Feedback Effects and Asset Prices*

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ABSTRACT

Feedback effects from asset prices to firm cash flows have been empirically documented. This finding raises a question for asset pricing: How are asset prices determined if price affects the fundamental value, which in turn affects the price? In this environment, by buying assets that others are buying, investors ensure high future cash flows for the firm and subsequent high returns for themselves. Hence, investors have an incentive to coordinate, which may generate self-fulfilling beliefs and multiple equilibria. Using insights from global games, we pin down investors' beliefs, analyze equilibrium prices, and find strong feedback leads to higher excess volatility.

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According to traditional valuation models, a firm's stock price is determined by its exogenously given cash flow. However, recent finance literature has questioned whether the cash flow is exogenous. For example, Subrahmanyam and Titman (2001) argue that a firm's stock price affects how the firm is perceived by its customers, suppliers, employees, lenders, and other stakeholders. In turn, these perceptions influence their purchase, supply, or investment decisions, which ultimately affect the firm's cash flow. This direct feedback from asset prices to asset cash flows also has been supported by recent empirical findings.[1]

In this paper, we allow for endogenous cash flows in an asset pricing model to address the following question: How do we determine asset prices in the presence of feedback effects? Answering this question is not an easy task since feedback effects generate incentives for investors to strategically coordinate their actions. For example, in the stock market, it may be optimal for some agents to buy when others are buying and sell when others are selling in order to affect the stock price and the subsequent cash flows. Such coordination entails forming beliefs about the actions of others. These beliefs may be self-fulfilling, introducing the possibility of multiple equilibria. Therefore, it is difficult to pin down investors' beliefs and solve for equilibrium prices in a model with feedback effects.

We overcome these difficulties by using the insights developed in Carlsson and van Damme (1993), and Morris and Shin (1998, 2002, 2003). These papers show that self-fulfilling beliefs arise from the common knowledge of the underlying fundamentals. They find when there is information heterogeneity among agents so that the fundamentals are no longer common knowledge, beliefs are uniquely determined.[2]

To generate information heterogeneity, we model an environment where a fraction of the investors have heterogenous private information and the rest are uninformed. Essentially, we use a rational expectation equilibrium (REE) model of asset prices, with an exogenous liquidity shock to prevent prices from being fully revealing, as in Hellwig (1980) and Grossman and Stiglitz (1980). To model feedback effects, we depart from these models by allowing informed investors to affect the asset value through their aggregate investment. Consequently, in our model, individual informed investors need to form beliefs about the size of the informed investor aggregate investment based on both their private information and stock prices before making the investment decision on the risky asset.[3]

We find that feedback effects are a significant source of excess volatility which we define as the sensitivity of price to non-fundamental shocks.[4] The comparative statics on excess volatility generate several empirical predictions. Some of these predictions explain existing empirical findings such as higher liquidity leads to lower excess volatility. Others are unique to our model. These unique predictions can be used to test whether feedback effects are strong enough to generate first-order asset pricing implications. For example, we predict stocks with higher feedback effects should exhibit higher excess volatility. This prediction suggests that stocks with higher institutional ownership will have higher excess volatility, since empirically we observe that institutional investors are on average better informed (and therefore have stronger coordination incentives). We also predict that more precise information (which could be proxied for by the inverse of the dispersion of analysts' forecasts) should lead to larger (less) excess volatility for illiquid (liquid) stocks.

The same forces that generate excess volatility may also lead to price multiplicity. Price multiplicity can be viewed as an extreme form of excess volatility since it induces price movements that are unrelated to fundamentals. We find that for multiplicity to occur, three conditions must be met. The first condition is intuitive; multiplicity requires a strong feedback effect. The second and third conditions, respectively, require that private signals must be precise and the exogenous liquidity must be low. These two conditions are also intuitive. A precise public signal leads to multiplicity by facilitating coordination. The precision of price as an endogenous public signal depends on both the precision of private signals and the level of liquidity.[5] We also calibrate the model using parameters commonly used in the literature and find price multiplicity is unlikely in real world financial markets.

This paper also contributes to the REE literature by identifying feedback effects as a source of price multiplicity. In REE models, asset prices clear the market and provide information regarding the underlying value of fundamentals. When asset prices clear the market, assets that many investors buy become expensive, and thus less desirable to other investors. The result that each investor wants to choose assets others are not choosing is what we call the "substitution effect." The opposite effect also arises because when an asset has a high price, it is likely that some informed investors have news that the future payoff of the asset will be high. The prospect of high future payoffs makes the asset more desirable to other investors. The result that high demand can push up the price and make other investors demand more is what we call the "information effect."[6] Price multiplicity arises when the information effect overwhelms the substitution effect causing demand for the asset to rise in the price level (i.e., a Giffen asset). The existing REE literature finds the non-linear learning of uninformed investors may lead to greater information effects from uninformed investors, resulting in price multiplicity (Gennotte and Leland (1990), Bhattacharya and

Spiegel (1991), Barlevy and Veronesi (2003), and Yuan (2005)). We find that feedback effects also lead to a heightened information effect, but for a different reason. In asset markets with feedback effects, prices are informative about both the fundamentals and the likelihood of coordination among informed investors. Our analysis shows that multiplicity occurs when the price is more informative of likelihood of coordination, rather than fundamentals. When private signals become more precise, prices aggregate across private signals and come close to fully revealing the fundamentals. In illiquid markets, this leads to a precise forecast of other informed investors' actions, resulting in price multiplicity. However, in liquid markets, even if private signals come close to fully revealing the fundamentals, it is difficult for individual informed investors to form sufficiently sharp beliefs regarding other informed investors' actions. Consequently, coordination on prices is difficult and price multiplicity does not arise. This finding is in line with the empirical observation that liquid asset markets in developed countries are remarkably stable.

We extend our model to allow uninformed investors to learn from asset prices. This extension introduces two competing effects on price multiplicity. On the one hand, uninformed investors increase their demand when the price reflects a higher likelihood of coordination among informed investors. This results in a higher price and thus a higher cost of coordination for informed investors, which makes price multiplicity unlikely. On the other hand, if uninformed investors exhibit a non-linear inference about the coordination component, price multiplicity can occur even when the informed investors' demand is downward sloping.

Our study relates to work of Angeletos and Werning (2006), Morris and Shin (2006), Hellwig, Mukherji, and Tsyvinski (2006), and Tarashev (2005). Angeletos and Werning (2006) are the first to introduce an endogenous public signal into a coordination game. They find when investors in a coordination game observe endogenous public signals formed in a separate market, multiplicity in equilibrium prices arises robustly. Morris and Shin (2006), who analyze the case where private signals are multidimensional but public signals are one-dimensional, show endogenous public signals may not restore common knowledge. Hellwig, Mukherji, and Tsyvinski (2006) and Tarashev (2005) consider the coordination problem within the currency market. Their focus is different from ours since they study speculative attacks while we analyze asset prices. More important, we differ from these studies by accounting for the fact that prices play a substitution role. In our model, investors want to coordinate to buy assets that others are buying but at the same time they would like to substitute away from assets that are too expensive. We show that this tradeoff moderates strategic complementarities that arise in pure coordination games.[7]

Our study also relates to the growing theoretical literature on feedback effects such as Leland (1992), Khanna, Slezak, and Bradley (1994), Dow and Gorton (1997), Boot and Thakor (1997), and Subrahmanyam and Titman (1999). These papers focus on how financial markets affect firms' investment and capital allocation decisions in presence of feedback. Recently, Goldstein and Guembel (forthcoming) find feedback effects may expose firms to market manipulation. Hirshleifer, Subrahmanyam, and Titman (forthcoming) illustrate how irrational traders can prevail when there is a feedback effect from asset prices to cash flows. Khanna and Sonti (2002) show that a "bubble-like" price movement may arise.

The remainder of the paper is organized as follows. In Section 2, the baseline model setup for an economy with one risky asset is developed. In Section 3, we study the coordination problem among heterogeneously informed investors who observe an endogenous public signal, the asset price. In Section 4, we explore the case when uninformed investors make inferences from asset prices. Section 5 concludes. All proofs are provided in the Appendix.

I. The Model Setup

In this section, we introduce the baseline model. We build on a noisy rational expectation equilibrium (REE) model of asset prices with informed and uninformed investors, as in Hellwig (1980) and Grossman and Stiglitz (1980), where a noisy demand shock prevents prices from fully revealing private information. We choose minimal departures from these models in studying the feedback effect. We first describe the assets in the model. We then introduce investors and the information structure.

Assets

We consider a one-period economy with two assets, a riskless bond and a risky asset. For simplicity, we use the bond as the numeraire. Hence, its price remains at one and the risk-free rate is zero. The risky asset can be thought of as a common stock, an equity claim on a firm. The risky asset has an aggregate supply of M, where $M > 0$. It has a risky terminal payoff which consists of two components, $\tilde{V} + f\left(X, \tilde{\theta}\right)$.

The first risky component of the dividend payoff, $\tilde{V}$, can be regarded as the payoff from the firm's regular, e.g., bricks-mortar, operations, where no R&D is needed. We let $\tilde{V} = \bar{V} + \sigma_v \tilde{\epsilon}_v$, where σ_v is a positive constant, and $\tilde{\epsilon}_v$ is a standard normal (with zero mean and unit volatility). For expositional convenience, we set $\bar{V} = 0$. However, our results can easily be extended to a non-zero $\bar{V}$. The second risky component of the dividend payoff, $f\left(X, \tilde{\theta}\right)$, comes from the stochastic payoff of the firm's new technology innovation. Here, $\tilde{\theta}$ is the fundamental value of the innovation and is drawn from the uniform distribution over the real line (an improper prior). X is the amount invested in the risky asset by informed investors.[8] We assume that $f\left(X, \tilde{\theta}\right)$ is positively related to both X and $\tilde{\theta}$, that is, $f_X > 0$, $f_{\tilde{\theta}} > 0$. This assumption captures the strategic complementarity among informed investors: The value of the equity is higher when more informed investors purchase it. $\tilde{\theta}$ and $\tilde{\epsilon}_v$ are independently distributed.

The dependence of the terminal value on X, captured by $f(X, \tilde{\theta})$, reflects the feedback effect. For example, if managers learn from informed investors in making real investment decisions, then their decisions, and in turn the terminal value of the risky asset, will be affected by the investment from informed investors, X, which aggregates heterogenous private information from informed investors.[9]

Investors

We assume that there are two types of investors in this economy: informed and uninformed investors.

Informed investors belong to a measure-one continuum, indexed by $i \in [0, 1]$. They have access to an information-production technology. This technology enables each informed investor to acquire a noisy private signal, $\tilde{s}_i$, at time 0 about $\tilde{\theta}$, the potential payoff of the new technology: $\tilde{s}_i = \tilde{\theta} + \sigma_s \tilde{\epsilon}_i$, where $\tilde{\epsilon}_i$ is uniformly distributed on $[-1, 1]$.[10] We denote the density of this uniform distribution on $[-1, 1]$ by h. Conditional on $\tilde{\theta}$, the signals are independently identically distributed across informed investors. We further assume that each informed investor is restricted to trade $x(i) \in [0, z]$, where z is a fixed number and $z \geq 1$. This position limit can be caused by limited capital and/or borrowing constraints faced by informed investors.[11] We denote the total demand from informed investors by $X = \int_0^1 x(i)\, di$. To study their strategic interaction, we assume informed investors are risk neutral and seek to maximize their expected profit. An informed investor's utility from buying $k \in [0, z]$ units of the asset is given by $\left(f\left(X, \tilde{\theta}\right) - P\right) k$, where $f\left(X, \tilde{\theta}\right)$ is the dividend payoff from the asset and P is the price of the asset. Because of risk neutrality, an informed investor either invests up to the position limit, z, or does not invest at all; therefore, the total demand, X, depends on the fraction of informed investors investing in the asset as well as the position limit, z.

Uninformed investors, occupying a measure-w continuum, are mean-variance investors with the same risk aversion parameter, ρ. They have the following aggregate demand curve for the risky asset:

$$L(P) = w \frac{(E(\tilde{V}) - P)}{\rho Var(\tilde{V})}. \tag{1}$$

According to this demand curve, uninformed investors provide liquidity in the market. When the price falls below $E(\tilde{V})$, uninformed investors will buy the asset. The slope of this demand curve is $w/(\rho Var(\tilde{V}))$, which we denote by $1/\lambda$.[12]

Finally, we assume there is a noise demand shock in the market, as in Grossman and Stiglitz (1980) and a host of other models in the asymmetric information literature. This assumption introduces noise in the information aggregation process and prevents the market-clearing price from fully revealing the fundamentals. More specifically, we assume that the noise demand is $\sigma_y \tilde{y}$, where $\sigma_y > 0$ and $\tilde{y}$ is a standard normal random variable independent of $\tilde{\epsilon}_v$, $\tilde{\theta}$, and $\tilde{\epsilon}_i$ for all i.[13] We use a standard measure of market liquidity which is the variance of the exogenous liquidity shock, σ_y^2.

