sorted into decile portfolios according to their BAIs. Table 5 reports the average monthly return, return over risk-free rate (excess return), and CAPM alpha, Fama-French alpha (1993), and Carhart alpha (1997) with corresponding Newey-West standard errors for the equal-weighted decile portfolios in the subsequent three-month testing period. The table also reports the performance spread between the top and bottom deciles. The sample period is from January 1981 to September 2006.

Decile	N	Average Return	Excess Return	CAPM Alpha	Fama-French Alpha	Carhart Alpha
1	305	0.0076	0.0031	-0.0035	-0.0032	-0.0030
		(0.0030)*	(0.0030)	(0.0012)**	(0.0012)**	(0.0012)*
2	305	0.0085	0.0039	-0.0023	-0.0022	-0.0023
		(0.0028)**	(0.0028)	(0.0009)**	(0.0009)*	(0.0009)**
3	305	0.0089	0.0043	-0.0017	-0.0015	-0.0018
		(0.0026)**	(0.0026)†	(0.0007)*	(0.0008)†	(0.0008)*
4	305	0.0086	0.0040	-0.0019	-0.0022	-0.0024
		(0.0026)**	(0.0026)	(0.0007)**	(0.0007)**	(0.0007)**
5	305	0.0088	0.0042	-0.0013	-0.0018	-0.0020
		(0.0023)**	(0.0023)†	(0.0006)*	(0.0006)**	(0.0006)**
6	305	0.0090	0.0044	-0.0011	-0.0014	-0.0014
		(0.0024)**	(0.0024)†	(0.0006)†	(0.0005)**	(0.0005)**
7	305	0.0096	0.0050	-0.0006	-0.0010	-0.0011
		(0.0024)**	(0.0024)*	(0.0007)	(0.0006)†	(0.0006)†
8	305	0.0107	0.0062	0.0007	0.0004	0.0005
		(0.0024)**	(0.0024)*	(0.0007)	(0.0006)	(0.0006)
9	305	0.0114	0.0068	0.0012	0.0012	0.0011
		(0.0026)**	(0.0026)**	(0.0009)	(0.0009)	(0.0009)
10	305	0.0110	0.0064	0.0005	0.0008	0.0013
		(0.0027)**	(0.0027)*	(0.0012)	(0.0012)	(0.0012)
Top 10% -	305	0.0034	0.0034	0.0040	0.0039	0.0043
Bottom 10%		(0.0019)†	(0.0019)†	(0.0020)*	(0.0021)†	(0.0019)*
Top 20% -	305	0.0031	0.0031	0.0037	0.0037	0.0039
Bottom 20%		(0.0016)*	(0.0016)*	(0.0016)*	(0.0018)*	(0.0016)*
Top 30% -	305	0.0027	0.0027	0.0033	0.0031	0.0034
Bottom 30%		(0.0013)*	(0.0013)*	(0.0014)*	(0.0015)*	(0.0014)*
Top 40% -	305	0.0023	0.0023	0.0028	0.0026	0.0028
Bottom 40%		(0.0012)†	(0.0012)†	(0.0012)*	(0.0014)†	(0.0013)*

Table 6: Value-Weighted Portfolio Returns Based on BAI.

At the end of each quarter, fund managers' beliefs regarding excess stock returns are elicited according to Result 2. For each fund, these beliefs are then correlated with the following month's excess returns to form its belief accuracy index (BAI). Table 6 reports the average monthly return, return over risk-free rate (excess return), CAPM alpha, Fama-French alpha (1993), and Carhart alpha (1997) with corresponding Newey-West standard errors for the value-weighted decile portfolios in the following three-month testing period. The table also reports the performance spread between the top and bottom deciles. The sample period is from January 1981 to September 2006.

Decile	N	Average Return	Excess Return	CAPM Alpha	Fama-French Alpha	Carhart Alpha
1	305	0.0073	0.0027	-0.0038	-0.0032	-0.0031
		(0.0029)*	(0.0030)	(0.0012)**	(0.0012)**	(0.0012)*
2	305	0.0078	0.0032	-0.0031	-0.0029	-0.0029
		(0.0028)**	(0.0028)	(0.0008)**	(0.0009)**	(0.0008)**
3	305	0.0086	0.0041	-0.0019	-0.0015	-0.0016
		(0.0026)**	(0.0026)	(0.0007)**	(0.0008)†	(0.0008)*
4	305	0.0081	0.0036	-0.0024	-0.0026	-0.0027
		(0.0026)**	(0.0026)	(0.0007)**	(0.0008)**	(0.0008)**
5	305	0.0092	0.0046	-0.0011	-0.0014	-0.0013
		(0.0024)**	(0.0024)†	(0.0006)†	(0.0007)*	(0.0007)†
6	305	0.0096	0.0050	-0.0005	-0.0004	-0.0006
		(0.0023)**	(0.0023)*	(0.0005)	(0.0005)	(0.0005)
7	305	0.0102	0.0056	-0.0000	-0.0001	-0.0002
		(0.0025)**	(0.0025)*	(0.0007)	(0.0007)	(0.0007)
8	305	0.0109	0.0063	0.0006	0.0007	0.0006
		(0.0026)**	(0.0026)*	(0.0007)	(0.0008)	(0.0008)
9	305	0.0108	0.0062	0.0005	0.0007	0.0003
		(0.0027)**	(0.0027)*	(0.0010)	(0.0011)	(0.0010)
10	305	0.0115	0.0070	0.0010	0.0013	0.0017
		(0.0027)**	(0.0027)*	(0.0012)	(0.0012)	(0.0012)
Top 10% -	305	0.0042	0.0042	0.0048	0.0045	0.0047
Bottom 10%		(0.0019)*	(0.0019)*	(0.0021)*	(0.0021)*	(0.0020)*
Top 20% -	305	0.0036	0.0036	0.0042	0.0041	0.0040
Bottom 20%		(0.0016)*	(0.0016)*	(0.0017)*	(0.0018)*	(0.0017)*
Top 30% -	305	0.0032	0.0032	0.0036	0.0035	0.0034
Bottom 30%		(0.0014)*	(0.0014)*	(0.0014)*	(0.0016)*	(0.0014)*
Top 40% -	305	0.0029	0.0029	0.0033	0.0032	0.0032
Bottom 40%		(0.0012)*	(0.0012)*	(0.0013)**	(0.0014)*	(0.0012)*

Table 7: Characteristics of Equal-Weighted Decile Portfolios Based on SBAI

At the end of each quarter, fund managers' beliefs regarding excess stock returns are elicited according to Result 1. For each fund, these beliefs are then correlated with the following month's excess returns to form its semi-belief accuracy index (SBAI). Funds are next sorted into decile portfolios according to their SBAIs. This table reports Carhart betas (1997) in the subsequent three-month period with corresponding Newey-West standard errors for the equal-weighted decile portfolios. It also reports the beta spread between the top and bottom deciles. The portfolio formation period is from January 1981 to January 2006.

