The Information Content of Revealed Beliefs in Portfolio Holdings

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Abstract

In this paper, we elicit heterogeneous fund manager beliefs on expected stock returns from funds' portfolio holdings at each quarter-end. Revealed beliefs are extracted by assuming that each fund manager aims to outperform a certain benchmark portfolio by choosing an optimal risk-return tradeoff. We then construct a measure of *differences* in beliefs among fund manages for each stock, the belief difference index (BDI). Specifically, we categorize funds into two groups, those with beliefs highly correlated with realized stock returns and those with beliefs less correlated. We then compute BDI as the difference in the average beliefs between these two groups. Sorting stocks based on BDI, we find that the annualized return difference between the top and bottom decile is about two to five percent. The predict of BDI significantly weakens for extremely small or large stocks, or when risk among stock returns is modeled using an identity or a diagonal matrix. These results indicate that 1) fund managers do adjust for risk when making portfolio decisions; 2) risk-return optimization is less used for small stocks by fund managers; and 3) there are less disagreements among managers about large stock returns.

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1 Introduction

Portfolio theory is a cornerstone of modern finance. Pioneered by the work of Markowitz (1952), it has led to a vast amount of research exploring optimal portfolios under various constraints (e.g., short-sales), frictions (e.g., heterogeneous information) and computational limits (e.g., estimation of the covariance matrix). This has resulted in a set of recipes for converting portfolio theory's raw ingredients—beliefs on the structure of stock returns—into its finished product, the optimal portfolio. In contrast, little attention has been devoted to the dual problem: extracting beliefs about the structure of expected stock returns from observed portfolio holdings. In this paper, we focus on the information embedded in the cross-sectional portfolio holdings of mutual fund managers, particularly the revealed heterogeneous fund manager beliefs about expected stock returns. We examine whether these revealed beliefs contain information about the skills of mutual fund managers and/or how they are embedded into the prices of common stocks.

To elicit fund managers' beliefs we make three assumptions. First, we assume that mutual fund managers possess heterogeneous beliefs. Second, we assume that each fund manager has a benchmark index. He wishes to outperform his benchmark with the minimum amount of risk subject to a performance target. This objective function is the same as that discussed in Roll (1992) and is commonly observed in practice. Last, we assume that the covariance matrix of asset returns is common knowledge among investors. This assumption is motivated by the empirical finding that estimating the second moment of a return-generating processes from historical data is considerably easier than estimating the first moment. The widely-implemented Black-Litterman model (1992) also adopts this assumption.

In our model, a mutual fund manager's portfolio holdings are the outcome of an optimization based on his beliefs about stock returns (which are specific to him) and about the covariance structure of these returns (which is common across investors). Therefore, his beliefs about stock returns can be easily backed out if the covariance structure is known. Empirically, we estimate the covariance matrix based on historical return data, which are observable to all investors. In estimating the covariance matrix, we use a multi-factor model. We motivate this by noting that multi-factor models are commonly used in the money management industry.

After backing out these revealed beliefs, we construct a measure of fund managers' stock picking ability by correlating each manager's revealed beliefs about stock returns with the subsequently realized returns. By construction, this correlation is not affected by investor-specific characteristics such as heterogenous risk-aversion and ensures that we capture the effect of heterogenous revealed beliefs. We then measure the *differences* in beliefs between the top thirty percent of fund managers ranked by this correlation and all the remaining fund managers, the belief difference index (BDI). We conjecture that ex post returns are more consistent with the beliefs of the best managers than with the beliefs of all other managers and the current prices are more consistent with the beliefs of the majority: the bottom seventy percent. Hence the differences in beliefs between these two groups of managers reveal information not embedded in the stock price: A large positive BDI statistic indicates that the positive information is not embedded into the stock price while a large negative BDI statistic suggests that the negative information is not embedded into the stock price. We sort stocks into deciles according to BDI and examine the subsequent three-month performance across the decile portfolios. The results show that, on average, stocks with higher BDI statistics outperform stocks with lower BDI statistics, indicating that revealed beliefs contain valuable information about future stock returns. We find the annualized performance spread between the top and bottom decile funds is about two to five percent, which is significant, both economically and statistically. These performance differences are not explained by variations in risk or style factors.

We also sort stocks into three groups: small, medium, and big according to their size at the end of each quarter and redo our analysis. We find that the significant performance difference between the top and bottom BDI deciles comes from the medium size group. This result suggests that the stock picking skills of fund managers are reflected mostly in medium size stocks.

Interestingly, when we replace the covariance matrix used in estimating revealed beliefs with an identity matrix or a diagonal matrix (that is, ignoring the idiosyncratic or the systematic risk in stock returns), the result on the BDI predicability is weaker. That is, by taking into account of the fact that fund managers believe stock returns exhibit risk and this risk is captured by idiosyncratic as well systematic components, the information content embedded in the cross-sectional portfolio holdings is sharper. This finding suggests that fund managers do care about risk when making portfolio decisions.

It is important to know whether there is information in fund holdings, in part because this information allows us to make some inferences about the degree to which the equity market is informationally efficient. One of the most frequently cited arguments for efficiency is the apparent lack of ability of mutual fund managers. However, Berk and Green (2004) show that managerial ability is consistent with a lack of performance persistence in equilibrium. Therefore, assessing managerial ability requires more powerful techniques than those which simply analyze historical fund returns. Our technique shows that many managers are able to forecast returns, that is, they possess stock picking abilities.[1]

Recently, there have been various attempts to investigate the information revealed by portfolio holdings for performance evaluation of portfolio managers. Grinblatt and Titman (1989); Daniel, Grinblatt, Titman, and Wermers (1997); Graham and Harvey (1996); Wermers (2000); Chen, Jegadeesh, and Wermers (2000); Ferson and Khang (2002); Cohen, Coval, and Pastor (2005); Kacperczyk, Sialm, and Zheng (2005); Kacperczyk, Sialm, and Zheng (2008); Cremers and Petajisto (2006); Kacperczyk and Seru (2007) and Breon-Drish and Sagi (2010) have made contributions along this line. Instead of future fund performance, our study extends this line of research by focusing on the implication of information revealed in cross-sectional portfolio holdings for future stock returns.

There have also been attempts to look beyond the information revealed in historical stock return data for future stock returns. Lo and Wang (2000; 2001) find that turnover satisfies an approximately linear k-factor structure and Goetzmann and Massa (2006) identify factors through a sample of net flows to nearly 1000 U.S. mutual funds over a year and a half period. Factors embedded in flow and turn-over data are shown to have valuable information for pricing stocks. Chen, Jegadeesh, and Wermers (2000) find that the consensus opinion of mutual industry (that is, the aggregate active trade of the mutual fund industry) reflects relative superior information about the value of the stock. Wermers, Yao, and Zhao (2007) find that stocks held by top ranked funds (according to measures such as Cohen, Coval, and Pastor (2005)) outperform the rest on average, indicating the investment value of mutual funds. Cohen, Polk, and Silli (2009) also find that top five stocks held by actively managed funds tend to outperform the market. Our paper is closely related to this line of research. Our paper complements the existing literature by formally proposing a method to extract the information embedded in the cross-sectional portfolio holding for fund managers' beliefs and study how the dispersion of these revealed beliefs (opinions) is related to the inefficiency of the market and future stock returns.

The remainder of this paper is organized as follows. In Section 2, we present our methodology for extracting beliefs from portfolio holdings. Section 3 provides the definition

[1]However, whether a manager can outperform the market also depends on whether he has superior market timing abilities, which this study is silent about.

of BDI. Section 4 describes the data used and the empirical implementation of the model. In Section 5, we construct BDI empirically and evaluate whether BDI has valuable information for predicting future stock returns. We conclude in Section 6.

2 Eliciting Fund Managers' Heterogeneous Beliefs

In this section we present a simple portfolio optimization model that highlights the theoretical foundations for eliciting portfolio managers' heterogeneous beliefs. Our objective here is to demonstrate how one can back out heterogeneous beliefs about future excess returns from observed portfolio holdings. To do so requires assumptions about the behavior of fund managers, specifically the nature of their portfolio optimization programs. Our assumptions amount to having mutual fund managers with heterogenous signals about future returns each following a simplified Black-Litterman portfolio optimization program (1992). Specifically, we first assume that a fund manager's performance is evaluated relative to some passive benchmark portfolio. Second, we assume that each manager's goal is to maximize expected returns while minimizing tracking error. Finally, we assume that each manager's beliefs about future returns are summarized by a posterior distribution which is obtained from combining the manager's private signal with the common prior.

Compared with the standard portfolio problem where the investor seeks to minimize return volatility for a given level of expected return (i.e. a Markowitz mean-variance framework),the fund manager in our setup seeks to minimize tracking error volatility for a given level of return *in excess of the benchmark return*. In other words, a fund manager is indifferent to the whims of his benchmark, as long as he can outperform it. As Roll (1992) points out, managers who implement this optimization program do not hold mean-variance efficient portfolios, yet tracking error criteria are widely used in practice. Thus, for our purposes, this appears to be a reasonable assumption.

In what follows, we first detail the return-generating process for risky and risk-free assets and the information structure among the fund managers. We then solve the fund manager's portfolio optimization problem. Finally, we show how a fund manager's beliefs about stock returns can be identified up to a constant given his (optimal) portfolio holdings and benchmark.

2.1 Returns and Information

In our setup the investment opportunity set consist of a risk-free asset with a constant return, r_f, and N risky assets where the ith asset's *excess return* (over the risk-free rate) is denoted by r_i. We write the N-vector of excess returns as $\mathbf{r} = [r_1, ..., r_N]'$. The excess returns of the risky assets are assumed to follow a normal distribution $\mathbf{r} \sim N(\boldsymbol{\mu}_0, \boldsymbol{\Sigma})$ where $\boldsymbol{\Sigma}$ is a full rank covariance matrix.

We assume that there are m mutual fund managers in this economy. Fund managers are assumed to possess common knowledge of the true covariance matrix, $\boldsymbol{\Sigma}$. This assumption is based on the relative ease of covariance estimation. However, the true mean of the returns, $\boldsymbol{\mu}_0$ is not known, neither to us, nor to the manager. This reflects that the means of returns are much more difficult to estimate (vis-as-vis covariances). We assume that each manager (manager m) comes up with his own best estimate of the true means, $\boldsymbol{\mu}_0$, and denote this estimate as $\boldsymbol{\mu}_m$. Further—for expositional purposes—we assume that each manager obtains his estimate, $\boldsymbol{\mu}_m$, using a Bayesian updating scheme[2]

In our Bayesian updating scheme, each manager has an identical prior on true means, $\boldsymbol{\mu}_0$:

$$p(\boldsymbol{\mu}_0) \sim N(\boldsymbol{\mu}_M, \tau\boldsymbol{\Sigma}) \tag{1}$$

where $\boldsymbol{\mu}_M$ denotes the expected returns for each security as implied by the CAPM and τ^{-1} is the precision of the prior. For simplicity, we assume that the covariance on the prior is similar to the (known) covariance of returns.

Each manager observes a signal vector, $\mathbf{s}_m$, about the future mean excess returns of stocks in his benchmark portfolio. This signal vector is a (noisy) observation of the true means, $\boldsymbol{\mu}_0$. Thus,

$$p(\mathbf{s}_m|\boldsymbol{\mu}_0) \sim N(\boldsymbol{\mu}_0, \tau_m\boldsymbol{\Sigma}), \tag{2}$$

where τ_m^{-1} is the precision of the manager's signal, which we shall refer to as the manager's *informedness*.[3] The manager's first problem is to come up with an assessment of the mean of returns, $\boldsymbol{\mu}_0$, given his information ($\mathbf{s}_m$). This amounts to finding the posterior distribution, $p(\boldsymbol{\mu}_0|\mathbf{s}_m)$, given the prior, $p(\boldsymbol{\mu}_0)$, and the conditional density of the signal, $p(\mathbf{s}_m|\boldsymbol{\mu}_0)$. Since we've assumed normality and a similar variance structure for the signal and the prior, it is

[2]This is the essence of the Black-Litterman framework.

