Hot and Cold Seasons in the Housing Market*

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Abstract

Every year housing markets in the United Kingdom and the United States experience systematic above-trend increases in both prices and transactions during the second and third quarters (the "hot season") and below-trend falls during the fourth and first quarters (the "cold season"). House price seasonality poses a challenge to existing models of the housing market. To explain seasonal patterns, this paper proposes a matching model that emphasizes the role of match-specific quality between the buyer and the house and the presence of thick-market effects in housing markets. It shows that a small, deterministic driver of seasonality can be amplified and revealed as deterministic seasonality in transactions and prices, quantitatively mimicking the seasonal fluctuations observed in the United Kingdom and the United States.

Key words: housing market, thick-market effects, search-and-matching, seasonality, house price fluctuations, match quality

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1 Introduction

A rich empirical and theoretical literature has been motivated by dramatic boom-to-bust episodes in regional and national housing markets.[1] Booms are typically defined as times when prices rise and there is intense trading activity, whereas busts are periods when prices and trading activity fall below trend.

While the boom-to-bust episodes motivating the extant work are relatively infrequent and their timing is hard to predict, this paper shows that in several housing markets, booms and busts are just as frequent and predictable as the seasons. Specifically, in most regions of the United Kingdom and the United States, each year a housing boom of considerable magnitude takes place in the second and third quarters of the calendar year (spring and summer, which we call the "hot season"), followed by a bust in the fourth and first quarters (fall and winter, the "cold season").[2] The predictable nature of house price fluctuations (and transactions) is confirmed by real estate agents, who in conversations with the authors observed that during the winter months there is less activity and prices are lower. Perhaps more compelling, publishers of house price indexes go to great lengths to produce seasonally adjusted versions of their indexes, usually the versions that are published in the media. As stated by some publishers:

> "House prices are higher at certain times of the year irrespective of the overall trend. This tends to be in spring and summer... We seasonally adjust our prices because the time of year has some influence. Winter months tend to see weaker price rises and spring/summer see higher increases all other things being equal." (From Nationwide House Price Index Methodology.)
>
> "House prices are seasonal with prices varying during the course of the year irrespective of the underlying trend in price movements. For example, prices tend to be higher in the spring and summer months." (From Halifax Price Index Methodology.)

The first contribution of this paper is to systematically document the existence and quantitative importance of these seasonal booms and busts.[3] For the United Kingdom as a whole, we find that the

[1] For example, see Stein (1995), Muellbauer and Murphy (1997), Genesove and Mayer (2001), Krainer (2001), Brunnermeier and Julliard (2008), Burnside, Eichenbaum, and Rebelo (2011), and the contributions cited therein.

[2] Since we use repeat-sale price indexes, changes in prices are not driven by changes in the characteristics of the houses transacted.

[3] Studies on housing markets have typically glossed over the issue of seasonality. There are a few exceptions, albeit they have been confined to only one aspect of seasonality (e.g., either quantities or prices) or to a relatively small geographical area. In particular, Goodman (1993) documents pronounced seasonality in *moving patterns* in the US, Case and Shiller (1989) find seasonality in Chicago house prices and—to a lesser extent—in Dallas. Hosios and Pesando

difference in annualized growth rates between hot and cold seasons is 6.5 percent for nominal house prices (5.5 percent for real prices) and 140 percent for the volume of transactions. For the United States as a whole, the corresponding differences are above 4.6 percent for nominal (and real) prices and 146 percent for transactions; US cities display higher seasonality, with differences in growth rates of 6.7 percent for (real) prices and 152 percent for transactions.[4]

The predictability and size of seasonal fluctuations in house prices pose a challenge to existing models of the housing market. As we argue in the web Appendix, in those models, anticipated changes in prices cannot be large: if prices are expected to be much higher in August than in December, then optimizing buyers will try to shift their purchases to the end of the year, narrowing down the seasonal price differential.[5,6] Our paper tries to answer the question of why presumably informed buyers do not try to buy in the lower-priced season and to shed light on the systematic seasonal pattern. (A lack of scope for seasonal arbitrage does not necessarily imply that most transactions should be carried out in one season nor that movements in prices and transactions should be correlated.) To offer answers to these questions, we develop a model for the housing market that more realistically captures the process of buying and selling houses and can generate seasonal patterns quantitatively comparable to those in the data.

The model builds on two elements of the housing market that we think are important for understanding seasonality in house prices. The first element is a search friction. Buyers and sellers potentially face two search frictions: one is locating a house for sale (or a potential buyer), and the other is determining whether the house (once found) is suitable for the buyer (meaning it is a sufficiently good match). The first friction is, in our view, less relevant in the housing market context because advertising by newspapers, real estate agencies, property web sites, and so on, can give sufficient information to buyers in order to locate houses that ex ante are in the acceptance set. But houses have many idiosyncratic features that can be valued differently by different buyers: two individuals

(1991) find seasonality in prices in the City of Toronto; the latter conclude "that individuals who are willing to purchase against the seasonal will, on average, do considerably better."

[4]The data for US cities corresponds to the 10-city Case-Shiller composite. Our focus on these two countries is largely driven by the reliability and quality of the data.

[5]The issue is most evident in frictionless models, where prices reflect the present discounted value of a (presumably long) stream of flow values. Thus, seasonality in rental flows or service costs has to be implausibly large to generate seasonality in house prices. More recent models of the housing market allow for search and matching frictions that lead to slightly more complex intertemporal non-arbitrage conditions and a somewhat modified relation between prices and flows. In the web Appendix, we study the canonical models in the literature and argue that these frictions alone cannot account for the high seasonality in the data, calling for an additional mechanism to explain the seasonal patterns.

[6]We note that house price seasonality does not appear to be driven by liquidity related to overall income. Income typically peaks in the last quarter, a period in which house prices and the volume of transactions fall below trend. There is also a seasonal peak in output in the second quarter, and seasonal recessions in the first and third quarters. (See Beaulieu and Miron (1992) and Beaulieu, Miron, and MacKie-Mason (1992)). House price seasonality thus is not in line with income seasonality: prices and transactions are above trend in the second and third quarters.

visiting the same house may attach different values to the property. We model this match-specific quality as a stochastic variable that is fully revealed after the buyer inspects the house. The second model's element is the notion that in a market with more houses for sale, a buyer is more likely to find a better match—what we refer to as "thick-market effect."[7] Specifically, we assume that in a market with more houses, the distribution of match-specific quality first-order stochastically dominates the distribution in a market with fewer houses.

Hence, our model starts from the premise that the utility potential buyers may derive from a house is fully captured by the match-specific quality between the buyer and the house. This match-specific quality is more likely to be higher in a market with more buyers and houses due to the thick-market effect. In a thick market (during the hot season), better matches are more likely to be formed and this increases the probability that a transaction takes place, resulting in a higher volume of transactions. Because better matches are formed, on average, prices will also be higher, provided that sellers have some bargaining power. This mechanism leads to a higher number of transactions and prices in the hot season when there are more buyers and sellers.

In the housing market this pattern is repetitive and systematic. The same half-year is a hot season and the same half-year is a cold season. The higher match-specific quality in the hot season can account for why potential buyers are willing to buy in the hot (high-price) season. But if our amplification mechanism is to explain seasonality, it has to answer two additional questions: one, why are some sellers willing to sell in the cold (low-price) season? In other words, why is there no complete "time agglomeration," whereby markets shut down completely in a cold season? Two, why is the pattern systematic—that is, why do hot and cold markets predictably alternate with the seasons?

To answer these two questions, we embedded the above mechanism into a seasonal model of the housing market and study how a deterministic driver of seasonality can be amplified and revealed as deterministic seasonality in transactions and prices due to the thick-market effects on the match-specific quality. By focusing on a periodic steady-state, we are studying a deterministic cycle in which agents are fully aware that they are in a market in which both transactions and prices fluctuate between high and low levels across the two seasons.

Our answer to the first question is related to the presence of search frictions in the form of match-specific quality. In the cold season any seller can decide whether to sell immediately or wait until the hot season, when presumably prospects might be more favorable on average. If a buyer then arrives

[7] The labor literature distinguishes the thick-market effects due to a faster arrival of offers and those due to the quality of the match. Our focus is entirely on the quality effect. See, for example, Diamond (1981), Petrongolo and Pissarides (2006) and Gautier and Teulings (2008).

and a match can be made, the seller has to decide whether to keep searching for a better offer or to sell at the potentially lower price. If he waits until the hot season, he can get, on average, a higher price, provided that he finds a buyer with a good match. There is, however, a probability that he will not find such buyer to make a transaction; the uncertainty created by this search friction is not present in a standard asset-pricing model, in which agents can always transact at market prices.

Our answer to the second question—why the hot and cold seasons are systematic—is related to our assumption about the desire to move house and the seasonal variations embedded within this decision. We claim that the arrival of the exogenous process by which households want to move (the "propensity to move") has a seasonal component. In the spring and summer months this propensity is higher because, for example, of the school calendar: families with school-age children may prefer to move in the summer, before their children start in new schools. These seasonal differences alone, however, cannot explain the full extent of seasonality we document: in the data, seasonality in houses for sale is much lower than seasonality in the volume of transactions; moreover, as Goodman (1993) documents, parents of school-age children account for less than a third of total movers.[8] Most of the explanatory power of the model is due to the thick-market effects on match-quality. We show that a slightly higher ex ante probability of moving in a given season (which increases the number of buyers and sellers) can trigger thick-market effects that make it appealing to all other existing buyers and sellers to transact in that season. This amplification mechanism can thus create substantial seasonality in the volume of transactions; the extent of seasonality in prices, in turn, increases with the bargaining power of sellers. Intuitively better matches in the hot season imply higher surpluses to be shared between buyers and sellers; to the extent that sellers have some bargaining power, this leads to higher house prices in the hot season. The calibrated model can quantitatively account for most of the seasonal fluctuations in transactions and prices in the United Kingdom and the United States.

The contribution of the paper can be summarized as follows. First, it systematically documents seasonal booms and busts in housing markets. Second, it develops a search-and-matching model that can quantitatively account for the seasonal patterns of prices and transactions observed in the United Kingdom and the United States. Understanding seasonality in house prices can serve as a first step to understanding how housing markets work and what the main mechanisms governing housing market fluctuations are. As such, it can help to put restrictions on the class of models needed to characterize housing markets. In other words, seasonality in house prices, what economists and publishers of house

[8] While weather conditions may make house search more convenient in the summer, it is unlikely that this convenience is worth so much money to the typical house buyer. Indeed, Goodman (1993) finds that seasonal moving patterns are similar across different regions. In addition, as we later report, cities with moderate weather throughout the year, such as Los Angeles and San Diego, also display strong seasonality in prices and transactions.

price indexes typically ignore or correct for, can contain relevant information to guide the development and selection of appropriate models for housing markets. Our analysis points to the presence of thick-and-thin market externalities; studying their interactions with other frictions at lower frequency might be a fruitful avenue for future research.

The paper is organized as follows. Section 2 reviews the related theoretical literature and discusses how the thick-and-thin market channel differs from and complements alternative explanations of housing market fluctuations. Section 3 presents the motivating empirical evidence and section 4 introduces the model. Section 5 presents the qualitative results and a quantitative analysis of the model; it then discusses additional implications of the model. Section 6 presents concluding remarks. The web Appendix presents supplementary empirical evidence supporting the model. It then studies the existing canonical models of the housing market and argues that they cannot account for the seasonality observed in the data. Next, it describes the efficiency properties of the model and generalizes the framework to study its robustness to different modelling assumptions; in particular it allows for differential moving costs as alternative triggers of seasonality; it studies different assumptions regarding the observability of the match quality, and different pricing mechanisms (including price posting by sellers). Finally, the web Appendix provides detailed micro-foundations for the thick-and thin-market effects. All analytical derivations and proofs are also collected in the web Appendix.

2 Related Theoretical Literature

The search-and-matching framework has been applied before to the study of housing markets (for example, see Wheaton (1990), Williams (1995), Krainer (2001), and Albrecht et al. (2007)). Recent work on housing market fluctuations, such as Novy-Marx (2009), Diaz and Jerez (2012) and Piazzesi and Schneider (2009), adopt an aggregate matching function (as in Pissarides (2000)) and focus on the role of market tightness (the ratio of the number of buyers to the number of sellers) in determining the probability of transactions taking place. These papers study the amplified response of housing markets to an unexpected shock. We instead focus on predictable cycles, with both sellers and buyers being fully aware of being in such periodic cycle. We distinguish the probability of making a contact and the probability that the house turns out to be a good match. The contact probability is always one in our model, but the match quality drawn is a random variable. In this sense, our setup is closest to Jovanovic (1979), which also emphasizes the stochastic nature of the match-specific quality for the labour market, and Krainer (2001) for the housing market. In contrast to previous models that focus on market tightness, transactions and prices in our set-up are governed by the distribution of match-specific quality.