Decile	N	Alpha	Market	SMB	HML	UMD
1	300	-0.0027	0.9895	0.2075	-0.0018	-0.0207
		(0.0008)**	(0.0326)**	(0.0769)**	(0.0586)	(0.0457)
2	300	-0.0018	0.9650	0.1828	0.0109	-0.0303
		(0.0006)**	(0.0180)**	(0.0396)**	(0.0442)	(0.0331)
3	300	-0.0016	0.9591	0.1410	0.0037	-0.0038
		(0.0006)**	(0.0171)**	(0.0347)**	(0.0428)	(0.0291)
4	300	-0.0011	0.9380	0.1532	0.0230	0.0066
		(0.0004)**	(0.0125)**	(0.0236)**	(0.0350)	(0.0214)
5	300	-0.0012	0.9440	0.1202	0.0156	-0.0087
		(0.0004)**	(0.0122)**	(0.0260)**	(0.0308)	(0.0199)
6	300	-0.0007	0.9420	0.1572	0.0287	-0.0041
		(0.0005)	(0.0148)**	(0.0211)**	(0.0294)	(0.0204)
7	300	-0.0006	0.9499	0.1430	0.0387	0.0047
		(0.0005)	(0.0120)**	(0.0200)**	(0.0286)	(0.0189)
8	300	-0.0009	0.9561	0.1332	0.0345	-0.0180
		(0.0005)†	(0.0119)**	(0.0235)**	(0.0333)	(0.0214)
9	300	-0.0001	0.9457	0.2103	0.0194	0.0211
		(0.0006)	(0.0136)**	(0.0267)**	(0.0381)	(0.0248)
10	300	0.0006	0.9431	0.3081	-0.0098	0.0526
		(0.0009)	(0.0237)**	(0.0487)**	(0.0646)	(0.0440)
Top 10% -	300	0.0033	-0.0465	0.1006	-0.0080	0.0733
Bottom 10%		(0.0014)*	(0.0493)	(0.1175)	(0.1096)	(0.0797)
Top 20% -	300	0.0025	-0.0329	0.0641	0.0002	0.0624
Bottom 20%		(0.0011)*	(0.0360)	(0.0838)	(0.0850)	(0.0601)
Top 30% -	300	0.0019	-0.0229	0.0401	0.0104	0.0369
Bottom 30%		(0.0009)*	(0.0282)	(0.0666)	(0.0706)	(0.0489)
Top 40% -	300	0.0016	-0.0142	0.0275	0.0117	0.0272
Bottom 40%		(0.0007)*	(0.0235)	(0.0536)	(0.0581)	(0.0403)

Table 8: Characteristics of Equal-Weighted Decile Portfolios Based on BAI

At the end of each quarter, fund managers' beliefs regarding excess stock returns are elicited according to Result 2. For each fund, these beliefs are then correlated with the following month's excess returns to form its belief accuracy index (BAI). Funds are next sorted into decile portfolios according to their BAIs. This table reports Carhart betas (1997) in the subsequent three-month period with corresponding Newey-West standard errors for the equal-weighted decile portfolios. It also reports the beta spread between the top and bottom deciles. The sample period is from January 1981 to September 2006.

Decile	N	Alpha	Market	SMB	HML	UMD
1	305	-0.0030	1.0435	0.2025	-0.0440	-0.0136
		(0.0012)*	(0.0434)**	(0.0738)**	(0.0750)	(0.0570)
2	305	-0.0023	1.0036	0.2035	-0.0111	0.0099
		(0.0009)**	(0.0237)**	(0.0504)**	(0.0477)	(0.0355)
3	305	-0.0018	0.9688	0.2616	-0.0124	0.0281
		(0.0008)*	(0.0218)**	(0.0279)**	(0.0429)	(0.0280)
4	305	-0.0024	0.9705	0.2326	0.0487	0.0230
		(0.0007)**	(0.0187)**	(0.0246)**	(0.0443)	(0.0275)
5	305	-0.0020	0.9199	0.2023	0.0785	0.0173
		(0.0006)**	(0.0146)**	(0.0205)**	(0.0392)*	(0.0206)
6	305	-0.0014	0.9189	0.1327	0.0436	-0.0037
		(0.0005)**	(0.0124)**	(0.0233)**	(0.0368)	(0.0205)
7	305	-0.0011	0.9432	0.1317	0.0641	0.0020
		(0.0006)†	(0.0199)**	(0.0328)**	(0.0433)	(0.0231)
8	305	0.0005	0.9123	0.1264	0.0438	-0.0124
		(0.0006)	(0.0145)**	(0.0337)**	(0.0421)	(0.0272)
9	305	0.0011	0.9216	0.1221	0.0015	0.0055
		(0.0009)	(0.0172)**	(0.0401)**	(0.0597)	(0.0377)
10	305	0.0013	0.9434	0.1316	-0.0421	-0.0526
		(0.0012)	(0.0243)**	(0.0460)**	(0.0763)	(0.0493)
Top 10% -	305	0.0043	-0.1001	-0.0710	0.0019	-0.0390
Bottom 10%		(0.0019)*	(0.0532)†	(0.0889)	(0.1346)	(0.0889)
Top 20% -	305	0.0039	-0.0910	-0.0762	0.0072	-0.0217
Bottom 20%		(0.0016)*	(0.0397)*	(0.0739)	(0.1105)	(0.0710)
Top 30% -	305	0.0034	-0.0795	-0.0959	0.0236	-0.0280
Bottom 30%		(0.0014)*	(0.0331)*	(0.0602)	(0.0915)	(0.0595)
Top 40% -	305	0.0028	-0.0665	-0.0971	0.0215	-0.0262
Bottom 40%		(0.0013)*	(0.0282)*	(0.0496)†	(0.0801)	(0.0498)

Table 9: Double Sorts Comparing SBAI with BAI

At the end of each quarter, fund managers' beliefs are extracted based on either Result 1 (to form semi-portfolio revealed beliefs) or Result 2 (to form portfolio revealed beliefs). These beliefs are correlated with the following month's excess returns to form SBAI and BAI respectively. In Panel A, funds are sorted into quintile portfolios according to the SBAI and then sorted within the quintiles according to the BAI. Panel B reverses the order. The portfolio formation period is from January 1981 to January 2006.