[3]Realistically, managers may have signals about only limited numbers of securities, or only about relative performance, or with different variance structure. We ignore this for the sake of tractability.

easily shown that the posterior distribution is:

$$p(\boldsymbol{\mu}_0|\mathbf{s}_m) \sim N(\boldsymbol{\mu}_m, \delta_m \boldsymbol{\Sigma}) \tag{3}$$

where $\boldsymbol{\mu}_m = \delta_m(\tau_m^{-1}\tilde{\mathbf{s}}_m + \tau^{-1}\boldsymbol{\mu}_M)$ and $\delta_m^{-1} = \tau^{-1} + \tau_m^{-1}$.

Having characterized the posterior beliefs of the fund managers, we examine managers' portfolio allocation problem. Let w_{m0} denote the percentage of wealth (or portfolio weight) invested by manager m in the risk-free asset and let $\mathbf{w}_m = [w_{m1}, \ldots, w_{in}]'$ denote the vector of his portfolio weights in each of the n risky assets. The portfolio weights satisfy the standard portfolio budget constraint:

$$w_{m0} + \mathbf{1}'\mathbf{w}_m = 1. \tag{4}$$

Manager m is assigned a benchmark portfolio against which he is judged. We denote manager m's benchmark portfolio weights with $\mathbf{q}_m = [q_{m1}, \ldots, q_{mn}]'$. We assume that the benchmark consists of only risky assets; consequently, benchmark weights must satisfy:

$$\mathbf{1}'\mathbf{q}_m = 1. \tag{5}$$

Fund managers are assumed to choose portfolio weights so as to maximize the expected return over the benchmark (i.e., active return) while minimizing tracking error volatility (i.e., active risk). We denote manager m's active return by z_m, where:

$$\begin{aligned} z_m &= (\mathbf{w}_m - \mathbf{q}_m)'(\mathbf{r} + \mathbf{1}r_0) + w_{m0}r_0 && (6) \\ &= (\mathbf{w}_m - \mathbf{q}_m)'\mathbf{r}. && (7) \end{aligned}$$

Conditional on his signal, manager m's expected active return is

$$E[z_m|\mathbf{s}_m] = (\mathbf{w}_m - \mathbf{q}_m)'\boldsymbol{\mu}_m \tag{8}$$

while his active risk is

$$Var[z_m|\mathbf{s}_m] = (\mathbf{w}_m - \mathbf{q}_m)'\delta_m\boldsymbol{\Sigma}(\mathbf{w}_m - \mathbf{q}_m). \tag{9}$$

Under the assumption of quadratic utility (or the usual equivalents), the manager's optimization can be stated as:

$$\max_{w_m} E[z_m|\mathbf{s}_m] - \gamma_m Var[z_m|\mathbf{s}_m] \tag{10}$$

where γ_m corresponds to manager m's effective risk aversion. The Lagrangian to the manager's problem is

$$L = E[z_m|\mathbf{s}_m] - \gamma_m Var[z_m|\mathbf{s}_m] - \lambda(\mathbf{1}'\mathbf{w}_m - 1) \tag{11}$$

$$= (\mathbf{w}_m - \mathbf{q}_m)'\boldsymbol{\mu}_m - \gamma_m\delta_m(\mathbf{w}_m - \mathbf{q}_m)'\boldsymbol{\Sigma}(\mathbf{w}_m - \mathbf{q}_m) - \lambda\mathbf{1}'(\mathbf{w}_m - \mathbf{q}_m) + \boldsymbol{\eta}'\mathbf{w}_m, \tag{12}$$

where λ is the Lagrange multiplier on the no borrowing constraint, and $\boldsymbol{\eta}$ is the vector of Lagrange multipliers for the no short sale constraints. The solution to the optimization problem is

$$\mathbf{w}_m - \mathbf{q}_m = (\gamma_m\delta_m)^{-1}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}_m + \lambda\mathbf{1} - \boldsymbol{\eta}), \tag{13}$$

To the extent that the short sale constraints are not binding ($\boldsymbol{\eta} \approx \mathbf{0}$), portfolio holdings reveal beliefs up to an affine transformation:

$$\boldsymbol{\mu}_m \approx \gamma_m\delta_m\boldsymbol{\Sigma}(\mathbf{w}_m - \mathbf{q}_m) - \lambda\mathbf{1}, \tag{14}$$

The covariance matrix of returns, $\boldsymbol{\Sigma}$ can be reasonably estimated using historical return data and we denote manager m's estimate of this matrix with $\boldsymbol{\Sigma}_m$. With some additional minor assumptions [4] we can state the following result:

Result 1 *Fund manager m's private beliefs on expected returns, $\boldsymbol{\mu}_m$, are revealed up to an affine transformation given: (a) the manager's portfolio weights $\mathbf{w}_m$, (b) the manager's benchmark portfolio $\mathbf{q}_m$, and (c) the manager's estimate of the covariance matrix of returns, $\boldsymbol{\Sigma}_m$:*

$$\hat{\boldsymbol{\mu}}_m = \boldsymbol{\Sigma}_m(\mathbf{w}_m - \mathbf{q}_m). \tag{15}$$

where $\hat{\boldsymbol{\mu}}_m$ denotes the vector of the manager's revealed beliefs*.*

These revealed beliefs have several useful properties. First, they are closely related to true beliefs in that $\boldsymbol{\mu}_m = a + b\hat{\boldsymbol{\mu}}_m$. Second, they are forward-looking and—for the most part—can be estimated using contemporanous observations on portfolio holdings.[5] Finally, they are multi-dimensional (i.e. we obtain revealed beliefs about multiple securities).

[4] The manager's problem was formulated with the covariance structure being fixed—we implicitly take it as time-varying but with sufficiently slow dynamics that it appears to be fixed in the manager's optimization problem. We also take some minor liberties by using the estimated covariance matrix where the original problem was formulated with the true covariance matrix known.

[5] We elaborate on the details of estimation in the next section. Quantities such as the covariance matrix are estimated using historical data, however the critical ingredient (the portfolio holdings) is contemporanous.

These features allow us to use revealed beliefs to construct a performance measure based on contemporanous—rather than historical—information.

In our setup, beliefs, $\boldsymbol{\mu}_m$, are more predictive of future returns, $\mathbf{r}$, the higher the precision of the manger's private signal. In the time series, this would be seen in a higher correlation between beliefs about some security and the ex-post realized returns of that security. However, true beliefs are not observed, only revealed beliefs are. Because reavealed beliefs are an affine transformation of true beliefs, a similar time-series relationship between future returns and *revealed beliefs* need not hold. The problem is that the translation component, a, and the scale component, b, may themselves be time-varying. According to Result 1, the scale component, b, is the product of total precision, and risk aversion: $b = \delta_m \gamma_m$. While it is conceivable that these quantities have some time-variation, we do not expect this to be a major issue in most times. More problematic is the translation component, $a = -\lambda \mathbf{1}$; it contains the Lagrange multiplier for the "no borrowing" constraint,

$$\lambda = \frac{\mathbf{1}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}_m}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}, \tag{16}$$

which scales with the overall level of expected returns: if the manager expects returns (for all securities) to be twice as high this quarter (vis-a-vis the previous quarter) and the no-borrowing constraint is binding in both quarters, then the translation term, a, will be twice as large this quarter. As previous researchers have noted (see for example Daniel, Grinblatt, Titman, and Wermers (1997)), fund manager skill—to the extent that it exists at all—is more likely to be found in "stock picking" rather than "market timing". The lack of "market timing" manifests itself in an inability to predict the overall level of returns or—equivalently—a noisy translation term, a. Consequently, the time series of a manager's reveleaed beliefs about a particular security's returns is not likely to be particularly informative about manager skill: such a time series will be necessarily confounded by the manager's (noisy) beliefs about market returns. For this reason, we will rely on the cross-section of a manager's beliefs at a single point in time to assess his skill.

3 Evaluating the Information Content of Revealed Beliefs

Our revealed beliefs, $\hat{\boldsymbol{\mu}}_m$, capture a fund manager's *ex-ante* beliefs on expected future stock returns. For a manager implementing a benchmark-tracking, mean-variance optimization framework, possessesing high quality information about future returns—informedness—will be evident through a measured correlation between the manager's revealed beliefs and the *ex-post* realized returns. Our goal is to construct a measure that identifies *which* managers are more or less informed at a given point in time. We propose that if our measure is successful, we should find that better-informed managers exhibit better future fund performance, and—more importantly—that the beliefs of the better-informed managers can identify securities where information has not been fully reflected in market prices.

As described in the previous section, measuring informedness from the time-series is problematic: managers' informedness is difficult to infer in the presence of noisy "market timing" signals. In addition, measures based on the time-series suffer from several additional issues. For one, the precision of a manager's signal—his informedness—may vary over time. Second, estimation from the time-series requires relatively long histories. Finally—although not formally modeled in our setup—a manager's precision *structure* may vary from time to time: for example, today a managaer may have a high precision signal about security X, while in the previous quarter his high precision signal was about security Y. Thus, we would prefer a cross-section-based estimate—one based on the overall accuracy of manager's beliefs at *a single point in time*—rather than one based on the time-series.

An intuitive candidate is the cross-sectional correlation: the correlation between the manager's revealed beliefs about the constituents of his portfolio and their realized returns in a subsequent period. The problem with this type of cross-sectional correlation is that—unlike the time-series—the cross-section cannot be assumed to be idependent and identically distributed. Consequently, the cross-sectional correlation estimate is generally not consistent for the quantity of interest—the informedness of the manager.

To mitigate this issue, we apply a linear transform to the revealed beliefs and realized returns and compute the sample correlation of the transformed vectors. The transformation we use is pre-multiplication by the Cholesky decomposition of the (estimated) covariance matrix, as is commonly employed in generalized least squares regression. This has the effect of partially "pre-whitening" our data (revealed belief–realized return tuples), resulting in independent—albeit not identically distributed observations.[6] The independence of the (transformed) observations permits the application of Markov's law of large numbers. We term the sample correlation of the transformed observations as the manager's "*belief accuracy index* (*BAI*)". This is made explicit in the following definition:

[6]Obtaining independence in this manner relies on the assumption of normally-distributed returns.

Definition 1 *Given the revealed beliefs of portfolio manager m, obtained from the manager's holdings at the end of month t, $\hat{\boldsymbol{\mu}}_{m,t}$, and given the excess one month returns for month $t+1$, $\mathbf{r}_{t+1}$; the manager's belief accuracy index (BAI) is defined as:*

$$BAI_{m,t} \equiv \widehat{\text{cor}}\left(S_t^{-1}\hat{\boldsymbol{\mu}}_{m,t}, S_t^{-1}\mathbf{r}_{t+1}\right) \tag{17}$$

where $\widehat{\text{cor}}(\cdot)$ is the sample correlation, and S_t is the Cholesky decomposition of the covariance matrix of stock returns, such that $S_t S_t' = \Sigma_t$ and where Σ_t is the covariance of stock returns at the end of month t.

Given fund managers' BAIs, we can categorize managers into two categories—"*informed*" and "*uninformed*"—based on their BAI scores. We consider a manager to be informed only if his BAI score is among the top twenty percent of BAI scores from all managers *at that time*. The remaining managers are deemed to be uninformed.

Definition 2 *Let I_t and U_t denote the set of informed and uninformed managers respectively as inferred from end of period t holdings:*

$$I_t \equiv \{m \in \{1...M\} : BAI_{m,t} \geq Q_{80}(BAI_{\cdot,t})\} \tag{18}$$

$$U_t \equiv \{m \in \{1...M\} : BAI_{m,t} < Q_{80}(BAI_{\cdot,t})\} \tag{19}$$

where $Q_{80}(\cdot)$ is the (80%) quantile function.