Our paper complements the seminal work by Krainer (2001) and Novy-Marx (2009), by highlighting a new mechanism that can account for some of the regularities observed in housing markets. Both Krainer (2001) and Novy-Marx (2009) also refer to "hot and cold" markets; however, in both studies the nature as well as the meaning of hot and cold markets is different than in our paper. The key idea in Novy-Marx (2009) is that, if for any reason the ratio of buyers to sellers (tightness) unexpectedly increases, houses can sell more quickly, decreasing the stock of sellers in the market. This in turn increases the relative number of buyers to sellers even more, amplifying the initial shock. As a result, the outside option of sellers improves, leading to higher prices. Thus, the entire amplification effect operates through market tightness. In our model, instead, market tightness plays no role; indeed, it is constant across all seasons. If an agent receives a shock that forces her to move, she becomes a potential buyer and a potential seller simultaneously and overall tightness does not change. The amplification mechanism in our model comes instead from the quality of the matches. In the hot season there are both more buyers and more sellers; the availability of a bigger stock of houses for sale improves the overall efficiency of the market, as buyers are more likely to find a better match. Put differently, our explanation relies on market thickness (the numbers of buyers and sellers) and its effect on the quality of matches, whereas Novy-Marx's hinges on tightness. This difference leads to crucially different predictions for the correlation between prices and transactions. In Novy-Marx (2009), the number of transactions in the housing market is not necessarily higher when prices are high. His model generates a positive correlation between prices and tightness, but not necessarily a positive correlation between prices and the volume of transactions, which is one of the salient features of housing markets (Stein 1995). Specifically, in Novy-Marx (2009), a large increase in the number of sellers and buyers that does not alter tightness would not alter prices at all, even if it substantially increases the number of transactions. Similarly, in his model, a decline in the number of sellers leads to an increase in tightness, lower volume of transactions, and higher prices, thus generating a negative comovement between prices and transactions. Instead, our model always generates a positive correlation between prices and transactions. As Wheaton (1990) has pointed out, moving houses most of the time means both selling a house and buying another one and hence, in this context, a model in which tightness plays a subdued role is appealing. In our model, a hot market is one with high prices, more buyers and sellers, and an unambiguously larger number of transactions.[9] Of course, in

[9]As Novy-Marx (2009), our model has predictions for average time on the market (TOM). Specifically, the model predicts that a house put up for sale in the cold season will take longer to sell. There is a difference, however, between

practice tightness and thickness of the market can operate simultaneously, and their role might vary at different frequencies. In this sense, our paper complements Novy-Marx (2009) and the existing literature focusing on tightness.

In our paper, "hot-and-cold markets" also are different from those in Krainer (2001), who studies the response of housing markets to an aggregate shock that affects the fundamental value of houses—his model cannot generate quantitatively meaningful fluctuations in prices unless the aggregate shock is very persistent. A deterministic cycle in Krainer's model is equivalent to setting the persistence parameter to zero, in which case his model predicts virtually no fluctuation in prices. Our set-up is different from Krainer (2001) in that it brings in thick-market effects which, due to their amplification, are able to generate quantitatively large fluctuations in transactions and prices.[10] In the web Appendix we expand on this point and argue that in the absence of a thick-market effect, existing models of the housing market are unable to account for the seasonality in the data.

Finally, we follow the literature (for example, see Wheaton 1990 and Krainer 2001) by assuming exogenous moving shocks. This essentially abstracts from the decision to dissolve a match, which would potentially require a role for school enrollments, marriages, job changes, and other socioeconomic determinants outside our model. The main potential contribution of allowing endogenous moving decision is to account for the seasonality in vacancies (homes for sale). Since we do not have data that is more fundamental (e.g. seasonality in shocks that change the match quality) than the observed seasonality in vacancies, we do not attempt to predict the seasonality in vacancies. Instead, in the calibration, we choose to match the seasonality in vacancies observed in the data, and study its effects on prices and transactions; thus the potential amplification mechanism through the endogenous moving decision is already embedded in the seasonality in vacancy.

3 Hot and Cold Seasons in the Data

In this section we study seasonality in housing markets in the United Kingdom and the United States at different levels of aggregation. The focus on these two countries is due to the availability of constant-

our mechanism and that in models emphasizing tightness. Our model predicts higher probability of a transaction and shorter average TOM for both buyers and sellers in the hot season. (We emphasize that the prediction is about the correlation between average time on the market and prices *over time*, not across sellers—or buyers—within a time period. See, for example, Krainer, 2001, and Diaz and Jerez, 2012.) Models that focus on market tightness predict an inverse relation between buyer's and seller's TOM (average TOM is short for buyers but long for sellers when tightness is high). Instead, our model predicts they move in the same direction. Empirical studies focus on sellers' TOM, largely because data on buyers' TOM is less easily observed. This prediction could potentially be tested empirically, as more data on the buyer's side are gathered.

[10] Unlike Krainer (2001), we also model the endogenous evolution of the number of vacancies and buyers over time.

quality house price series.[11] As already noted, publishers of house price indexes produce both seasonally adjusted (SA) and non-seasonally adjusted (NSA) series. This is also the case for transactions. In our analysis, we use exclusively the (raw) NSA series to compute the extent of seasonality.[12] In what follows, we first describe the data sources and assess the degree of seasonality in the data. Next, we discuss the behavior other variables related to the housing market. Finally, we provide empirical evidence motivating the mechanism we propose.

3.1 Data

United Kingdom

As a source for house price data, we use the repeat-price index based on Case-Shiller (1987)'s method, produced by the Land Registry for England and Wales. The repeat-sale index measures average price changes in repeat sales of the same properties; as such, the index is designed to control for the characteristics of the homes sold.[13] The index is constructed at different levels of geographic aggregation and starts in 1995:Q1. In the interest of space, we discuss here the results for the main planning regions and in the web Appendix we report the results at finer levels of disaggregation.[14] To compute real price indexes, we later deflate the house price indexes using the NSA retail price index (RPI) provided by the U.K. Office for National Statistics.

For transactions, we use the data on sales volumes also published by the Land Registry.

United States

We use two sources for house prices in the United States. The first is the Federal Housing Finance Agency (FHFA), which took over the Office of Federal Housing Enterprise Oversight; we focus on the

[11] Constant-quality indexes mitigate concerns with compositional changes in the types of houses transacted across seasons. Results for other countries show qualitatively similar seasonal patterns, but we are less confident about the comparability of the data.

[12] In the web Appendix we show the implied seasonal patterns based on the publishers' in-house adjustments.

[13] The approach significantly limits the extent to which changes in the composition of the sample of houses transacted can influence the price index. Specifically, using information on the values of the same physical units at two points in time controls for differences in housing attributes across properties in the sample.

[14] Ther are two other sources providing quality-adjusted NSA house price indexes: one is the Department of Communities and Local Government and the other is Halifax, one of the country's largest mortgage lenders. Both sources report regional price indexes based on hedonic regressions. The results are consistent across all sources (see web Appendix.) Other house price publishers, such as the Nationwide Building Society (NBS), report quality adjusted data but they are already SA (the NSA data are not publicly available). The NBS, however, reports in its methodology description that June is generally the strongest month for house prices and January is the weakest; this justifies the seasonal adjustment they perform in the published series. In a somewhat puzzling paper, Rosenthal (2006) argues that seasonality in the NBS data is elusive; we could not, however, gain access to the NSA data to assess which of the two conflicting assessments (the NBS's or Rosenthal's) was correct. We should perhaps also mention that Rosenthal (2006) also reaches very different conclusions from Muellbauer and Murphy (1997) with regards to lower-frequency movements.

repeat-sale purchase-only index, which starts in 1991:Q1. The second source is Standard and Poor's (S&P) Case-Shiller price series for major U.S. cities, which starts in 1987:Q1. To compute real price indexes, we use the NSA consumer price index (CPI) provided by the U.S. Bureau of Labor Statistics.[15]

Data on the number of transactions at the regional level come from the National Association of Realtors (NAR), and correspond to the number of sales of existing single-family homes. Data for U.S. cities come from S&P and correspond to sales pair counts on which the repeat-price index is based.

3.2 Extent of Seasonality

We focus our study on deterministic seasonality, which is easier to understand (and to predict) for buyers and sellers (unlikely to be all econometricians), and hence most puzzling from a theoretical point of view. In the United Kingdom and the United States, prices and transactions in both the second and third quarters are above trend, while in both first and fourth quarters they are below trend. For ease of exposition, we group data into two broadly defined seasons—second and third quarter, or "hot season," and fourth and first quarter, or "cold season." (We use interchangeably the terms "hot season" and "summer" to refer to the second and third quarters and "cold season" and "winter" to refer to the first and fourth quarters.)

In the next set of figures, we depict in dark (red) bars the average (annualized) price increase from winter to summer, $\ln\left(\frac{P_S}{P_W}\right)^2$, where P_S is the price index at the end of the hot season and P_W is the price at the end of the cold season. Correspondingly, we depict in light (blue) bars the average (annualized) price increase from summer to winter $\ln\left(\frac{P_{W'}}{P_S}\right)^2$, where $P_{W'}$ is the price index at the end of the cold season in the following year. We plot similar figures for transactions.

The extent of seasonality for each geographical unit can then be measured as the difference between the two bars. This measure nets out lower-frequency fluctuations affecting both seasons. In the model we later present, we use a similar metric to gauge the extent of seasonality.

3.2.1 Housing Market Seasonality in the United Kingdom

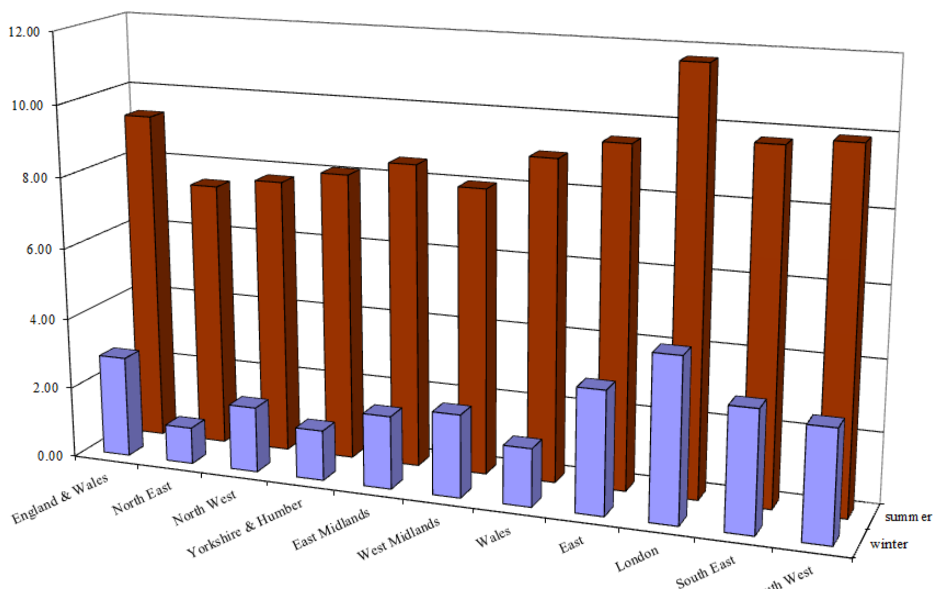
Nominal and Real House Prices

Figure 1 reports the average annualized percent price increases in the summer and winter from 1996 through to 2012 using the regional price indexes provided by the Land Registry. During the period analyzed, the average nominal price increases in the winter were around 3 percent in all regions except

[15] There is little seasonality in the U.S. CPI, a finding first documented by Barsky and Miron (1989), and hence the seasonal patterns in nominal and real housing prices coincide.

for London. In the summer, the average growth rates were above 8 percent in all regions, except for the North East and North West. As shown in the graph, the differences in growth rates across the two broad seasons are generally very large and economically significant, with an average of 6.5 percent for all regions. Similar seasonal patterns emerge with other sources of constant-quality prices going back to 1983.[16] While the average growth rates differ across different time periods, the seasonal pattern appears extremely robust.

Figure 1: Average Annualized House Price Changes in Summer and Winter, by Region



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. Land Registry, 1996-2012.

The seasonal pattern of real house prices (that is, house prices relative to the NSA aggregate price index) depends also on the seasonality of aggregate inflation. In the United Kingdom, overall price inflation during this period displayed a small degree of seasonality. The difference in overall inflation rates across the two seasons, however, can hardly "undo" the differences in nominal house price inflation, implying a significant degree of seasonality also in real house prices (see Figure $A2$ in the web Appendix). Netting out the effect of overall inflation reduces the differences in growth rates between winters and summers to a country-wide average of 5.5 percent.[17]

Number of Transactions

Seasonal fluctuations in house prices are accompanied by qualitatively similar fluctuations in the

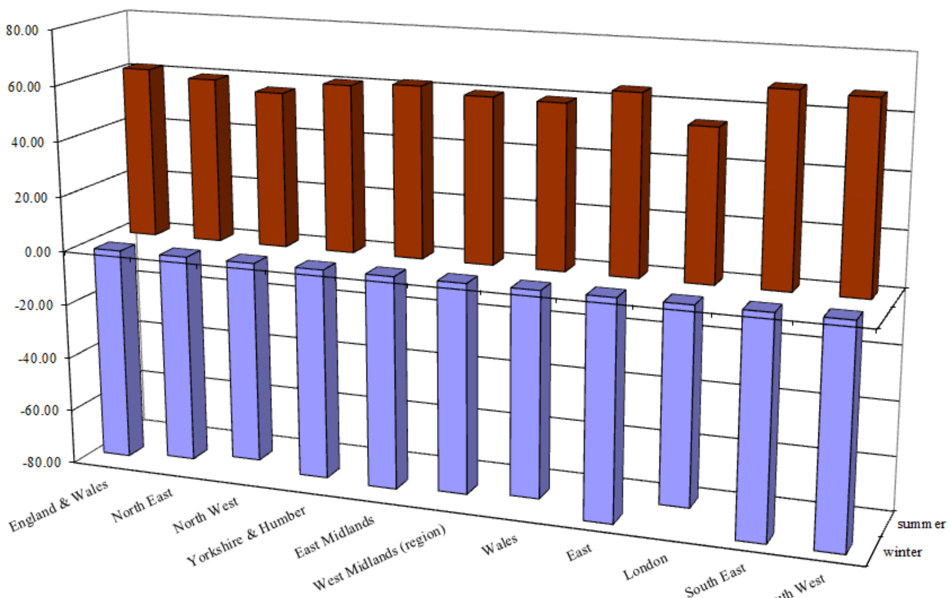
[16] See also Figure $A1$ in the web Appendix for results based on alternative sources.