Panel A: Sorting funds first by SBAI and then by BAI

Quintile of SBAI	Quintile of BAI					
	1	2	3	4	5	5-1
	300	300	300	300	300	300
1	0.0075	0.0081	0.007	0.008	0.01	0.0025
	(0.0033)*	(0.0028)**	(0.0028)*	(0.0025)**	(0.0024)**	(0.0019)
2	0.0075	0.0088	0.0091	0.0093	0.0109	0.0034
	(0.0030)*	(0.0026)**	(0.0024)**	(0.0024)**	(0.0025)**	(0.0016)*
3	0.0076	0.0092	0.0088	0.0102	0.0111	0.0036
	(0.0027)**	(0.0026)**	(0.0024)**	(0.0024)**	(0.0025)**	(0.0016)**
4	0.0088	0.01	0.0091	0.0098	0.0108	0.002
	(0.0027)**	(0.0025)**	(0.0024)**	(0.0025)**	(0.0026)**	(0.0015)**
5	0.0101	0.0099	0.0116	0.0116	0.0119	0.0018
	(0.0026)**	(0.0026)**	(0.0028)**	(0.0027)**	(0.0029)**	(0.0015)

Panel B: Sorting funds first by BAI and then by SBAI

Quintile of BAI	Quintile of SBAI					
	1	2	3	4	5	5-1
	300	300	300	300	300	300
1	0.0078	0.0078	0.0082	0.0074	0.0092	0.0015
	(0.0031)*	(0.0029)**	(0.0029)**	(0.0028)**	(0.0028)**	(0.0011)
2	0.0083	0.0084	0.0091	0.0098	0.0092	0.0009
	(0.0028)**	(0.0026)**	(0.0026)**	(0.0026)**	(0.0027)**	(0.0011)
3	0.0085	0.0086	0.0089	0.0089	0.01	0.0015
	(0.0024)**	(0.0023)**	(0.0024)**	(0.0024)**	(0.0026)**	(0.0012)
4	0.0092	0.0099	0.0097	0.0109	0.0115	0.0022
	(0.0024)**	(0.0024)**	(0.0024)**	(0.0025)**	(0.0026)**	(0.0011)*
5	0.0107	0.0111	0.011	0.0114	0.0114	0.0007
	(0.0025)**	(0.0025)**	(0.0026)**	(0.0027)**	(0.0028)**	(0.0011)

Table 10: Robustness Check of Equal-Weighted Portfolio Returns Based on BAI

At the end of each quarter, fund managers' beliefs regarding excess stock returns are elicited according to Result 2 where $k = 20$. For each fund, these beliefs are then correlated with the following month's excess returns to form its belief accuracy index (BAI). Funds are sorted into decile portfolios according to their BAIs. Table 10 reports the average month return, return over risk-free rate (excess return), CAPM alpha, Fama-French alpha (1993), and Carhart alpha (1997) with corresponding Newey-West standard errors for the equal-weighted decile portfolios in the subsequent three-month testing period. The table also reports the performance spread between the top and bottom deciles. The sample period is from January 1981 to September 2006.

Decile	N	Average Return	Excess Return	CAPM Alpha	Fama-French Alpha	Carhart Alpha
1	305	0.0068	0.0023	-0.0043	-0.0037	-0.0036
		(0.0029)*	(0.0029)	(0.0013)**	(0.0014)**	(0.0014)**
2	305	0.0081	0.0035	-0.0024	-0.0023	-0.0025
		(0.0026)**	(0.0026)	(0.0008)**	(0.0008)**	(0.0008)**
3	305	0.0082	0.0037	-0.0022	-0.0024	-0.0023
		(0.0025)**	(0.0025)	(0.0007)**	(0.0007)**	(0.0006)**
4	305	0.0091	0.0046	-0.0012	-0.0012	-0.0012
		(0.0024)**	(0.0025)†	(0.0006)†	(0.0006)*	(0.0006)*
5	305	0.0092	0.0047	-0.0010	-0.0011	-0.0012
		(0.0024)**	(0.0024)†	(0.0005)†	(0.0005)*	(0.0005)*
6	305	0.0095	0.0050	-0.0007	-0.0011	-0.0014
		(0.0024)**	(0.0024)*	(0.0006)	(0.0004)*	(0.0005)**
7	305	0.0102	0.0057	-0.0000	-0.0004	-0.0003
		(0.0025)**	(0.0025)*	(0.0006)	(0.0005)	(0.0005)
8	305	0.0103	0.0058	0.0001	-0.0001	-0.0002
		(0.0025)**	(0.0025)*	(0.0007)	(0.0006)	(0.0006)
9	305	0.0111	0.0066	0.0008	0.0005	0.0005
		(0.0027)**	(0.0027)*	(0.0010)	(0.0009)	(0.0009)
10	305	0.0113	0.0068	0.0008	0.0011	0.0014
		(0.0028)**	(0.0028)*	(0.0012)	(0.0012)	(0.0012)
Top 10% -	305	0.0045	0.0045	0.0050	0.0048	0.0050
Bottom 10%		(0.0020)*	(0.0020)*	(0.0021)*	(0.0023)*	(0.0021)*
Top 20% -	305	0.0037	0.0037	0.0042	0.0038	0.0040
Bottom 20%		(0.0016)*	(0.0016)*	(0.0017)*	(0.0019)*	(0.0017)*
Top 30% -	305	0.0032	0.0032	0.0036	0.0033	0.0034
Bottom 30%		(0.0013)*	(0.0013)*	(0.0014)*	(0.0016)*	(0.0014)*
Top 40% -	305	0.0027	0.0027	0.0029	0.0027	0.0027
Bottom 40%		(0.0011)*	(0.0011)*	(0.0011)**	(0.0013)*	(0.0012)*

Table 11: Robustness Check of Value-Weighted Portfolio Returns Based on BAI. At the end of each quarter, fund managers' beliefs on excess stock returns are extracted based on the algorithm in Section 2.4 where $k = 20$. For each fund, these beliefs are then correlated with the following month's excess returns to form its BAI. Funds are sorted into decile portfolios according to their BAIs. Table 11 reports the average monthly return, return over risk-free rate (excess return), CAPM alpha, Fama-French alpha (1993), and Carhart alpha (1997) with corresponding Newey-West standard errors for value-weighted decile portfolios in the following three-month testing period. The table also reports the performance spread between the top and bottom deciles. The sample period is from January 1981 to September 2006.