The relative exclusivity of our "informed manager club" is deliberate. Truly informed managers will often be mis-classified by our procedure (our rate of false negatives is likely to be high). This is a classic trade-off: by reducing the rate of false positives, we pay with more false negatives. However, we are much more concerned about false positives and so apply a relatively high standard. The intutition is that our goal is to establish *existence* of superior information among fund managers rather than to measure accurately its full extent. Thus, we do not need to cast an especially wide net. On the other hand, we cannot make the club arbitrarily small; later in this section, we introduce the notion of informed manager's "consensus" belief: estimating such consensus beliefs will require non-trivial numbers of "informed" managers. To fully formalize these notions would require the introduction of a model of information diffusion; as this is not central to our analysis, we rely on the intuitive argument.[7]

[7]The bar also needs to be high due to more practical considerations. Lack of informedness and chance are

Our basic premise is this: informed managers have more accurate beliefs about future realized stock returns than their uninformed peers. This is not mere tautology. While it is true that having "accurate beliefs" (high BAI_t, as measured on $t+1$) is exactly what lands a manager in the "informed-manager club", it is by no means certain that the beliefs of members of this club will be any more accurate than of their "uninformed" peers at time $t+2$. We conjecture that at the time BAI is measured ($t+1$), not all the beliefs of informed managers have fully "borne fruit". That is, managers with high BAIs were correct about a number of stocks (by construction), but not about all stocks: some stocks performed worse than the informed managers had believed, while others fared better. Our contention is that although some of these apparently "fruitless" beliefs were due to chance (i.e. noise in the signal), some were due to delays in uninformed investors' acquisition of information. That is, with time informed managers will be proven right about some of these apparently fruitless beliefs as well. It is precisely this—that the beliefs of managers who have tended to be correct recently continue to have predictive power—that we test in this paper.

A characterization of this story is that that current stock prices incorporate the information available to uninformed investors. Informed investors possess access to a stream of information that is not immediately reflected in prices. Informed investors trade on this information, which is eventually revealed to the uninformed investors and only then is it fully reflected in prices.[8] This characterization is consistent with an equilibrium of the type proposed by Grossman and Stiglitz (1980), where the cost of obtaining information deters a certain fraction of investors from obtaining it. Since the less-informed investors' beliefs simply reflect current market prices, the difference between the informed investors' beliefs and those of the less-informed investors measures how much information held by informed investors is yet to be embedded into the stock price.

For any stock, the difference between the informed and less-informed beliefs constitute

not the only way to a low BAI score: a manager may score badly because he does not actually use a portfolio optimization program of the sort we have assumed for him. Possible deviations range from tracking some broad market index rather than an equity-based benchmark, longer or shorter horizons, different covariance estimation techniques, all the way to complete abandonment of the mean-variance framework (e.g. simple stock-picking or market timing). For these "deviant" managers, our procedure will not reveal beliefs or much of anything else (besides noise). As it is certain that such deviant managers exists, we have to make room for them; we make room for them by expanding the uninformed category.

[8] We are assuming here that the information of better informed managers takes longer than one month to reach the rest of the market. If this were not the case, then the beliefs of high BAI managers should not have predictive power.

a measure of "unpriced" information. This difference should predict future non-systematic price movement if information available to the informed investors is eventually learned by the less-informed mass of market participants. Thus, differences in beliefs between the two groups of investors reveal information not yet reflected in the stock price: a large positive difference indicates that the positive information is not embedded into the stock price while a large negative difference suggests that the negative information is not embedded into the stock price. We refer to this measure as the "*belief difference index (BDI)*":

Definition 3 *Let s index the set of exchange-traded securities, and let $\hat{\boldsymbol{\mu}}_{m,t,s}$ denote the revealed belief of mananager m about security s computed from holding data available at the end of period t. Then the time-t belief difference index for security s is defined as:*

$$BDI_{t,s} \equiv \underset{\{m \in I_t\}}{\widehat{\text{avg}}} \left(R(\hat{\boldsymbol{\mu}}_{m,t,s})\right) - \underset{\{m \in U_t\}}{\widehat{\text{avg}}} \left(R(\hat{\boldsymbol{\mu}}_{m,t,s})\right) \tag{20}$$

where $R(\cdot)$ denotes the rank function, $\widehat{\text{avg}}(\cdot)$ denotes the sample average (over managers), I_t and U_t are the set of informed and uninformed managers (defined in Equation (2))

Note that with BDI, we are interested in comparing *differences* in revealed beliefs across fund managers. Because revealed beliefs are an affine transformation of true beliefs ($\boldsymbol{\mu}_m = a_m + b_m\hat{\boldsymbol{\mu}}_m$), we must account for potential differences in the translation (a_m) and scale (b_m) components across managers. We do so by ranking the revealed beliefs of each manager and averaging the ranks.[9] Normalizing revealed beliefs in this way allows us to focus on managers' *relative* beliefs (i.e. a manager's view on security X vis-a-vis security Y) by removing the effect of manager-specific beliefs about the market as a whole (a_m) as well as the effect of manager's risk aversion and precision (b_m).

4 Data and Methods

In the previous section we have implicitly assumed the availability of revealed beliefs as well as our knowledge of features of the distribution of returns (i.e. covariances). In this section we describe our sample, we operationalize the extraction of revealed beliefs from mutual fund stock holdings, and finally, we operationalize the calculation of our informedness-based measures based on those beliefs.

[9] This amounts to a very simple form of rank aggregation. It is best suited to situations where all managers hold the same securities in their portfolios. This is clearly not the case in our data. However, we do not expect that application of more sophisticated rank aggregation methods to materially affect our results.

4.1 Sample

Our sample consists of observations on mutual fund portfolio holdings as well as the observed returns of those holdings. It covers mutual fund holdings from the first quarter of 1980 to the final quarter of 2009. In constructing the sample, we employ four primary databases: the Thomson-Reuters Mutual Fund Holdings database, the CRSP stock daily return file, the CRSP stock monthly return file, and the CRSP mutual fund monthly return file.

The Thomson-Reuters Mutual Fund Holdings database [10] comprises mandatory SEC filings as well as voluntary disclosures from all registered U.S. mutual funds. Mutual fund holdings are typically available on a quarterly basis. Wermers (2000) describes this database in more detail. This database does not provide extensive coverage of non-equity and foreign holdings: thus we shall restrict our attention to domestic, all-equity funds.

To construct the sample, we begin with the quarterly fund holdings obtained from Thomson-Reuters. In order to eliminate foreign and non-equity funds we remove those fund-quarters where the mutual fund's Investment Object Code (IOC) is reported as something other than: aggressive growth, growth, growth and income, unclassified, or missing. We remove all observations where the reported number of shares held is missing, where the CUSIP of the security is missing, or where the CUSIP cannot be matched to the CRSP return file. We also eliminate any funds that cannot be matched to a fund tracked in the CRSP mutual fund file.[11]. We eliminate any fund-quarters where the fund's equity holdings amount to less than \$5 million or where the fund holds fewer than 20 stocks. Finally, we remove those fund-quarters where Thomson-Reuters imputed the holdings using reports from previous quarters.[12] Table 6 presents a year by year summary of the sample.

4.2 Extracting Beliefs

From Result 1, the extraction of fund managers' revealed beliefs requires three elements: the manager's portfolio holdings, the manager's covariance matrix, the manager's benchmark portfolio, and the manager's performance target (the expected active return). If all these

[10]This database was formerly known as CDA/Spectrum.

[11]Matching the Thomson-Reuters holdings database to the CRSP mutual fund file is done using the MFLINKS tables provided by Wermers (2000) The coverage of the MFLINKS tables is not complete; however, for domestic equity funds (the focus of our enquiry), the coverage is believed to be exhaustive.

[12]This corresponds to observations where the $RDATE$ is from an earlier quarter than indicated by the $FDATE$.

elements were observable, the extraction of beliefs would be streightforward. One need simply reverse the manager's portfolio optimization "program" and use its outputs (holdings), and "known" inputs (covariances, benchmark, and risk aversion) to determine the remaining unkown inputs (beliefs). Of course, some of the required "known" quantities cannot be directly observed and so we resort to estimating them instead. In this section we describe our methodology for constructing these estimates.

Portfolio Holdings

Our inability to observe portfolio holdings ($\mathbf{w}_{m,t}$) is the major constraint on our methodology. Ideally, we would observe holdings continually. In reality, fund managers only report holdings on a quarterly basis.[13] Thus—in the best case—our methodology is limited to extracting beliefs on a quarterly basis which limits the power of our tests.

Another issue is that some of the manager's holdings do not correspond to domestic equity issues for which we have readily available return data. These might be foreign securities, ADRs, bonds, commercial paper, etc.. In theory, these securities play a role in the manager's optimization problem and would thus need to be incorporated into our methodology. However, without return data we are unable to estimate the covariances of these securities. Hence, we ignore them in our analysis.

A similar problem stems from the fact that the portfolio holdings reports do not include easily identifiable information about the manager's holding of the the risk-free asset (various cash equivalents). Ignoring risk-free holdings results in a biased estimate of beliefs. In our setup, high holdings of risk-free assets correspond to beliefs about market-wide downturn. That said, this is another issue we shall simply ignore.

We expect that the bias that is introduced by ingoring non-equity assets and risk-free positions will not be large. For one, it is unusual to observe domestic equity funds that hold large non-equity positions. This is entirely consistent with the optimization problem we've posited. In the banchmark-tracking portfolio optimization, more risk averse manager will not hold more of the risk-free asset; rather, he will deviate less from the benchmark portfolio. In our framework, holdings of large cash position are consistent with a strong belief that the market is overpriced rather than a high degree of risk-aversion. A large cash position would be indicative of a manager who is engaged in market timing (and who is convinced that the market is overpriced). We doubt fund managers engage in significant market timing

[13] Earlier in the sample period, even lower reporting frequencies were common

strategies, if for no other reason than because the evidence presented in Daniel, Grinblatt, Titman, and Wermers (1997) suggests they are not especially good at it. In any event, to the extent that these non-equity positions are important, ignorning them can only undermine our efforts to find predictive power in our measures.

Finally, we must consider that the reported fund holdings do not accurately represent the true holdings of the mutual fund manager. In particular, significant evidence exists that mutual fund managers engage in "window-dressing" activities around the end of the quarter[14]. Again, anything that impedes our ability to observe the manager's "true" holdings will only work against us. However, since window-dressing activity typically involves the purchase of well-known securities that have done well in the recent past, one could argue that window-dressing itself may generate mechanical return continuation (or return reversal) relationships that could account for the results of our analysis. Although we ignore window-dressing in the main analysis, we shall address this issue in our robustness checks.

To summarize, we estimate holdings as follows,

Definition 4 *Let $\mathbf{p}_{m,t}$ represent manager m's time t report of (dollar) holdings of CRSP securities (as reported by Thomson-Reuters). Then, manager m's portfolio weights are estimated as*

$$\hat{\mathbf{w}}_{m,t} \equiv \frac{\mathbf{p}_{m,t}}{\mathbf{1}'\mathbf{p}_{m,t}}. \tag{21}$$

The weights are only defined for those quarters where the fund manager has reported holdings[15].

Covariance Matrix

Our second challenge is obtaining a plausible estimate of the *manager's* covariance matrix. Estimating covariance matrices from financial data series is always problematic. The culprit is the relatively small number of observations (T) given the large number of securities (N). Typically, N is on the order of a few thousand while decades of monthly data only yield Ts that are on the order of a few hundred. Unless $T >> (N^2 + N)/2$, the conventional covariance estimator will tend to produce a singular matrix whose eigenvalues bear little resemblance to the originals.[16] In principle, the solution is to use very high high frequency

[14]see Lakonishok, Shleifer, Thaler, and Vishny (1991); Musto (1999); Meier and Schaumburg (2004).

[15]In terms of Thomson-Reuters fields, we require that the the report date ($RDATE$) occurs in the quarter ending on the file date ($FDATE$). In other words, we exclude fund-quarters with imputed holdings.

[16]Schäfer and Strimmer (2005) provide some simulation results that demonstrate the severity of the problem.

returns; however—as Lo and MacKinlay (1990) have shown—small stocks may not react to common market news for days or even weeks. Such non-synchronous trading effects preclude the use of returns at arbitrarily high frequencies. Typical methods for addressing this problem include factor models and shrinkage estimation (or some combination of both).

At this point, it is important to emphasize our primary goal. It is not to find the best estimate of the *true* covariance matrix. Primarily, our goal is to find the best estimate of the *manager's estimate* of the covariance matrix. In our analysis, the use of this matrix is largely limited to its role in the manager's optimization problem: we prefer knowledge of the manager's estimate to knowledge of the truth.

The fund manager's problem is to obtain an estimate that is accurate and *well-behaved.* By well-behaved, we mean that it can be inverted—the manager needs the inverese to implement the optimization program. Of the various estimation methods alluded to earlier, the multiple factor approach is one of the most popular. This approach yields well-behaved estimators and is simple to implement. Perhaps most importantly, it is likely to be used by mutual fund managers. Risk models sold to the mutual fund industry by vendors like MSCI Barra feature multi-factor covariance matrix estimation.