[17] We also looked at more disaggregated data, using the Halifax series, distinguishing between first-time buyers and former-owner occupiers, as well as purchases of newly built houses versus existing houses. Seasonal patterns are qualitatively similar across the various groups, but tend to be quantitatively stronger for former-owner occupiers and existing houses. The results are reported in Table $A1$ in the web Appendix.

Our model, by abstracting from construction, will speak more directly to the evidence on existing houses, and, as it will become clear, former-owner occupiers.

number of transactions, as illustrated in Figure 2.[18] As the figure shows, the number of transactions increases sharply in the summer term and accordingly declines in the winter term. The average difference in growth rates during this period, our metric for seasonality) was 139 percent.

Figure 2: Average Annualized Changes in Transactions in Summer and Winter, by Region



Note: Annualized growth rates of the number of transactions in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. Land Registry, 1996-2012.

3.2.2 Statistical Significance of the Differences between Summer and Winter

We test the statistical significance of the differences in growth rates across seasons, $\left[\ln\left(\frac{P_S}{P_W}\right)^2 - \ln\left(\frac{P_{W'}}{P_S}\right)^2\right]$, using a t-test on the equality of means.[19] Table 1 reports the average differences in growth rates across seasons and standard errors, together with the statistical significance. The first two columns show the results for seasonality in nominal house prices; the third and fourth columns show the corresponding results for real house prices and the last two columns show the results for the volume of sales.[20] The differences in price changes across seasons are quite sizable for most

[18] A different dataset from the Council of Mortgage Lenders going back to 1983 (and to 1974 for some regions) show similar seasonal patterns. See Figure *A*3 in the web Appendix.

[19] The test on the equality of means is equivalent to the t-test on the slope coefficient from a regression of annualized growth rates on a dummy variable that takes value 1 if the observation falls on the second and third quarter and 0 otherwise. The dummy coefficient captures the annualized difference across the two seasons, regardless of the frequency of the data (provided growth rates are annualized). To see this note that the annualized growth rate in, say, the hot season, $\ln\left(\frac{P_S}{P_W}\right)^2$, is equal to the average of annualized quarterly growth rates in the summer term: $\ln\left(\frac{P_S}{P_W}\right)^2 = 2\ln\left(\frac{P_3}{P_1}\right) = \frac{1}{2}\left[4\ln\left(\frac{P_3}{P_2}\right) + 4\ln\left(\frac{P_3}{P_2}\right)\right]$, where the subindices indicate the quarter, and, correspondingly, $2\ln\left(\frac{P_{1'}}{P_3}\right) = \frac{1}{2}\left[4\ln\left(\frac{P_{1'}}{P_4}\right) + 4\ln\left(\frac{P_4}{P_3}\right)\right]$. Hence a regression with quarterly (or semester) data on a summer dummy will produce an unbiased estimate of the average difference in growth rates across seasons. We use quarterly data to exploit all the information and gain on degrees of freedom.

[20] Tables *A*2*a* and *A*2*b* in the web Appendix shows the results at geographically more disaggregated levels and Table *A*3 shows the corresponding information at the regional level using aternative datasets.

regions, in the order of 6 to 7 percent on average in nominal terms and 5 to 6 percent on average in real terms; from a statistical point of view, the results are significant at the 10 percent level (or lower). For transactions, the differences reach 139 percent for the country as a whole, and are statistically significant at the 1 percent level. Taken together, the data point to a strong seasonal cycle in all regions, with a large increase in transactions and prices during the summer relative to the winter.

Table 1: Difference in Annualized Percentage Changes in U.K. House Prices and Transactions between Summer and Winter, by Region 1996-2012

	Nominal house price		Real house price		Volume of Sales	
Region	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
England & Wales	6.472***	(2.272)	5.547**	(2.489)	139.249***	(15.603)
North East	6.439**	(2.874)	5.514*	(3.092)	134.992***	(17.647)
North West	5.897**	(2.528)	4.972*	(2.734)	128.950***	(15.864)
Yorks & Humber	6.683**	(2.595)	5.757**	(2.806)	136.786***	(16.588)
East Midlands	6.473**	(2.497)	5.548**	(2.708)	139.083***	(16.572)
West Midlands	5.686**	(2.290)	4.761*	(2.515)	135.452***	(15.745)
Wales	7.346***	(2.579)	6.420**	(2.809)	133.254***	(16.580)
East	6.050**	(2.412)	5.125*	(2.604)	144.768***	(16.116)
London	7.129***	(2.467)	6.204**	(2.624)	124.953***	(14.981)
South East	6.336**	(2.413)	5.410**	(2.592)	152.763***	(15.740)
South West	6.798***	(2.507)	5.873**	(2.709)	150.323***	(16.772)

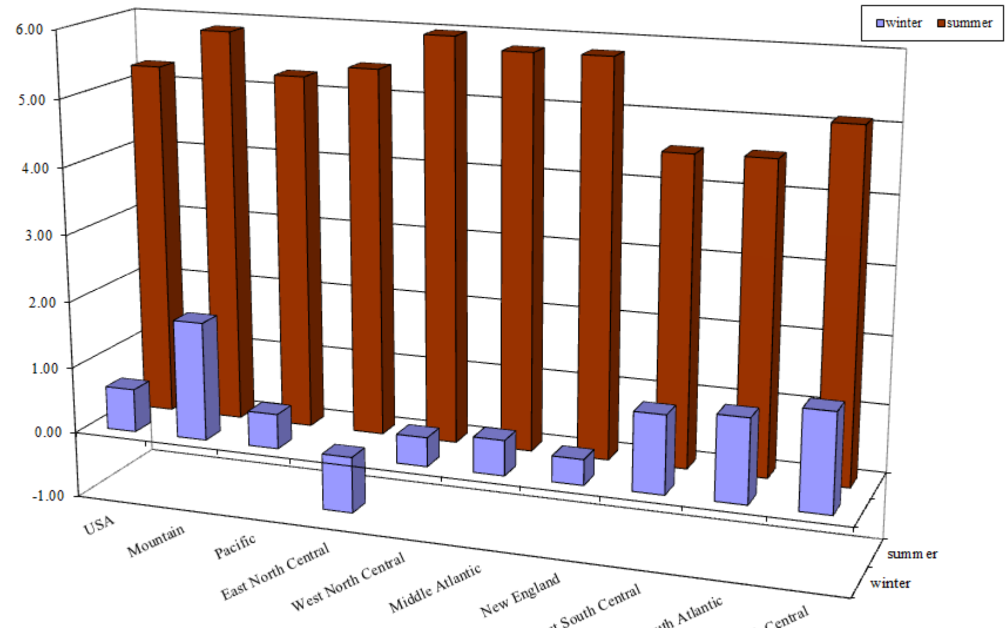
Note: The Table shows the average differences (and standard errors), by region for 1995-2012.
*Significant at 10%; **significant at 5%; ***significant at 1%. Source: Land Registry Repeat Sale Index.

Housing Market Seasonality in the United States

Nominal and Real House Prices

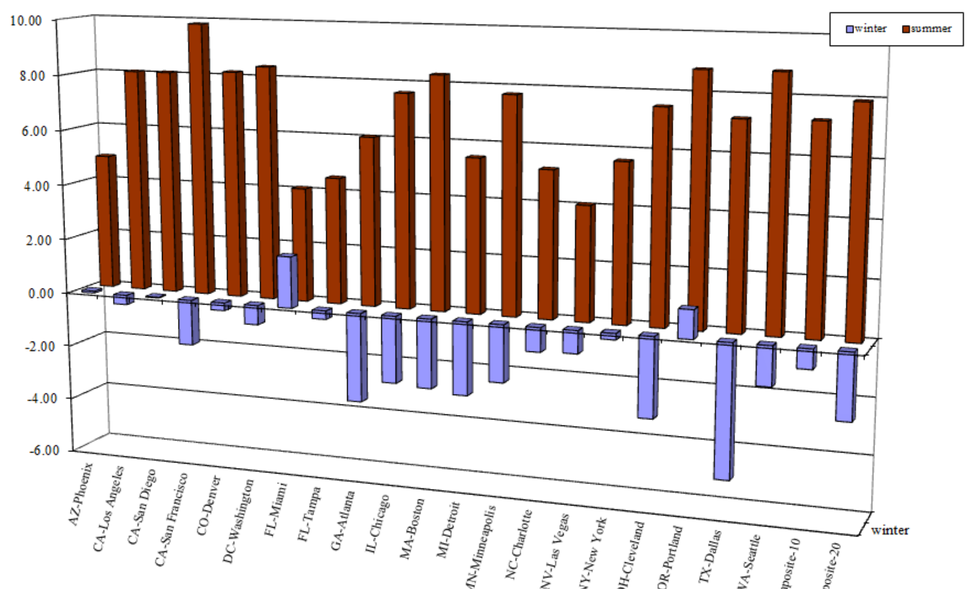
Figure 3 illustrates the annualized nominal house price increases for different regions from FHFA and Figure 4 shows the plot using the S&P's Case-Shiller indexes for major cities. As shown, for most US regions the seasonal pattern is qualitatively similar to that in the United Kingdom, albeit the extent of seasonality is somewhat smaller averaging 4.6 percent for nominal prices and 4.8 percent for real prices. For some of the major U.S. cities, however, the degree of seasonality is comparable to and even higher than that in the United Kingdom, as illustrated in Figure 4.

Figure 3: Average Annualized U.S. House Price Increases in Summer and Winter, by Region



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions. FHFA, 1991-2012.

Figure 4: Average Annualized U.S. House Price Changes in Summer and Winter, by City



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) by U.S. city. S&P Case and Shiller, 1987-2012. (For some cities, the index starts later.)

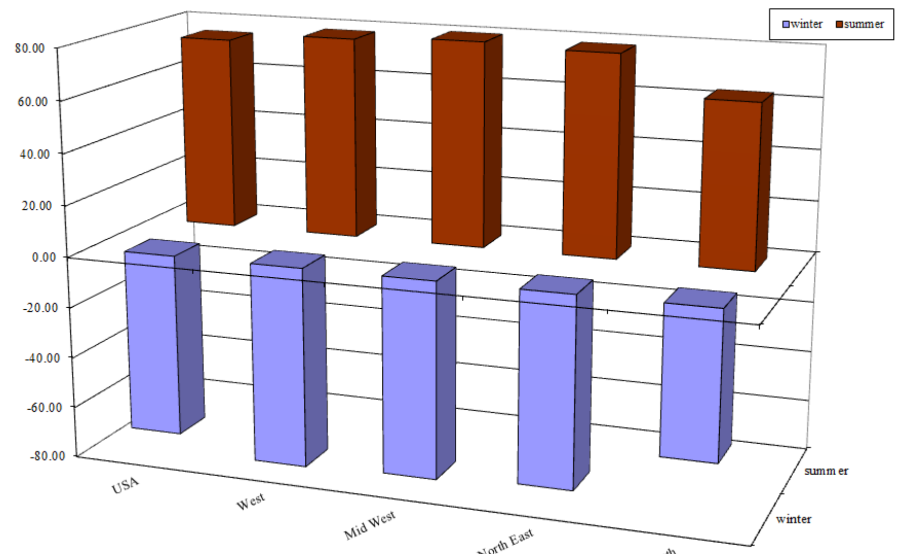
Transactions

Figure 5 shows the annualized growth rates in the number of transactions from 1991 through to 2012 for main census regions; the data come from National Association of Realtors (NAR).[21] As was the case for the United Kingdom, the seasonality of US transactions is overwhelming: the volume of

[21] The series actually starts in 1989, but we use 1991 for comparability with the FHFA-census-level division price series; adding these two years does not change the results.