II. Asset Prices in the Presence of Feedback Effects

In this section, we first characterize the equilibrium solution with feedback effects. We show that informed investors' beliefs and strategies are uniquely determined when their information set contains the private signal, $\tilde{s}$, and price, P. Next, to illustrate the properties of equilibrium prices, we study the sensitivity of aggregate demand to price changes and price sensitivity to non-fundamental shocks (i.e., excess volatility). We show that feedback effects strengthen the information effect because the price is informative not only of the fundamentals but also the likelihood of coordination among informed investors; hence, feedback effects are a significant source of excess volatility. We then derive comparative statics results for excess volatility and find higher excess volatility when feedback effects are stronger, and, for illiquid assets, when private information is more precise. Lastly, we demonstrate price multiplicity arises when the strengthened information effect exceeds the substitution effect and establish conditions for price multiplicity and uniqueness. When calibrating the model using parameter values commonly adopted in the literature to mimic the reality, we find price multiplicity is an extreme case.

A. Equilibrium with Feedback Effects

First, we introduce the definition of equilibrium. In this definition, the feedback effect, $f(X, \tilde{\theta})$, is of a general form.[14]

DEFINITION 1: [Equilibrium] An equilibrium consists of a price function, $P(\tilde{\theta}, \tilde{y})$, strategies, $\pi(\tilde{s}_i, P) : \mathbb{R}^2 \rightarrow [0, 1]$, and the corresponding aggregate demands, $L(P)$ and $X(P, \tilde{\theta})$, such that:

- For informed agent i, $\pi(\tilde{s}_i, P) \in argmax_\pi z\pi E\left[f\left(X\left(P, \tilde{\theta}\right), \tilde{\theta}\right) - P|\tilde{s}_i = s_i, P\right]$.
- Uninformed investor demand, $L(P)$, is given by $w(E(\tilde{V}) - P)/(\rho Var(\tilde{V}))$.
- The market clearing condition is satisfied: $X\left(P, \tilde{\theta}\right) + L(P) + \sigma_y \tilde{y} = M$.

A *monotone equilibrium (with cutoff strategies)* is an equilibrium where $\pi(\tilde{s}_i, P) = 1$ if $\tilde{s}_i \geq g(P)$ for some function $g(P)$, and $\pi(\tilde{s}_i, P) = 0$ otherwise. In words, in a monotone equilibrium an informed investor buys the asset if and only if her private signal exceeds a cutoff $g(P)$.

Recall that informed investor i's signal is $\tilde{s}_i = \tilde{\theta} + \sigma_s \tilde{\epsilon}_i$, where $\tilde{\epsilon}_i$ is uniformly distributed on $[-1, 1]$. Consider a situation where all informed investors follow a cutoff strategy, that is, they buy if $\tilde{s}_i \geq g(P)$, or equivalently, if $\tilde{\epsilon}_i \geq (g(P) - \tilde{\theta})/\sigma_s$. Their aggregate demand can be characterized by considering three cases. First, if $\tilde{\theta} < g(P) - \sigma_s$, all informed investors receive signals below $g(P)$ and their aggregate demand is zero. Second, if $\tilde{\theta} > g(P) + \sigma_s$, all informed investors receive signals above $g(P)$ and they each demand z units of the asset. Since the size of informed investors is normalized to one, their aggregate demand in this case is z. Finally, if $g(P) - \sigma_s \leq \tilde{\theta} \leq g(P) + \sigma_s$, the proportion of informed investors who receive signals above $g(P)$ is $\left(1 - \left(g(P) - \tilde{\theta}\right)/\sigma_s\right)/2$, and their aggregate demand is z times this proportion. Thus, in a monotone equilibrium informed investors' aggregate demand is given by:

$$X\left(P, \tilde{\theta}\right) = \begin{cases} 0 & \text{if } \tilde{\theta} < g(P) - \sigma_s \\ \frac{z}{2}\left(1 - \frac{g(P) - \tilde{\theta}}{\sigma_s}\right) & \text{if } g(P) - \sigma_s \leq \tilde{\theta} \leq g(P) + \sigma_s \\ z & \text{if } g(P) + \sigma_s < \tilde{\theta} \end{cases}. \tag{2}$$

Substituting the uninformed investor aggregate demand $L(P)$ into the market clearing condition we obtain:

$$P = \lambda X + \lambda \sigma_y \tilde{y} - \lambda M, \tag{3}$$

where λ measures the price impact of a marginal change in aggregate demand. Combining Equations (2) and (3), we see that market clearing prices satisfy

$$P = \begin{cases} \lambda\sigma_y\tilde{y} - \lambda M & \text{if } \tilde{\theta} < g(P) - \sigma_s \\ \frac{\lambda z}{2}\left(1 - \frac{g(P) - \tilde{\theta}}{\sigma_s}\right) + \lambda\sigma_y\tilde{y} - \lambda M & \text{if } g(P) - \sigma_s \leq \tilde{\theta} \leq g(P) + \sigma_s \\ \lambda z + \lambda\sigma_y\tilde{y} - \lambda M & \text{if } g(P) + \sigma_s < \tilde{\theta} \end{cases}. \tag{4}$$

From Equation (4), we observe that the market clearing prices are uninformative about $\tilde{\theta}$ when $\tilde{\theta} < g(P) - \sigma_s$ or when $g(P) + \sigma_s < \tilde{\theta}$. However, for intermediate values of the asset fundamentals, that is when $g(P) - \sigma_s \leq \tilde{\theta} \leq g(P) + \sigma_s$, we find:

$$\tau \equiv \left(\frac{2\sigma_s}{\lambda z}P + \frac{2\sigma_s}{z}M + g\left(P\right) - \sigma_s\right) = \tilde{\theta} + \frac{2\sigma_s\sigma_y}{z}\tilde{y}, \tag{5}$$

which is observable to informed investors and is uncorrelated with their private signals conditional on $\tilde{\theta}$. Hence, τ is a sufficient statistic for the information in P in this intermediate region. It is distributed normally with a mean of $\tilde{\theta}$ and a standard deviation of $2\sigma_s\sigma_y/z$.

Note that the precision of τ, or the market clearing price, P, as a public signal for the fundamentals, $\tilde{\theta}$, is endogenously determined. It decreases with the variance of the exogenous private signal, σ_s^2, and the variance of the noise demand, σ_y^2. However, it increases with the size of the informed investors' position limit, z. Given the information conveyed in P, we can solve for the informed investors' cutoff strategy $g(P)$. Through the market clearing condition, we can solve for the equilibrium price(s). The following proposition characterizes the equilibrium.

Proposition 1:[The Monotone Equilibrium] Consider the game with incomplete information described in this section and its equilibrium as described in Definition 1. In this game,

- the informed investors' monotone equilibrium strategies are uniquely determined; that is, there is a unique function $g : \mathbb{R} \to \mathbb{R}$ such that the equilibrium strategies of informed investors are given by $\pi\left(\tilde{s}, P\right) = 1$ if $\tilde{s} \geq g(P)$ and 0 otherwise;

- in any monotone equilibrium, given a market clearing price P, the informed investors' aggregate demand, $X(P, \tilde{\theta})$, is uniquely characterized by Equation (2) and the uninformed investors' demand is given uniquely by $L(P) = -P/\lambda$;

- the equilibrium price $P(\tilde{\theta}, \tilde{y})$ satisfies Equation (4).

Proposition 1 indicates that, as in other REE models, a given price leads to a unique demand from informed investors for a realization of the fundamental. However, equilibrium prices may not be unique in our model. Multiplicity of equilibrium prices arises whenever the aggregate demand from both informed and uninformed investors, $X(P,\tilde{\theta})+L(P)$, has a backward-bending region where it is increasing in price (or equivalently, whenever Equation (4) has multiple solutions).[15]

In the following subsections, we provide more intuition for the equilibrium by analyzing the case where the dividend payoff is linear in X and $\tilde{\theta}$.

B. *Decomposing the Information Effect*

To understand the properties of equilibrium prices in our model, we decompose the information and substitution effects of prices by examining the linear case where $f(X,\tilde{\theta}) = \alpha X+\tilde{\theta}$. We start with a lemma that provides an explicit solution for the informed investors' equilibrium strategy.

LEMMA 1:[Equilibrium Cutoff Strategy] The monotone equilibrium cutoff strategy, $g(P)$, when the dividend payoff function is $f(X,\tilde{\theta}) = \alpha X+\tilde{\theta}$, is unique and is characterized by the following equation:

$$g(P) = P+\sigma_s - \left(\alpha+\frac{2\sigma_s}{z}\right)\left(M+\frac{P}{\lambda} - E\left(\sigma_y\tilde{y}\,\middle|\, M+\frac{P}{\lambda}-z \le \sigma_y\tilde{y} \le M+\frac{P}{\lambda}\right)\right). \quad (6)$$

To understand this result, the following illustration is helpful. Consider the informed investor who receives the cutoff signal, $\hat{s}=g(P)$. This investor must be indifferent between investing and staying out, which implies:

$$E[\alpha X+\tilde{\theta}|\mathcal{F}] - P = 0, \quad (7)$$

where $\mathcal{F} = \{\hat{s}, P\}$. Given her estimate of the amount of the risky asset held by informed investors, $E[X|\mathcal{F}]$, her estimate of $E[\tilde{\theta}|\mathcal{F}]$ can be computed using Equation (2):

$$E[\tilde{\theta}|\mathcal{F}] = \sigma_s\left(\frac{2E[X|\mathcal{F}]}{z}-1\right)+\hat{s}. \quad (8)$$

Substituting Equation (8) into Equation (7) and using the market clearing condition, we obtain:

$$\begin{aligned} g(P) &= \hat{s} = P+\sigma_s - \left(\alpha+\frac{2\sigma_s}{z}\right)E[X|\mathcal{F}] \quad (9)\\ &= P+\sigma_s - \left(\alpha+\frac{2\sigma_s}{z}\right)\left(M+\frac{P}{\lambda} - E\left[\sigma_y\tilde{y}|M+\frac{P}{\lambda}-z \le \sigma_y\tilde{y} \le M+\frac{P}{\lambda}\right]\right), \end{aligned}$$

which is equivalent to Equation (6) in Lemma 1.

To understand Lemma 1 intuitively, we next examine the price sensitivity of the informed investors' cutoff strategy. From Equations (8) and (9), this is given by:

$$\frac{\partial g(P)}{\partial P} = \underbrace{1}_{\text{Substitution Effect}} - \underbrace{\left(\underbrace{\alpha \frac{\partial E[X|\mathcal{F}]}{\partial P}}_{\text{Coordination Component}} + \underbrace{\frac{\partial E[\tilde{\theta}|\mathcal{F}]}{\partial P}}_{\text{Fundamental Component}} \right)}_{\text{Information Effect}}. \tag{10}$$

The terms in Equation (10) illustrate the standard substitution and information effects of price on the informed investors' optimal investment strategy. To see this, suppose that the asset price increases by one unit. Normally an informed investor would not purchase the risky asset unless her signal increased by one unit. This is the standard substitution effect. However, the increase in price also may signal a higher likelihood of coordination in investment (i.e., the coordination component of the dividend payoff) and better fundamentals (i.e., the fundamental component of the dividend payoff). Due to this information effect, she may purchase the risky asset with a lower signal even though the price has increased, i.e., $g(P)$ may be decreasing in P. To see this we write $\partial g(P)/\partial P$ as:

$$\frac{\partial g(P)}{\partial P} = 1 - \left(\alpha + \frac{2\sigma_s}{z}\right)\left(\frac{1 - \Lambda(M + P/\lambda, z, \sigma_y)}{\lambda}\right), \tag{11}$$

where

$$\Lambda(M + P/\lambda, z, \sigma_y) \equiv \lambda \frac{\partial E\left[\sigma_y \tilde{y} \,\middle|\, M + \frac{P}{\lambda} - z \leq \sigma_y \tilde{y} \leq M + \frac{P}{\lambda}\right]}{\partial P}.$$

The function, Λ, plays an important role in the inference problem faced by an informed investor. Specifically, conditional on a given price, Λ is the expected fraction of a marginal price change that is caused by noise trading. Consequently, $1 - \Lambda$ is the expected fraction of a marginal price change that is caused by aggregate informed trading.[16]

Equation (11) shows that the cutoff function, $g(P)$, is decreasing in P, when the feedback effect, α, is large enough, or when the private signals become noisy enough. Figure 1 illustrates the latter point showing that as σ_s gets larger, the decreasing region in $g(P)$ becomes more pronounced. Intuitively, with poor quality private signals, informed investors depend more on public information, P, to make inferences about fundamental values, which results in a heightened information effect. This is the flip side of the intuition provided in the existing coordination game literature, where more precise private information leads to less severe coordination problems, since agents depend more on their private information to make investment decisions (Morris and Shin (2003)).