Decile	N	Average Return	Excess Return	CAPM Alpha	Fama-French Alpha	Carhart Alpha
1	305	0.0065	0.0020	-0.0045	-0.0038	-0.0038
		(0.0029)*	(0.0029)	(0.0013)**	(0.0014)**	(0.0013)**
2	305	0.0074	0.0029	-0.0033	-0.0030	-0.0029
		(0.0026)**	(0.0027)	(0.0008)**	(0.0009)**	(0.0008)**
3	305	0.0081	0.0035	-0.0024	-0.0025	-0.0023
		(0.0025)**	(0.0026)	(0.0007)**	(0.0008)**	(0.0008)**
4	305	0.0092	0.0047	-0.0011	-0.0006	-0.0007
		(0.0024)**	(0.0024)†	(0.0006)†	(0.0007)	(0.0008)
5	305	0.0098	0.0053	-0.0003	-0.0002	-0.0005
		(0.0025)**	(0.0025)*	(0.0006)	(0.0006)	(0.0006)
6	305	0.0095	0.0049	-0.0007	-0.0009	-0.0010
		(0.0024)**	(0.0024)*	(0.0005)	(0.0005)†	(0.0006)†
7	305	0.0105	0.0060	0.0002	0.0001	0.0001
		(0.0025)**	(0.0025)*	(0.0006)	(0.0006)	(0.0007)
8	305	0.0105	0.0059	0.0001	0.0001	-0.0002
		(0.0026)**	(0.0026)*	(0.0008)	(0.0007)	(0.0007)
9	305	0.0109	0.0064	0.0006	0.0002	0.0002
		(0.0026)**	(0.0026)*	(0.0009)	(0.0009)	(0.0008)
10	305	0.0109	0.0064	0.0002	0.0005	0.0007
		(0.0028)**	(0.0029)*	(0.0013)	(0.0013)	(0.0012)
Top 10% -	305	0.0043	0.0043	0.0047	0.0043	0.0045
Bottom 10%		(0.0020)*	(0.0020)*	(0.0021)*	(0.0023)†	(0.0021)*
Top 20% -	305	0.0040	0.0040	0.0043	0.0038	0.0038
Bottom 20%		(0.0016)*	(0.0016)*	(0.0018)*	(0.0019)*	(0.0017)*
Top 30% -	305	0.0034	0.0034	0.0037	0.0034	0.0032
Bottom 30%		(0.0014)*	(0.0014)*	(0.0015)*	(0.0016)*	(0.0014)*
Top 40% -	305	0.0029	0.0029	0.0031	0.0027	0.0026
Bottom 40%		(0.0012)*	(0.0012)*	(0.0012)*	(0.0013)*	(0.0012)*

Table 12: Double Sorts Comparing SBAI and BAI with GT (Grinblatt and Titman (1989))
At the end of each quarter, fund managers' beliefs are extracted based on either Result 1 (to form semi-portfolio revealed beliefs) or Result 2 (to form portfolio revealed beliefs). These beliefs are correlated with the following month's excess returns to form semi-portfolio revealed belief accuracy index (SBAI) and portfolio revealed belief accuracy index (BAI) respectively. Similarly, at the end of each quarter, change of portfolio weights of stocks in a given fund's portfolio positions are observed and correlated with the following month's excess returns to form a fund's GT measure (as in Grinblatt and Titman (1989)). In Panel A, funds are sorted into quintile portfolios according to the GT measure and then sorted within the quintiles according to SBAI. In Panel B, funds are sorted into quintile portfolios according to the GT measure and then sorted within the quintiles according to BAI. The portfolio formation period (sample period) is from January 1981 to January 2006 (September 2006) for SBAI (BAI).

Panel A: Sorting funds first by GT and then by SBAI						
Quintile of GT	Quintile of SBAI					
	1	2	3	4	5	5-1
1	0.0074	0.0080	0.0092	0.0086	0.0094	0.0020
	(0.0029) *	(0.0028) **	(0.0027) **	(0.0027) **	(0.0027) **	(0.0012)
2	0.0093	0.0093	0.0092	0.0093	0.0100	0.0005
	(0.0026) **	(0.0026) **	(0.0024) **	(0.0025) **	(0.0026) **	(0.0010)
3	0.0093	0.0095	0.0102	0.0101	0.0113	0.0022
	(0.0026) **	(0.0025) **	(0.0024) **	(0.0024) **	(0.0025) **	(0.0011) *
4	0.0087	0.0101	0.0097	0.0106	0.0121	0.0034
	(0.0026) **	(0.0025) **	(0.0024) **	(0.0024) **	(0.0027) **	(0.0013) **
5	0.0092	0.0097	0.0106	0.0107	0.0124	0.0032
	(0.0028) **	(0.0026) **	(0.0026) **	(0.0026) **	(0.0029) **	(0.0014) *
Panel B: Sorting funds first by GT and then by BAI						
Quintile of GT	Quintile of BAI					
1	0.0078	0.0090	0.0084	0.0080	0.0098	0.0020
	(0.0030) **	(0.0029) **	(0.0027) **	(0.0026) **	(0.0027) **	(0.0017)
2	0.0094	0.0086	0.0097	0.0096	0.0102	0.0008
	(0.0027) **	(0.0027) **	(0.0024) **	(0.0025) **	(0.0025) **	(0.0014)
3	0.0081	0.0093	0.0107	0.0101	0.0110	0.0032
	(0.0027) **	(0.0025) **	(0.0024) **	(0.0024) **	(0.0025) **	(0.0015) *
4	0.0082	0.0094	0.0092	0.0102	0.0110	0.0031
	(0.0027) **	(0.0024) **	(0.0024) **	(0.0025) **	(0.0025) **	(0.0015) *
5	0.0082	0.0100	0.0100	0.0107	0.0126	0.0044
	(0.0029) **	(0.0027) **	(0.0026) **	(0.0027) **	(0.0028) **	(0.0017) **

Table 13: Double Sorts Comparing SBAI and BAI with FundRank (SSYZ (2007))

At the end of each quarter, fund managers' beliefs are extracted based on either Result 1 (to form semi-portfolio revealed beliefs) or Result 2 (to form portfolio revealed beliefs). These beliefs are correlated with the following month's excess returns to form semi-portfolio revealed belief accuracy index (SBAI) and portfolio revealed belief accuracy index (BAI) respectively. Similarly, at the end of each quarter, the FundRank measure of Shi, Stoffman, Yuan, and Zhi (2007) is formed using the quarter-end holding data. In Panel A, funds are sorted into quintile portfolios according to the FundRank measure and then sorted within the quintiles according to the SBAI. In Panel B, funds are sorted into quintile portfolios according to the FundRank measure and then sorted within the quintiles according to the BAI. The portfolio formation (sample) period is from January 1981 to January 2006 (September 2006) for SBAI (BAI).