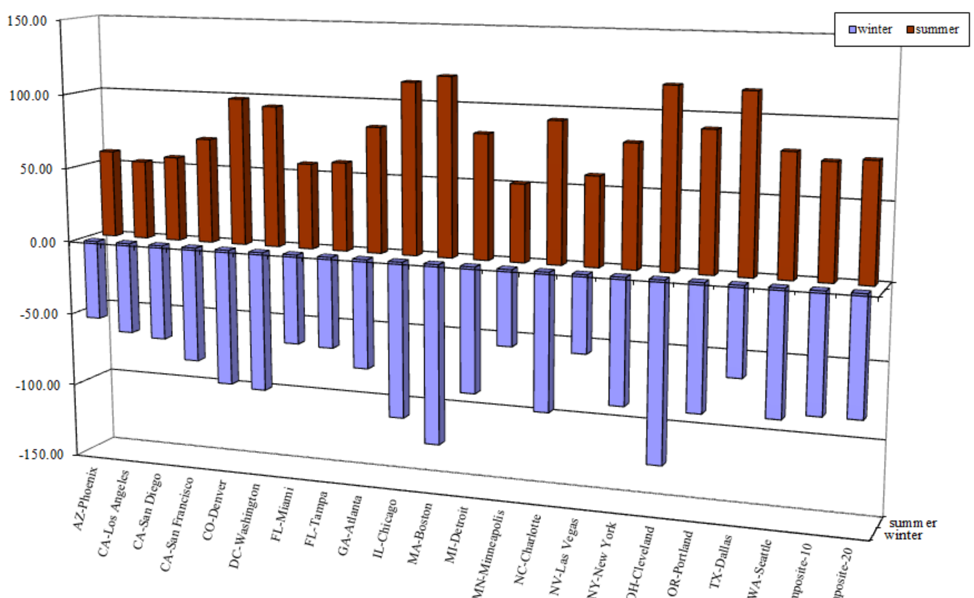
sales rises sharply in the summer and falls in the winter.

Figure 5: U.S. Average Annualized Increases in Transactions in Summer and Winter, by Region



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions. NAR 1991-20012.

Figure 6: U.S. Average Annualized Changes in Transactions in Summer and Winter, by City



Note: Annualized growth rates of the volume of transactions in summers (second and third quarters) and winters (fourth and first quarters) by U.S. city. S&P Case and Shiller, 1987-2012. (For some cities, the index starts later.)

Statistical Significance of the Differences between Summer and Winter

We summarize the differences in growth rates across seasons and report the results from a test on mean differences in Tables 2 and 3. Table 2 shows the results for prices using FHFA's Census-division levels and for transactions using NAR's Census-level data. Table 3 shows the results using S&P's Case-Shiller city-level data on prices and transactions.

Table 2: Difference in Annualized Percentage Changes in U.S. House Prices and Transactions between Summer and Winter, by Region, 1991-2012

		Nominal house price		Real house price		Volume of Sales	
Region	Division	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
USA	USA	4.632***	(1.532)	4.828***	(0.324)	146.460***	(13.704)
West	Mountain	4.081*	(2.278)	4.437***	(0.285)	120.458***	(15.934)
	Pacific	4.747	(3.053)	4.982***	(0.298)		
Mid West	East North Central	6.283***	(1.332)	6.493***	(0.616)	156.637***	(15.735)
	West North Central	5.540***	(1.117)	5.592***	(0.379)		
North East	Middle Atlantic	5.281***	(1.579)	5.514***	(0.181)	156.499***	(11.181)
	New England	5.433***	(1.892)	5.818***	(0.315)		
South	East South Central	3.380***	(1.070)	3.721***	(0.475)	152.671***	(13.038)
	South Atlantic	3.307*	(1.946)	3.449***	(0.332)		
	West South Central	3.635***	(0.763)	3.668***	(0.196)		

Note: The Table shows the average differences (and standard errors), by region for 1991:Q1-2012:Q1. *Significant at 10%; **significant at 5%; ***significant at 1%. Sources: For prices, FHFA Purchase Only Repeat Sale Index. For volume, NAR Existing single family home sales series.

Table 3: Difference in Annualized Percentage Changes in U.S. House Prices and Transactions between Summer and Winter, by City, 1987-2012

	Nominal house price		Real house price		Volume of Sales	
City	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
AZ-Phoenix	4.817	(3.229)	3.575	(3.208)	111.580***	(10.186)
CA-Los Angeles	8.398***	(2.768)	7.177**	(2.781)	113.885***	(10.824)
CA-San Diego	8.039***	(2.581)	6.818***	(2.583)	120.378***	(13.264)
CA-San Francisco	11.485***	(2.925)	10.264***	(2.875)	147.059***	(10.884)
CO-Denver	8.435***	(1.296)	7.214***	(1.336)	189.180***	(8.811)
DC-Washington	9.087***	(2.045)	7.866***	(2.037)	187.102***	(7.376)
FL-Miami	2.234	(2.620)	1.013	(2.637)	116.339***	(7.193)
FL-Tampa	4.831**	(2.201)	3.61	(2.184)	119.349***	(8.108)
GA-Atlanta	9.233***	(2.057)	7.878***	(1.993)	155.706***	(10.020)
IL-Chicago	10.039***	(1.894)	8.818***	(1.921)	216.916***	(11.983)
MA-Boston	10.799***	(1.519)	9.577***	(1.617)	237.146***	(6.763)
MI-Detroit	8.118***	(2.547)	6.763**	(2.579)	164.965***	(24.750)
MN-Minneapolis	9.780***	(2.363)	8.538***	(2.413)	100.873***	(19.367)
NC-Charlotte	6.081***	(0.989)	4.860***	(0.888)	183.521***	(11.357)
NV-Las Vegas	4.875	(3.077)	3.654	(3.057)	109.396***	(15.120)
NY-New York	5.846***	(1.641)	4.625***	(1.751)	163.048***	(10.150)
OH-Cleveland	10.354***	(1.214)	9.133***	(1.211)	235.867***	(10.829)
OR-Portland	7.787***	(1.683)	6.566***	(1.592)	173.961***	(11.023)
TX-Dallas	11.925***	(1.552)	9.671***	(1.478)	173.849***	(19.268)
WA-Seattle	10.201***	(1.902)	8.882***	(1.875)	161.596***	(11.479)
Composite-10	7.955***	(1.918)	6.734***	(1.952)	152.570***	(6.476)
Composite-20	10.261***	(3.263)	8.007**	(3.296)	154.212***	(7.801)

Note: The Table shows the average differences (and standard errors), by region for 1987-2012. *Significant at 10%; **significant at 5%; ***significant at 1%. Sources: S&P Case-Shiller Price Index and Sales pair counts. Some of the series start after 1987. The composite-20 index starts only in 2000.

For the United States as a whole, the differences in annualized growth rates in nominal prices are in the order of 4.6 percent (4.8 percent for real prices) and statistically significant at standard levels. There is some variation across regions, with some displaying low seasonality (South) and others (Mid West and North East) displaying significant levels of seasonality. Interestingly, the Case-Shiller index for U.S. cities displays even higher levels of seasonality, comparable to and even higher than the levels observed in UK regions, with some variation across cities. The 10-city composite index shows a statistically significant seasonality of 7.2 percent for nominal prices and 6.7 percent for real prices.

The seasonality in the volume of transactions is comparable to (or higher than) that in the United Kingdom, with an average difference in growth rates across seasons of 146 percent for the US as a whole and of 152.6 percent for the 10-city composite.

3.3 Other housing variables and cross-market comparisons

As part of our study of house markets, we also analyzed data on rental prices, but we were unable to identify a seasonal pattern in either country.[22] This is in line with anecdotal evidence suggesting that rents are sticky. Similarly, interest rates did not exhibit a seasonal pattern in the last four decades of data.[23] In the interest of space, we do not report the results in the paper. In the model we present later, we will work under the assumption that rents and interest rates are aseasonal.

The data description makes it evident that seasonal cycles are present across most of the United Kingdom and the United States, although with some heterogeneity with regards to intensity. In particular, though most U.S. cities display strong seasonality, cities such as Miami and Las Vegas show little (or statistically insignificant) variation over the season. Given the data limitations (20 observations on price seasonality corresponding to the cities in the Case-Shiller data), it would be virtually impossible to draw causal links from the potential triggers of seasonality because winters are

[22] We studied the average registered private rents collected by U.K. Housing and Construction Statistics. We run regressions using as dependent variables both the rent levels and the log of rents on a dummy variable taking a value of 1 in the second and third quarters and 0 otherwise, detrending the data in different ways. We found no evidence of deterministic seasonality. For the United States, the Bureau of Labor Statistics (BLS) provides two series that can serve as proxies: one is the NSA series of owner's equivalent rent and the second is the NSA rent of primary residence; both series are produced for the construction of the CPI and correspond to averages over all U.S. cities. For each series, we run similar regressions as for the UK. The results yielded no discernible pattern of seasonality. We take this as only suggestive as, of course, the data are not as clean and detailed as we would wish.

[23] We investigated seasonality in different interest rate series published by the Bank of England: the repo (base) rate, an average interest rate charged by the four major U.K. banks before the crisis (Barclays Bank, Lloyds Bank, HSBC, and National Westminster Bank), and a weighted average standard variable mortgage rate from banks and Building Societies. None of the interest rate series displays seasonality. For the United States, we studied data on mortgage rates produced by the Board of Governors of the Federal Reserve System, corresponding to contract interest rates on commitments for fixed-rate first mortgages; the data are quarterly averages beginning in 1972 and the original data are collected by Freddie Mac. Consistent with the findings of Barsky and Miron (1989) and the evidence from the United Kingdom, we did not find any significant deterministic seasonality.

mild in these cities and there is a larger population of elderly people, factors which are intimately related. We note, though, that the mildness of a winter per se does not straightforwardly predict aseasonality, as cities such as Los Angeles, San Diego, or San Francisco display strong seasonality in prices, despite their benign weather. A perhaps more likely trigger of seasonality is the school calendar.[24] As noted earlier, however, only a small portion of the population of potential home buyers have school-age children.[25] One of the model's implications is that even slight differences in the "fundamentals" of the seasons have the potential to trigger thick-market effects with large swings in the volume of transactions and prices. Hence, in equilibrium, most people end up transacting in the summer. This is consistent with the data, illustrated in Figure $A4$ in the web Appendix, which shows that people in different life-cycle stages (not just parents of school age children) tend to move in the summer—a regularity originally noted by Goodman (1991).

We also note that U.S. cities tend to display more seasonality than the United States as a whole, a pattern that, as we shall explain, can be rationalized by our model. In particular, the model predicts that there should be more price seasonality in markets in which sellers have higher bargaining power. The cross-city evidence appears consistent with this prediction in that price seasonality is positively correlated with the price-to-rent ratio (which, in turn increases with the degree of bargaining power in our model).[26]

Some may argue that cities by their sheer size, are likely to be "thicker" throughout the year and hence seasonal differences in thickness are relatively unimportant. Anecdotal evidence, however, suggests that even within cities, housing markets can be highly segmented, as people tend to search in relatively narrow neighborhoods and geographic areas (e.g., to be close to school, jobs, families). Thus, for example, London or Washington DC as a whole are not the relevant sizes of the local housing market, and it would be improper to use these cities as boundaries to define market thickness (e.g., for those familiar with London's geography and social structure, people searching in South Kensington will never search in the East End). In other words, seemingly large cities may mask a collection of relatively smaller and segmented housing markets that can see significant changes in thickness throughout the year. A limitation of the data is hence that we cannot meaningfully compare thickness across geographic units.

Finally, we note that seasonality appears to be slightly higher during the recent crisis, although

[24] There is a positive correlation between seasonality and the ratio of school age children to elderly people in a city. However, the results seems entirely driven by Miami and Tampa.

[25] The fraction of movers with children between 6 and 17 years old is 0.22 according to the American Housing Survey 1999.

[26] The correlation between seasonality and the price-to-rent ratio in the data is about 0.3. Data on Price-to-rent ratios come from the 2009 New York Times index.

it is too early to draw general conclusions. As we gather more data (and cycles) over time, we may be able to discern whether this is indeed a systematic pattern. This would be consistent with our model: during cyclical busts, the incentives to transact during the summer (the thick market) are even stronger, the chances to find a better match are relatively higher.

3.4 Match Quality and Seasons in the Data

The key idea at the core of the model we propose is that, due to the thick-market effect, the average quality of matches formed in the summer is higher than in the winter. We use individual household data from the American Housing Survey (AHS) to check the empirical plausibility of this idea. Though the quality of the match is house-owner specific and not directly observable to the econometrician, we consider three proxies that should be correlated with it.

The first proxy is the duration of the match. The premise is that, in practice, if the house is a good fit for the household, the household will tend to stay longer; in other words, the duration of stay should be indicative of the quality of the match. (In the labour literature the duration of the employment relationship is often used as a proxy for the quality of the match.) We hence ask whether in the data, matches formed in the hot season tend to last longer. And we find that this indeed the case. The results are summarized in Table 4, which shows (poisson) regressions of the number of years of stay on a dummy variable that takes the value 1 if the household head moved in the summer season. (The results are similar if we use instead the season in which the house was bought.)[27] As the table shows, on average, the duration of stay increases by 3.3 to 4.3 percent when households move in the summer. The results are robust to a number of controls, including the age of the house, the family income, the size of the household, the number of households older than 18, as well as regional fixed effects, the urban/suburban/rural status of the location and the heating and cooling degree days.