[INSERT FIGURE 1 ABOUT HERE]

Next, we aggregate informed and uninformed investors' demand to study the price sensitivity of aggregate demand. From Equation (2), we see that for intermediate values of asset fundamentals, that is, when $\tilde{\theta} \in [g(P) - \sigma_s, g(P) + \sigma_s]$, $\partial X(P, \tilde{\theta})/\partial P = (-z/(2\sigma_s))(\partial g(P)/\partial P)$, the substitution and information effects are magnified by $z/(2\sigma_s)$ when they are aggregated across all informed investors.[17] Moreover, since uninformed investors do not learn from prices, their demand reflects only the substitution effect. They decrease their demand by $1/\lambda$ units when prices increase by one unit, that is, $\partial L(P)/\partial P = -1/\lambda$. Therefore, the following equation describes the price sensitivity of the overall aggregate demand curve in this region:

$$\frac{\partial(X(P,\tilde{\theta}) + L(P)))}{\partial P} = -\underbrace{\left(\underbrace{\left(\frac{1}{\lambda}\right)}_{\text{(from Uninformed)}} + \underbrace{\left(\frac{z}{2\sigma_s}\right)}_{\text{(from Informed)}} \right)}_{\text{Substitution Effect}} + \underbrace{\left(\underbrace{\frac{z\alpha}{2\sigma_s}\left(\frac{1-\Lambda(M+P/\lambda, z, \sigma_y)}{\lambda}\right)}_{\text{Coordination Component}} + \underbrace{\left(\frac{1-\Lambda(M+P/\lambda, z, \sigma_y)}{\lambda}\right)}_{\text{Fundamental Component}} \right)}_{\text{Information Effect (from Informed)}} \quad (12)$$

The decomposition in the above equation highlights the fact that feedback effects strengthen the information effect. A price increase in the presence of feedback effects not only signals a larger asset fundamental, but also a higher likelihood of investor coordination (e.g., the coordination component in Equation (12)). The latter effect is larger when the feedback effects (α) are stronger, the investment level (z) is higher, the distribution of investor signals (σ_s) is less dispersed, and the market liquidity (σ_y) is smaller. In the next three subsections, we study the implications of this strengthened information effect on excess volatility for asset prices and price multiplicity.

C. *Excess Volatility and Feedback Effects*

In this subsection, we show that feedback effects are a significant source of excess volatility when equilibrium price is unique. To do this, we use comparative statics relating excess volatility to the strength of coordination incentives (α), the dispersion of private information

(σ_s) and the level of liquidity (σ_y). To proxy for excess volatility, we use price sensitivity to non-fundamental shocks, $\partial P/\partial \tilde{y}$. The following lemma provides an explicit expression for this term.

LEMMA 2:[Price Sensitivity to Non-Fundamental Shocks] For intermediate values of the asset fundamentals, that is, when $g(P) - \sigma_s \leq \tilde{\theta} \leq g(P) + \sigma_s$, the sensitivity of price to external noise demand shocks is given by $\frac{\partial P}{\partial \tilde{y}} = \lambda \sigma_y / \left(1 + \frac{\lambda z}{2\sigma_s} \frac{\partial g(P)}{\partial P}\right)$.[18]

Using the expression for excess volatility given in the previous lemma, the next proposition generates empirically testable comparative statics for excess volatility.

PROPOSITION 2:[Excess Volatility]

(i) As the coordination incentive for informed investors increases, excess volatility increases (i.e., $\frac{\partial}{\partial \alpha} \frac{\partial P}{\partial \tilde{y}} > 0$).

(ii) As private information becomes more precise, excess volatility of price increases if and only if the feedback effect is strong and the market is sufficiently illiquid (i.e., $\frac{\partial}{\partial \sigma_s} \frac{\partial P}{\partial \tilde{y}} < 0 \Leftrightarrow \alpha > \lambda$ and $\sigma_y \leq \overline{\sigma}_y$, where $\overline{\sigma}_y$ satisfies $\lambda - \alpha(1 - \Lambda(M + P/\lambda, z, \overline{\sigma}_y)) = 0$).

(iii) As liquidity decreases, change in excess volatility of price is ambiguous, but when the feedback effect is strong, as the market becomes extremely illiquid, excess volatility of price increases (i.e., for $\alpha > \lambda$ and σ_y close to zero, $\frac{\partial}{\partial \sigma_y} \frac{\partial P}{\partial \tilde{y}} < 0$).

The main message of this proposition is that feedback effects are a significant source of excess volatility, especially when they are strong. This proposition together with our earlier results on price multiplicity yields several unique testable hypotheses that we discuss below.

First, part (i) of Proposition 2 shows that as the coordination incentive increases so does excess volatility.[19] This provides several hypotheses regarding the relationship between feedback effects and the cross-sectional stock return excess volatility. For example, institutional investors are typically better informed and therefore have stronger coordination incentives. This observation, together with part (i) of the above proposition, leads to an indirect test of whether feedback effects have a first-order effect on asset prices. Namely, it suggests that stocks with larger institutional ownership should have higher excess return volatilities. Following earlier literature (e.g., Shiller (1981) and LeRoy and Porter (1981)), excess volatility can be proxied by the volatility of stock returns in excess of the volatility of dividends or firm-level idiosyncratic volatility.

Second, parts (ii) and (iii) show that liquidity plays an important role in determining *when* feedback effects lead to excess volatility. Part (ii) of Proposition 2 indicates that when feedback effects are strong and the market is illiquid, more precise private information leads to higher excess volatility. This result may appear counter-intuitive, since more precise information should in general reduce volatility due to external noise shocks. However, in our setting a more precise private signal (or a less volatile noise demand) not only is more informative about the fundamental, but also leads to easier coordination, especially when the market is illiquid. This result also provides some unique testable predictions. For example, illiquid stocks with large institutional ownership should exhibit higher excess volatility as the dispersion of analysts' forecasts (which is a proxy for the noise level of the private signal) decreases, whereas the opposite should be true for liquid stocks.

Finally, part (iii) of Proposition 2 shows that higher liquidity does not necessarily decrease excess volatility, except for illiquid stocks with strong feedback effects. There is some evidence that liquidity is negatively related to excess volatility. Our result qualifies this finding and suggests conditions under which we should expect this relationship to be more pronounced.

Since price multiplicity can be viewed as another source of excess volatility, that is, volatility that is not caused by shocks to the asset fundamentals, in the next subsection, we establish conditions for price multiplicity and uniqueness.

D. Price Multiplicity and Feedback Effects

Price multiplicity occurs for some realizations of noise demand ($\tilde{y}$) if the aggregate demand, $X(P, \tilde{\theta}) + L(P)$, has a backward-bending region. The decomposition of price sensitivity of the aggregate demand in Equation (12) shows that aggregate demand has a backward-bending region when the aggregate information effect dominates the aggregate substitution effect. This happens only when the coordination component of the aggregate information effect is significantly large. We can observe this phenomenon algebraically from Equation (12). The substitution effect exhibited by the uninformed investors' demand, $1/\lambda$, is always larger than the fundamental component of the aggregate information effect. Thus, the key determinant of multiplicity in equilibrium prices is the balance of the substitution effect created from informed investor demand and the coordination component of the information effect (again from informed investor demand). Therefore, the channel that leads to price multiplicity is *not* the informed investors' inference about the fundamental component *but* rather about the coordination component.

The following proposition explicitly characterizes the conditions for the unique equilibrium price together with the limiting results.

PROPOSITION 3:[Price Multiplicity and Uniqueness] The following are equivalent:

(i) the equilibrium price is unique;

(ii) the aggregate information effect is smaller than the aggregate substitution effect;

(iii) $(\alpha + 2\sigma_s/z)\Lambda(M + P/\lambda, z, \sigma_y) > \alpha - \lambda$ for all P.

Moreover, as σ_y approaches zero, there are realizations of noise trading such that multiple equilibrium prices will occur. On the other hand, as σ_y approaches ∞, there is always a unique equilibrium price. As σ_s approaches ∞, the equilibrium price is always unique. However, as σ_s approaches zero, the equilibrium price is unique if and only if $\sigma_y \geq \overline{\sigma}_y$ where $\overline{\sigma}_y$ satisfies $\lambda - \alpha(1 - \Lambda(M + P/\lambda, z, \overline{\sigma}_y)) = 0$.[20]

From part (iii) of Proposition 3, it is clear that multiple equilibrium prices can occur only when the coordination incentive or the feedback effect, α, of a marginal change in the aggregate informed demand is stronger than the cost of coordination, that is, its price impact, λ. When $\alpha < \lambda$, the equilibrium price is always uniquely determined. To distinguish between these two cases, we use the following definition:

DEFINITION 2:[Strong vs. Weak Feedback] The feedback effect is *strong* when $\alpha > \lambda$ and *weak* otherwise.

Proposition 3 shows that when the feedback effect is weak, there is a unique equilibrium regardless of the precision of the private and public signals. However, when the feedback effect is strong, price multiplicity may arise. Figure 2 illustrates an example. In this example, the feedback is strong and the aggregate demand function has a backward bending region when the private signal is precise. In this case, for certain realizations of noise demand, price multiplicity arises.

[INSERT FIGURE 2 ABOUT HERE]

The limiting results in Proposition 3 highlight the channel through which price multiplicity arises in our model. The limiting results for *public* signals are intuitive. For example, when σ_y approaches ∞, price is an extremely noisy signal and thus the information effect vanishes, resulting in a unique equilibrium price. When σ_y approaches zero, the price fully reveals the fundamentals and thus the information effect dominates the substitution effect, resulting in multiple equilibrium prices.

By contrast, the limiting results for *private* signals are more subtle. When the noise in the private signal, σ_s, approaches ∞, the distribution of informed investors' signals becomes dispersed and coordination among informed investors becomes more difficult. Therefore, the coordination component of the information effect vanishes, leading to a domination of the substitution effect and a unique equilibrium price. However, when σ_s approaches zero, price multiplicity may or may not occur, depending on the liquidity of the market. In a liquid market, i.e., when σ_y is large, the price is less informative about informed investors' aggregate demand (X), even though it almost fully reveals $\tilde{\theta}$. Hence, coordination is difficult even when private signals are very precise. This coordination difficulty leads to a unique equilibrium price. Conversely, in an illiquid market, sharp inferences about informed investors' demand at a given price are possible, especially for a very small σ_s, resulting in multiple equilibrium prices. This is in contrast to findings in both Morris and Shin (2003) and Angeletos and Werning (2006).[21] The former finds a unique equilibrium and the latter finds price multiplicity regardless of liquidity levels. Comparisons across the limiting results of these studies are illustrated in Figure 3. The differences arise because in Morris and Shin (2003) the precision level of the public signal is exogenously given, and in Angeletos and Werning (2006) price does not have a substitution role in the coordination game.

[INSERT FIGURE 3 ABOUT HERE]

It is important to emphasize that the source of multiplicity in this setup is different from existing noisy REE models. For example, Yuan (2005) has shown that the information effect is always dominated by the substitution effect in a standard Grossman-Stigliz setup (1980), which implies that the equilibrium is unique. However, if there exists an additional source of uncertainty, such as borrowing or short-sales constraints (as in Yuan (2005) or in Barlevy and Veronesi (2003) together with distributions with large extreme tails), or programming trading status (as in Gennotte and Leland (1990)), the information effect (from uninformed investors) may dominate the substitution effect (from uninformed investors), leading to multiple equilibria. That is, in existing Grossman-Stigliz models, the non-linear inference by uninformed investors may give rise to multiple equilibrium prices only if there is an additional source of uncertainty. By contrast, in the current setup, multiplicity arises due to the strategic interaction among heterogeneously informed investors.

E. Price Multiplicity: Calibration Results

The existing literature argues that more informative prices may facilitate greater coordination, thus playing a destabilizing role. By accounting for the fact that prices play a substitution role as well, we find that when the substitution effect dominates the information effect, unique equilibrium is obtained. Hence, asset prices may have a limited role in aggregating private information, especially in a liquid market.

To gauge which effect is likely to prevail in actual financial markets, we calibrate the model adopting parameter values that are used in the literature. We run two sets of calibrations. In one set of calibrations, the parameter values follow those in Gennotte and Leland (1990) and in the other follow Yuan (2005). In these papers these parameters are chosen so that the risky asset can be interpreted as a stock market portfolio with an expected return of 6% and a standard deviation of about 20%.[22]

For each of these calibrations, we find the smallest ratio of precision of the price to the precision of the private signal above which the aggregate demand has an upward-sloping region. In other words, if the ratio of the precisions exceeds this cutoff value, price multiplicity may occur for some realizations of the noise demand. Since multiplicity can only occur when $\alpha > \lambda$, we experiment with values for α that are 5, 10, and 20 times λ. We find in our calibrations that the cutoff ratio is 2.44, 3.69, and 5.16 respectively using parameters in Yuan (2005) and 5.66, 8.65, and 20.66 respectively using parameters in Gennotte and Leland (1990). These calibration results suggest that prices have to be extremely informative for price multiplicity to occur. For comparison, this ratio is 0.0001 in Yuan (2005) and 0.55 in Gennotte and Leland (1990).

III. Learning from Prices by Uninformed Investors

In this section, we consider the case where all investors, including uninformed investors, condition their demand on the price of the risky asset. Thus, asset prices both coordinate informed investors' beliefs and transmit information to uninformed investors. By considering this case, we essentially endogenize the price impact of a marginal change in aggregate informed demand. Recall that, such price impact is measured by λ in the previous section, where it is constant and is determined by unconditional moments of the asset value. By contrast, when uninformed investors infer the asset value from asset prices, the price impact of a marginal change in aggregate informed demand varies with the asset price. When observing a high price, uninformed investors rationally infer a higher likelihood of informed coordination and a better fundamental value and increase their demand accordingly. This, in turn, increases the price impact of a marginal change in aggregate informed demand and consequently makes coordination among them more costly.

In the rest of this section, we keep all the assumptions on the information structure of informed investors, but to make uninformed investors' inference problem well defined, we assume that $\tilde{\theta}$ is uniformly drawn from the interval, $[\underline{\theta}, \overline{\theta}]$, rather than from the whole real line. The following definition describes the corresponding equilibrium concept.