Panel A: Sorting funds first by FundRank and then by SBAI						
Quintile of FundRank	Quintile of SBAI					
	1	2	3	4	5	5-1
1	0.0064	0.0085	0.0095	0.0085	0.0100	0.0036
	(0.0027) *	(0.0025) **	(0.0024) **	(0.0024) **	(0.0026) **	(0.0013) **
2	0.0081	0.0091	0.0092	0.0095	0.0101	0.0020
	(0.0027) **	(0.0025) **	(0.0024) **	(0.0025) **	(0.0026) **	(0.0013)
3	0.0094	0.0088	0.0094	0.0100	0.0103	0.0009
	(0.0027) **	(0.0026) **	(0.0025) **	(0.0025) **	(0.0026) **	(0.0012)
4	0.0081	0.0095	0.0092	0.0097	0.0106	0.0025
	(0.0027) **	(0.0026) **	(0.0025) **	(0.0026) **	(0.0027) **	(0.0012) *
5	0.0089	0.0089	0.0093	0.0108	0.0120	0.0031
	(0.0029) **	(0.0027) **	(0.0027) **	(0.0027) **	(0.0029) **	(0.0016) †
Panel B: Sorting funds first by FundRank and then by BAI						
Quintile of FundRank	Quintile of BAI					
1	0.0063	0.0084	0.0080	0.0084	0.0104	0.0041
	(0.0029) *	(0.0026) **	(0.0025) **	(0.0023) **	(0.0025) **	(0.0017) *
2	0.0078	0.0086	0.0090	0.0096	0.0097	0.0019
	(0.0029) **	(0.0026) **	(0.0024) **	(0.0024) **	(0.0026) **	(0.0017)
3	0.0084	0.0089	0.0090	0.0098	0.0109	0.0025
	(0.0028) **	(0.0026) **	(0.0024) **	(0.0024) **	(0.0026) **	(0.0016)
4	0.0076	0.0087	0.0085	0.0096	0.0114	0.0038
	(0.0029) **	(0.0027) **	(0.0024) **	(0.0024) **	(0.0026) **	(0.0016) *
5	0.0088	0.0089	0.0084	0.0107	0.0116	0.0028
	(0.0030) **	(0.0028) **	(0.0027) **	(0.0026) **	(0.0027) **	(0.0019)

Table 14: Double Sorts Comparing SBAI and BAI with CS (DGTW (1997))

At the end of each quarter, fund managers' beliefs are extracted based on either Result 1 (to form semi-portfolio revealed beliefs) or Result 2 (to form portfolio revealed beliefs). These beliefs are correlated with the following month's excess returns to form semi-portfolio revealed belief accuracy index (SBAI) and portfolio revealed belief accuracy index (BAI) respectively. Similarly, at the end of each quarter, the Characteristic Selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997) is formed using the quarter-end holding data and the month after return data. In Panel A, funds are sorted into quintile portfolios according to the CS measure and then sorted within the quintiles according to the SBAI. In Panel B, funds are sorted into quintile portfolios according to the CS measure and then sorted within the quintiles according to the BAI. The portfolio formation period (sample period) is from January 1981 to January 2006 (September 2006) for BAI (SBAI).

Panel A: Sorting funds first by CS and then by SBAI						
Quintile of CS	Quintile of SBAI					
	1	2	3	4	5	5-1
1	0.0091	0.0104	0.0121	0.0120	0.0135	0.0045
	(0.0032) **	(0.0032) **	(0.0030) **	(0.0032) **	(0.0032) **	(0.0015) **
2	0.0115	0.0107	0.0109	0.0121	0.0130	0.0016
	(0.0027) **	(0.0027) **	(0.0027) **	(0.0028) **	(0.0029) **	(0.0012)
3	0.0106	0.0117	0.0113	0.0110	0.0134	0.0028
	(0.0028) **	(0.0029) **	(0.0027) **	(0.0027) **	(0.0028) **	(0.0014) *
4	0.0106	0.0119	0.0120	0.0118	0.0134	0.0028
	(0.0029) **	(0.0029) **	(0.0027) **	(0.0028) **	(0.0030) **	(0.0015) †
5	0.0120	0.0116	0.0111	0.0132	0.0146	0.0026
	(0.0033) **	(0.0031) **	(0.0031) **	(0.0032) **	(0.0035) **	(0.0018)
Panel B: Sorting funds first by CS and then by BAI						
Quintile of CS	Quintile of BAI					
1	0.0006	0.0040	0.0063	0.0073	0.0131	0.0125
	(0.0033)	(0.0033)	(0.0033) †	(0.0032) *	(0.0034) **	(0.0022) **
2	0.0070	0.0090	0.0091	0.0116	0.0157	0.0087
	(0.0029) *	(0.0030) **	(0.0027) **	(0.0027) **	(0.0028) **	(0.0016) **
3	0.0091	0.0107	0.0128	0.0139	0.0184	0.0094
	(0.0031) **	(0.0027) **	(0.0026) **	(0.0027) **	(0.0028) **	(0.0018) **
4	0.0120	0.0134	0.0143	0.0172	0.0192	0.0071
	(0.0030) **	(0.0028) **	(0.0028) **	(0.0028) **	(0.0029) **	(0.0020) **
5	0.0160	0.0189	0.0203	0.0227	0.0254	0.0094
	(0.0034) **	(0.0034) **	(0.0033) **	(0.0032) **	(0.0032) **	(0.0023) **

Appendix A
An Alternative Model: n Non-Redundant Assets

In this section I present a portfolio choice model where all n risky assets are non-redundant. This is the only difference with the model presented in Section 2 of the paper. This simple departure turns out to simplify the mathematics a lot. One may argue that this model may not be as empirically relevant as the model presented in Section 2 since many risky assets in reality are redundant. However, in either case, the main results of the paper remain the same.