We model the covariance structure in stock returns using 53 factors. We start with the three Fama French (1993) factors: excess market return ($MKTRF$), small-minus-big (SMB), and high-minus-low (HML). To this we add momentum, or up-minus-down factor (UMD) of Carhart (1997). Finally, we also include the returns on the 49 industry portfolios available from Ken French's website. Thus, the data generating process for excess returns is taken to be

$$\tilde{\mathbf{r}}_t = \alpha_t + \mathbf{b}_t\tilde{\mathbf{f}}_t + \tilde{\mathbf{e}}_t$$

where $\tilde{\mathbf{f}}_t$ is the return to the factors, with covariance matrix $\mathbf{\Omega}_t$, and $\tilde{\mathbf{e}}_t$ is the ($i.i.d.$) vector of idiosyncratic returns with zero mean and a diagonal variance structure:

$$Var[\tilde{\mathbf{e}}_t] = \mathbf{D}_t \equiv diag(\sigma^2_{t,1}, \ldots, \sigma^2_{t,N}).$$

The $N \times 53$ matrix $\mathbf{b}_t$ consists of a stack of (time-varying) factor loading row-vectors—one for each of the N securities. Idiosyncratic returns are uncorrelated with the factor returns, hence the covariance of returns is:

$$\mathbf{\Sigma}_t = \mathbf{b}_t\mathbf{\Omega}_t\mathbf{b}'_t + \mathbf{D}_t. \tag{22}$$

To estimate the factor covariance matrix, we use the sample covariance estimator denoted by $\hat{\mathbf{\Omega}}_t$. Estimates of the factor loadings, $\hat{\mathbf{b}}_t$, are obtained by regressing each security's returns on the contemporanous returns of the 53 factors; we stack the 53 coefficient estimates from each of the N regressions to construct $\hat{\mathbf{b}}_t$. To estimate the N diagonal elements of $\mathbf{D}_t$, we use the root mean squared error from each of the factor-loadings; we call this estimate $\hat{\mathbf{D}}_t$. The resulting estimator of $\mathbf{\Sigma}_t$ takes an analogous form:

$$\hat{\mathbf{\Sigma}}_t = \hat{\mathbf{b}}_t \hat{\mathbf{\Omega}}_t \hat{\mathbf{b}}_t' + \hat{\mathbf{D}}_t. \tag{23}$$

It would be optimisitic to suppose that the covariance in security returns remains constant for our sample period, so we have allowed for time variation in specifying our model. For estimation purposes, we use five year, rolling windows to construct all the estimates ($\hat{\mathbf{b}}_t$, $\hat{\mathbf{\Omega}}_t$ and $\hat{\mathbf{D}}_t$). In order to maintain a sufficient number of observations in the short (five year) time-series, we use weekly return data. We follow the convention and calculate weekly (Thursday to Wednesday) stock (and factor) returns from the daily return files. Using weekly returns is a compromise: estimates based on high frequency data are subject to non-synchronous trading effects. Covariance estimates that do not take this into account understate the degree of co-movement. By using weekly returns, we increase the number of observations four-fold without incurring the brunt of the non-synchronous trading bias.

To summarize, we estimate a covariance estimate, $\hat{\mathbf{\Sigma}}_t$, each quarter based on a 53 factor model using the previous five years of weekly return data. For the purposes of our analysis, we assume that each mutual fund manager obtains the same matrix for use in his portfolio optimization problem.

Benchmark Portfolio

So far in our discussion of belief extraction we have treated the benchmark portfolio, $\mathbf{q}_{m,t}$, as given. However, as alluded to earlier, this information is not directly observable, and must be estimated.

While there exists data on reported mutual fund benchmarks, these data are not available for all the funds-quarters in our sample. Futhermore, there is no guarantee that such reports are accurate: funds may publicly claim one benchmark, while evaluating the manager based on another. Given this, our approach is to let the holdings data speak: if the fund holds, or has recently (in the last five years) held some security, then that security is considered to be part of its benchmark. We set the benchmark weights based on market capitalization, thus the fund's benchmark is a value-weighed index of securities in which the fund has shown any interest in the last five years. Formally,

Definition 5 *Let* $\mathbf{c}_t \equiv [c_{1,t}, ..., c_{N,t}]$ *be the vector of end-of-period* t *market-capitalizations for all CRSP firms, and* $\mathbf{H}_{m,t} \equiv diag(h_{1,m,t}, ..., h_{N,m,t})$ *be an* $N \times N$ *diagonal matrix where each diagonal element corresponds to a CRSP security and indicates whether manager* m *has reported holding that security in the last five years:*

$$h_{i,m,t} = \begin{cases} 1 & \exists \tau \in [t-60, t] : |w_{i,m,\tau}| > 0 \\ 0 & otherwise \end{cases} \tag{24}$$

Then, for a quarter ending on t*, manager* m*'s (estimated) benchmark portfolio is*

$$\hat{\mathbf{q}}_{m,t} \equiv \frac{\mathbf{H}_{m,t}\mathbf{c}_t}{\mathbf{1}'\mathbf{H}_{m,t}\mathbf{c}_t} \tag{25}$$

while the (estimated) set of benchmark portfolio securities is

$$\hat{B}_{m,t} \equiv \{i \in \{1...N\} : h_{i,m,t} = 1\} \tag{26}$$

Our benchmark selection methodology will include all securities that are part of the true benchmark. This follows from the premise that the manager is mean-variance optimizing. If that is the case, then the solution to the optimization problem will inevitably suggest some non-zero position for every security. Although one could argue that the manager will not hold certain negative positions due to short-sale constraints, in our setting this is not a major issue. Unlike a mean-variance investor, our fund manager will rarely run up against a short-sale constraint; this is because to our fund managers, any underweighing of a security relative to the benchmark is effectively a short position.[17]

Estimation of Revealed Beliefs, BAI and BDI

We calculate revealed beliefs for each manager as described in Result 1 with one caveat. For each manager we restrict the universe of available securities to those that are in his estimated benchmark (the set $\hat{B}_{m,t}$). In other words, the portfolio and benchmark weight vectors—$\mathbf{w}_{m,t}$ and $\mathbf{q}_{m,t}$ respectively—as well as the covariance matrix, $\hat{\mathbf{\Sigma}}_{m,t}$, only include only those securities

[17]Note that Result 1 indicates that managers have no revealed beliefs on stocks that are not in their benchmarks. That is. if a manager does not hold a stock *and* the stock is not in his benchmark, then we regard this manager as having no view on this stock.

where the benchmark weight is non-zero.[18]. The manager solves the optimization problem as if securities outside his benchmark were unavailable. Consequently, revealed beliefs are only obtained for those securities that are part of the manager's (estimated) benchmark. Since securities that are "available" to the managers would end up in the manager's portfolio, this approach is most natural[19]

We only observe funds' holdings data as of the end of a calendar quarter; consequently, our estimates of *revealed belief*, $\hat{\mu}_{m,t}$, are based on holdings data from quarter-end month t (March, June, September, and December). We calculate the fund managers' belief accuracy indices, $BAI_{m,t}$, using the realized returns from the subsequent month ($t+1$, or April, July, October, and January), as per Definition 1. The definition of BAI incorporates the covariance of returns, $\mathbf{\Sigma}_t$: to operationalize the calculation we use the 53 factor-model estimate of the covariance, $\hat{\mathbf{\Sigma}}_{m,t}$, from Equation (23) that we used in calculating the manager's revealed beliefs.

After calculating the managers belief accuracy indices, we sort managers into informed and informed sets based on their accuracy score. At time $t+1$, for each security s, we calculate the belief difference index, $BDI_{s,t}$ as the difference in average revealed beliefs about security s between the informed and uninformed managers as described in Definition 3. As not all managers have revealed beliefs on all securities, the average belief in the BDI calculation is taken to be average belief among those managers that have a belief.

Our hypothesis is that the belief difference index can predict future returns. We conjecture that this is so because information incorporated into the beliefs of the relatively few informed managers will eventually be incorporated into the beliefs of the many uninformed managers (and other investors). Our approach to test this is to form equal-weight decile stock portfolios based on stocks' BDI scores. We expect that the stocks in the top deciles will outperform those in the lower deciles.

[18]To be more precise, the manager's estimated covariance matrix is taken to be a principal minor of our 53-factor estimate:

$$\hat{\mathbf{\Sigma}}_{m,t} \equiv \left[\hat{\mathbf{\Sigma}}_t\right]_{\hat{B}_{m,t}}$$

[19]The alternative would be to include all securities in the vectors and in the covariance matrix. This alternative is less reasonable because fund managers often have mandates to invest in a limited investment universe, e.g. certain industry stocks and therefore have little incentive to acquire information about the stocks outside their investment universe.

The timing of our tests is free from look-ahead bias: we rely only on historical data in all aspects of portfolio construction (i.e. all covariance estimates, factor estimates, and index calculations). Furthermore, we only use lagged mutual fund portfolio holdings because we must wait one month before we can construct our BAI and BDI ideces. For some quarter end date t (March, June, September, and December) we wait until $t+1$ (April, July, October, January) before we infer beliefs, calculate $BAI_{m,t}$ and $BDI_{s,t}$, and form the decile portfolios; we evaluate the performance of the decile portfolios using monthly returns at $t+2$, $t+3$, and $t+4$. This is summarized in Figure 6.

5 Results

The primary aim of our empirical analysis is to determine the extent to which the structure of fund holdings—as summarized by our BDI measures—can be used to predict stock returns. High BDI stocks are those stocks that are favored by informed managers; we expect these stocks to outperform as the positive information available to the better informed manager is gradually incorporated into the beliefs of the general (uninformed) investor population. Indeed, we find that this seems to be the case. In the remainder of this section, we detail our findings.

5.1 Revealed Beliefs and Future Stock Returns

As described in the previous section, to test the predictability of BDI, one month into each quarter end (e.g. January 31) we calculate BDI using portfolios data from the previous quarter end (e.g. December 31). We sort the stocks into deciles based on BDI. We construct equal-weight decile portfolios. We evaluate performance using the subsequent three months of returns (e.g. February, March, and April). Portfolios are rebalanced monthly.

The performance measures we consider include excess returns, alphas from the one factor CAPM model of Sharpe (1963), three-factor model of Fama and French (1993), and four-factor model of Carhart (1997), as well as the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997). For the various measures of alpha, we run standard time-series regressions. We use the Newey-West standard errors to deal with possible auto-correlation in the error terms.

The risk- and style-adjusted net returns as well as factor loadings[20] for each equal-weight

[20]For brevity we report only the loadings from the four factor regression. Unreported three factor and

decile portfolio are reported in Table 2. The lowest (highest) BDI stocks are in *Portfolio -5* (*Portfolio +5*). In the same table we also include similar statistics for the long-short portfolio (*Portfolio +5/-5*) which is long is the highest decile, and short in lowest decile.

From the second column in Table 2, excess returns (returns over the risk-free rate) can be seen to be—for the most part—monotonically increasing as one moves from low to high BDI portfolios. Stocks in the lowest deciles earn average excess monthly returns of only 30bp per month, while those in the highest decile earn average excess returns of 79bp. The 49bp difference is statistically significant at the 5% level.

A similar pattern is evident in the CAPM alphas tallied in the third column: a general upward trend is easyly discerned. The 51bp difference in alphas between the top and bottom decile is similar to the difference in excess returns, and is again significant at the 5% level.

Including adjustments for additional known risk-factors does not alter the pattern significantly. In column four and five we report alphas from time-series regressions of characteristic selectivity as well as the alphas from the standard Carhart/Fama-French four factor regressions.[21] Both approaches correct for the usual factors that are known to explain the cross section of stock returns. In both cases the top BDI decile significantly outperforms the lowest BDI decile by 50–54bp; here, the differences are statistically significant at the 1% level. Thus it appears that the information of informed fund managers generates positive returns and does so without reliance on well-known investment strategies (alternatively, without loading on known risk factors).

A BDI-based investment strategy generates returns of approximately 5%–6% per year. Further, much of the returns are driven by stocks in the highest BDI deciles. Investing in the top BDI decile (*Portfolio +5*)) produces a risk-adjusted return of 30–40bp per month or approximately 4% per year. This is important as it may be costly—or impossible—to short certain stocks. It appears that a significant portion of the return predictability that we document is driven by informed managers choosing stocks that outperform the market and

CAPM loadings are very similar.