DEFINITION 3:[Equilibrium with Learning by Uninformed Investors] An equilibrium consists of a price function, $P(\tilde{\theta}, \tilde{y})$, strategies, $\pi(\tilde{s}_i, P) : \left[\underline{\theta} - \sigma_s, \overline{\theta} + \sigma_s\right] \times \mathbb{R} \to [0, 1]$, and the corresponding aggregate demands, $X(P, \tilde{\theta})$ and $L(P)$, such that:

- For informed agent i, $\pi(\tilde{s}_i, P) \in argmax_{\pi} z\pi E\left[f(X\left(P, \tilde{\theta}\right), \tilde{\theta}) - P|\tilde{s}_i = s_i, P\right]$.
- Uninformed investor demand, $L(P)$, is given by

$$L(P) = w\frac{E[\tilde{V} + f(X(P, \tilde{\theta}), \tilde{\theta})|P] - P}{\rho Var[\tilde{V} + f(X(P, \tilde{\theta}), \tilde{\theta})|P]}; \tag{13}$$

- The market clearing condition is satisfied: $X\left(P, \tilde{\theta}\right) + L(P) + \sigma_y \tilde{y} = M$.

Before presenting the equilibrium solution, we first note a technical problem that occurs when $\tilde{\theta}$ is close to the upper or lower bounds. This problem is typical for global games. Suppose that informed investors follow a cutoff strategy. At a given price, consider the payoff of an informed investor, assuming that her signal is the cutoff signal. As her signal increases, the agent will believe that the asset, on average, has a higher fundamental. However, close to the boundaries, an additional countervailing effect appears. For example, as the distance between this signal and the upper boundary, $\overline{\theta}$, falls below σ_s, the informed agent believes that fewer informed investors will buy the asset. If the signal is close to the upper boundary, $\overline{\theta}$, then this agent will believe that no matter what the true fundamental is, fewer than half of the informed investors will buy the asset. Therefore, close to the boundary, the payoff from buying the asset may in fact decrease as the signal increases, which may lead to equilibrium multiplicity. Since this is a technical problem that appears only close to the boundaries, we assume that in a small neighborhood of $\underline{\theta}$ ($\overline{\theta}$), the informed investors will receive an arbitrarily negative (positive) private payoff. The following proposition provides a characterization of the monotone cutoff equilibrium as this neighborhood vanishes, and the informed and uninformed investors face the same dividend function in the limit.

Proposition 4:[Monotone Equilibrium with Learning by Uninformed Investors] Suppose that, for the uninformed investors, the dividend function is given by $f(X,\tilde{\theta})$, and for the informed investors, it is given by $f_\xi(X,\tilde{\theta})$, which is equal to $f(X,\tilde{\theta})$ if $\underline{\theta}+\xi \leq \tilde{\theta} \leq \overline{\theta}-\xi$, $-\infty$ if $\tilde{\theta} < \underline{\theta}+\xi$ and ∞ if $\tilde{\theta} > \overline{\theta}-\xi$. Consider the game of incomplete information described in this section and its equilibrium as described in Definition 3. In this game,

- as ξ approaches zero, there exists a unique monotone equilibrium strategy for informed investors; that is, there is a unique function $g:\mathbb{R}\rightarrow\left[\underline{\theta}+\sigma_s,\overline{\theta}-\sigma_s\right]$ such that the equilibrium strategies of informed investors are given by $\pi\left(\tilde{s},P\right)=1$ if $\tilde{s}\geq g(P)$ and 0 otherwise.

- in any monotone equilibrium, the informed investors' aggregate demand, $X(P,\tilde{\theta})$, is uniquely characterized by Equation (2). For a sufficiently large σ_v, uninformed investor demand, $L(P)$, is uniquely characterized by Equation (13).

- and the equilibrium price $P(\tilde{\theta},\tilde{y})$ satisfies

$$P=\begin{cases}\lambda\sigma_y\tilde{y}-\lambda\hat{M}(P) & \text{if } \tilde{\theta}<g(P)-\sigma_s\\ \frac{\lambda z}{2}\left(1-\frac{g(P)-\tilde{\theta}}{\sigma_s}\right)+\lambda\sigma_y\tilde{y}-\lambda\hat{M}(P) & \text{if } g(P)-\sigma_s\leq\tilde{\theta}\leq g(P)+\sigma_s\\ \lambda z+\lambda\sigma_y\tilde{y}-\lambda\hat{M}(P) & \text{if } g(P)+\sigma_s<\tilde{\theta}\end{cases}, \quad (14)$$

  where $\hat{M}(P)=M-L(P)-P/\lambda$.

This proposition shows that, even when uninformed investors learn from the price, an equilibrium in cutoff strategies exists and the informed investors' equilibrium strategies are uniquely determined. Moreover, the informed investors' aggregate demand is characterized by the same equation as before, Equation (2). The main difference between the two cases is that the price sensitivity of the uninformed investors' demand is no longer constant, but varies with price.

To see this difference, the right side of Equation (13) depends implicitly on $L(P)$. We show in the Appendix that $L(P)$ is a solution to a complicated algebraic equation which is difficult to express in closed-form, but can easily be solved numerically. Once we have a solution for $L(P)$, Proposition 4 provides a simple procedure with which to solve for equilibrium prices. Specifically, using $L(P)$, we first compute $\hat{M}(P)$. Next, we consider a fictitious economy where uninformed investors do not learn from prices and the asset supply is given by $\hat{M}(P)$. We use this to solve for the equilibrium strategy, $g(P)$, of informed investors. Finally, given $\hat{M}(P)$ and $g(P)$, we solve Equation (14) to find the market clearing prices.

Now we illustrate this procedure in the linear example where $f(X,\tilde{\theta}) = \alpha X + \tilde{\theta}$. The following lemma characterizes the equilibrium strategies of informed investors.

Lemma 3:[Equilibrium Cutoff Strategy with Learning by Uninformed Investors] When the dividend payoff function is $f(X,\tilde{\theta}) = \alpha X + \tilde{\theta}$, the equilibrium cutoff strategy, $g(P)$, is unique. As ξ goes to zero, the cutoff strategy is characterized by:

$$g(P) = P + \sigma_s - \left(\alpha + \frac{2\sigma_s}{z}\right)\left(\hat{M}(P) + \frac{P}{\lambda} - E\left(\sigma_y \tilde{y} \,\middle|\, \hat{M}(P) + \frac{P}{\lambda} - z \le \sigma_y \tilde{y} \le \hat{M}(P) + \frac{P}{\lambda}\right)\right), \quad (15)$$

when $\underline{\theta} + \sigma_s \le g(P) \le \overline{\theta} - \sigma_s$.[23]

Note the above equilibrium strategies are the same as those in Equation (6) in Lemma 1, with M replaced by $\hat{M}(P)$. This is because the asset supply in the fictitious economy is $\hat{M}(P)$. The endogenous price impact is demonstrated in Equation (15). Since $\hat{M}(P) = M - L(P) - P/\lambda$, $g(P)$ increases with $L(P)$. Intuitively, this implies that a larger uninformed demand creates a higher cutoff value for informed investors. Thus, coordination is more difficult when uninformed investors increase their demand. Panel (a) of Figure 4 illustrates a numerical example where the uninformed investors' demand may increase in the asset price while the informed investors' demand and the aggregate demand are both downward-sloping. This suggests that, as price increases, coordination may become more difficult due to the additional price impact induced by increased uninformed investor demand.

[INSERT FIGURE 4 ABOUT HERE]

However, when uninformed investors make inferences based on price, there is an additional source for multiplicity. The following equation describes the price sensitivity of aggregate demand in the intermediate region where $\tilde{\theta} \in [g(P) - \sigma_s, g(P) + \sigma_s]$ and illustrates the additional information effect from uninformed investors:

$$\begin{aligned}\frac{\partial(X(P,\tilde{\theta}) + L(P))}{\partial P} = & -\underbrace{\left(\underbrace{\left(\frac{1}{\lambda}\right)}_{\text{(from Uninformed)}} + \underbrace{\left(\frac{z}{2\sigma_s}\right)}_{\text{(from Informed)}}\right)}_{\text{Substitution Effect}} + \underbrace{\left(\frac{\partial L(P)}{\partial P} + \frac{1}{\lambda}\right)}_{\substack{\text{Information Effect} \\ \text{(from Uninformed)}}} \\ & + \underbrace{\left(\underbrace{\frac{z\alpha}{2\sigma_s}\left(\frac{1 - \Lambda(\hat{M}(P) + P/\lambda, z, \sigma_y)}{\lambda}\right)}_{\text{Coordination Component}} + \underbrace{\left(\frac{1 - \Lambda(\hat{M}(P) + P/\lambda, z, \sigma_y)}{\lambda}\right)}_{\text{Fundamental Component}}\right)}_{\text{Information Effect (from Informed)}}\end{aligned} \quad (16)$$

To illustrate the importance of this additional effect, we provide a numerical example in Panel (b) of Figure 4. In this example, informed investors, as an aggregate, do not treat the asset as a Giffen good, yet through the uninformed investors' information effect, aggregate demand has a backward-bending region. The following corollary reflects this numerical example.

COROLLARY 1:[Backward-Bending Uninformed Demand] When the dividend payoff function is $f(X, \tilde{\theta}) = \alpha X + \tilde{\theta}$, the aggregate uninformed investor demand may increase in the observed price even if the aggregate informed investor demand is downward sloping.

Corollary 1 identifies another channel for multiplicity and excess volatility in the market: the nonlinear inference of uninformed investors in the presence of feedback effects.

IV. Conclusion

In this paper, we present an REE framework to analyze the properties of asset prices when feedback effects exist. Specifically, we solve for equilibrium prices in a setting where an asset's cash flow is endogenously determined by the amount of investment made by informed investors. In this setting, informed investors have strategic incentives to coordinate their investments. Our results show that in the presence of feedback effects, the asset price is informative of both the fundamentals and the likelihood of coordination among informed investors. By distinguishing the coordination and fundamental components of the information effect in relation to the counter-veiling substitution effect, we highlight sources of volatility in asset markets.

Our findings contribute to the existing finance literature in several ways. First, we identify a new source of volatility and multiplicity: the strategic coordination among informed investors. This is different from the existing finance literature, which shows that price multiplicity arises from the non-linear learning of uninformed investors. Second, we find that liquidity plays an important role in determining whether feedback effects lead to excess volatility. This is because in a liquid market, regardless of the precision of the private signals, it is difficult for informed investors to form precise forecasts of others' investment decisions and to invest when others invest. On the other hand, in an illiquid market, price multiplicity may occur when private information becomes extremely precise. This is because it is easier to forecast aggregate informed investors' demand at a given price, rather than because the prices are fully revealing of the fundamentals. Finally, we contribute to the existing finance literature by providing theoretical foundations for generating new testable hypotheses regarding how feedback effects relate to excess volatility in cross-sectional stock returns. These tests allow us to investigate when feedback effects are empirically important enough to affect asset prices.

In this paper, we focus on the properties of asset prices without committing to a specific form of the feedback effect. Building on the analysis in the paper, we argue that our model may have interesting dynamic implications for financial markets across different stages of the business cycle. For example, the feedback effect may arise because an increase in stock price has eased the financing constraint for firms and thus enabled these firms to increase investments (Sunder (2005) and Baker, Stein, and Wurgler (2003)). Alternatively, the feedback effect may arise because managers can learn from the information in the stock price about the prospect of their own firms (Dow and Gorton (1997) and Subrahmanyam and Titman (1999)).[24] This information, in turn, can guide managers in making corporate decisions, such as investments, and hence may affect the value of the firm. Indeed, information production is often shown to be more active when economic fundamentals are strong (Van Nieuwerburgh and Veldkamp (2006)). Hence, in both feedback cases, we expect a strong feedback effect during the boom (positive) as well as the bust (negative) over the business cycle. However, for either feedback effect to generate price multiplicity and extreme volatility in the financial markets, the market has to be illiquid enough and the market informed participants have to hold sufficiently precise information. Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Suarez and Sussman (1997) all present models where productivity shocks are amplified due to credit constraints. However, our model indicates that, when private information is heterogenous, the amplification of the fundamentals could be large or small.

A final interesting application of our model could be in the housing market. This is a market characterized by (1) borrowing constraints due to high initial down-payment, (2) illiquidity due to high transaction costs, and (3) private information. Our model predicts housing markets, unlike other financial markets, are more likely to amplify fundamental shocks and to exhibit distinct boom and bust patterns.

Appendix A
Proofs

Proof of Proposition 1

In the Appendix, to lighten the notation, we refer to $\tilde{\theta}$ as θ. For a given private signal, $\tilde{s}_i = s$, θ is distributed uniformly on $[s - \sigma_s, s + \sigma_s]$. The following equation expresses conditional expectation of the risky asset's dividend payoff for an informed investor who observes private signal s and price P:

$$\int_{-\infty}^{\infty} f\left(X\left(P,\theta\right),\theta\right) h\left(\theta\left|s,P\right.\right) d\theta = \int_{-\infty}^{g(P)-\sigma_s} f\left(X\left(P,\theta\right),\theta\right) h\left(\theta\left|s,P\right.\right) d\theta \tag{A1}$$
$$+\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} f\left(X\left(P,\theta\right),\theta\right) h\left(\theta\left|s,P\right.\right) d\theta + \int_{g(P)+\sigma_s}^{\infty} f\left(X\left(P,\theta\right),\theta\right) h\left(\theta\left|s,P\right.\right) d\theta,$$

where $h\left(\theta\left|s,P\right.\right)$ is the density of θ conditional on $\tilde{s}_i = s$ and P.