A.1 Assets

To develop the model, I first focus on a standard portfolio allocation problem. In this problem, the available investment opportunities consist of a riskless asset with a constant return, r_f, and n non-redundant risky assets where ith asset's *excess return* over the risk-free rate (r_f) is denoted as $\tilde{r}_i$. The n risky assets have the following variance-covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma^2_{1,1} & \dots & \sigma^2_{1,n} \\ \vdots & \ddots & \vdots \\ \sigma^2_{n,1} & \dots & \sigma^2_{n,n} \end{bmatrix}, \tag{A1}$$

which is assumed to be full rank. I assume that the spectral decomposition of the Σ matrix has the following form:

$$\Sigma = \begin{bmatrix} \mathbf{b}_1 & \cdots & \mathbf{b}_n \end{bmatrix} \begin{bmatrix} \sigma^2_{f_1} & & \\ & \ddots & \\ & & \sigma^2_{f_n} \end{bmatrix} \begin{bmatrix} \mathbf{b}'_1 \\ \vdots \\ \mathbf{b}'_n \end{bmatrix}, \tag{A2}$$

where Σ has n eigenvectors. Let Σ_f denote the following:

$$\begin{bmatrix} \sigma^2_{f_1} & & \\ & \ddots & \\ & & \sigma^2_{f_n} \end{bmatrix}.$$

Then, the spectral structure of the Σ matrix can be re-written as:

$$\Sigma = \mathbf{b}\Sigma_f\mathbf{b}'. \tag{A3}$$

A.2 Beliefs

I assume that there are m investors in this economy. These investors possess a common knowledge of Σ, but are heterogeneously informed about the risky assets' excess returns. As mentioned in the introduction, this assumption is motivated by the fact that the second moments can be better estimated than the first moments by using high-frequency historical return data, as shown in the empirical asset pricing literature.

I use μ_{mi} to denote investor m's belief of asset i's expected excess return. Investor m's belief of the n assets' expected excess returns can be written as $\mu_m = [\mu_{m1}, ..., \mu_{mn}]'$, and total m investors' beliefs on n assets can be written as $\mu = [\mu_1, ..., \mu_m]'$, which is $m \times n$ matrix.

To explore the spectral decomposition of the Σ matrix, I decompose μ on the same basis by projecting μ to the subspace V spanned by $\mathbf{b}$, namely,

$$\mu = \hat{\mu}\mathbf{b}', \tag{A4}$$

where $\hat{\mu}$ is a $m \times n$ matrix and $\hat{\mu} = [\hat{\mu}_1, ..., \hat{\mu}_m]'$. That is, $\hat{\mu}_m$, a $n \times 1$ vector, describes investor m's belief on n eigenfactors.

With this characterization, I make one assumption regarding the belief structure. Specifically, I assume that the column vectors of $\hat{\mu}$ are orthonormal. The orthogonality assumption is basically a rationality assumption since the columns of $\hat{\mu}$ reflect beliefs about different orthogonal eigenfactors. This assumption simplifies the derivation for investors' beliefs, as shown later.

A.3 Investor Portfolio Optimization Problem

Let w_{m0} denote the percentage of wealth (or portfolio weight) invested by investor m in the riskless asset and $\mathbf{w}_m = [w_{m1}, \cdots, w_{mn}]'$ denote the vector of portfolio weights in each of the n risky assets by investor m. The portfolio weights satisfy the following equation:

$$w_{m0} + \sum_{i=1}^{n} w_{mi} = 1, \tag{A5}$$

where $\mathbf{w} = [\mathbf{w}_1, ..., \mathbf{w}_m]'$ and is a $m \times n$ matrix.

In this economy, investors choose portfolio weights to obtain a standard mean-variance optimization for expected returns. Investor m, conditional on his beliefs, chooses his portfolio weights, $\mathbf{w}_m$, to maximize the following:

$$\max_{\{\mathbf{w}_m\}} \left(\mathbf{w}_m^{'} \mu_m - \frac{1}{2} \gamma \mathbf{w}_m^{'} \Sigma \mathbf{w}_m \right), \tag{A6}$$

where γ is assumed to be the same across investors.

The first-order condition for investor m yields a mean-variance efficient portfolio:

$$\mathbf{w}_m = \frac{1}{\gamma} \Sigma^{-1} \mu_m. \tag{A7}$$

Finally, the matrix of optimal portfolio weights by all m investors in this economy can be written as:

$$\mathbf{w} = \frac{1}{\gamma} \mu \Sigma^{-1}. \tag{A8}$$

A.4 Heterogeneous Beliefs Revealed in Portfolio Holdings

For a given Σ, investor private beliefs can be immediately revealed by $\mathbf{w}$. This result is stated in the following lemma, which is immediate from Equation (A8).

Lemma 2 *Once observing a portfolio holding matrix* $\mathbf{w}$*, investor private beliefs on expected return of risky assets for a given* Σ *can be computed as*

$$\mu = \gamma \mathbf{w} \Sigma. \tag{A9}$$

I will demonstrate next that the common belief on Σ among investors, which is forward-looking, can also be revealed in $\mathbf{w}$. To do so, let us first denote the spectral decomposition of $\mathbf{w}^{'}\mathbf{w}$ as:

$$\mathbf{w}^{'}\mathbf{w} = \mathbf{u} e \mathbf{u}' = \begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_k & \cdot & \mathbf{u}_n \end{bmatrix} \begin{bmatrix} e_1 & & \\ & \ddots & \\ & & e_k \end{bmatrix} \begin{bmatrix} \mathbf{u}_1' \\ \vdots \\ \mathbf{u}_k' \\ \cdot \\ \mathbf{u}_n' \end{bmatrix}. \tag{A10}$$

The next proposition shows how $\mathbf{w}$ reveals Σ up to a constant.

Proposition A1 $\Sigma = a \mathbf{u} e^{-\frac{1}{2}} \mathbf{u}'$ *where* a *is a constant.*

PROOF OF PROPOSITION A1:

Since the column vectors of $\hat{\mu}$ are orthogonal. The following is immediate from Equation (A8):

$$\mathbf{w}^{'}\mathbf{w} \;=\; \frac{1}{\gamma^2} \Sigma^{-1} \mathbf{b} \hat{\mu}' \hat{\mu} \mathbf{b}' \Sigma^{-1} = \frac{1}{\gamma^2} \Sigma^{-1} \mathbf{b} \mathbf{b}' \Sigma^{-1}. \tag{A11}$$

To obtain $\mathbf{b}$, note that since $\Sigma = \mathbf{b}\Sigma_f\mathbf{b}'$, Equation (A11) can be written as:

$$\mathbf{w}'\mathbf{w} = \frac{1}{\gamma^2}(\mathbf{b}\Sigma_f\mathbf{b}')^{-1}\mathbf{b}\mathbf{b}'(\mathbf{b}\Sigma_f\mathbf{b})^{-1} = \frac{1}{\gamma^2}\mathbf{b}\Sigma_f^{-2}\mathbf{b}' \tag{A12}$$

where $1/\gamma^2$ is a constant. Therefore, $\mathbf{b} = a_1 \begin{bmatrix}\mathbf{u}_1 & \cdots & \mathbf{u}_k & \cdot & \mathbf{u}_n\end{bmatrix}$ and $\Sigma_f = a_2 e^{-\frac{1}{2}}$ where a_1 and a_2 are two constants. The rest follows since $\Sigma = \mathbf{b}\Sigma_f\mathbf{b}'$. ■

Since $\mathbf{b}$ and Σ are revealed by portfolio matrix, I term them "*portfolio (revealed) betas*" and "*portfolio (revealed) variance-covariance matrix*" respectively. After obtaining $\mathbf{b}$ and Σ, the investor beliefs, μ, can be computed by Equations (A8). I term these beliefs "*portfolio (revealed) beliefs*" because these beliefs are revealed by corresponding portfolio holdings.