As a second (inverse) proxy for the quality of the match, we consider the number of repairs and additions made to a house during the first two years after its purchase, which we interpret as inversely related to the quality of the original match. We then ask whether the number of repairs and alterations depends on the season in which the match was formed. The results are summarized in Table 5, which shows (poisson) regressions of the number of repairs and alterations on a dummy variable which takes the value one if the household head moved in the summer. Consistent with our hypothesis, we find that the number of repairs and additions is about 10 percent lower when the household moved in the

[27] The regressions use data on households for which we observe a full duration spell—that is, households who report the date of the last move and the date of the move previous to that. The data correspond to the AHS 1999.

summer than when the household moved in the winter (as before, using the season in which the house was bought does not alter the results). The regression results are robust to the same geographic, house- and household-specific characteristics described above.

Finally, and related to the previous idea, we use as third proxy the cost of repairs and additions incurred on a house during the first two years after its purchase, which we also interpret as an inverse proxy for the quality of the match. The results are described in Table 6. We find that on average, the cost of repairs and alterations (relative to the value of the house) are 15 percent lower when the house was bought in the summer. [28]

In all, the micro evidence appears consistent with the idea that matches formed in the summer tend to be of better quality.

Table 4. Duration of the Match and Season in which Match was Formed

	Dependent variable: length of stay (in years)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Moved into unit in the Summer	0.035*	0.033*	0.033*	0.034*	0.037*	0.041**	0.043**	0.043**
	[0.020]	[0.020]	[0.020]	[0.020]	[0.020]	[0.020]	[0.020]	[0.020]
Year unit was built		0.001	0.001	0.000	0.000	0.000	0.000	0.000
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Family income (in US$1,000)		0.002***	0.002***	0.002***	0.002***	0.002***	0.002***	0.002***
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Number of Persons in household					0.034***	0.059***	0.059***	0.059***
					[0.007]	[0.008]	[0.008]	[0.008]
Number of adults +18 in household						-0.090***	-0.090***	-0.090***
						[0.017]	[0.017]	[0.017]
Region fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Central city/suburban status fixed effects	No	No	No	Yes	Yes	Yes	Yes	Yes
Average heating/cooling degree days controls	No	No	No	No	No	No	Yes	Yes
CMSA fixed effects	No	No	No	No	No	No	No	Yes
Observations	6,885	6,885	6,885	6,885	6,885	6,885	6,885	6,885

Note: Poisson regression. The dependent variable is the duration of stay (in years). Moved into unit in the Summer takes the value 1 if household head moved in the spring or summer (in the previous move). Sample includes all respondents for whom we observe a full duration spell. Robust standard errors in brackets. Significance: *** p<0.01, ** p<0.05, * p<0.1. Central city/suburban status categories: 1) central city of MSA; 2) inside MSA, but not in central city-urban; 3) inside MSA, but not in central city-rural; 4) outside MSA, urban; 5) outside MSA, rural. Heating/cooling degree days categories: 1) Coldest: 7,001+ heating degree days and < 2,000 cooling degree days; 2) Cold: 5,500-7,000 heating degree days and < 2,000 cooling degree days; 3) Cool: 4,000-5,499 heating degree days and < 2,000 cooling degree days; 4) Mild: < 4,000 heating degree days and < 2,000 cooling degree days; 5) Mixed: 2,000-3,999 heating degree days and 2,000+ cooling degree days; 6) Hot: < 2,000 heating degree days and 2,000+ cooling degree days.

[28] Since the regressions in Table 6 are in logs, the summer effect is obtained as: $-15\% = [\exp(-0.17) - 1] * 100\%$.

Table 5. Number of Repairs and Alterations and Season in which Match was Formed

Dependent variable: Number of repairs an alterations within two years after move relative to property value								
Moved into unit in the Summer	-0.102**	-0.108**	-0.110**	-0.110**	-0.112**	-0.110**	-0.114**	-0.109**
	[0.047]	[0.046]	[0.045]	[0.046]	[0.046]	[0.045]	[0.045]	[0.045]
Year unit was built		-0.013***	-0.014***	-0.014***	-0.014***	-0.015***	-0.015***	-0.015***
		[0.001]	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]
Family income (in US$1,000)			0.002***	0.002***	0.002***	0.002***	0.002***	0.002***
			[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Number of Persons in household				0.042**	0.055***	0.054***	0.057***	0.057***
				[0.017]	[0.018]	[0.018]	[0.017]	[0.017]
Number of adults +18 in household					-0.044	-0.04	-0.039	-0.04
					[0.035]	[0.035]	[0.036]	[0.035]
Region fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Central city/suburban status fixed effects	No	No	No	Yes	Yes	Yes	Yes	Yes
Average heating/cooling degree days controls	No	No	No	No	No	No	Yes	Yes
CMSA fixed effects	No	No	No	No	No	No	No	Yes
Observations	5,982	5,982	5,982	5,982	5,982	5,982	5,982	5,982

Note: Poisson regression. The dependent variable is the number of repairs and alterations within the last two years relative to the property value. Sample includes all respondents who moved in or after 1997. Moved into unit in the Summer takes the value 1 if household head moved in the spring or summer (in the last move). Robust standard errors in brackets. Significance: *** p<0.01, ** p<0.05, * p<0.1. Central city/suburban status categories: 1) central city of MSA; 2) inside MSA, but not in central city-urban; 3) inside MSA, but not in central city-rural; 4) outside MSA, urban; 5) outside MSA, rural. Heating/cooling degree days categories: 1) Coldest: 7,001+ heating degree days and < 2,000 cooling degree days; 2) Cold: 5,500-7,000 heating degree days and < 2,000 cooling degree days; 3) Cool: 4,000-5,499 heating degree days and < 2,000 cooling degree days; 4) Mild: < 4,000 heating degree days and < 2,000 cooling degree days; 5) Mixed: 2,000-3,999 heating degree days and 2,000+ cooling degree days; 6) Hot: < 2,000 heating degree days and 2,000+ cooling degree days.

Table 6. Costs of Repairs and Alterations and Season in which Match was Formed

Dependent variable: (log) of cost of repairs an alterations within two years after move relative to property value								
Moved into unit in the Summer	-0.179**	-0.188***	-0.185***	-0.181**	-0.181**	-0.173**	-0.168**	-0.178**
	[0.071]	[0.071]	[0.071]	[0.071]	[0.071]	[0.070]	[0.070]	[0.071]
Year unit was built		-0.008***	-0.007***	-0.007***	-0.007***	-0.009***	-0.009***	-0.009***
		[0.001]	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]	[0.002]
Family income (in US$1,000)			-0.001*	-0.001	-0.001	0.000	0.000	0.000
			[0.001]	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]
Number of Persons in household				-0.052**	-0.058*	-0.061**	-0.058*	-0.063**
				[0.025]	[0.030]	[0.030]	[0.030]	[0.030]
Number of adults +18 in household					0.021	0.024	0.021	0.023
					[0.067]	[0.068]	[0.067]	[0.067]
Region fixed effects	3051	3051	3051	3051	3051	3051	3051	3051
Central city/suburban status fixed effects	No	No	No	Yes	Yes	Yes	Yes	Yes
Average heating/cooling degree days controls	No	No	No	No	No	No	Yes	Yes
CMSA fixed effects	No	No	No	No	No	No	No	Yes
Observations	3,051	3,051	3,051	3,051	3,051	3,051	3,051	3,051

Note: The dependent variable is the (log of the) costs of repairs and alterations within the last two years relative to the property value. Sample includes all respondents who moved in or after 1997. Moved into unit in the Summer takes the value 1 if household head moved in the spring or summer (in the last move). Robust standard errors in brackets. Significance: *** p<0.01, ** p<0.05, * p<0.1. Central city/suburban status categories: 1) central city of MSA; 2) inside MSA, but not in central city-urban; 3) inside MSA, but not in central city-rural; 4) outside MSA, urban; 5) outside MSA, rural. Heating/cooling degree days categories: 1) Coldest: 7,001+ heating degree days and < 2,000 cooling degree days; 2) Cold: 5,500-7,000 heating degree days and < 2,000 cooling degree days; 3) Cool: 4,000-5,499 heating degree days and < 2,000 cooling degree days; 4) Mild: < 4,000 heating degree days and < 2,000 cooling degree days; 5) Mixed: 2,000-3,999 heating degree days and 2,000+ cooling degree days; 6) Hot: < 2,000 heating degree days and 2,000+ cooling degree days.

4 A Search-and-Matching Model for the Housing Market

We have mentioned that the predictability and size of the seasonal variation in house prices pose a challenge to existing models of the housing market. In the web Appendix we study the canonical models in the literature and argue that they cannot account for the seasonality we see in the data, calling for a different mechanism to explain seasonal patterns. In this section we develop a search-and-matching model for the housing market with two key elements, "match-specific quality" and "thick-market effects." We then show that the model can generate seasonal fluctuations comparable to those in the data.

4.1 The Model Economy

The economy is populated by a unit measure of infinitely lived agents, who have linear preferences over housing services and a non-durable consumption good. Each period agents receive a fixed endowment of the consumption good which they can either consume or use to buy housing services. An agent can only enjoy housing services by living in one house at a time, that is, he can only be "matched" to one house at a time. Agents who are not matched to a house seek to buy one ("buyers").

There is a unit measure of housing stock. Correspondingly, each period a house can be either matched or unmatched. A matched house delivers a flow of housing services of quality ε to its owner. The quality of housing services ε is match-specific, and it reflects the suitability of a match between a house and its owner. In other words, for any house, the quality of housing services is idiosyncratic to the match between the house and the potential owner. For example, a particular house may match a buyer's taste perfectly well, while at the same time being an unsatisfactory match to another buyer. Hence, ε is not the type of house (or of the seller who owns a particular house). This is consistent with our data, which control for houses' characteristics, but not for the quality of a match.[29]

We assume that in a market with many houses for sale, a buyer is more likely to find a better match, what we refer to as the "thick-market effect." As in Diamond (1981), we model this idea by assuming that the match-specific quality ε follows a distribution $F(\varepsilon, v)$, with positive support and finite mean, and

$$F(., v') \leq F(., v) \Leftrightarrow v' > v, \tag{1}$$

where v denotes the stock of houses for sales. In words, when the stock of houses v is larger, a random match-quality draw from $F(\varepsilon, v)$ is likely to be higher. The web Appendix provides detailed

[29] Repeat-sale indexes do not control for the quality of a match, which is not observed by data collectors.

micro-foundations for this assumption.[30]

Unmatched houses are "for sale" and are owned by "sellers;" sellers receive a flow u from any unmatched house they own, where the flow u is common to all sellers.

4.2 Seasons and Timing

There are two seasons, $j = s, w$ (for summer and winter); each model period is a season, and the two seasons alternate. At the beginning of a period j, an existing match between a homeowner and his house breaks with probability $1 - \phi^j$, and the house is put up for sale, adding to the stock of houses for sales, denoted by v^j. The homeowner whose existing match has broken becomes simultaneously a seller and a buyer, adding to the pool of buyers, denoted by b^j. In our baseline model, the parameter ϕ^j is the only (ex ante) difference between the seasons.[31] We focus on periodic steady states with constant v^s and v^w. Since a match is between one house and one agent, and there is a unit measure of agents and a unit measure of houses, it is always the case that the mass of houses for sale equals the mass of buyers: $v^j = b^j$.

Our objective is to investigate how such deterministic driver of seasonality can be amplified and revealed as seasonality in transactions and prices in the housing market due to the thick-market effects on the match-specific quality. By focusing on the periodic steady-state, we are studying a deterministic cycle and agents are aware that they are in such a cycle with ϕ^j, transactions, and prices fluctuating between high and low levels across the two seasons.

During each period, every buyer meets with a seller and every seller meets with a buyer. Upon meeting, the match-specific quality between the potential buyer and the house is drawn from a distribution $F(\varepsilon, v)$. If the buyer and seller agree on a transaction, the buyer pays a price (discussed later) to the seller, and starts enjoying the housing services ε. If not, the buyer looks for a house again next period, the seller receives the flow u, and puts the house up for sale again next period.[32] An agent can hence be either a matched homeowner or a buyer, and, at the same time, he could also be a seller. Sellers also may have multiple houses to sell.[33]

[30] Heuristically, one way to interpret our assumption is as follows. Suppose the buyer samples n units of vacant houses when the stock of vacancies is v. As long as the number of units sampled n increases in v, the maximum match quality ε in the sample will be "stochastically larger." In other words, for any underlying distribution of match quality, the distribution of the maximum in a sample of size n will first-order stochastically dominate the distribution of the maximum in a smaller sample $n' < n$. As such, F can be interpreted as the distribution of the sample maximum. In the web Appendix, we offer rigorous micro-foundations for this assumption.

[31] This difference could be determined, for example, by the school calendar or summer marriages, among other factors, exogenous to our model. In the web Appendix we discuss seasonal transaction costs as an alternative driver of seasonality.

[32] In the web Appendix we relax the assumption that if the transaction does not go through, the buyer and seller need to wait for next period to transact with other agents.

[33] In the web Appendix, we show that the probability of owning multiple house is quantitatively small.