[21]We follow the procedure detailed in Daniel, Grinblatt, Titman, and Wermers (1997). For each one of our decile portfolios we construct a corresponding value-weighted characteristic-matched portfolio. We then regress the decile portfolio's returns *in excess of* the returns of the corresponding characteristic-matched portfolio on the four factors of Fama and French (1993) and Carhart (1997). We include the four factors as an additional control and omitting them does not materially affect the estimates. All data used to construc the characteristic-matched portfolios was kindly provided by Russ Wermers and obtained from his website: `http://www.rhsmith.umd.edu/faculty/rwermers/`.

not simply avoiding stocks that underperform.[22] This suggests that the information we have identified contains significant amount of "good news" unknown to the market rather than reflecting more generally-available but non-actionable knowledge.[23]

We now turn to the features of the the decile portfolios. In columns six thru nine of Table 2 we present the estimated factor loadings from the four-factor regressions. The loading on the market factor ($\hat{\beta}_{mkt}$) do not show any pronounced deviation from unity. The loadings on the small minus big factor ($\hat{\beta}_{SMB}$) follow a "U"-shaped pattern, with the most extreme deciles having the highest loadings while the loadings on the high-minus-low and momentum factors ($\hat{\beta}_{HML}$ and $\hat{\beta}_{UMB}$ respectively) follow an inverse "U" shape. That is, stocks in the middle deciles—those stocks on which the informed and uninformed managers agree—are bigger, have higher book-to-market, and have performed better in the past than the stocks on which the two groups of managers disagree. Informed and uniformed managers disagree most about small, growth stocks, with poor recent performance. This is entirely consistent with intuition.

Based on our information-based conjecture, we expect managers' superior information to relate primarily to smaller firms. Large firms are widely-held, widely scrutinized, and employ large numbers in their operations. For such firms, there are many ways for information to be revealed to the market. On the other hand, in smaller firms, there are fewer "loose lips" through which information can be disclosed. We investigate this further by considering the predictive power of BDI for stocks in different market-capitalization brackets. Each quarter, we sort stocks in the CRSP universe by market capitalization into one of three groups: Small Cap, Medium Cap, and Large Cap. We repeat our previous analysis for subsamples of stocks in each market-capitalization group. For example, in the Small Cap analysis, all our decile portfolios contain equal numbers of stocks from the Small Cap group. In Table 3 we report the results for each of the three groups; for brevity we only present the estiamtes for the long-short portfolios. The table clearly shows a monotonically decreasing alphas as one one moves to higher market capitalization groups: the BDI strategy performs best on smaller

[22] To some extent this is by construction: the securities in the manager's benchmark that the manager never purchases (presumably due to negative information) will not appear in our estimate of his benchmark portfolio. Therefore, a manager who has a very large aversion to particular security may appear to us to have no opinion.

[23] For example, it could be that a large majority of market participants are pessimistic and believe that that a certain security is over-priced. In the presence of absolute short-sale constraints, their beliefs play no role, and price is determined by the optimistic minority.

stocks. The pattern is consistent for all our measures of portfolio performance. If we consider only the smallest third of the CRSP universe, the strategy generates returns between 57bp and 72bp per month (depending on measure), all significant at either the 1% or 5% level.

5.2 Robustness

The results of our main analysis suggest that a BDI-based strategy produces large positive alphas at conventional significance levels. However, some have noted that alphas in mutual fund performance studies tend to have complex, non-normal distributions arising from the heteroegeneity in manager's risk-taking.[24] Although we have focused on the performance of a strategy based on information aggregated from multiple mutual fund managers, there remains a possibility that the mechanics of our strategy induces similar non-normalities and complexities in the distribution of alphas.

In Figure 6 we plot various diagnostics for the key regression (the long-short portfolio performance) of our main analysis. Panel B of Figure 6 plots the quantiles of the regression residuals against the Gaussian quantiles. The panel shows significant departures from normality: the residuals show pronounced fat tails. Given our relatively short sample (348 monthly return observations), such deviations from normality may lead some to question the validity of our standard errors estimates. To address this issue, we estimate confidence intervals for our estimates using non-parametric bootstrap.

We perform 2,000 bootstrap iterations of our main analysis. For each one of the iterations, we draw (with replacement) 348 observations from the time-series of decile portfolio and Fama-French factor returns. We do not use block-based sampling as Panels C and D of Figure 6 show no evidence of auto-correlation in the errors. With each bootstrapped sample, we repeat the regressions presented in Table 2. From this we retain the distribution of estimates. In Table 4 we present the average estimates, the 95% confidence intervals for each estimate, as well as indicators of significance at the conventional levels. The confidence intervals from the bootstrap are symmetric, and very similar to those implied by the Newey-West estimates; significance levels remain largely unchanged. It appears that the Newey-West error estimates are not unreasonable.

In addition to non-normality, Panel A of Figure 6 reveals that there is some heteroskedasticity in the residuals, particularly around the new millenium. To determine the extent to

[24]For an example, see Kosowski, Timmermann, Wermers, and White (2006)

which this affects our results, we repeat our main analysis for various subsamples, where each subsample excludes one five year period. In Table 5 we present the long-short portfolio returns for each of these subsamples. The sub-sample estimates are very similar to each other and to the full-sample results: they do not suggest that any particular period is driving the results.

In addition to the above statistical issues, we briefly consider alternative (mechanical) explanations for our results. One possibility is that managers engage in window-dressing by tailoring their top holdings to reflect recent winners. To the extent that such recent winners exhibit some systematic performance continuation (or reversal), one could argue that our approach is indirectly capturing these effects. To deal with this possibility we repeat our main analysis excluding managers' top ten holdings. Specifically, in calculating BDI for each security, we only include the beliefs of those managers for whom the security was not a top ten holding. The estimates from this analysis are presented in Table 6—they are nearly identical to those from the complete sample. Thus, it does not appear that manager's top holdings play a significant role in our results.

Another possibility is that fund manager holdings (i.e. portfolio weights), and not "revealed beliefs" are the true driver to our results. In this scenario, "informed" managers know (relatively speaking) which securities will perform well in the future—they load up on these securities and ignore the covariance of returns. To explore this possibility, we repeat our primary analysis using manager portfolio weights in place of revealed beliefs and present the results in Table 7. While the long-short BDI strategy based on untransformed portfolio weights generates statistically significant positive returns, the returns from such a strategy are approximately half as large as those from a long-short BDI strategy based on revealed beliefs. Along the same lines, it could be that managers are picking stocks with high expected returns and low idiosyncratic volatility. Table 8 presents the results of an analysis based on such an assumption: again, the long-short BDI strategy is successful, but alphas are considerably lower than those derived under the full mean-variance assumption. This suggests that fully accounting for the managers' optimization problem (i.e. considering covariances) improves the identification of underpriced securities.

5.3 Revealed Beliefs and Future Fund Returns

Thus far, we have focused on future stock returns conditional on the degree to which our "informed" managers agree on the stock being under-appreciated by the rest of the market. If such "informed" managers exist, their superior information should also be reflected in superior fund performance. Thus, why not focus on the funds?

The first problem is one of risk-aversion. An informed fund manager will trade off higher expected returns from tilting his portfolio to reflect the information against his desire to track his benchmark. Concern about benchmark tracking puts a limit on the size of the manager's bet and this reduces the sensitivity of fund returns to informedness. Thus, the power of statistical tests based on fund returns is limited.

Similarly, to the extent that informedness is expected to show up in fund returns, it is expected to show up in gross returns. Fund managers' ability to capture the gains accruing from informedness may eliminate most—if not all—excess performance in net returns. As the actual time series of fees incurred by each fund is not available,[25] gross fund returns are not observable at monthly frequencies.

A more fundamental problem is the definition of "performance" itself. For benchmark-tracking mean variance optimizing managers, an intuitive measure of performance is Jensen's alpha (Jensen (1968)). However, as has been often noted,[26] this measure is very sensitive to the choice of benchmark. As the manager's true benchmark is not known, there is considerable potential for bias in the estimation of performance with Jensen's alpha. Similar issues arise with alternative measures such as Sharpe or information ratios. In addition to this, there is the issue of whether beating a benchmark with well-known passive strategies (such as holding value stocks or past winners) represents performance.

By focusing on the returns of securities preferred by informed managers, we have largely avoided these issues. Here, we briefly consider fund performance, with our interest limited to the relationship between BAI and the typical performance measures.

In Table 9 we present the results of our standard portfolio performance analysis using fund returns. Here we regress BAI quintile portfolio net returns on the usual risk factors. In addition to the issues just alluded to, this approach does not aggregate managers' beliefs, it simply aggregates their net performance. As expected, portfolio performance shows a clear upward pattern as one moves to higher BAI quintiles. The informed funds (those in the

[25]Only annual summaries are provided in the CRSP mutual fund file.

[26]See for example, Roll (1978).

highest BAI quintile) outperform the rest by between 2–10bp per month depending on the measure; the smaller differences are not significant at conventional levels. A similar pattern emerges in benchmark-adjusted performance measures. Table 10 shows that Jensen's alpha, Sharpe ratio, and information ratio are almost uniformly increasing with BAI quintile.

The patterns in the performance measures suggests that high BAI managers have some superior infromation; however funds with superior information do not generate significantly higher returns for their investors. Although investors are unlikely to profit from investing in informed funds, the information held by informed funds appears to be able to generate significant abnormal returns.

6 Conclusion

This paper examines the information content of revealed beliefs of mutual fund managers. The revealed beliefs are backed out by reverse-engineering fund manger's portfolio optimization problem. Specifically, to elicit the revealed beliefs, we assume that each manager rationally optimizes over the risk return tradeoff relative to his own benchmark portfolio. The key idea is that managers tilt their portfolios toward stocks with better risk-return tradeoffs according to their private beliefs. Hence, by observing fund managers' holdings, one can determine whether their beliefs on future returns are accurate.

Based on these revealed beliefs, we measure the differences in beliefs between minority informed funds (those with higher correlated revealed beliefs with subsequent realized returns) and the rest, which is BDI. The evidence in this paper suggests that high BDI stocks (i.e. those favored by informed managers) outperform those with low BDI even after adjusting for the usual risk factors. Theoretically, the BDI measure contributes to the general finance literature by demonstrating it is possible to extract information contained in cross-sectional fund holdings by exploiting a portfolio optimization framework. Empirically, the result on the predictability of BDI over future stock returns suggest that in addition to adjusting for risk in their portfolio allocation decisions, professional money managers have access to informative (and unpriced) signals about future stock performance.

More fundamentally, the paper makes a unique contribution to the finance literature by introducing a revealed preference approach to measuring investor expectations. That is, instead of estimating investor expectation regarding risk and returns from historical returns, we show that it can be useful to back out investors' expectations regarding returns (and potentially risks) from their portfolio holdings. This revealed belief is forward-looking, inherently dynamic, and heterogenous among investors. This approach may be of great empirical importance for future work. For example, various strands of the market microstructure literature are built on the assumption that investors are heterogeneously or asymmetrically informed. Without a concrete measure of investor beliefs, most empirical tests of these theories are based on equilibrium price patterns, which can suffer from endogeneity and measurement error. Having a relatively direct measure of investor beliefs can help researchers identify the degree of information asymmetry at a point in time or among a set of investors. Similarly, the asset pricing literature often involves estimation of dynamically changing expected returns. The information provided in portfolio holdings on investor belief about expected future returns may improve existing estimation techniques, and hence have important implications for empirical asset pricing.

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Figure 1: Timeline

This figure shows the data-dependency time-line for our procedure. The time index, t, is in months and represents the last day of the month. The blocks below the time-line represent estimated quantities: the horizontal range of each block denotes the date range of data needed to construct the estimate. The vertical position of the block denotes dependencies among the estimates: each estimated qauntity can depend on quantities appearing above it. For example, when t is December 31, 1991 the figure indicates that our estimates of manager m's portfolio holdings, $\hat{\mathbf{w}}_{m,t}$ depend only on data from December 31, 1991. On the other hand, our estimate of manager m's benchmark index, $\hat{\mathbf{q}}_{m,t}$ depends on historical data *up to* December 31st (as well as the portfolio holdings).