Appendix B

An Alternative Model: n Assets, k Eigen-factors and Idiosyncratic Risks

In this section I present a portfolio choice model where investors do not demand any returns when exposed to certain types of risks. This is the only difference with the model presented in Section 2 of the paper. Specifically, here, I decompose the variance-covariance matrix into two components. Investors form beliefs of expected returns when exposed to the first component but demand zero expected returns when exposed to the second component. Intuitively, this decomposition can be thought of as decomposing the risk into a systematic component and an idiosyncratic component. This simple departure turns out to be quite mathematically involved as demonstrated later. The appeal of this model versus the one in Section 2 of the paper is due to consideration that empirically investors may not demand returns for exposing to certain types of risks. However, the main results of the paper do not change.

B.1 Assets

In this economy, the available investment opportunities consist of a risk-free asset with a constant return, r_f, and n risky assets where ith asset's *excess return* over the risk-free rate (r_f) is denoted as $\tilde{r}_i$. The n risky assets have the following variance-covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma^2_{1,1} & \dots & \sigma^2_{1,n} \\ \vdots & \ddots & \vdots \\ \sigma^2_{n,1} & \dots & \sigma^2_{n,n} \end{bmatrix}, \tag{B1}$$

which is assumed to be full rank. I assume that the spectral decomposition of the Σ matrix has the following form:

$$\Sigma = \underbrace{\begin{bmatrix} \mathbf{b}_1 & \cdots & \mathbf{b}_k \end{bmatrix} \begin{bmatrix} \sigma^2_{f_1} & & \\ & \ddots & \\ & & \sigma^2_{f_k} \end{bmatrix} \begin{bmatrix} \mathbf{b}'_1 \\ \vdots \\ \mathbf{b}'_k \end{bmatrix}}_{\hat{\Sigma}} + \Sigma^{\perp}, \tag{B2}$$

where $\hat{\Sigma}$ has k eigenvectors and $\Sigma^{\perp}$ has $n-k$ eigenvectors. Let Σ_f denote the following:

$$\begin{bmatrix} \sigma^2_{f_1} & & \\ & \ddots & \\ & & \sigma^2_{f_k} \end{bmatrix}.$$

Then, Equation (B2) can be written as:

$$\Sigma = \mathbf{b}\Sigma_f\mathbf{b}' + \Sigma^{\perp}. \tag{B3}$$

Corresponding to the above spectral structure, we have the orthogonal decomposition $\mathbb{R}^n = \hat{V} \oplus V^{\perp}$, where $\hat{V}$, denoting a subspace in $\mathbb{R}^n$, is the span of $\{\mathbf{b}\}_{l=1,...,k}$, and $V^{\perp}$ is its orthogonal complement.[10]

Intuitively, this decomposition can be thought of as decomposing the risk into a systematic component (characterized by k factors) and an idiosyncratic component; or alternatively, as we demonstrate later, a part of the risk that investors can form beliefs of expected return on, and a part of risk of which investors do not demand expected return when exposing to.

B.2 Beliefs

I assume that there are m investors in this economy. These investors possess a common knowledge of Σ, but are heterogeneously informed about the risky assets' excess returns. As mentioned in the introduction, this assumption is motivated by the fact that the second moments can be better estimated than the first moments by using high-frequency historical return data, as shown in the empirical asset pricing literature.

I use μ_{mi} to denote investor m's information or belief of asset i's expected excess return. Investor m's belief of the n assets' expected excess returns can be written as $\mu_m = [\mu_{m1}, ..., \mu_{mn}]'$, and total m investors' beliefs on n assets can be written as $\mu = [\mu_1, ..., \mu_m]'$, which is $m \times n$ matrix.

To explore the spectral decomposition of the Σ matrix, I decompose μ on the same basis by projecting μ to the subspaces V and $V^{\perp}$, namely,

$$\mu = \hat{\mu}\mathbf{b}' + \mu^{\perp}, \tag{B4}$$

where $\hat{\mu}$ is a $m \times k$ matrix and $\hat{\mu} = [\hat{\mu}_1, ..., \hat{\mu}_m]'$. That is, $\hat{\mu}_m$, a $k \times 1$ vector, describes investor m's belief on k factors.

With this characterization, I make two important assumptions regarding the belief structure. First, I assume that the column vectors of $\hat{\mu}$ are orthonormal. This assumption is

[10] Aït-Sahalia, Cacho-Diaz, and Hurd (2007) use this decomposition to solve the consumption-portfolio selection problem of an investor facing both Brownian and jump risks. Technically, this decomposition is useful to deal with idiosyncratic risks (for example, jump risks or asset-specific risks when the number of assets (n) is greater than the number of factors (k)). When $n = k$, the decomposition is not necessary and the derivation of the results in the paper is less involved.

motivated by the fact that $\hat{\mu}$ reflect beliefs on different eigenfactors. This assumption is important for extracting the variance-covariance matrix, as shown later. Second, I assume $\mu^{\perp} = \mathbf{0}$, that is, investors receive information about expected return on the asset's systematic risk component and no such information except the prior regarding the asset's idiosyncratic risk component.[11] Again, this assumption, as shown later, simplifies derivation on individual's heterogeneous beliefs.

B.3 Investor Portfolio Optimization Problem

Let w_{m0} denote the percentage of wealth (or portfolio weight) invested by investor m in the riskless asset and $\mathbf{w}_m = [w_{m1}, \cdots, w_{mn}]'$ denote the vector of portfolio weights in each of the n risky assets by investor m. The portfolio weights satisfy the following equation:

$$w_{m0} + \sum_{i=1}^{n} w_{mi} = 1, \tag{B5}$$

where $\mathbf{w} = [\mathbf{w}_1, ..., \mathbf{w}_m]'$ and is a $m \times n$ matrix.