4.3 The Homeowner

The value of living in a house with match quality ε starting in season s is given by:

$$H^s(\varepsilon) = \varepsilon + \beta\phi^w H^w(\varepsilon) + \beta(1-\phi^w)[V^w + B^w],$$

where $\beta \in (0,1)$ is the discount factor. With probability $(1-\phi^w)$ the homeowner receives a moving shock and becomes both a buyer and a seller (putting his house up for sale), with continuation value $(V^w + B^w)$, where V^j is the value of a house for sale to the seller and B^j is the value of being a buyer in season $j = s, w$, as defined later. With probability ϕ^w the homeowner keeps receiving housing services of quality ε and stays in the house. The formula for $H^w(\varepsilon)$ is perfectly isomorphic to $H^s(\varepsilon)$; in the interest of space we omit here and throughout the paper the corresponding expressions for season w. The value of being a matched homeowner can be therefore re-written as:

$$H^s(\varepsilon) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon + \frac{\beta(1-\phi^w)(V^w+B^w) + \beta^2\phi^w(1-\phi^s)(V^s+B^s)}{1-\beta^2\phi^w\phi^s}, \tag{2}$$

which is strictly increasing in ε. The first term that enters the housing value $H^s(\varepsilon)$ is the effective (adjusted for moving probabilities) present discounted value of staying in a house with match quality ε and the second term contains the values in the event that the match may dissolve in any future summer or winter.

4.4 Market Equilibrium

We focus on the case in which both seller and buyer observe the quality of the match, ε, which is drawn from $F^j(\varepsilon) \equiv F(\varepsilon, v^j)$; we derive the results for the case in which the seller cannot observe ε in the web Appendix. If the transaction goes through, the buyer pays the seller a mutually agreed price, and starts enjoying the housing services flow in the same season j. If the transaction does not go through, the buyer receives zero housing services and looks for a house again next season. This will be the case, for example, if buyers searching for a house pay a rent equal to the utility they derive from the rented property—what is key is that the rental property is not owned by the same potential seller with whom the buyer meets. On the seller's side, when the transaction does not go through, he receives the flow u in season j and puts the house up for sale again next season. The flow u can be interpreted as a net rental income received by the seller. Again, what is key is that the tenant is not the same potential buyer who visits the house.

4.4.1 Reservation Quality

The total surplus of a transaction is:

$$S^s(\varepsilon) = H^s(\varepsilon) - [\beta(B^w + V^w) + u]. \tag{3}$$

Intuitively, a new transaction generates a new match of value $H^s(\varepsilon)$; if the transaction does not go through, the buyer and the seller obtain βB^w and $(\beta V^w + u)$, respectively. Since ε is observable and the surplus is transferrable, a transaction goes through as long as the total surplus $S^s(\varepsilon)$ is positive. Given $H^s(\varepsilon)$ is increasing in ε, a transaction goes through if $\varepsilon \geqslant \varepsilon^s$, where the reservation ε^s is defined by:

$$\varepsilon^s =: H^s(\varepsilon^s) = \beta(B^w + V^w) + u, \tag{4}$$

and $1 - F^s(\varepsilon^s)$ is thus the probability that a transaction is carried out. Since the reservation quality ε^s is related to the total surplus independently of how the surplus is divided between the buyer and the seller, we defer the discussion of equilibrium prices to Section 27. Using the expression of housing value $H^s(\varepsilon)$ in (2), equation (4) becomes:

$$\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^s = u - \frac{\beta^2\phi^w(1-\phi^s)}{1-\beta^2\phi^w\phi^s}(B^s + V^s) + \frac{1-\beta^2\phi^s}{1-\beta^2\phi^w\phi^s}\beta\phi^w(B^w + V^w). \tag{5}$$

The Bellman equation for the sum of values is:

$$B^s + V^s = \beta(B^w + V^w) + u + [1 - F^s(\varepsilon^s)] E^s [S^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s], \tag{6}$$

where $E^s[.]$ indicates the expectation is taken with respect to distribution $F^s(.)$. The sum of values in season s covers the outside option, $\beta(B^w + V^w) + u$ (the flow u plus the option value of buying and selling next season) and, with probability $[1 - F^s(\varepsilon^s)]$, on the expected surplus from a transaction for sellers and buyers. Solving this explicitly and using the expression for $S^j(\varepsilon)$, $j = s, w$ in (20):

$$B^s + V^s = \frac{u}{1-\beta} + \frac{(1+\beta\phi^w) h^s(\varepsilon^s) + \beta(1+\beta\phi^s) h^w(\varepsilon^w)}{(1-\beta^2)(1-\beta^2\phi^w\phi^s)}, \tag{7}$$

where $h^s(\varepsilon^s) \equiv [1 - F^s(\varepsilon^s)] E[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s]$ is the expected surplus of quality above threshold ε^s.

The equilibrium values $\varepsilon^s, \varepsilon^w, (B^s + V^s)$, and $(B^w + V^w)$ in (5) and (7) depend on equilibrium vacancies v^s and v^w, which we now derive.

4.4.2 Stock of houses for sale

In any season s, the law of motion for the stock of houses for sale (and for the stock of buyers) is

$$v^s = (1-\phi^s)\left[v^w\left(1-F^w\left(\varepsilon^w\right)\right)+1-v^w\right]+v^wF^w\left(\varepsilon^w\right)$$

where the first term corresponds to houses that received a moving shock and hence were put for sale this season and the second term corresponds to vacancies from last period that did not find a buyer. The expression simplifies to

$$v^s = 1-\phi^s+v^wF^w\left(\varepsilon^w\right)\phi^s. \tag{8}$$

The equilibrium quantities $\left(B^s+V^s, B^w+V^w, \varepsilon^s, \varepsilon^w, v^s, v^w\right)$ jointly satisfy equations (5), (7), and (8) together with the isomorphic equations for the other season. They are independent of how the total surplus is shared across buyers and sellers, that is, independent of the exact price-setting mechanism. We hence discuss seasonality in vacancies and transactions first, before we specify the particular price-setting mechanism.

5 Model-Generated Seasonality

In the baseline model seasonality is driven by the higher moving probability in the summer: $1-\phi^s > 1-\phi^w$. As shown earlier, the equilibrium quantities $\left(B^s+V^s, B^w+V^w, \varepsilon^s, \varepsilon^w, v^s, v^w\right)$ jointly satisfy six equations. Before jumping directly to the quantitative results we discuss the underlying mechanisms through which a higher probability of relocating in the summer leads to a larger stock of vacancies and a higher expected return for buyers and sellers, i.e. $v^s > v^w$ and $B^s+V^s > B^w+V^w$; hence, this section aims at making the model's mechanics more explicit.

It is important to reiterate that our notion of seasonality is not a cross-steady states comparison, that is, we are not comparing a steady-state with a high probability of moving houses to another steady-state with a low probability of moving. Instead, the seasonal values we derive are equilibrium values along a periodic steady state where agents take into account that the economy is fluctuating deterministically between the summer and the winter seasons.

Using (8), the stock of houses for sale in season s is given by:

$$v^s = \frac{1-\phi^s+\phi^sF^w\left(\varepsilon^w\right)\left(1-\phi^w\right)}{1-F^s\left(\varepsilon^s\right)F^w\left(\varepsilon^w\right)\phi^s\phi^w}. \tag{9}$$

The ex ante higher probability of moving in the summer $(1-\phi^s > 1-\phi^w)$ clearly has a direct positive effect on v^s, and this effect also dominates quantitatively when we calibrate the model to match the average duration of stay in a house.[34] Thus, this implies $v^s > v^w$. The probability of moving is exogenous in our model and we calibrate it so as to match the seasonality in vacancies. Our main interest is to predict the seasonality in transactions and prices.

To that aim, we first take a somewhat tedious but useful detour to comment on the seasonality of the sum of values $(B^j + V^j)$. Intuitively, a higher stock of vacancies in the summer implies higher expected returns to a buyer and a seller in the summer because of better matches through the thick-market effect. These higher expected returns in the summer, however, also raise the outside options of a buyer and a seller in the winter. Higher outside options make both the buyer and the seller more demanding and tend to increase the reservation quality in the winter. In equilibrium, however, the overall effect on reservation quality is ambiguous.[35] More formally, the higher stock of vacancies in the summer, $v^s > v^w$, implies a higher expected surplus quality $h^s(.)$ for any given cutoff through the thick-market effects as in (1).[36] Given $\phi^w > \phi^s$, it thus follows from equation (7) that $B^s + V^s > B^w + V^w$ if the two equilibrium cutoffs ε^s and ε^w are close. In other words, the expected return $(B^j + V^j)$ is higher in the summer as long as the thick-market effect dominates a potentially offsetting equilibrium effect from the reservation quality. Quantitatively, the two cutoffs turn out to be close for reasonable parametrizations of the model and hence the thick-market effect indeed dominates.

[34] More specifically, the numerator is a weighted average of 1 and $F^w(\varepsilon^w)(1-\phi^w)$, with $1-\phi^s$ being the weight assigned to 1 in the equation for v^s. Since $1-\phi^s > 1-\phi^w$, the equation for v^s assigns a higher weight on 1. Since $F^w(\varepsilon^w)(1-\phi^w) < 1$, higher weight on 1 leads to $v^s > v^w$; this is because $F^w(\varepsilon^w)(1-\phi^w)$ is virtually aseasonal as there are two opposite effects: $F^w(\varepsilon^w) > F^s(\varepsilon^s)$ and $(1-\phi^w) < (1-\phi^s)$ that tend to largely cancel each other.

[35] Note, using (4), that lower outside options $(B^w + V^w)$ imply a lower housing value for the marginal transaction in the summer,

$$H^s(\varepsilon^s) < H^w(\varepsilon^w). \tag{10}$$

This does not necessarily imply a lower reservation quality in the summer, $\varepsilon^s < \varepsilon^w$ because the ranking of $H^s(\varepsilon)$ and $H^w(\varepsilon)$ depends on the level of ε. To see this, note from (2), that $H^j(\varepsilon)$ is linear in ε for $j = s, w$. Given $\phi^w > \phi^s$, $H^s(.)$ is steeper than $H^w(.)$. The difference in the intercepts between $H^s(.)$ and $H^w(.)$ is proportional to:

$$\beta\left[(1-\phi^w)(1-\beta\phi^s)(B^w + V^w) - (1-\phi^s)(1-\beta\phi^w)(B^s + V^s)\ \right],$$

which is negative when $B^s + V^s > B^w + V^w$. Therefore, $H^s(.)$ and $H^w(.)$ must cross once at $\hat{\varepsilon}$. Thus if the equilibrium reservation quality in the summer is sufficiently high, $\varepsilon^s > \hat{\varepsilon}$, then $H^s(\varepsilon^s) > H^w(\varepsilon^s)$. Therefore, in order for inequality (10) to hold, we must have $\varepsilon^w > \varepsilon^s$. In this case, a lower outside option in the summer leads to a lower cutoff. On the other hand, if the equilibrium reservation quality in the summer is sufficiently low, $\varepsilon^s < \hat{\varepsilon}$, then $H^s(\varepsilon^s) < H^w(\varepsilon^s)$; in this case, the inequality $\varepsilon^w > \varepsilon^s$ is no longer required for inequality (10) to hold. In sum, the two equilibrium cutoffs cannot be ranked.

[36] To see this, rewrite $h^s(x) = \int_x [1 - F^s(\varepsilon)]\, d\varepsilon$ using integration by parts.

5.1 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s \left[1 - F^s\left(\varepsilon^s\right)\right]. \tag{11}$$

(An isomorphic expression holds for Q^w). From (11), it is evident that a larger stock of vacancies in the summer, $v^s > v^w$, has a direct positive effect on the number of transactions in the summer relative to winter. Furthermore, if the probability of a transaction is also higher in the summer, then transactions will be more seasonal than vacancies. This amplification effect, which follows from the first-order stochastic dominance of $F^s(.)$ over $F^w(.)$, is indeed present in our quantitative exercise.[37] Intuitively, a higher stock of vacancies leads to better matches through the thick-market effect, resulting in a higher transaction probability.[38]

5.2 Seasonality in Prices

As discussed earlier, results on seasonality in vacancies and transactions are independent of the exact price-setting mechanism, i.e. how the surplus is shared between a buyer and seller. Let $S_v^s(\varepsilon)$ and $S_b^s(\varepsilon)$ be the surpluses of a transaction to the seller and to the buyer, respectively, in season s, when the match quality is ε and the price is $p^s(\varepsilon)$:

$$S_v^s(\varepsilon) \equiv p^s(\varepsilon) - \left(u + \beta V^w\right), \tag{12}$$

$$S_b^s(\varepsilon) \equiv H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w. \tag{13}$$

The value functions for the buyer and the seller in season s are, respectively:

$$V^s = \beta V^w + u + \left[1 - F^s\left(\varepsilon^s\right)\right] E^s\left[S_v^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right], \tag{14}$$

$$B^s = \beta B^w + \left[1 - F^s\left(\varepsilon^s\right)\right] E^s\left[S_b^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right]. \tag{15}$$

[37] As said, there could be an additional effect if the cutoffs are highly seasonal. For example, if $\varepsilon^w > \varepsilon^s$, there will be even lower volume of transactions in the winter. This is because the outside option for both buyers and sellers is to wait and transact in the next season. Therefore, a higher outside option in the winter makes both buyers and sellers more demanding in the winter and hence less likely to transact, yielding an even smaller number of transactions.