First, we consider the first term on the right side of Equation (A1). Using Bayes rule, this term can be written as:

$$\begin{aligned}&\int_{-\infty}^{g(P)-\sigma_s} f\left(X\left(P,\theta\right),\theta\right) h\left(\theta\left|s,P\right.\right) d\theta \\ = \;& prob\left(\theta < g\left(P\right) - \sigma_s\left|s,P\right.\right) \int_{-\infty}^{g(P)-\sigma_s} f\left(X\left(P,\theta\right),\theta\right) h\left(\theta\left|s,P,\theta < g\left(P\right)-\sigma_s\right.\right) d\theta.\end{aligned}$$

Note that

$$prob\left(\theta < g\left(P\right) - \sigma_s\left|s,P\right.\right) = \frac{\left(\min\left\{s+\sigma_s, g\left(P\right)-\sigma_s\right\} - \left(s-\sigma_s\right)\right)}{2\sigma_s}.$$

Moreover, by Equation (4), price is uninformative about θ conditional on $\theta < g(P) - \sigma_s$. Thus, the posterior is uniform over this range, and

$$h\left(\theta\left|s,P,\theta < g\left(P\right)-\sigma_s\right.\right) = \frac{1}{\min\left\{s+\sigma_s, g(P)-\sigma_s\right\} - \left(s-\sigma_s\right)}.$$

Combining we obtain:

$$\int_{-\infty}^{g(P)-\sigma_s} f\left(X\left(P,\theta\right),\theta\right) h\left(\theta\left|s,P\right.\right) d\theta = \frac{1}{2\sigma_s}\int_{s-\sigma_s}^{\min\{s+\sigma_s, g(P)-\sigma_s\}} f\left(X\left(P,\theta\right),\theta\right) d\theta.$$

Similarly the second term on the right side of Equation (A1) can be written as:

$$\begin{aligned}&\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} f\left(X\left(P,\theta\right),\theta\right) h\left(\theta\left|s,P\right.\right) d\theta = prob\left(g\left(P\right)-\sigma_s \le \theta \le g\left(P\right)+\sigma_s\right) \\ &\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} f\left(X\left(P,\theta\right),\theta\right) h\left(\theta\left|s,P,g\left(P\right)-\sigma_s \le \theta \le g\left(P\right)+\sigma_s\right.\right) d\theta.\end{aligned}$$

Let

$$\tau \equiv \left(\frac{2\sigma_s}{\lambda z} P + \frac{2\sigma_s}{z} M + g(P) - \sigma_s \right).$$

Note that, in this region, $\tau = \theta + (2\sigma_s \sigma_y / z)\tilde{y}$. Hence, τ is a sufficient statistic for the information conveyed by the equilibrium clearing price, P. Thus,

$$h(\theta \,|s, P, g(P) - \sigma_s \leq \theta \leq g(P) + \sigma_s) = h(\theta \,|s, \tau, g(P) - \sigma_s \leq \theta \leq g(P) + \sigma_s)$$

$$= \begin{cases} \dfrac{\phi\left(\frac{\theta-\tau}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y}}{\int_{[s-\sigma_s, s+\sigma_s]\cap[g(P)-\sigma_s, g(P)+\sigma_s]} \phi\left(\frac{\theta-\tau}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y} d\theta} & \text{if } \theta \in [s-\sigma_s, s+\sigma_s] \cap [g(P)-\sigma_s, g(P)+\sigma_s] \\ 0 & \text{otherwise} \end{cases}.$$

Thus,

$$\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} f(X(P,\theta), \theta)\, h(\theta \,|s, P)\, d\theta = \frac{||[s-\sigma_s, s+\sigma_s] \cap [g(P)-\sigma_s, g(P)+\sigma_s]||}{2\sigma_s}$$

$$\frac{\int_{[s-\sigma_s, s+\sigma_s]\cap[g(P)-\sigma_s, g(P)+\sigma_s]} f(X(P,\theta),\theta)\, \phi\left(\frac{\theta-\tau}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y} d\theta}{\int_{[s-\sigma_s, s+\sigma_s]\cap[g(P)-\sigma_s, g(P)+\sigma_s]} \phi\left(\frac{\theta-\tau}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y} d\theta},$$

where $||[s-\sigma_s, s+\sigma_s] \cap [g(P)-\sigma_s, g(P)+\sigma_s]||$ is the length of the interval.
Finally, we can write the third term on the right side of Equation (A1) as:

$$\int_{g(P)+\sigma_s}^{\infty} f(X(P,\theta), \theta)\, h(\theta \,|s, P)\, d\theta = \frac{1}{2\sigma_s} \int_{\max\{s-\sigma_s, g(P)+\sigma_s\}}^{s+\sigma_s} f(X(P,\theta), \theta)\, d\theta.$$

To solve for the cutoff strategy, we consider the agent who receives the cutoff signal, $s = g(P)$. For this agent, the first and the third terms are 0, so the indifference condition becomes:

$$\frac{\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} f\left(\frac{z}{2}\left(1 - \frac{g(P)-\theta}{\sigma_s}\right), \theta\right) \phi\left(\frac{\theta-\tau}{2\sigma_s\sigma_y/z}\right) d\theta}{\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} \phi\left(\frac{\theta-\tau}{2\sigma_s\sigma_y/z}\right) d\theta} = P.$$

To find the cutoff value(s) $g(P)$ for a given P, we need to find those values of κ that satisfy:

$$\frac{\int_{\kappa-\sigma_s}^{\kappa+\sigma_s} f\left(\frac{z}{2}\left(1 - \frac{\kappa-\theta}{\sigma_s}\right), \theta\right) \phi\left(\frac{\theta-\kappa-2\sigma_s P/(\lambda z)-2\sigma_s M/z+\sigma_s}{2\sigma_s\sigma_y/z}\right) d\theta}{\int_{\kappa-\sigma_s}^{\kappa+\sigma_s} \phi\left(\frac{\theta-\kappa-2\sigma_s P/(\lambda z)-2\sigma_s M/z+\sigma_s}{2\sigma_s\sigma_y/z}\right) d\theta} = P.$$

Next we show that there is a unique κ that satisfies the above equation. Using a change of variables, $x = \frac{\theta-\kappa}{\sigma_s}$, we can rewrite this equation as:

$$\frac{\int_{-1}^{1} f\left(\frac{z}{2}(1+x), \kappa+\sigma_s x\right) \phi\left(\frac{x-2P/(\lambda z)-2M/z+1}{2\sigma_y/z}\right) dx}{\int_{-1}^{1} \phi\left(\frac{x-2P/(\lambda z)-2M/z+1}{2\sigma_y/z}\right) dx} = P. \tag{A2}$$

Since $f(X,\theta)$ is increasing in θ, it is also increasing in κ for a given P. So the left side is increasing in κ if and only if

$$\int_{-1}^{1} \phi\left(\frac{x - 2P/(\lambda z) - 2M/z + 1}{2\sigma_y/z}\right) dx > 0,$$

which is clearly true.

We have just shown that, for a given P, there is a unique signal, κ, that makes an informed investor indifferent between acquiring the asset or not. Moreover, since $f(X,\theta)$ is increasing in θ, for a given P, the payoff from acquiring the asset is strictly positive if $s > \kappa$ and strictly negative if $s < \kappa$. Therefore, an informed investors buys if and only if her private signal exceeds $g(P) = \kappa$, which completes the proof of the first bullet point in Proposition 1. The second and third bullet points follow immediately from the first one.

Proof of Lemma 1

Substituting $f(X,\theta) = \alpha X + \theta$ in the indifference condition in Equation (A2) and rearranging the terms, we obtain:

$$g(P) = \kappa = P - \frac{\alpha z}{2} - \frac{\int_{-1}^{1} (\frac{\alpha z}{2} + \sigma_s) x \phi\left(\frac{x-2P/(\lambda z)-2M/z+1}{2\sigma_y/z}\right) dx}{\int_{-1}^{1} \phi\left(\frac{x-2P/(\lambda z)-2M/z+1}{2\sigma_y/z}\right) dx}.$$

To complete the proof, we use a change of variables $u = (2P/(\lambda z) + 2M/z - 1 - x) / (2\sigma_y/z)$, and rearrange one more time to obtain:

$$\begin{aligned} g(P) &= P - \frac{\alpha z}{2} - \frac{\int_{\frac{P/\lambda+M-z}{\sigma_y}}^{\frac{P/\lambda+M}{\sigma_y}} (\frac{\alpha z}{2} + \sigma_s)\left(2P/(\lambda z) + 2M/z - 1 - 2\sigma_y u/z\right) \phi(u)\, du}{\int_{\frac{P/\lambda+M-z}{\sigma_y}}^{\frac{P/\lambda+M}{\sigma_y}} \phi(u)\, du} \qquad \text{(A3)} \\ &= P + \sigma_s - \left(\alpha + \frac{2\sigma_s}{z}\right)\left(M + \frac{P}{\lambda} - \sigma_y E\left[u \middle| \frac{P/\lambda + M - z}{\sigma_y} \le u \le \frac{P/\lambda + M}{\sigma_y}\right]\right). \end{aligned}$$

This completes the proof of Lemma 1.

Proofs of Lemma 2 and Proposition 2

The sensitivity of price to external noise demand shocks in the intermediate region can be computed from Equations (4) and (11) as:

$$\frac{\partial P}{\partial \tilde{y}} = \frac{\lambda \sigma_y}{1 + \frac{\lambda z}{2\sigma_s}\frac{\partial g(P)}{\partial P}} = \frac{\lambda \sigma_y}{\frac{\lambda z - \alpha z(1-\Lambda(M+P/\lambda, z, \sigma_y))}{2\sigma_s} + \Lambda(M + P/\lambda, z, \sigma_y)}.$$

We start by proving part (iii). First, we compute the derivative $\frac{\partial}{\partial \sigma_s}\frac{\partial P}{\partial \tilde{y}}$ and note that it has the same sign as $\lambda - \alpha(1 - \Lambda(M + P/\lambda, z, \sigma_y))$. Suppose $\alpha > \lambda$. By Lemma 2, $\Lambda(M + P/\lambda, z, \sigma_y))$ is increasing in σ_y and goes to zero as σ_y goes to zero and goes to one as σ_y goes to ∞. Therefore, $\frac{\partial}{\partial \sigma_s}\frac{\partial P}{\partial \tilde{y}} < 0$ if and only if $\sigma_y < \overline{\sigma}_y$, where $\overline{\sigma}_y$ satisfies $\lambda - \alpha(1 - \Lambda(M + P/\lambda, z, \overline{\sigma}_y)) = 0)$. In all other cases, $\frac{\partial}{\partial \sigma_s}\frac{\partial P}{\partial \tilde{y}} > 0$.

Next, we prove part(ii). First we compute the derivative

$$\begin{aligned}\frac{\partial}{\partial \sigma_y}\frac{\partial P}{\partial \tilde{y}} &= \frac{2\lambda\sigma_s}{\lambda z - \alpha z + (\alpha z + 2\sigma_s)\Lambda(M + P/\lambda, z, \sigma_y)} \\ &\quad - \frac{2\lambda\sigma_s\sigma_y(\alpha + 2\sigma_s)}{(\lambda z - \alpha z + (\alpha z + 2\sigma_s)\Lambda(M + P/\lambda, z, \sigma_y))^2}\frac{\partial \Lambda(M + P/\lambda, z, \sigma_y)}{\partial \sigma_y}.\end{aligned}$$

Note that the second term is always negative. When $\alpha > \lambda$, the first term becomes negative as σ_y approaches zero. Thus, $\frac{\partial}{\partial \sigma_y}\frac{\partial P}{\partial \tilde{y}} < 0$ for $\alpha > \lambda$ and σ_y close to zero.

Part (i) follows immediately by taking the appropriate derivative and noticing that $0 \leq \Lambda(M + P/\lambda, z, \sigma_y) \leq 1$.

The following technical lemma summarizes key properties of the function Λ that we use extensively in the analysis.

Lemma A1: The function $\Lambda(M + P/\lambda, z, \sigma_y)$ is increasing in σ_y. It approaches 0 as σ_y approaches 0 when $-\lambda M \leq P \leq -\lambda(M - z)$, and approaches 1 as σ_y approaches ∞.

Proof of Lemma A1

The following two lemmas are useful in this proof as well as the price sensitivity analysis. The first of these two lemmas generalizes a result from Burdett (1996) from right-truncated distributions to doubly-truncated distributions.

Lemma A2: Suppose that f is a log-concave and differentiable density function on $\mathbb{R}$ and $h < k$. Then the truncated variance, $Var(u|h \leq u \leq k)$, computed using the density f, is decreasing in h and increasing in k.