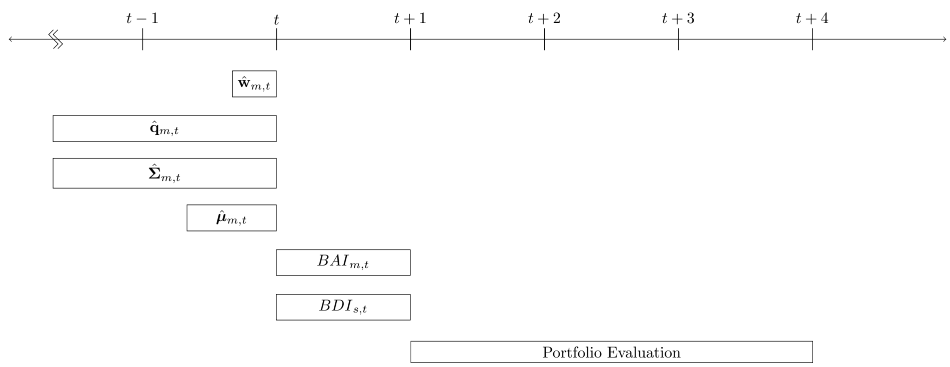

Figure 2: Residual Diagnostics

The following four panels show diagnostics for the residuals estimated in the regression of the +5/-5 portfolio returns on the four Carhart factors. Panel A presents the time-series of residuals. In Panel B, the residual quantiles are plotted against Gaussian quantiles. Panel C and D plot the residuals' auto-correlation and partial auto-correlation at various lags. The overall picture is one of heteroskedastic fat-tailed errors; there is little indication of significant auto-correlation.

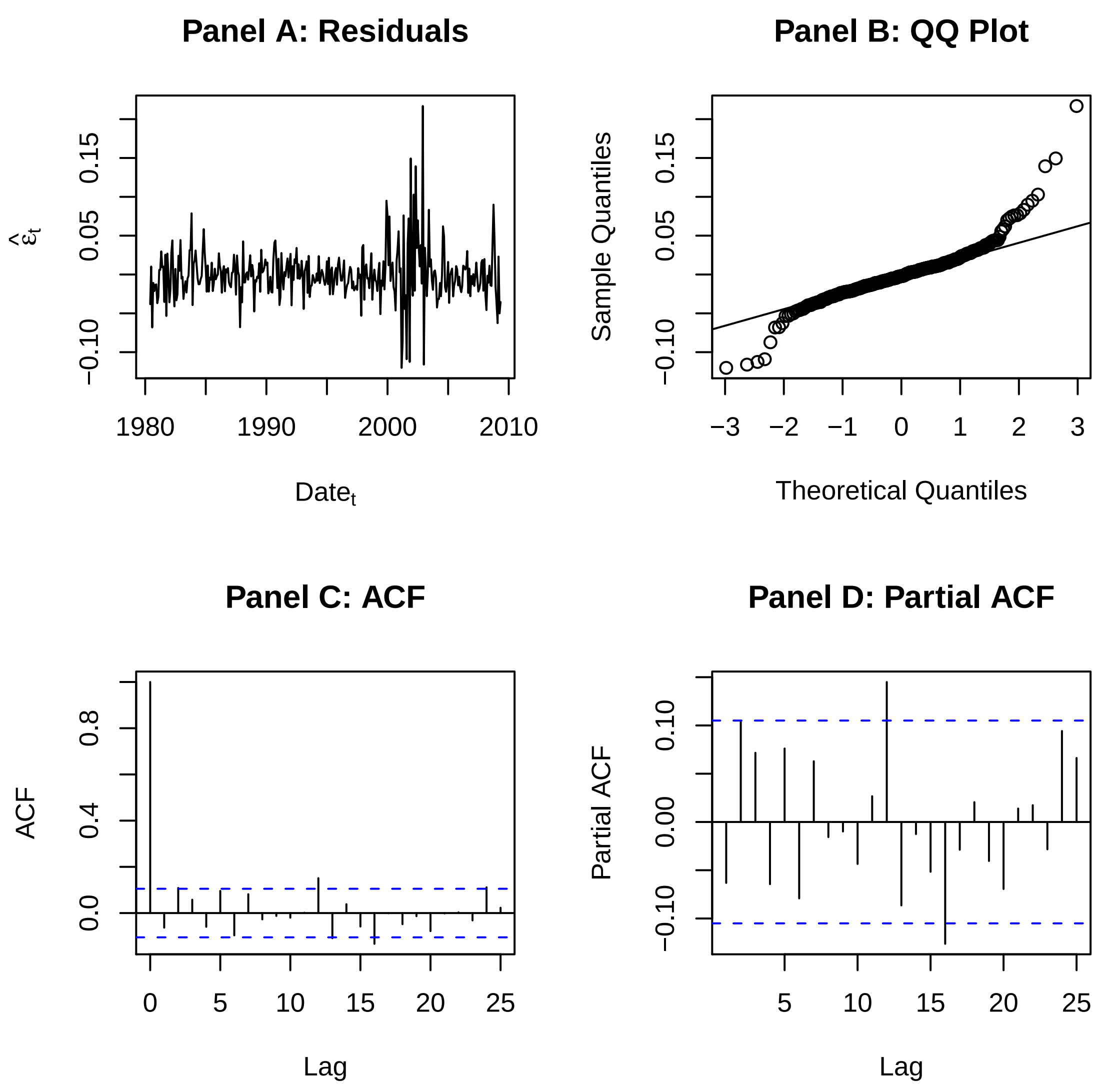

Table 1: Summary Statistics

We present descriptive statistics for every year in our sample. Starting with mutual fund holdings data provided by Thomson Financial, we eliminate non-equity funds (based on IOC code), funds that do not appear in the CRSP mutual fund monthly file, funds that hold fewer than 20 stocks, and funds that have less than $5 million in equity holdings. We report the number of funds, the number of distinct stocks held by the funds in our sample, the total market capitalization of the mutual funds in our sample, as well as the funds' average market capitalization. All statistics are calculated as of the end of the fourth quarter.

Year	# Funds	# Stocks	# Stocks (Fund Avg.)	MktCap ($B) (Total)	MktCap ($B) (Avg.)
1980	251	2147	61	37.06	0.15
1981	243	2300	61	32.70	0.13
1982	205	2427	63	37.62	0.18
1983	255	3108	70	55.11	0.22
1984	277	3234	68	57.67	0.21
1985	291	3481	72	78.54	0.27
1986	338	3750	75	90.56	0.27
1987	388	3771	77	105.59	0.27
1988	401	4019	84	112.20	0.28
1989	430	3959	81	130.04	0.30
1990	463	3673	82	133.66	0.29
1991	549	3948	90	207.66	0.38
1992	646	4122	99	271.39	0.42
1993	750	5532	118	359.82	0.48
1994	892	5985	120	415.43	0.47
1995	896	6320	122	581.17	0.65
1996	1284	6956	124	906.86	0.71
1997	1216	7014	125	1148.65	0.94
1998	1291	6318	119	1453.62	1.13
1999	1476	6098	117	1911.79	1.30
2000	1259	6133	123	1610.21	1.28
2001	1210	5513	116	1437.03	1.19
2002	1200	5298	123	1132.39	0.94
2003	1193	5060	128	1661.91	1.39
2004	1360	5029	134	2461.05	1.81
2005	1199	5097	134	2582.10	2.15
2006	1215	5075	129	2870.17	2.36
2007	1145	5187	144	2952.29	2.58
2008	1070	4890	147	1689.74	1.58

Table 2: BDI Decile Portfolios

One month into each quarter we sort stocks into deciles based on the difference in opinion between *informed* and *uninformed* fund managers. For each manager, we calculate the manager's (normalized) belief on returns for stocks in his benchmark. We then seperately calculate the average belief about each stock among the informed and uninformed managers. For a given stock, the difference in beliefs about that stock between the informed and uninformed managers is used to assign the stock into one of ten deciles. Stocks assigned to the +5 decile are stocks that informed managers expect to have much higher returns than do the uninformed managers. Stocks assigned to the -5 decile are expected to have higher returns by the uniformed managers. We report (in basis points per month) the average excess returns ($\bar{r}_{ex}$), CAPM alpha (α_1), DGTW characteristic selectivity alpha (α_{CS}), and the Carhart alpha (α_4), along with the loandings on the four Carhart factors for each of the ten (equal-weight) decile portfolios ($-5, ..., +5$), as well as a long-short portfolio long in the +5 decile, and short the -5 decile ($+5/-5$). Standard error estimates appear in brackets below each estimate. Next to each α estimate, we indicate with superscripts whether a two-sided t-test of the null hypothesis $H_0 : \alpha = 0$ is rejected at 10% (†), 5% (‡), or 1% (*) level.

Portfolio	$\hat{\bar{r}}_{ex}$	$\hat{\alpha}_1$	$\hat{\alpha}_{CS}$	$\hat{\alpha}_4$	$\hat{\beta}_{mkt}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{UMD}$
-5	30	-30	-12	-21	1.13	0.091	0.726	-0.238
	[42.24]	[18.66]	[10.70]	[12.68]†	[0.04]	[0.07]	[0.08]	[0.04]
-4	53	-4	20	7	1.09	0.105	0.602	-0.272
	[36.94]	[16.57]	[9.32]‡	[10.28]	[0.03]	[0.06]	[0.07]	[0.04]
-3	66	10	23	18	1.08	0.136	0.525	-0.243
	[34.46]†	[14.76]	[7.24]*	[8.79]‡	[0.02]	[0.06]	[0.08]	[0.04]
-2	63	9	13	10	1.07	0.160	0.480	-0.169
	[33.61]†	[15.41]	[7.22]†	[9.11]	[0.03]	[0.07]	[0.08]	[0.03]
-1	66	12	17	8	1.08	0.220	0.471	-0.135
	[32.72]‡	[13.94]	[6.68]‡	[7.86]	[0.02]	[0.07]	[0.08]	[0.03]
+1	68	16	13	12	1.04	0.201	0.486	-0.125
	[32.80]‡	[14.64]	[6.46]†	[8.56]	[0.03]	[0.07]	[0.06]	[0.04]
+2	77	24	17	19	1.06	0.237	0.533	-0.130
	[33.20]‡	[15.17]	[6.23]*	[8.98]‡	[0.03]	[0.07]	[0.08]	[0.03]
+3	82	28	24	22	1.06	0.228	0.650	-0.138
	[33.77]‡	[16.18]†	[7.62]*	[10.01]‡	[0.03]	[0.06]	[0.06]	[0.03]
+4	77	21	26	22	1.07	0.154	0.721	-0.183
	[35.88]‡	[16.96]	[9.05]*	[10.94]‡	[0.02]	[0.05]	[0.05]	[0.04]
+5	79	21	38	33	1.04	0.004	0.844	-0.237
	[38.86]‡	[19.23]	[12.31]*	[15.16]‡	[0.03]	[0.08]	[0.05]	[0.07]
+5/-5	49	51	50	54	-0.09	-0.088	0.117	0.001
	[18.87]‡	[19.96]‡	[18.55]*	[19.90]*	[0.05]	[0.12]	[0.09]	[0.10]

Notes: $N = 348$. Standard errors calculated using a Newey-West (Bartlett) kernel with a bandwidth of five months. The DGTW characteristic selectivity (CS) alphas (α_{CS}) are obtained by regressing the CS returns on the four Carhart factors; not surprising, these estimates are very similar to the (unreported) raw CS returns.

Table 3: BDI Decile Portfolios (by Market Capitalization)

At the end of each quarter we sort stocks based on the market capitalization into one of three groups: Small Cap, Medium Cap, and Large Cap. Each of the three groups contains one third of the stocks in the CRSP universe. For each one of the three groups, one month into each quarter, we sort stocks into deciles based on the difference in opinion between *informed* and *uninformed* fund managers. For each manager, we calculate the manager's (normalized) belief on returns for stocks in his benchmark. We then seperately calculate the average belief about each stock among the informed and uninformed managers. For a given stock, the difference in beliefs about that stock between the informed and uninformed managers is used to assign the stock into one of ten deciles. Stocks assigned to the +5 decile are stocks that informed managers expect to have much higher returns than do the uninformed managers. Stocks assigned to the -5 decile are expected to have higher returns by the uniformed managers. We report (in basis points per month) the average excess returns ($\bar{r}_{ex}$), CAPM alpha (α_1), DGTW characteristic selectivity alpha (α_{CS}), and the Carhart alpha (α_4), along with the loandings on the four Carhart factors for each of the ten (equal-weight) decile portfolios ($-5, ..., +5$), as well as a long-short portfolio long in the +5 decile, and short the -5 decile ($+5/-5$). Standard error estimates appear in brackets below each estimate. Next to each α estimate, we indicate with superscripts whether a two-sided t-test of the null hypothesis $H_0 : \alpha = 0$ is rejected at 10% (†), 5% (‡), or 1% (*) level.