[38] Our model predicts higher probability of transactions in the hot season, thus faster sale and shorter average time on the market for both buyers and sellers. Though we do not have high-frequency data on time on the market to assess seasonality, at lower frequencies, average time to sell tends to be shorter when prices are high (see Krainer, 2001 and Diaz and Jerez, 2012), a relation that is consistent with our mechanism.

A seller can count on his outside option, $\beta V^w + u$ (the flow u plus the option value of selling next season) and, with probability $[1 - F^s(\varepsilon^s)]$, on the expected surplus from a transaction for sellers. A buyer counts on her outside option, βB^w (the option value of buying next season), and, with the same probability, on the expected surplus for buyers. The two Bellman equations (14) and (15) describe the incentives of buyers and sellers in any season s. They will only agree to a transaction if they obtain a positive surplus from the exchange. In particular, (14) shows why a seller would agree to sell in the winter season, even though the average price is higher in the summer. A positive surplus in the winter, $p^w(\varepsilon) - (u + \beta V^s) > 0$, already takes into account the potential higher price in the summer and therefore the higher value of being a seller in the summer (V^s).

We now consider the case in which prices are determined by Nash bargaining. The price maximizes the Nash product:

$$\max_{p^s(\varepsilon)} [S_v^s(\varepsilon)]^\theta [S_b^s(\varepsilon)]^{1-\theta} \quad s.t. \quad S_v^s(\varepsilon), S_b^s(\varepsilon) \geqslant 0;$$

where θ denotes the bargaining power of the seller. The solution implies

$$\frac{S_v^s(\varepsilon)}{S_b^s(\varepsilon)} = \frac{\theta}{1-\theta}, \tag{16}$$

which simplifies to (see web Appendix E):

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + (1-\theta)\frac{u}{1-\beta}, \tag{17}$$

a weighted average of the housing value for the matched homeowner and the present discounted value of the flow u. In other words, the price guarantees the seller the proceeds from the alternative usage of the house $(\frac{u}{1-\beta})$ and a fraction θ of the social surplus generated by the transaction $\left[H^s(\varepsilon) - \frac{u}{1-\beta}\right]$.

The average price of a transaction is:

$$P^s \equiv E^s[p^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s] = (1-\theta)\frac{u}{1-\beta} + \theta E^s[H^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s], \tag{18}$$

which is increasing in the conditional expected surplus of housing services for transactions exceeding the reservation ε^s. Since u is aseasonal, house prices are seasonal if $\theta > 0$ and the surplus to the seller is seasonal. Moreover, the extent of seasonality is increasing in θ. Intuitively, the source of seasonality is coming from higher average match quality in a thicker market. The higher match quality generates higher utility to the buyer. This will show up as a higher price only if the seller has some bargaining power to extract a fraction of the surplus generated from the match. To see this in equations, rewrite

$E^s[H^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s]$ as the sum of two terms:

$$E^s[H^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s] = H^s(\varepsilon^s) + E^s[S^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s]. \tag{19}$$

The first term, $H^j(\varepsilon^j)$, the housing value of the marginal transaction, tends to reduce the average price in the summer since $H^s(\varepsilon^s) < H^w(\varepsilon^w)$. The second term, $E^s[S^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s]$, is the expected surplus of a transaction, which tends to increase the average price in the summer due to higher match-quality. To see this second term more clearly, observe from (3) and (4) that

$$S^s(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}(\varepsilon - \varepsilon^s), \tag{20}$$

thus

$$E^s[S^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s] = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s} E^s[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s].$$

The average housing value will thus be higher in the summer for two reasons. First, the probability of staying is higher in the winter, $\phi^w > \phi^s$. Second, and more important, given the assumption of first-order stochastic dominance, a higher stock of vacancies $v^s > v^w$ increases the likelihood of drawing a higher match-quality $[1 - F^s(\varepsilon)] \geqslant [1 - F^w(\varepsilon)] \ \ \forall\varepsilon$. This generally leads to a higher conditional surplus in the hot season: $E^s[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s] \geqslant E^w[\varepsilon - \varepsilon^w \mid \varepsilon \geqslant \varepsilon^w]$.[39]

Given that θ affects P^s only through the equilibrium mass of vacancies (recall the reservation quality ε^s is independent of θ), it follows that the extent of seasonality in prices is increasing in θ. Since (18) holds independently of the steady state equation for v^s and v^w, this result is independent of what drives $v^s > v^w$. Note finally that the extent of seasonality in prices is decreasing in the size of the (aseasonal) flow u.

[39] To see this, rewrite the conditional surplus using integration by parts:

$$E^s[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s] = \frac{\int_{\varepsilon^s}(1 - F^s(\varepsilon))\, d\varepsilon}{1 - F^s(\varepsilon^s)}. \tag{21}$$

Putting aside the issue of the equilibrium cutoffs ε^s and ε^w (which are are quantitatively close), it follows from equation (21) that the conditional surplus is higher in the hot season, $E^s[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s] \geqslant E^w[\varepsilon - \varepsilon^w \mid \varepsilon \geqslant \varepsilon^w]$, unless the increase in the likelihood of drawing a particular level of match quality ε dominates the sum of the increase in likelihood of drawing all match qualities higher than ε, i.e. unless $\frac{1-F^s(\varepsilon)}{1-F^w(\varepsilon)} > \frac{\int_\varepsilon (1-F^s(\varepsilon))d\varepsilon}{\int_\varepsilon (1-F^w(\varepsilon))d\varepsilon}$. We cannot rule out this possibility in general, but this case does not arise in our calibration exercise. More formally, we could impose a "uniform" stochastic ordering (see Keilson and Sumita, 1982) as a sufficient condition to rule out this case. But as said, such assumption is not necessary for obtaining higher prices in the hot season.

5.2.1 Comparison to a Standard Asset-Pricing Approach

It is useful to compare the price mechanism in our setup with that in a standard asset pricing approach. Equation (14) can be compared to the no-arbitrage condition in asset pricing. Substituting the expression for the surplus into (14), we obtain

$$V^s = \left[1 - F^s\left(\varepsilon^s\right)\right] P^s + F^s\left(\varepsilon^s\right)\left(\beta V^w + u\right)$$

The equation expresses the value of a seller as a weighted average of the market price P^s and the continuation value $(\beta V^w + u)$, with the weights given, correspondingly, by the probabilities that the transaction goes through or not. Without the search friction, a buyer will always purchase the house at the market price P^s, thus the probability of a transaction is one. In that case, the value for being a seller is $V^s = P^s$. Moreover, the surplus of a transaction is zero in a competitive equilibrium (with perfect arbitrage), so the Bellman equation (14) is equivalent to

$$P^s = \beta P^w + u = \beta\left(\beta P^s + u\right) + u \Longrightarrow P^s = \frac{u}{1-\beta},$$

and $P^s = P^w$. In other words, without the model's friction, seasonality in moving probabilities ϕ^s will not be transmitted into seasonality in prices.[40]

Our price index P^j, $j = s, w$ is the average price of transactions in season j. The seasonality in price indexes, $P^s > P^w$, is due to the thick market effect, whereby matches are more likely to be better in the hot season (with a higher stock of houses for sale). In what follows we focus on discussing the mechanism from the seller's perspective (a similar argument can be put forward from a buyer's perspective). The price index P^j is not the price that every seller receives. More specifically, consider a seller in the winter who is meeting with a buyer that has a match-specific quality equal to ε. He has to decide whether to sell now at an agreed price or to wait until the summer, where the average price is P^s. Notice that the seller is not comparing P^w to P^s in his decision because what is relevant for him is not the average price P^w but rather $p^w(\varepsilon)$, which is determined between him and the buyer

[40] Notice that with the search friction, $P^s \neq \frac{u}{1-\beta}$. From

$$V^s = \beta V^w + u + \left[1 - F^s\left(\varepsilon^s\right)\right] E^s\left[S_v^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right]$$

substitute the expression for V^w and obtain:

$$V^s = \frac{u}{1-\beta} + \frac{\left[1 - F^s\left(\varepsilon^s\right)\right] E^s\left[S_v^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right] + \beta\left[1 - F^w\left(\varepsilon^w\right)\right] E^w\left[S_v^w(\varepsilon) \mid \varepsilon \geqslant \varepsilon^w\right]}{1-\beta^2}$$

where the expected surpluses are strictly positive.

with quality match ε. The equilibrium value functions (14) and (15) ensure that a transaction will take place as long as the surplus is positive. The option of being able to sell at a possibly higher price in the summer has already been incorporated into the equilibrium surpluses (12) and (13), which in turn pin down the equilibrium price $p^w(\varepsilon)$ as in (17). So even though the price of a transaction for a specific ε might be higher in the hot season, it does not follow that a seller will only transact in the summer because of the stochastic nature of ε. By not transacting at $p^w(\varepsilon)$, a seller may end up with an even lower $p^s(\tilde{\varepsilon})$ in the summer if he meets a buyer with a lower match quality $\tilde{\varepsilon}$, or with no transaction at all if the match quality $\tilde{\varepsilon}$ is too low. So the corresponding arbitrage condition for the seller to decide whether to wait until the hot season has to consider both the probability of transacting in the summer and the distribution of the match quality conditional on transacting. In contrast, in a standard asset-pricing model with deterministic seasons, a seller can always transact (with certainty) at market prices. The choice of whether to sell in the current season or in the next depends exclusively on the flow of benefits (or costs) of owning the house for one season relative to the expected seasonal appreciation.

5.3 Quantitative Analysis

In this section we calibrate the model to study its quantitative implications.

5.3.1 Parameter values

We assume the distribution of match-quality $F(\varepsilon, v)$ follows a uniform distribution on $[0, v]$. When $v^s > v^w$ (which will follow from $\phi^w > \phi^s$), this implies first-order stochastic ordering, $F^s(.) \leqslant F^w(.)$.

We set the discount factor β so that the implied annual real interest rate is 6 percent, as calculated by Blake (2011) for the United States. (The rate might be slightly higher for the United Kingdom, though we use the same to ease cross-country comparability.)

We calibrate the average probability of staying in the house, $\phi = (\phi^s + \phi^w)/2$, to match survey data on the average duration of stay in a given house, which in the model is given by $\frac{1}{1-\phi}$. The median duration in the United States from 1993 through 2005, according to the American Housing Survey, was 18 semesters; the median duration in the United Kingdom during this period, according to the Survey of English Housing was 26 semesters. The implied (average) moving probabilities $(1-\phi)$ per semester are hence 0.056 and 0.038 for the United States and the United Kingdom, respectively. Because there is no direct data on the ex-ante ratio of moving probabilities between seasons, $(1-\phi^s)/(1-\phi^w)$, we use a range of $(1-\phi^s)/(1-\phi^w)$ from 1.1 to 1.5.[41] This implies a difference in staying probabilities between seasons, $\phi^w - \phi^s$, ranging from 0.004 to 0.015 in the United Kingdom and 0.005 to 0.022 in the United States. One way to pin down the level of $(1-\phi^s)/(1-\phi^w)$ is to use data on inventories (or homes for sale), which correspond to the vacancies v^j in our model. The data are available at quarterly frequency for the United States from the NAR (for the United Kingdom, data on vacancies only exist at yearly frequency). Seasonality in inventories was 28 percent during $1991-2012$.[42] As will become clear from the results displayed below, the ratio that exactly matches seasonality in US vacancies is $(1-\phi^s)/(1-\phi^w) = 1.25$. The reader may want to view this as a deep parameter and potentially use it also for the UK, under the assumption that the extent of seasonality in ex-ante moving probabilities does not vary across countries.

We calibrate the flow value u to match the implied average rent-to-price ratio received by the seller. In the UK, the average gross rent-to-price ratio is roughly around 5 percent per year, according to Global Property Guide.[43] For the US, Davis et al. (2008) argue that the ratio was around 5 percent prior to 1995 when it started falling, reaching 3.5 percent by 2005. In our model, the u/P ratio (where P stands for the average price, absent seasonality) corresponds to the net rental flow received by the seller after paying taxes and other relevant costs; it is accordingly lower than the gross rent-to-price ratio. As a benchmark, we choose u so that the net rent-to-price ratio is equal to 3 percent per year (or 1.5 percent per semester), equivalent to assuming a 40 percent income tax on rent).[44] To obtain the value of u, which, as we said, is aseasonal in the data, we use the equilibrium equations in the model without seasonality, that is, the model in which $\phi^s = \phi^w = \phi$. From (18) and (5), the average

[41] The two surveys mentioned also report the main reasons for moving. Around 30 percent of the respondents report that living closer to work or to their children's school and getting married are the main reasons for moving. These factors are of course not entirely exogenous, but they can carry a considerably exogenous component; in particular, the school calendar is certainly exogenous to housing market movements (see Goodman 1993 and Tucker, Long, and Marx 1995 on seasonal mobility). In all, the survey evidence supports our working hypothesis that the *ex ante* probability to move is higher in the summer (or, equivalently the probability to stay is higher in the winter).