Proof of Lemma A2

From Proposition 4 in Burdett (1996), it follows immediately that variance is increasing in k for a fixed h. In applying the proposition, we need only to notice that, for a left-truncated distribution, we can replace negative infinity with h everywhere and the argument goes through exactly. The fact that, for a fixed k, the variance is decreasing in h, follows immediately from applying this result to the density $\tilde{f}(x) = f(-x)$.

Lemma A3: Let $E\left(u \,\middle|\, \frac{a+x}{b} \geq u \geq \frac{x}{b}\right)$ be the truncated expectation of the standard normal distribution with $a, b > 0$ and let $\Lambda(a + x, a, b) \equiv b\frac{\partial E\left(u \,\middle|\, \frac{a+x}{b} \geq u \geq \frac{x}{b}\right)}{\partial x}$. Then,

(i) $0 < \Lambda(a + x, a, b) \leq 1$.

(ii) $\Lambda(a + x, a, b)$ is increasing in b if $x \leq 0 \leq a + x$.

(iii) $\Lambda(a + x, a, b)$ goes to one as $b \to \infty$. When $x < 0 < a + x$, $\Lambda(a + x, a, b)$ goes to zero as $b \to 0$.

Proof of Lemma A3

Taking the derivative of $E\left(u\,\middle|\,\frac{a+x}{b} \geq u \geq \frac{x}{b}\right)$ with respect to x and multiplying by b, we obtain:

$$\Lambda(a+x,a,b) = \frac{\left[\left(\frac{a+x}{b}\right)\phi\left(\frac{a+x}{b}\right) - \frac{x}{b}\phi\left(\frac{x}{b}\right)\right]\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du - \left[\phi\left(\frac{a+x}{b}\right) - \phi\left(\frac{x}{b}\right)\right]\int_{\frac{x}{b}}^{\frac{a+x}{b}}u\phi\left(u\right)du}{\left(\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du\right)^2}$$

$$= \frac{\phi\left(\frac{a+x}{b}\right)\left[\frac{a}{b}\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du - \int_{\frac{x}{b}}^{\frac{a+x}{b}}\left(u-\frac{x}{b}\right)\phi\left(u\right)du\right] + \phi\left(\frac{x}{b}\right)\left[\int_{\frac{x}{b}}^{\frac{a+x}{b}}\left(u-\frac{x}{b}\right)\phi\left(u\right)du\right]}{\left(\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du\right)^2}.$$

Note that $\int_{\frac{x}{b}}^{\frac{a+x}{b}}\left(u-\frac{x}{b}\right)\phi\left(u\right)du$ is strictly positive and $\frac{a}{b}\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du > \int_{\frac{x}{b}}^{\frac{a+x}{b}}\left(u-\frac{x}{b}\right)\phi\left(u\right)dz$. Therefore, the denominator is strictly positive. This proves that $\Lambda(a+x,a,b) > 0$.

Next, using integration by part, we see the following is true.

$$\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi(u)du = \frac{a+x}{b}\phi\left(\frac{a+x}{b}\right) - \frac{x}{b}\phi\left(\frac{x}{b}\right) + \int_{\frac{x}{b}}^{\frac{a+x}{b}}u^2\phi(u)du.$$

Using this, as well as the fact that

$$\left(\phi\left(\frac{x}{b}\right) - \phi\left(\frac{a+x}{b}\right)\right) = \int_{\frac{x}{b}}^{\frac{a+x}{b}}u\phi(u)du,$$

we simplify the expression of $\Lambda(a+x,a,b)$ as

$$\Lambda(a+x,a,b) \;=\; 1 - \frac{\int_{\frac{x}{b}}^{\frac{a+x}{b}}u^2\phi(u)du}{\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi(u)du} + \left(\frac{\int_{\frac{x}{b}}^{\frac{a+x}{b}}u\phi(u)du}{\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi(u)du}\right)^2 = 1 - Var\left(u|\frac{x}{b} \leq u \leq \frac{a+x}{b}\right).$$

It immediately follows that $\Lambda(a+x,a,b)$ is no greater than one, proving part (i). Moreover, by Lemma A2, when $x \leq 0 \leq a+x$, the truncated variance $Var\left(u|\frac{x}{b} \leq u \leq \frac{a+x}{b}\right)$ is decreasing in b, thus $\frac{\partial\Lambda(a+x,a,b)}{\partial b} \geq 0$. This proves part (ii).

Next, we show the limiting results in part (iii). To identify the lower and upper bounds of $\Lambda(a+x,a,b)$, we first note that:

$$\frac{\frac{a}{b}\phi\left(\frac{x}{b}\right)}{\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du} = \frac{\left[\left(\frac{a+x}{b}\right)\phi\left(\frac{a+x}{b}\right) - \frac{x}{b}\phi\left(\frac{x}{b}\right)\right]\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du - \left[\phi\left(\frac{a+x}{b}\right) - \phi\left(\frac{x}{b}\right)\right]\int_{\frac{x}{b}}^{\frac{a+x}{b}}\frac{a+x}{b}\phi\left(u\right)du}{\left(\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du\right)^2}$$

and

$$\frac{\frac{a}{b}\phi\left(\frac{a+x}{b}\right)}{\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du} = \frac{\left[\left(\frac{a+x}{b}\right)\phi\left(\frac{a+x}{b}\right) - \frac{x}{b}\phi\left(\frac{x}{b}\right)\right]\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du - \left[\phi\left(\frac{a+x}{b}\right) - \phi\left(\frac{x}{b}\right)\right]\int_{\frac{x}{b}}^{\frac{a+x}{b}}\frac{x}{b}\phi\left(u\right)du}{\left(\int_{\frac{x}{b}}^{\frac{a+x}{b}}\phi\left(u\right)du\right)^2}.$$

When $\phi\left(\frac{a+x}{b}\right) > \phi\left(\frac{x}{b}\right)$,

$$\frac{\frac{a}{b}\phi\left(\frac{x}{b}\right)}{\int_{\frac{x}{b}}^{\frac{a+x}{b}} \phi(u)\, du} \leq \Lambda(a+x, a, b) \leq \frac{\frac{a}{b}\phi\left(\frac{a+x}{b}\right)}{\int_{\frac{x}{b}}^{\frac{a+x}{b}} \phi(u)\, du}.$$

When $\phi\left(\frac{x}{b}\right) > \phi\left(\frac{a+x}{b}\right)$, the inequalities above are reversed.

In either case, applying L'Hopital's rule, we show both the lower and the upper bounds of $\Lambda(a+x, a, b)$ approach one as $b \to \infty$.

Next, we show the limiting result as b goes to zero. When $x < 0 < a + x$, the truncated variance $Var\left(u | \frac{x}{b} \leq u \leq \frac{a+x}{b}\right)$ approaches one and $\Lambda(a+x, a, b)$ approaches 0. This concludes the proof.

Lemma A1 now follows from parts (ii) and (iii) of Lemma A3.

Proof of Proposition 3

From Equations (2) and (A3), we learn that the slope of the informed investors' aggregate demand function in the intermediate region of $-1 \leq (g(P) - \theta)/\sigma_s \leq 1$ is

$$\frac{\partial X(P, \theta)}{\partial P} = \left(-\frac{z}{2\sigma_s}\right)\left(1 - \frac{\alpha z + 2\sigma_s}{\lambda z}\left(1 - \Lambda(M + P/\lambda, z, \sigma_y)\right)\right).$$

Since the slope of the uninformed investors' aggregate demand curve is $-1/\lambda$, the slope of the aggregate demand curve of the economy in the intermediate region is

$$\frac{\partial(X(P, \theta) + L(P))}{\partial P} = \frac{(\alpha - \lambda)z}{2\lambda\sigma_s} - \frac{\alpha z + 2\sigma_s}{2\lambda\sigma_s}\Lambda(M + P/\lambda, z, \sigma_y).$$

Therefore, the necessary and sufficient condition for a unique equilibrium price is $(\alpha + 2\sigma_s/z)\Lambda(M + P/\lambda, z, \sigma_y) > \alpha - \lambda$.

When σ_y goes to infinity, that is, when the public signal is extremely noisy, $\Lambda(M + P/\lambda, z, \sigma_y) = 1$, by Lemma A3. This implies that both the informed investors' demand curve and the aggregate demand curve of the economy are downward sloping and the equilibrium price is unique.

When σ_y goes to zero, that is, when the public signal is extremely informative in the region where $-\lambda M < P < -\lambda(M - z)$, $\Lambda(M + P/\lambda, z, \sigma_y) = 0$ by Lemma A3. This implies that both the informed investors' demand curve and the aggregate demand curve of the economy have backward-bending regions and there are multiple equilibrium prices.

Proof of Proposition 4

Given a private signal $\tilde{s} = s$, a price P and the corresponding equilibrium demand of uninformed investors $L(P)$, the informed investors' inference problem can be solved as in the proof of Proposition 1, by replacing the asset supply M with $\hat{M}(P)$ everywhere. The only difficulty arises when s is close to the boundaries, in particular when $s < \underline{\theta} + \sigma_s$ or $s > \overline{\theta} - \sigma_s$. In these cases, we need to adjust the formulas appropriately.

Suppose that the informed investors' dividend function is given by $f_\xi(X, \theta)$. Recall that $f_\xi(X, \theta) = -\infty$ if $\theta < \underline{\theta} + \xi$ and $f_\xi(X, \theta) = \infty$ if $\theta > \overline{\theta} - \xi$. Everywhere else $f_\xi(X, \theta)$ is the same as $f(X, \theta)$ and satisfies our earlier assumptions, in particular it is increasing in both arguments. As in the proof of Proposition 1, we next consider an agent who receives the cutoff signal $s = \kappa$. We denote this agent's utility from buying the asset by $W(\kappa)$, which can be written (after using a change of variables) as:

$$W(\kappa) = \frac{\int_L^U f_\xi\left(\frac{z}{2}(1+x), \kappa + \sigma_s x\right) \phi\left(\frac{x - 2P/(\lambda z) - 2M/z + 1}{2\sigma_y/z}\right) dx}{\int_L^U \phi\left(\frac{x - 2P/(\lambda z) - 2M/z + 1}{2\sigma_y/z}\right) dx}, \tag{A4}$$

where

$$L = \begin{cases} (\underline{\theta} - \kappa)/\sigma_s & \text{if } \underline{\theta} - \sigma_s < \kappa < \underline{\theta} + \sigma_s \\ -1 & \text{if } \underline{\theta} + \sigma_s < \kappa < \overline{\theta} + \sigma_s \end{cases} \quad \text{and} \quad U = \begin{cases} 1 & \text{if } \underline{\theta} - \sigma_s < \kappa < \overline{\theta} - \sigma_s \\ (\overline{\theta} - \kappa)/\sigma_s & \text{if } \overline{\theta} - \sigma_s < \kappa < \overline{\theta} + \sigma_s \end{cases}.$$

It is easy to see that the expression in Equation (A4) is increasing in κ for $\underline{\theta} + \sigma_s + \xi \leq \kappa \leq \overline{\theta} - \sigma_s - \xi$. If $\underline{\theta} - \sigma_s \leq \kappa < \underline{\theta} + \sigma_s + \xi$, then the expression in Equation (A4) is $-\infty$. Similarly, if $\overline{\theta} - \sigma_s - \xi < \kappa \leq \overline{\theta} + \sigma_s$, then the expression in Equation (A4) is ∞.

Next, we construct $g(P)$. Now, suppose that $P < W(\underline{\theta} + \sigma_s + \xi)$. If an agent receives the signal $\underline{\theta} + \sigma_s + \xi$, by construction her payoff from buying the asset is less than the price. Thus she does not buy the asset. The same is true for all agents with signals that are less than $\underline{\theta} + \sigma_s + \xi$. However, an agent with a higher signal expects to make ∞ by buying the asset, so she will buy the asset. Therefore, in this range, the cutoff is given by $g(P) = \underline{\theta} + \sigma_s + \xi$. By a symmetric argument, $g(P) = \overline{\theta} - \sigma_s - \xi$ if $P > W(\overline{\theta} - \sigma_s - \xi)$. Thus, we can write the (unique) cutoff strategy $g(P)$ as:

$$g(P) = \begin{cases} \underline{\theta} + \sigma_s + \xi & \text{if } P < W(\underline{\theta} + \sigma_s + \xi) \\ W^{-1}(P) & \text{if } W(\underline{\theta} + \sigma_s + \xi) \leq P \leq W(\overline{\theta} - \sigma_s - \xi) \\ \overline{\theta} - \sigma_s - \xi & \text{if } P > W(\overline{\theta} - \sigma_s - \xi) \end{cases}.$$

This completes the proof of the first bullet point in Proposition 4.