Portfolio	$\hat{\bar{r}}_{ex}$	$\hat{\alpha}_1$	$\hat{\alpha}_{CS}$	$\hat{\alpha}_4$	$\hat{\beta}_{mkt}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{UMD}$
Small Cap +5/-5	57	58	68	72	-0.09	-0.266	-0.048	0.001
	[19.97]*	[19.72]*	[26.35]‡	[22.14]*	[0.05]	[0.10]	[0.07]	[0.08]
Medium Cap +5/-5	53	56	42	50	-0.07	0.099	0.128	0.003
	[21.36]‡	[22.65]‡	[25.40]†	[25.50]‡	[0.08]	[0.12]	[0.11]	[0.10]
Large Cap +5/-5	30	34	-9	11	-0.04	0.071	0.226	0.215
	[26.90]	[29.97]	[22.02]	[26.03]	[0.10]	[0.21]	[0.20]	[0.13]

Notes: $N = 348$. Standard errors calculated using a Newey-West (Bartlett) kernel with a bandwidth of five months. The DGTW characteristic selectivity (CS) alphas (α_{CS}) are obtained by regressing the CS returns on the four Carhart factors; not surprising, these estimates are very similar to the (unreported) raw CS returns.

Table 4: BDI Decile Portfolios (Bootstrap)

We apply non-parametric bootstrap to the analysis outlined in Table 2. For each one of our 2000 bootstrap iterations, we draw (with replacement) 348 observations from the time-series of portfolio and Fama-French factor returns. For each bootstrapped sample, we repeat the regressions described in Table 2. As before, we report (in basis points per month) the average excess returns ($\hat{\bar{r}}^*_{ex}$), CAPM alpha ($\hat{\alpha}^*_1$), DGTW characteristic selectivity alpha ($\hat{\alpha}^*_{CS}$), and the Carhart alpha ($\hat{\alpha}^*_4$), along with the loandings on the four Carhart factors for each of the ten (equal-weight) decile portfolios ($-5, ..., +5$), as well as a long-short portfolio long in the +5 decile, and short the -5 decile ($+5/-5$). Bootstrap estimates of each parameter's 95% confidence interval appear in brackets below each estimate. Next to each confidence interval estimate, we indicate with superscripts whether zero falls outside of the 90% (†), 95% (‡), or 99% (*) confidence interval.

Portfolio	$\hat{\bar{r}}^*_{ex}$	$\hat{\alpha}^*_1$	$\hat{\alpha}^*_{CS}$	$\hat{\alpha}^*_4$	$\hat{\beta}^*_{mkt}$	$\hat{\beta}^*_{HML}$	$\hat{\beta}^*_{SMB}$	$\hat{\beta}^*_{UMD}$
-5	30	-30	-11	-22	1.13	0.088	0.735	-0.232
	$[-40,97]$	$[-64,6]$	$[-35,12]$	$[-48,4]$	$[1.05,1.21]^*$	$[-0.05,0.23]$	$[0.62,0.87]^*$	$[-0.32,-0.14]$
-4	54	-4	20	6	1.09	0.103	0.606	-0.265
	$[-11,115]$	$[-32,26]$	$[-1,40]^\dagger$	$[-17,30]$	$[1.03,1.15]^*$	$[-0.04,0.25]$	$[0.51,0.71]^*$	$[-0.36,-0.16]$
-3	65	9	21	17	1.08	0.134	0.528	-0.238
	$[5,125]^\ddagger$	$[-17,36]$	$[4,39]^\ddagger$	$[-2,36]^\dagger$	$[1.03,1.13]^*$	$[0.02,0.25]^\ddagger$	$[0.44,0.64]^*$	$[-0.32,-0.15]$
-2	63	9	12	10	1.07	0.159	0.488	-0.166
	$[4,118]^\ddagger$	$[-15,31]$	$[-3,27]$	$[-8,27]$	$[1.03,1.12]^*$	$[0.05,0.27]^*$	$[0.41,0.59]^*$	$[-0.24,-0.10]$
-1	66	12	17	8	1.08	0.215	0.479	-0.132
	$[9,122]^\ddagger$	$[-10,33]$	$[3,31]^\ddagger$	$[-8,24]$	$[1.03,1.12]^*$	$[0.11,0.31]^*$	$[0.38,0.59]^*$	$[-0.18,-0.08]$
+1	68	16	12	12	1.04	0.199	0.492	-0.126
	$[11,123]^\ddagger$	$[-7,38]$	$[-1,26]^\dagger$	$[-4,28]$	$[0.99,1.09]^*$	$[0.11,0.29]^*$	$[0.42,0.58]^*$	$[-0.18,-0.07]$
+2	78	25	18	19	1.06	0.234	0.542	-0.128
	$[18,132]^\ddagger$	$[1,49]^\ddagger$	$[6,29]^*$	$[3,35]^\ddagger$	$[1.01,1.11]^*$	$[0.15,0.32]^*$	$[0.45,0.65]^*$	$[-0.19,-0.07]$
+3	82	28	24	22	1.06	0.224	0.654	-0.137
	$[24,140]^*$	$[1,55]^\ddagger$	$[11,37]^*$	$[5,40]^*$	$[1.01,1.12]^*$	$[0.13,0.31]^*$	$[0.57,0.75]^*$	$[-0.20,-0.07]$
+4	78	21	25	21	1.07	0.151	0.723	-0.181
	$[17,144]^\ddagger$	$[-10,50]$	$[8,42]^*$	$[1,43]^\ddagger$	$[1.02,1.12]^*$	$[0.07,0.23]^*$	$[0.63,0.81]^*$	$[-0.26,-0.10]$
+5	80	22	38	33	1.04	0.007	0.844	-0.235
	$[12,145]^\ddagger$	$[-13,60]$	$[16,61]^*$	$[5,61]^\ddagger$	$[0.99,1.10]^*$	$[-0.10,0.12]$	$[0.75,0.94]^*$	$[-0.34,-0.13]$
+5/-5	49	51	49	55	-0.09	-0.083	0.110	-0.002
	$[15,86]^*$	$[16,88]^*$	$[16,86]^*$	$[18,97]^*$	$[-0.19,0.02]$	$[-0.29,0.12]$	$[-0.06,0.26]$	$[-0.17,0.14]$

Notes: Here, our reported parameter estimates are the average estimates from all bootstrap replications, hence there are minor deviations from estimates reported in Table 2. Confidence intervals are quantiles from the bootstrap distribution of parameter estimates.

Table 5: BDI +5/-5 Portfolio (Performance under Various Subsamples)

We repeat the analysis of Table 2 seven times, each time excluding observations from a different five-year date range. We report (in basis points per month) the average excess returns ($\bar{r}_{ex}$), CAPM alpha (α_1), DGTW characteristic selectivity alpha (α_{CS}), and the Carhart alpha (α_4), along with the loandings on the four Carhart factors for the long-short portfolio long in the +5 decile, and short the −5 decile. Standard error estimates appear in brackets below each estimate. Next to each α estimate, we indicate with superscripts whether a two-sided t-test of the null hypothesis $H_0 : \alpha = 0$ is rejected at 10% (†), 5% (‡), or 1% (*) level. Out results appear to be fairly consistent across time periods.

Excluded Years	$\hat{\bar{r}}_{ex}$	$\hat{\alpha}_1$	$\hat{\alpha}_{CS}$	$\hat{\alpha}_4$	$\hat{\beta}_{mkt}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{UMD}$
1980 – 1982	55	58	57	63	-0.10	-0.102	0.118	-0.003
	[19.67]*	[20.82]*	[19.26]*	[20.66]*	[0.05]	[0.13]	[0.09]	[0.10]
1983 – 1987	47	48	47	52	-0.09	-0.121	0.137	-0.002
	[21.18]‡	[22.27]‡	[20.42]‡	[21.74]‡	[0.06]	[0.13]	[0.09]	[0.10]
1988 – 1992	45	46	48	50	-0.08	-0.083	0.152	-0.013
	[21.68]‡	[22.77]‡	[21.04]‡	[22.33]‡	[0.06]	[0.12]	[0.09]	[0.10]
1993 – 1997	55	57	55	60	-0.09	-0.091	0.120	0.002
	[21.62]‡	[22.47]‡	[20.72]*	[22.13]*	[0.05]	[0.14]	[0.09]	[0.10]
1998 – 2002	48	53	43	47	-0.09	0.067	0.071	0.037
	[18.45]‡	[19.30]*	[20.17]‡	[22.51]‡	[0.06]	[0.11]	[0.09]	[0.13]
2003 – 2007	52	54	49	55	-0.08	-0.085	0.127	0.020
	[20.84]‡	[21.69]‡	[19.50]‡	[21.49]‡	[0.05]	[0.13]	[0.09]	[0.11]
2008 – 2010	50	51	55	59	-0.06	-0.091	0.120	-0.036
	[18.87]*	[20.78]‡	[19.88]*	[21.15]*	[0.06]	[0.13]	[0.09]	[0.11]

Notes: Since our sample runs from 1980–2010, the first and last row of the table include slightly more observations.

Table 6: BDI Decile Portfolios (Excluding Manager's "Top Ten")

We repeat our main analysis exactly as described in Table 2, except for one modification: in calculating each stock's BDI, we do not consider the beliefs of fund managers for whom the stock is one of the top ten holdings. By doing so, we eliminate the most obvious channel through which window-dressing activity could influence are results. For each BDI decile portfolio, we report (in basis points per month) the average excess returns ($\bar{r}_{ex}$), CAPM alpha (α_1), DGTW characteristic selectivity alpha (α_{CS}), and the Carhart alpha (α_4), along with the loandings on the four Carhart factors for each of the ten (value-weight) decile portfolios ($-5, ..., +5$), as well as a long-short portfolio long in the $+5$ decile, and short the -5 decile ($+5/-5$). Standard error estimates appear in brackets below each estimate. Next to each α estimate, we indicate with superscripts whether a two-sided t-test of the null hypothesis $H_0 : \alpha = 0$ is rejected at 10% (†), 5% (‡), or 1% (*) level.

Portfolio	$\hat{\bar{r}}_{ex}$	$\hat{\alpha}_1$	$\hat{\alpha}_{CS}$	$\hat{\alpha}_4$	$\hat{\beta}_{mkt}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{UMD}$
-5	33	-27	-9	-19	1.12	0.087	0.736	-0.229
	[42.17]	[18.55]	[10.60]	[12.91]	[0.04]	[0.07]	[0.07]	[0.05]
-4	50	-8	17	3	1.09	0.114	0.581	-0.262
	[36.72]	[16.20]	[8.47]‡	[9.59]	[0.03]	[0.07]	[0.07]	[0.04]
-3	62	6	23	16	1.08	0.102	0.511	-0.243
	[34.65]†	[15.12]	[8.84]*	[9.25]†	[0.03]	[0.06]	[0.08]	[0.04]
-2	65	11	16	12	1.07	0.183	0.481	-0.173
	[32.90]‡	[14.81]	[6.26]‡	[8.58]	[0.02]	[0.07]	[0.08]	[0.03]
-1	66	13	14	9	1.07	0.232	0.462	-0.136
	[32.94]‡	[13.97]	[6.36]‡	[8.10]	[0.03]	[0.06]	[0.07]	[0.03]
+1	67	14	13	13	1.05	0.176	0.465	-0.140
	[32.74]‡	[14.49]	[6.93]†	[8.83]	[0.03]	[0.08]	[0.08]	[0.04]
+2	77	24	19	19	1.08	0.241	0.537	-0.141
	[34.01]‡	[16.16]	[7.40]‡	[9.78]†	[0.03]	[0.07]	[0.07]	[0.04]
+3	82	28	27	23	1.06	0.229	0.670	-0.142
	[33.57]‡	[16.05]†	[7.52]*	[9.31]‡	[0.03]	[0.05]	[0.06]	[0.03]
+4	74	18	23	20	1.06	0.166	0.743	-0.200
	[35.94]‡	[17.07]	[9.80]‡	[11.37]†	[0.03]	[0.05]	[0.05]	[0.05]
+5	80	21	41	34	1.04	0.000	0.841	-0.242
	[39.80]‡	[19.74]	[12.86]*	[15.54]‡	[0.03]	[0.08]	[0.06]	[0.07]
+5/-5	46	48	50	53	-0.08	-0.087	0.105	-0.013
	[18.97]‡	[20.01]‡	[19.09]*	[20.07]*	[0.05]	[0.12]	[0.08]	[0.10]

Notes: $N = 348$. Standard errors calculated using a Newey-West (Bartlett) kernel with a bandwidth of five months. The DGTW characteristic selectivity (CS) alphas (α_{CS}) are obtained by regressing the CS returns on the four Carhart factors; not surprising, these estimates are very similar to the (unreported) raw CS returns.