In this economy, investors choose portfolio weights to obtain a standard mean-variance optimization for expected returns. Investor m, conditional on his beliefs, chooses his portfolio weights, $\mathbf{w}_m$, to maximize the following:

$$\max_{\{\mathbf{w}_m\}} \left(\mathbf{w}_m^{'} \mu_m - \frac{1}{2} \gamma \mathbf{w}_m^{'} \Sigma \mathbf{w}_m \right), \tag{B6}$$

where γ is assumed to be the same across investors.

To explore the spectral decomposition of the Σ matrix and the belief structure specified in Equation (B4), I look for the optimal portfolio weights on the same basis (i.e., projecting the portfolio weights, $\mathbf{w}_m$, to the subspaces V and $V^{\perp}$) which is of the following form:

$$\mathbf{w}_m = \hat{\mathbf{w}}_m + \mathbf{w}_m^{\perp}. \tag{B7}$$

The optimization problem separates into:

$$(\hat{\mathbf{w}}_m, \mathbf{w}_m^{\perp}) = \arg \max_{\{\hat{\mathbf{w}}_m, \mathbf{w}_m^{\perp}\}} \left(\hat{\mathbf{w}}_m' \mathbf{b} \hat{\mu}_m - \frac{1}{2} \gamma \hat{\mathbf{w}}_m^{'} \hat{\Sigma} \hat{\mathbf{w}}_m \right) + \left(\mathbf{w}_m^{\perp} \mu_m^{\perp} - \frac{1}{2} \gamma \mathbf{w}_m^{\perp\prime} \Sigma^{\perp} \mathbf{w}_m^{\perp} \right). \tag{B8}$$

The first-order condition for investor m yields a mean-variance efficient portfolio:

$$\mathbf{w}_m^{\perp} = 0, \text{ and} \tag{B9}$$

$$\mathbf{w}_m = \hat{\mathbf{w}}_m = \frac{1}{\gamma} \hat{\Sigma}^{\dagger\prime} \mathbf{b} \hat{\mu}_m, \tag{B10}$$

[11] This assumption is reasonable considering that idiosyncratic risks can be diversified away.

where † denotes Moore-Penrose generalized inverse. Finally, the matrix of optimal portfolio weights by all m investors in this economy can be written as:

$$\mathbf{w} = \frac{1}{\gamma}\hat{\mu}\mathbf{b}'\hat{\Sigma}^{\dagger}. \tag{B11}$$

B.4 Heterogeneous Beliefs Revealed in Portfolio Holdings

For a given $\hat{\Sigma}$, investor private beliefs can be immediately revealed by $\mathbf{w}$. This result is stated in the following lemma, which is immediate from Equation (B11).

Lemma 3 *Once observing a portfolio holding matrix* $\mathbf{w}$*, investor private beliefs on expected return of risky assets for a given* $\hat{\Sigma}$ *can be computed as*

$$\mu = \hat{\mu}\mathbf{b}' = \gamma\mathbf{w}\hat{\Sigma}. \tag{B12}$$

I will demonstrate next that the common belief on Σ among investors, which is forward-looking, can also be revealed in $\mathbf{w}$. To do so, let us first denote the spectral decomposition of $\mathbf{w}'\mathbf{w}$ as:

$$\mathbf{w}'\mathbf{w} = \mathbf{u}e\mathbf{u}' = \begin{bmatrix}\mathbf{u}_1 & \cdots & \mathbf{u}_k\end{bmatrix}\begin{bmatrix} e_1 & & \\ & \ddots & \\ & & e_k \end{bmatrix}\begin{bmatrix}\mathbf{u}_1' \\ \vdots \\ \mathbf{u}_k'\end{bmatrix}. \tag{B13}$$

The next proposition shows how $\mathbf{w}$ reveals $\hat{\Sigma}$ up to a constant.

Proposition B1 $\hat{\Sigma} = a\mathbf{u}e^{-\frac{1}{2}}\mathbf{u}'$ *where a is a constant.*

PROOF OF PROPOSITION B1:

Since the column vectors of $\hat{\mu}$ are orthogonal, the following is immediate from Equation (B11):

$$\mathbf{w}'\mathbf{w} \;=\; \frac{1}{\gamma^2}\hat{\Sigma}^{\dagger\prime}\mathbf{b}\hat{\mu}'\hat{\mu}\mathbf{b}'\hat{\Sigma}^{\dagger} = \frac{1}{\gamma^2}\hat{\Sigma}^{\dagger\prime}\mathbf{b}\mathbf{b}'\hat{\Sigma}^{\dagger}. \tag{B14}$$

To obtain $\mathbf{b}$, note that since $\hat{\Sigma} = \mathbf{b}\Sigma_f\mathbf{b}'$, Equation (B14) can be written as:

$$\begin{aligned}\mathbf{w}'\mathbf{w} &= \frac{1}{\gamma^2}(\mathbf{b}(\Sigma_f\mathbf{b}'))^{\dagger\prime}\mathbf{b}\mathbf{b}'((\mathbf{b}\Sigma_f)\mathbf{b})^{\dagger} = \frac{1}{\gamma^2}(\Sigma_f\mathbf{b}')^{\dagger}\mathbf{b}^{\dagger\prime}\mathbf{b}\mathbf{b}'\mathbf{b}'^{\dagger}(\mathbf{b}\Sigma_f)^{\dagger} \\ &= \frac{1}{\gamma^2}(\Sigma_f\mathbf{b}')^{\dagger}(\mathbf{b}\Sigma_f)^{\dagger} = \frac{1}{\gamma^2}\mathbf{b}'^{\dagger}\Sigma_f^{-2}\mathbf{b}^{\dagger} = \frac{1}{\gamma^2}\mathbf{b}\Sigma_f^{-2}\mathbf{b}'\end{aligned} \tag{B15}$$

where $1/\gamma^2$ is a constant. The last three equalities are obtained by repeatedly using the facts that 1) $\mathbf{b}^{\dagger} = \mathbf{b}'$ (because $\mathbf{b}$ are orthogonal) and 2) a property of Moore-Penrose generalized inverse: If $rank(A) = rank(B)$, then $(AB)^{\dagger} = B^{\dagger}A^{\dagger}$ (Theorem 5.9 in Schott (2005)). The last equality gives a spectral decomposition of $\mathbf{w}^{'}\mathbf{w}$. Therefore, $\mathbf{b} = a_1 \begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_k \end{bmatrix}$ and $\Sigma_f = a_2 e^{-\frac{1}{2}}$ where a_1 and a_2 are two constants. The rest follows since $\hat{\Sigma} = \mathbf{b}\Sigma_f\mathbf{b}'$.■