The uninformed investors' inference problem is similar to that of the informed investors except that the uninformed investors' information set contains only the price P, and their dividend function is given by $f(X, \theta)$. To simplify some of the expressions that follow we sometimes refer to $f(X(P,\theta),\theta)$ as ν. The following equation expresses the uninformed investors' conditional expectation of the risky asset's dividend payoff $E[\tilde{V}+\nu|P]$:

$$\int_{\underline{\theta}}^{\overline{\theta}} \nu h\left(\theta\left|P\right.\right)d\theta=\int_{\underline{\theta}}^{g(P)-\sigma_s} \nu h\left(\theta\left|P\right.\right)d\theta+\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} \nu h\left(\theta\left|P\right.\right)d\theta+\int_{g(P)+\sigma_s}^{\overline{\theta}} \nu h\left(\theta\left|P\right.\right)d\theta \quad \text{(A5)}$$

where $h\left(\theta\left|P\right.\right)$ is the density of θ conditional on P.[25] The sum of the first and the third terms in Equation (A5) can be written as

$$\frac{1}{\overline{\theta}-\underline{\theta}}\left(\int_{\underline{\theta}}^{g(P)-\sigma_s} f\left(X\left(P,\theta\right),\theta\right)d\theta+\int_{g(P)+\sigma_s}^{\underline{\theta}} f\left(X\left(P,\theta\right),\theta\right)d\theta\right).$$

Next we consider the second term in Equation (A5). Using Bayes rule, we rewrite this term as

$$\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} f\left(X\left(P,\theta\right),\theta\right)h\left(\theta\left|P\right.\right)d\theta=prob\left(g\left(P\right)-\sigma_s\leq\theta\leq g\left(P\right)+\sigma_s\right)$$
$$\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} f\left(X\left(P,\theta\right),\theta\right)h\left(\theta\left|P,g\left(P\right)-\sigma_s\leq\theta\leq g\left(P\right)+\sigma_s\right.\right)d\theta.$$

Let $\varphi\equiv\left(\frac{2\sigma_s}{\lambda z}P+\frac{2\sigma_s}{z}\hat{M}(P)+g\left(P\right)-\sigma_s\right)$. Note that, in this region, $\varphi=\theta+(2\sigma_s\sigma_y/z)\tilde{y}$. Hence, φ is a sufficient statistic for the information conveyed by the equilibrium clearing price, P. Thus,

$$h\left(\theta\left|P,g\left(P\right)-\sigma_s\leq\theta\leq g\left(P\right)+\sigma_s\right.\right)=h\left(\theta\left|\varphi,g\left(P\right)-\sigma_s\leq\theta\leq g\left(P\right)+\sigma_s\right.\right)$$
$$=\begin{cases}\dfrac{\phi\left(\frac{\theta-\varphi}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y}}{\int_{g(P)-\sigma_s}^{g(P)+\sigma_s}\phi\left(\frac{\theta-\varphi}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y}d\theta} & \text{if } \theta\in\left[g\left(P\right)-\sigma_s,g\left(P\right)+\sigma_s\right] \\ 0 & \text{otherwise}\end{cases}.$$

Thus,

$$\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} f\left(X\left(P,\theta\right),\theta\right)h\left(\theta\left|P\right.\right)d\theta=\frac{\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} f\left(X\left(P,\theta\right),\theta\right)\phi\left(\frac{\theta-\varphi}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y}d\theta}{\int_{g(P)-\sigma_s}^{g(P)+\sigma_s}\phi\left(\frac{\theta-\varphi}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y}d\theta}.$$

Adding this term to the first and third terms gives the expected dividend payoff of the asset conditional on the price. The conditional variance of the dividend payoff is $Var[\tilde{V}+\nu|P]=\sigma_v^2+Var[\nu|P]$, where $Var[\nu|P]$ is given by

$$\frac{1}{\overline{\theta}-\underline{\theta}}\left(\int_{\underline{\theta}}^{g(P)-\sigma_s}\left(f\left(X\left(P,\theta\right),\theta\right)\right)^2d\theta+\int_{g(P)+\sigma_s}^{\underline{\theta}}\left(f\left(X\left(P,\theta\right),\theta\right)\right)^2d\theta\right)+$$
$$\frac{\int_{g(P)-\sigma_s}^{g(P)+\sigma_s}\left(f\left(X\left(P,\theta\right),\theta\right)\right)^2\phi\left(\frac{\theta-\varphi}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y}d\theta}{\int_{g(P)-\sigma_s}^{g(P)+\sigma_s}\phi\left(\frac{\theta-\varphi}{2\sigma_s\sigma_y/z}\right)\frac{z}{2\sigma_s\sigma_y}d\theta}-\left(E[\tilde{V}+\nu|P]\right)^2.$$

Since the uninformed investors are mean-variance maximizers, their demand $L(P)$ equals $(w(E[\tilde{V}+\nu|P]-P)/(\rho Var[\tilde{V}+\nu|P])$, which is a function of $L(P)$ itself. Thus, the equilibrium $L(P)$ is a fixed point of

$$l = w\frac{E[\tilde{V} + \nu|P, l] - P}{\rho Var[\tilde{V} + \nu|P, l]}.$$

To complete the proof, we show that the fixed point is unique if σ_v is large enough. We will show this in several steps. First, note that $(w(E[\tilde{V} + \nu|P, l] - P)/(\rho Var[\tilde{V} + \nu|P, l])$ is bounded, so when l is a very large positive (negative) number it is above (below) the right side. Moreover the right side is a continuous function of l, so a fixed point exists. To see that the fixed point is unique, we will show that $(w(E[\tilde{V} + \nu|P, l] - P)/(\rho Var[\tilde{V} + \nu|P, l])$ is decreasing in l. To this end, we first show that an increase in l results in a distribution over θ that first-order stochastically dominates the earlier one. To see this, note that a larger l leads to a smaller φ and changes the distribution only in the range $\theta \in [g(P) - \sigma_s, g(P) + \sigma_s]$, where it equals $\phi\left(\frac{\theta-\varphi}{2\sigma_s\sigma_y/z}\right)/(\int_{g(P)-\sigma_s}^{g(P)+\sigma_s} \phi\left(\frac{\theta-\varphi}{2\sigma_s\sigma_y/z}\right) d\theta)$. We need to show that $Prob(\theta \leq x)$ is decreasing in φ. Taking the derivative with respect to φ, we see that this is true if and only if

$$-\int_{g(P)-\sigma_s}^{x} \phi'(\theta - \varphi)d\theta \int_{g(P)-\sigma_s}^{g(P)+\sigma_s} \phi(\theta - \varphi)d\theta + \int_{g(P)-\sigma_s}^{g(P)+\sigma_s} \phi'(\theta - \varphi)d\theta \int_{g(P)-\sigma_s}^{x} \phi(\theta - \varphi)d\theta < 0.$$

Plugging $\phi'(x) = -x\phi(x)$ into this expression and rearranging terms, we can rewrite it as

$$\frac{\int_{g(P)-\sigma_s}^{x}(\theta - \varphi)\phi(\theta - \varphi)d\theta}{\int_{g(P)-\sigma_s}^{x} \phi(\theta - \varphi)d\theta} < \frac{\int_{x}^{g(P)+\sigma_s}(\theta - \varphi)\phi(\theta - \varphi)d\theta}{\int_{x}^{g(P)+\sigma_s} \phi(\theta - \varphi)d\theta},$$

which is clearly true for all $x \in (g(P) - \sigma_s, g(P) + \sigma_s)$. Therefore, $(\partial E[\tilde{V} + \nu|P, l])/\partial l < 0$. To complete the proof, note that the change in $(w(E[\tilde{V} + \nu|P, l] - P)/(\rho Var[\tilde{V} + \nu|P, l]))$ can be written as

$$\frac{w}{\rho}\frac{\frac{\partial E[\tilde{V}+\nu|P,l]}{\partial l}(\sigma_v^2 + Var[\nu|P, l]) - \frac{\partial Var[\nu|P,l]}{\partial l}((E[\tilde{V} + \nu|P, l] - P))}{(\sigma_v^2 + Var[\nu|P, l])^2},$$

which is negative for large enough σ_v^2. So the fixed point is unique, which completes the proof.

Proof of Lemma 3

The proof of this lemma is omitted since it is very similar to the proof of Lemma 1.

Appendix B

Feedback Effects Between Asset Price and Asset Value

Here we consider an extension of our model where the feedback effect occurs between the asset price and the dividend payoff. Thus we replace $f(X,\theta)$ by $f(P,\theta)$ in the model. We assume that $f(P,\theta)$ is increasing in both P and θ. One motivation for this extension is empirical evidence suggesting that even random movements in price can affect the asset value. For example, Hirshleifer, Subrahmanyam, and Titman (forthcoming) argue security price affects the firm's cash flows if the firm's stakeholders make economic decisions based on security price. Gilchrist, Himmelberg, and Huberman (forthcoming) analyze how real investments react to the "bubble" component in prices, as measured by analysts' forecast dispersion.

In this version of the model, for general dividend payoff functions, both the definition of equilibrium and Proposition 1 hold essentially unchanged. In the remainder of this appendix, we consider the linear case, $f(P,\theta) = \eta P + \theta$, and compare the results with the corresponding results where the feedback effect occurs through informed investors' aggregate investment. The following lemma provides the equilibrium cutoff strategy for the linear case:

LEMMA B1: The equilibrium cutoff strategy, $\hat{g}(P)$, when the dividend payoff function is $f(P,\theta) = \eta P + \theta$, is unique and is characterized by the following equation

$$\hat{g}(P) = P + \sigma_s - \eta P - \frac{2\sigma_s}{z}\left(M + \frac{P}{\lambda} - E\left(\sigma_y \tilde{y} \,\middle|\, M + \frac{P}{\lambda} - z \leq \sigma_y \tilde{y} \leq M + \frac{P}{\lambda}\right)\right). \quad \text{(B1)}$$

Proof of Lemma B1 *The proof is similar to the proof of Lemma 1, so we omit the details. Essentially, we substitute $f(P,\theta) = \alpha P + \theta$ in the indifference condition in Equation (A2), and then use a change of variables.*

To compare $\hat{g}(P)$ with $g(P)$, we first examine the price sensitivity of $\hat{g}(P)$:

$$\begin{aligned} \frac{\partial \hat{g}(P)}{\partial P} &= \underbrace{1}_{\text{Substitution Effect}} - \underbrace{\left(\underbrace{\eta}_{\text{Coordination Component}} + \underbrace{\frac{\partial E[\theta|\mathcal{F}]}{\partial P}}_{\text{Fundamental Component}} \right)}_{\text{Information Effect}} \\ &= 1 - \eta - \frac{2\sigma_s}{z}\left(\frac{1 - \Lambda(M + P/\lambda, z, \sigma_y)}{\lambda}\right). \end{aligned} \quad \text{(B2)}$$

When the feedback effect is through the informed investors' aggregate investment, investors need to infer the informed investors' aggregate investment from the asset price. Comparing the coordination component of the information effect in Equations (10) and (B2), we see that this inference is no longer necessary when the feedback effect occurs directly through price. Consequently, the coordination component is constant and equals η, which is the sensitivity of the dividend payoff to price. An increase in price now has a *constant* positive feedback to the asset value at a rate η. In particular, when $\eta > 1$, the coordination component of the information effect is always larger than the substitution effect, and $\hat{g}(P)$, is strictly decreasing everywhere. This means that, for a given fundamental, θ, no informed investor buys the asset when its price is extremely low and all informed investors buy when the price is extremely high. This happens because the feedback effect is so strong that it always dominates the price impact. We view this scenario highly unrealistic; hence, the reasonable parameter values for η should be strictly less than one.

Next, we examine conditions for uniqueness of the equilibrium price when the feedback effect occurs directly from price. To do this, we derive the sensitivity of aggregate demand to price:

$$\begin{aligned}\frac{\partial(X(P,\theta)+L(P)))}{\partial P} = & -\underbrace{\left(\underbrace{\left(\frac{1}{\lambda}\right)}_{\text{(from Uninformed)}} + \underbrace{\left(\frac{z}{2\sigma_s}\right)}_{\text{(from Informed)}}\right)}_{\text{Substitution Effect}} \\ & + \underbrace{\left(\underbrace{\frac{z\eta}{2\sigma_s}}_{\text{Coordination Component}} + \underbrace{\left(\frac{1-\Lambda(M+P/\lambda,z,\sigma_y)}{\lambda}\right)}_{\text{Fundamental Component}}\right)}_{\text{Information Effect (from Informed)}}\end{aligned} \tag{B3}$$

The following proposition describes the condition.

PROPOSITION B1: The equilibrium price is unique if and only if $\sigma_s\Lambda(M + P/\lambda, z, \sigma_y) > z\lambda(\eta - 1)/2$ for all P.

Proof of Proposition B1 *Follows immediately from Equation (B3).*

It is immediate to see that multiplicity can occur only when η falls outside the reasonable range. When $\eta < 1$, the equilibrium price is uniquely determined.

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Notes

[1]For example, Durnev, Morck, and Yeung (2004), Luo (2005), and Chen, Goldstein, and Jiang (forthcoming) find evidence that when managers learn from stock prices in making real investment decisions, the information content of stock price may affect a firm's cash flow. Fulghieri and Lukin (2001), Sunder (2005), and Baker, Stein, and Wurgler (2003) find evidence that feedback can also result from the effect of the stock price on the firm's access to capital, when lenders learn from stock prices when making investment decisions. There is also evidence that even random movements in stock prices affect firms' real investment decisions (Gilchrist, Himmelberg, and Huberman (forthcoming); Polk and Sapienza (2002)).

[2]However, it is not apparent whether these results apply in the financial market because, as argued by Atkeson (2000) and Angeletos and Werning (2006), prices as endogenous public signals may aggregate private information and thus restore common knowledge about the fundamentals among investors.