Table 7: BDI Decile Portfolios (Identity Variance Matrix))

We repeat the analysis of Table 2 but substitute the identity matrix for the estimated covariance matrix in the managers' optimization problem; equivalently, we assume managers maximize returns while minimizing variance under an assumption of independent and identically distributed returns. As before, we sort stocks into deciles with stocks assigned to the +5 decile being the stocks that informed managers expect to have much higher returns than do the uninformed managers.

We report (in basis points per month) the average excess returns ($\bar{r}_{ex}$), CAPM alpha (α_1), DGTW characteristic selectivity alpha (α_{CS}), and the Carhart alpha (α_4), along with the loandings on the four Carhart factors for each of the ten decile portfolios ($-5, ..., +5$), as well as a long-short portfolio long in the +5 decile, and short the -5 decile ($+5/-5$). Standard error estimates appear in brackets below each estimate. Next to each α estimate, we indicate with superscripts whether a two-sided t-test of the null hypothesis $H_0 : \alpha = 0$ is rejected at 10% (†), 5% (‡), or 1% (*) level.

Portfolio	$\hat{\bar{r}}_{ex}$	$\hat{\alpha}_1$	$\hat{\alpha}_{CS}$	$\hat{\alpha}_4$	$\hat{\beta}_{mkt}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{UMD}$
-5	66	14	5	2	1.03	0.227	0.636	-0.057
	[33.86]†	[13.85]	[5.76]	[7.65]	[0.02]	[0.04]	[0.04]	[0.03]
-4	59	4	10	4	1.09	0.190	0.580	-0.182
	[35.20]†	[12.80]	[6.33]	[7.10]	[0.01]	[0.04]	[0.06]	[0.03]
-3	57	1	14	4	1.11	0.170	0.538	-0.199
	[33.73]†	[13.16]	[5.49]‡	[6.70]	[0.02]	[0.05]	[0.07]	[0.03]
-2	60	1	17	8	1.13	0.167	0.564	-0.239
	[35.27]†	[14.81]	[6.95]‡	[7.78]	[0.02]	[0.05]	[0.06]	[0.03]
-1	62	3	20	17	1.12	0.107	0.572	-0.300
	[36.66]†	[14.71]	[6.92]*	[8.33]‡	[0.02]	[0.05]	[0.08]	[0.03]
+1	62	3	25	15	1.11	0.094	0.586	-0.264
	[35.95]†	[15.64]	[8.12]*	[9.40]	[0.03]	[0.05]	[0.06]	[0.03]
+2	67	8	23	20	1.10	0.111	0.598	-0.280
	[36.85]†	[16.13]	[8.98]‡	[10.09]‡	[0.02]	[0.04]	[0.07]	[0.03]
+3	74	17	22	22	1.08	0.132	0.652	-0.209
	[35.89]‡	[16.13]	[6.53]*	[9.29]‡	[0.02]	[0.04]	[0.06]	[0.03]
+4	79	24	28	27	1.04	0.123	0.656	-0.168
	[34.89]‡	[16.11]	[6.68]*	[9.13]*	[0.02]	[0.04]	[0.06]	[0.02]
+5	83	31	25	25	0.99	0.127	0.741	-0.079
	[36.23]‡	[17.15]†	[7.58]*	[9.89]‡	[0.02]	[0.04]	[0.04]	[0.03]
+5/-5	17	17	21	23	-0.04	-0.100	0.105	-0.022
	[10.19]†	[9.71]†	[8.99]‡	[10.31]‡	[0.02]	[0.05]	[0.04]	[0.03]

Notes: $N = 348$. Standard errors calculated using a Newey-West (Bartlett) kernel with a bandwidth of five months. The DGTW characteristic selectivity (CS) alphas (α_{CS}) are obtained by regressing the CS returns on the four Carhart factors; not surprising, these estimates are very similar to the (unreported) raw CS returns.

Table 8: BDI Decile Portfolios (Idiosyncratic Variance Matrix)

We repeat the analysis of Table 2 substituting the diagonal matrix of idosyncratic variances for the estimated covariance matrix in the calculation of managers' beliefs; that is, we assume that managers maximize returns while minimizing idisyncratic variance. As before, we sort stocks into deciles with stocks assigned to the +5 decile being the stocks that informed managers expect to have much higher returns than do the uninformed managers.

We report (in basis points per month) the average excess returns ($\bar{r}_{ex}$), CAPM alpha (α_1), DGTW characteristic selectivity alpha (α_{CS}), and the Carhart alpha (α_4), along with the loandings on the four Carhart factors for each of the ten decile portfolios ($-5, ..., +5$), as well as a long-short portfolio long in the +5 decile, and short the -5 decile ($+5/-5$). Standard error estimates appear in brackets below each estimate. Next to each α estimate, we indicate with superscripts whether a two-sided t-test of the null hypothesis $H_0 : \alpha = 0$ is rejected at 10% (†), 5% (‡), or 1% (*) level.

Portfolio	$\hat{\bar{r}}_{ex}$	$\hat{\alpha}_1$	$\hat{\alpha}_{CS}$	$\hat{\alpha}_4$	$\hat{\beta}_{mkt}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{UMD}$
-5	53	-7	8	-5	1.12	0.045	0.804	-0.134
	[38.74]	[15.84]	[7.33]	[8.86]	[0.03]	[0.03]	[0.05]	[0.03]
-4	57	0	13	4	1.09	0.158	0.647	-0.203
	[34.64]†	[15.73]	[6.32]‡	[8.61]	[0.02]	[0.04]	[0.07]	[0.02]
-3	57	0	8	4	1.11	0.179	0.553	-0.214
	[34.85]	[13.95]	[6.14]	[7.30]	[0.02]	[0.04]	[0.07]	[0.02]
-2	63	7	20	13	1.08	0.189	0.559	-0.253
	[34.16]†	[14.22]	[6.78]*	[7.74]†	[0.02]	[0.05]	[0.07]	[0.03]
-1	67	11	23	20	1.09	0.170	0.525	-0.267
	[34.80]†	[14.56]	[6.32]*	[8.26]‡	[0.02]	[0.05]	[0.06]	[0.03]
+1	63	9	14	11	1.07	0.230	0.474	-0.205
	[32.63]†	[14.04]	[5.32]*	[7.00]	[0.02]	[0.04]	[0.06]	[0.03]
+2	61	6	13	8	1.07	0.193	0.545	-0.200
	[33.76]†	[14.96]	[7.68]†	[8.54]	[0.02]	[0.06]	[0.07]	[0.03]
+3	79	24	27	26	1.05	0.144	0.571	-0.167
	[33.39]‡	[14.10]†	[6.30]*	[8.23]*	[0.02]	[0.05]	[0.07]	[0.03]
+4	81	27	22	25	1.05	0.136	0.645	-0.129
	[35.52]‡	[15.22]†	[6.67]*	[8.98]*	[0.02]	[0.05]	[0.05]	[0.02]
+5	87	30	39	30	1.07	0.090	0.816	-0.140
	[39.15]‡	[18.33]	[9.55]*	[11.39]*	[0.03]	[0.05]	[0.05]	[0.03]
+5/-5	34	37	32	35	-0.05	0.045	0.012	-0.006
	[9.06]*	[9.09]*	[10.38]*	[11.29]*	[0.04]	[0.05]	[0.03]	[0.03]

Notes: $N = 348$. Standard errors calculated using a Newey-West (Bartlett) kernel with a bandwidth of five months. The DGTW characteristic selectivity (CS) alphas (α_{CS}) are obtained by regressing the CS returns on the four Carhart factors; not surprising, these estimates are very similar to the (unreported) raw CS returns.

Table 9: BAI Quntile Performance

One month into each quarter ($t + 1$) we compute each fund's *belief accuracy index* (BAI). We then track the performance of funds in the different BAI quintiles for the next three months ($t + 2 ... t + 4$). Funds in the highest BAI quintile (Q1) had the lowest BAI score, while funds in the highest BAI quintile (Q5) had the highest. We report (in basis points per month) the average excess returns ($\bar{r}_{ex}$), CAPM alpha (α_1), DGTW characteristic selectivity alpha (α_{CS}), and the Carhart alpha (α_4), along with the loandings on the four Carhart factors for each of the BAI Percentiles, as well as a long-short portfolio long in the top 20% of (*informed*) funds, and short in the remaining (*uninformed*) funds. All returns are computed on an equal-weighted basis (each fund has equal weight). Standard error estimates appear in brackets below each estimate. Next to each α estimate, we indicate with superscripts whether a two-sided t-test of the null hypothesis $H_0 : \alpha = 0$ is rejected at 10% (†), 5% (‡), or 1% (*) level.

BAI Q'ile	$\hat{\bar{r}}_{ex}$	$\hat{\alpha}_1$	$\hat{\alpha}_{CS}$	$\hat{\alpha}_4$	$\hat{\beta}_{mkt}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{UMD}$
Q1	43	-4	26	-6	0.96	0.046	0.116	-0.020
	[19.15]‡	[3.83]	[2.18]*	[3.84]	[0.01]	[0.03]	[0.04]	[0.02]
Q2	44	-4	29	-5	0.96	0.004	0.146	-0.003
	[19.13]‡	[3.33]	[2.20]*	[3.08]	[0.01]	[0.02]	[0.02]	[0.01]
Q3	50	2	30	-1	0.97	0.007	0.179	0.015
	[19.26]*	[3.50]	[2.23]*	[2.93]	[0.01]	[0.02]	[0.02]	[0.01]
Q4	50	2	32	-2	0.97	-0.003	0.186	0.030
	[19.20]*	[4.04]	[2.31]*	[3.17]	[0.01]	[0.02]	[0.02]	[0.01]
Q5	56	9	31	4	0.95	0.022	0.205	0.032
	[19.52]*	[5.53]†	[2.40]*	[4.25]	[0.01]	[0.03]	[0.02]	[0.02]
+Q5/-Q1..4	9	10	2	7	-0.02	0.008	0.048	0.027
	[5.60]†	[5.63]†	[2.34]	[5.29]	[0.01]	[0.04]	[0.04]	[0.02]

Notes: $N = 348$. Standard errors calculated using a Newey-West (Bartlett) kernel with a bandwidth of five months. The DGTW characteristic selectivity (CS) alphas (α_{CS}) are obtained by regressing the CS returns on the four Carhart factors. To calculate CS, we use the stock performance of reported fund holdings, for all other measures the fund performance (from the CRSP mutual fund file) is used.

Table 10: BAI Quintile Benchmark-Adjusted Performance

One month into each quarter ($t+1$) we compute each fund's *belief accuracy index* (BAI). We then track the performance of funds in the different BAI quintiles for the next three months ($t+2...t+4$). All funds have equal weight. Funds in the highest BAI quintile (Q1) had the lowest BAI score, while funds in the highest BAI quintile (Q5) had the highest. We report each quintile's Jensen's alpha (in basis points per month), Sharpe ratio and information ratio.

BAI Q'ile	Jensen's-α	Sharpe Ratio	Info. Ratio
Q1	88	0.46	0.31
Q2	84	0.44	0.32
Q3	87	0.47	0.36
Q4	80	0.45	0.35
Q5	85	0.50	0.40

Notes: $N = 348$. Fund returns are from the CRSP mutual fund file. Each quintile's benchmark portfolio is constructed by summing the consituent funds' benchmark portfolios (and normalizing). Each fund's benchmark is estimated as the value-weighted portfolio of securities that have been held by the fund at any point in the previous five years.