[3]In this paper, we do not commit to a specific form of the feedback effect since feedback may arise for various reasons, as demonstrated in the empirical literature. Rather, we focus on the asset pricing implications of the feedback effect. In the main text, we study the case where feedback occurs between the informed investors' aggregate investment and the asset value. In Appendix B, we consider an alternative specification where the feedback effect occurs between the asset price and the asset value. The asset pricing implications are robust to this extension.

[4]There is a large literature on excess volatility starting with Shiller (1981) and LeRoy and Porter (1981). This literature finds that stock prices move too much to be justified by changes in subsequent dividends, or put differently, the volatility of stock returns is excessive relative to the volatility of fundamentals. Our definition of excess volatility follows the definition in this literature.

[5]It is worth noting that for a given equilibrium price, agents' equilibrium beliefs and strategies are uniquely determined. Yet, the converse is not necessarily true. Given agents' uniquely determined equilibrium beliefs and strategies, there may be multiple equilibrium prices that clear the market. This is in contrast to herding models of the feedback effect. In these models, beliefs are self-fulfilling and multiple equilibria are robust.

[6]The dual roles of equilibrium prices are formally introduced by Admati (1985).

[7]An emerging and active field of research applies the insight in Carlsson and van Damme (1993) and Morris and Shin (1998, 2002, 2003) to a study of multiple equilibria in financial markets, although these models do not include prices. Examples include: bank runs (Goldstein and Pauzner (2005)); liquidity (Morris and Shin (2005); Plantin (2004)); and information acquisition (Chamley (2003)).

[8]A detailed description of agents in this economy is provided in the next section.

[9]This supposition is supported by the empirical evidence in Durnev, Morck, and Yeung (2004), Luo (2005), and Chen, Goldstein, and Jiang (forthcoming).

[10]Uniform distribution is assumed for the noise term for ease of exposition and is not crucial for the results.

[11]The specific size of this position limit on asset holdings is not crucial for our results. What is crucial is that informed investors cannot take unlimited positions; if they do, strategic interaction among informed investors will become immaterial.

[12]Essentially we assume that uninformed investors are either unaware of the feedback component, or expect not to be compensated for holding the idiosyncratic risk, i.e., $\tilde{\theta}$, of a particular firm. This allows us to capture the fact that uninformed investors are liquidity providers and enables us to focus on the strategic interaction among informed investors. From a modeling perspective, we could specify an upward-sloping asset supply curve directly rather than introduce uninformed agents. However, we choose to specify uninformed investors' preferences explicitly to allow for an extension where they learn from prices, as outlined in Section 4.

[13]The results are essentially the same when we assume that the noise demand follows a fatter tail distribution, such as a double exponential.

[14]In Appendix B, we extend the model by analyzing an alternative specification of the feedback effect, $f(P, \tilde{\theta})$.

[15]Since, by Proposition 1, $L(P)$ is always decreasing in P, it is necessary but not sufficient for multiplicity to arise that the demand from informed investors, $X(P, \tilde{\theta})$, has a backward-bending region.

[16]The meaning of Λ can also be seen by taking the derivative of the market clearing condition with respect to P which results in $\lambda\frac{\partial E[X|\mathcal{F}]}{\partial P}+\lambda\frac{\partial E[\sigma_y\tilde{y}|\mathcal{F}]}{\partial P}=1$. The terms on the left side represent, in expectation, the percentage of a marginal price change that comes from changes in informed demand and noise demand.

[17]Intuitively, the multiplier $z/(2\sigma_s)$ arises because private signals of informed investors become more dispersed with a large σ_s and a change in the price would move only a few informed investors from one side of the cutoff to the other, limiting their impact at the aggregate level. Moreover, the price impact of the informed investors as a group is further limited since the investment level of each of them is capped by z. Therefore, with dispersed private signals and constrained investment positions, it is possible that the price sensitivity of the aggregate informed demand is small, even when the price sensitivity of the cutoff strategies is extremely large (as indicated by the graphs in Figures 1 and 2).

[18]Outside the intermediate region, price sensitivity to non-fundamental shocks is constant.

[19]The comparative statics in parts (i) and (ii) of Proposition 2 apply both when the equilibrium price is unique and when it is not, and thus are not only due to multiplicity.

[20]There exists a unique $\overline{\sigma}_y > 0$ that satisfies $\lambda-\alpha(1-\Lambda(M+P/\lambda,z,\overline{\sigma}_y))=0$. To see this note that by Lemma A1, $\Lambda(M+P/\lambda,z,\sigma_y))$ is increasing in σ_y and approaches zero as σ_y approaches zero and one as σ_y approaches ∞.

[21]When comparing our multiplicity vs. uniqueness results with those of Morris and Shin (2003), one must keep in mind that our results refer to price multiplicity and theirs to multiplicity in equilibrium strategies.

[22]For the calibrations following Gennotte and Leland (1990), we use 0.28 for the standard deviation of the fundamental, and 0.63 for the standard deviation of the private signal. For the calibrations following Yuan (2005), we use 0.20 for the standard deviation of the fundamental, and 0.08 for the standard deviation of the private signal. In both cases, the position limit of informed investor is set to one. The ratio of uninformed to informed investors is 5.67 in Yuan (2005) and 49 in Gennotte and Leland (1990). This ratio reflects the 15% and 2% informed holdings of the stock market used respectively in these papers. Using this ratio as well the standard deviation of the fundamental, we obtain λ.

[23]If P is such that $g(P)$ computed from Equation (15) is more (or less) than $\overline{\theta}-\sigma_s$ (or,

$\underline{\theta} + \sigma_s$), then $g(P)$ is set to $\overline{\theta} - \sigma_s$ (or $\underline{\theta} + \sigma_s$).

[24] The idea behind this theory is that stock prices aggregate information from many different participants who do not have channels for communication with the firm outside the trading process. Thus, stock prices may contain some information that managers do not have. This information is more likely to be about the demand for the firm's products, or about strategic issues, such as competition with other firms.

[25]The conditional expectation can always be separated into these three regions because $\underline{\theta} + \sigma_s \leq g(P) \leq \overline{\theta} - \sigma_s$ for all P.

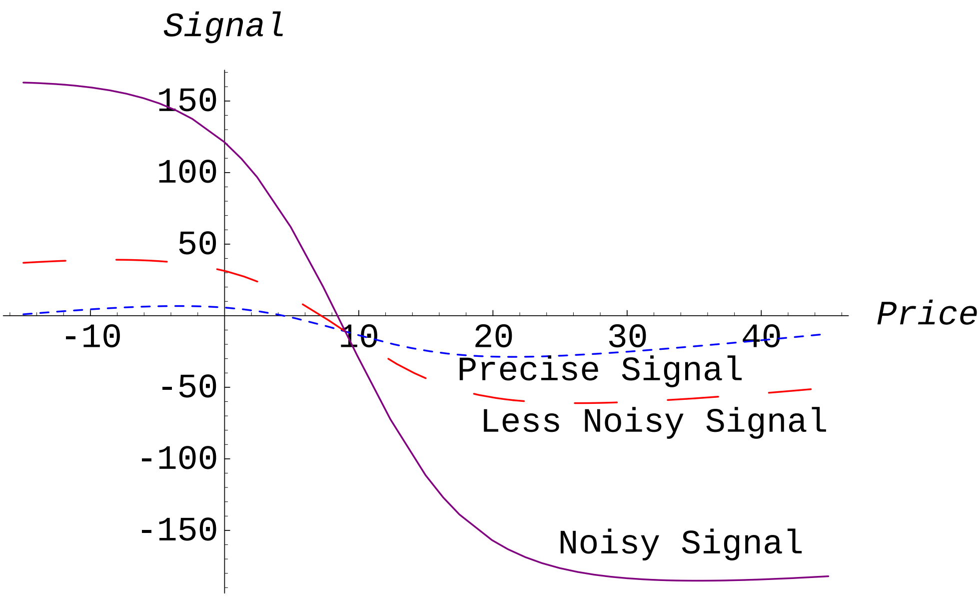

Figure 1. Equilibrium Cutoff Strategies. The dotted line, the dashed line, and the solid line in the graph represent the equilibrium cutoff value, $g(P)$, when informed investors' signal is precise ($\sigma_s = 20$), less noisy ($\sigma_s = 60$), and noisy ($\sigma_s = 600$), respectively. Informed investors would purchase the risky asset if their signals are above the cutoff value. The parameters are chosen to illustrate a case where $\sigma_y = 4$, $\alpha = 2$, $z = 20$, $\lambda = 1$, and $M = 1$.

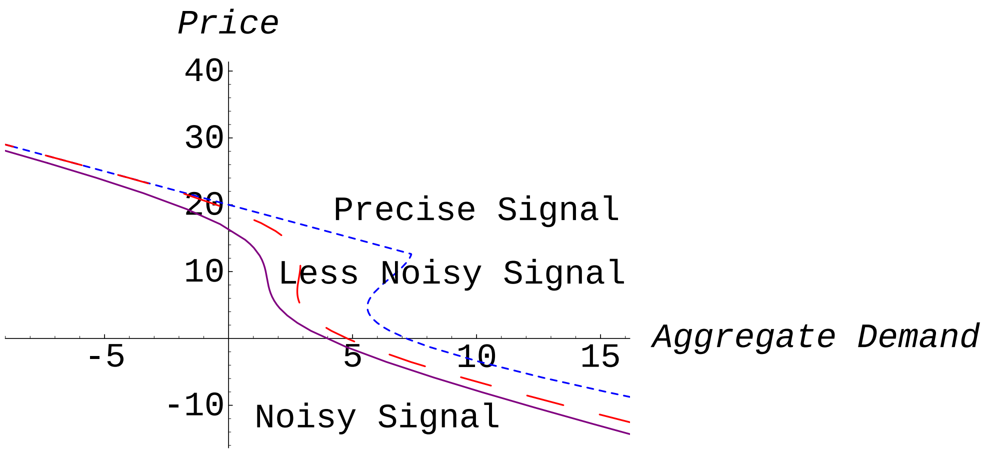

Figure 2. Backward-bending Aggregate Demands. The dotted line, the dashed line, and the solid line in the graph each represent the aggregate demand, $X(P, \tilde{\theta}) + L(P)$, when the signal is precise ($\sigma_s = 20$), less noisy ($\sigma_s = 60$), and noisy ($\sigma_s = 200$), respectively. The parameters are chosen to illustrate the case where $\sigma_y = 4$, $\alpha = 2$, $z = 20$, $\lambda = 1$, $M = 1$, and $\tilde{\theta} = 0$.

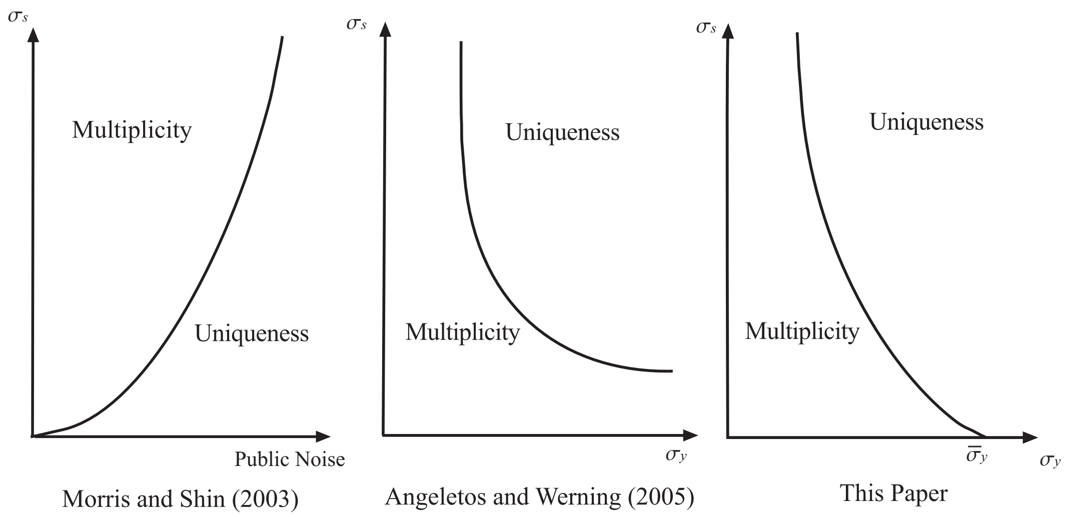

Figure 3. Regions of Multiplicity and Uniqueness. The σ_s measures the exogenous noise in the private information. The σ_y measures the exogenous noise trading. The lines in the left, middle, and right panels delineate the boundary between the multiple equilibria region and the unique equilibrium region in Morris and Shin (2003), Angeletos and Werning (2006), and this paper, respectively.

(a) Only uninformed demand is backward bending. Here $w = 0.5$ and $\tilde{\theta} = -22.7273$.

(b) Uninformed and aggregate demand are backward bending. Here and $w = 6$ and $\tilde{\theta} = -24.6465$.

Figure 4. The dotted line, the dashed line, and the solid line in the graphs represent the uninformed investor demand, $L(P)$, the informed investor demand, $X(P)$, the aggregate demand, $X(P, \tilde{\theta}) + L(P)$, respectively. The parameters are chosen to illustrate the case where $\sigma_y = 7$, $\alpha = 2$, $z = 20$, $M = 1$, $\tilde{\theta} = 0$, $\sigma_v = 1$, $\sigma_s = 70$, $\rho = 1$, $\underline{\theta} = -100$, and $\overline{\theta} = 100$.