[42] We use the inventory series provided by NAR. As a measure of seasonality we use, as before, the difference in annualized growth rates in vacancies between broadly defined summers and winters. As an alternative definition of vacancies we also looked at vacant houses' data from the US Census Bureau. Vacancy is computed as the sum of houses for sale at the beginning of the season relative to the stock of houses. The degree of seasonality in this series, using the same metric is 31 percent.

[43] Data for the United Kingdom and other European countries can be found in http://www.globalpropertyguide.com/Europe/United-Kingdom/price-rent-ratio

[44] In principle, other costs can trim down the 3-percent u/P ratio, including maintenance costs, and inefficiencies in the rental market that lead to a higher wedge between what the tenant pays and what the landlord receives; also, it might not be possible to rent the house immediately, leading to lower average flows u. Note that lower values of u/p lead to even higher seasonality in prices and transactions for any given level of seasonality in moving shocks.

price and the reservation quality ε^d in the absence of seasonality are (see web Appendix):

$$P = \frac{u}{1-\beta} + \theta \frac{\left[1 - \beta F\left(\varepsilon^d\right)\right] E\left[\varepsilon - \varepsilon^d \mid \varepsilon \geqslant \varepsilon^d\right]}{(1-\beta)(1-\beta\phi)}, \tag{22}$$

and

$$\frac{\varepsilon^d}{1-\beta\phi} = \frac{u + \frac{\beta\phi}{1-\beta\phi}\int_{\varepsilon^d} \varepsilon dF(\varepsilon)}{1-\beta\phi F\left(\varepsilon^d\right)}. \tag{23}$$

We hence substitute $u = 0.015 \cdot P$ in the aseasonal model (equivalent to an annual rent-to-price ratio of 3 percent) for $\theta = 1/2$ (when sellers and buyers have the same bargaining power) and find the equilibrium value of P given the calibrated values for β and $F(.)$. We then use the implied value of $u = 0.015 \cdot P$ as a parameter.[45]

Finally, in reporting the results for prices we vary the seller's bargaining power parameter θ from 0 to 1.

5.3.2 The Extent of Seasonality

Given the calibrated values of u, β, and ϕ discussed above, Table 7 displays the extent of seasonality in vacancies and transactions generated by the model for different values of the ratio of moving probabilities (recall that seasonality in vacancies and transactions is independent of the bargaining power of the seller, θ). As throughout the paper, our metric for seasonality is the annualized difference in growth rates between the two seasons. Column (1) shows the ratio of moving probabilities, $\frac{1-\phi^s}{1-\phi^w}$. Columns (2) and (5) show the implied difference in moving probabilities between the two seasons for the United States and the United Kingdom, $[(1-\phi^s) - (1-\phi^w)]$. (Recall that, because the average stay in a house differs across the two countries, a given ratio can imply different values for $\phi^w - \phi^s$, as the average probability of stay ϕ differs.) Columns (3) and (4) show the extent of seasonality in vacancies and transactions for an average stay of 9 years (as in the United States) and Columns (6) and (7) show the corresponding figures for an average stay of 13 years (as in the United Kingdom)

[45] We also calibrated the model using different values of u for different θ (instead of setting $\theta = 1/2$), keeping the ratio u/P constant. Results are not significantly different under this procedure, but the comparability of results for different values of θ becomes less clear, since u is not kept fixed.

Table 7. Seasonality in vacancies and transactions for different $\frac{1-\phi^s}{1-\phi^w}$.

Ratio of moving probabilities between seasons (1)	***Average moving probability: 0.0556 Stay of 9 years (U.S.)***			***Average moving probability: 0.0385 Stay of 13 years (U.K.)***		
	Implied seasonal *difference* in moving probabilities (2)	Vacancies (3)	Transactions (4)	Implied seasonal *difference* in moving probabilities (5)	Vacancies (6)	Transactions (7)
1.10	0.005	12%	49%	0.004	11%	48%
1.20	0.010	23%	94%	0.007	21%	93%
1.25	0.012	28%	115%	0.009	25%	113%
1.30	0.014	33%	136%	0.010	30%	133%
1.40	0.019	42%	174%	0.013	38%	171%
1.50	0.022	51%	211%	0.015	45%	207%

The first point to note is the large amplification mechanism present in the model: For any given level of seasonality in vacancies, seasonality in transactions is at least four times bigger. Second, the Table shows that a small absolute difference in the probability to stay between the two seasons can induce large seasonality in transactions. Third, if we constrain ourselves to $\frac{1-\phi^s}{1-\phi^w} = 1.25$ to match the data on vacancies for the United States, this implies a level of seasonality in transactions of about 115 percent in the United States (the empirical counterpart is 146 percent). For the United Kingdom, ideally we would like to recalibrate the ratio $\frac{1-\phi^s}{1-\phi^w}$ to match its seasonality in vacancies; however, as said, the data are only available at yearly frequency. Using the same ratio $\frac{1-\phi^s}{1-\phi^w} = 1.25$ as a parameter for the United Kingdom would yield a seasonality in vacancies of 25 percent (the difference with the United States is due to the longer duration of stay in the United Kingdom). This in turn would imply a degree of seasonality in transactions of 113 percent (the empirical counterpart is 139.) Note that, for a given ratio $\frac{1-\phi^s}{1-\phi^w}$, the model generates more seasonality in transactions in the United States than in the United Kingdom (as in the data) because a given ratio implies a higher difference in moving probabilities $[(1-\phi^s)-(1-\phi^w)]$ in the United States than in the United Kingdom, as the average stay is shorter in the former.

Seasonality in prices, as expressed earlier, depends also on the bargaining power of the seller, θ. Figure 7 plots the model-generated seasonality in prices for different θ and $\frac{1-\phi^s}{1-\phi^w}$, assuming an average stay of 13 years (as in the United Kingdom), and Figure 8 shows the corresponding plot for an average stay of 9 years (as in the United States). As illustrated, seasonality increases with both θ and $\frac{1-\phi^s}{1-\phi^w}$. If, as before, we take $\frac{1-\phi^s}{1-\phi^w} = 1.25$ as given, the exercise implies that to match real-price seasonality in the United Kingdom (of about 5.5 percent), the bargaining power coefficient θ needs to be around 0.8 percent. The corresponding value for the United States as a whole, with real-price seasonality of 4.8 percent, is 0.73 percent.[46]

Figure 7: Seasonality in Prices for Different θ and $\frac{1-\phi^s}{1-\phi_w}$. United Kingdom

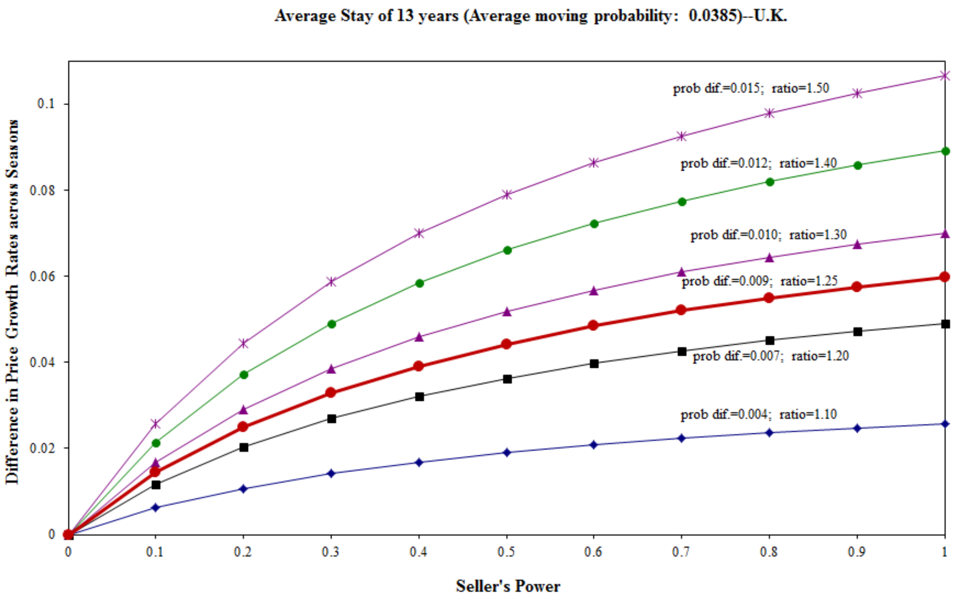
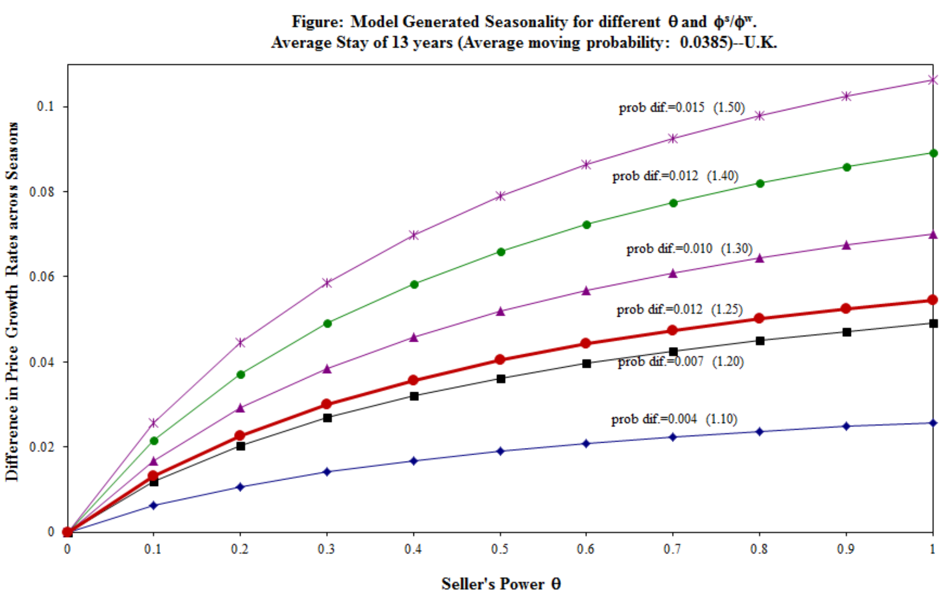


Figure 8. Seasonality in Prices for Different θ and $\frac{1-\phi^s}{1-\phi_w}$. United States



In all, though stylized, the model can generate seasonal fluctuations quantitatively comparable to

[46] A somewhat higher bargaining power of sellers in the United Kingdom appears plausible. First, population density in the United Kingdom is higher than in the United States making land relatively scarcer, and potentially conferring home owners more power in price negotiations (this should also be true in denser U.S. cities). Second, anecdotal evidence suggests that land use regulations are particularly stringent in the United Kingdom (see OECD Economic Outlook 2005). Finally, as discussed earlier, price seasonality in U.S. cities is positively correlated with price-to-rent ratios (which, within the model, increases with sellers' power).

those in the data. Together with the results from our study of alternative models in the web Appendix and the micro evidence supporting the mechanism, we conclude that thick-market effects on quality offer a plausible explanation for the seasonal patterns in the data.

6 Concluding Remarks

Using data from the United Kingdom and the United States, this paper documents seasonal booms and busts in housing markets. It argues that the predictability and high extent of seasonality in house prices cannot be quantitatively reconciled with existing models in the housing literature.

To explain the empirical patterns, the paper presents a search-and-matching model emphasizing two elements of the housing market. The first is a match-specific component: buyers have different idiosyncratic preferences over houses. The second is the notion that in a market with more houses for sale, a buyer is more likely to find a better match, which we refer to as the thick-market effect. With these two elements, the model generates an amplification mechanism such that a small (deterministic) difference in the propensity to relocate across seasons can result in large seasonal swings in house prices and the volume of transactions. When calibrated using data from the United States and the United Kingdom, the model can quantitatively account for most of the seasonal fluctuations in prices and transactions observed in the data. The idea that matches formed in the summer are of better quality—the idea underlying the model's mechanism—is consistent with empirical evidence presented in the paper.

The model sheds light on a new mechanism governing fluctuations in housing markets that is also likely to operate at lower frequencies. In particular, the thick-market effect at the core of the model's propagation mechanism does not depend on the frequency of the shocks. Lower frequency shocks associated with either business-cycle shocks of with less frequent booms and busts in housing markets could also be propagated through the same mechanism to amplify fluctuations in transactions and prices.

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Appendix For Online Publication

Hot and Cold Seasons in the Housing Market, by Ngai and Tenreyro.

The Appendix is organized as follows. Section A provides supporting figures and tables and a supplementary description of housing price seasonality. Section B studies alternative models of the housing market. It starts with the simplest (frictionless) model, carrying out back-of-the-envelope calculations using the implied asset-pricing relations. It then examines the main canonical models in the housing market. Section C provides micro-foundations for the key assumption in the model, that is, the stochastic dominance of distribution functions for match qualities with higher vacancies. Section D discusses the efficiency properties of the model and studies its robustness to different modelling assumptions; in particular, it studies the case with moving costs and their role as alternative triggers of seasonality, and a different searching procedure, that allows the buyer and seller to contemplate their second-best offers. Section E presents all the derivations and proofs. Section F presents the model when the quality of the match is not observed by the seller and investigates different pricing mechanisms, including price posting by the seller. Section G describes additional statistics generated by the model.

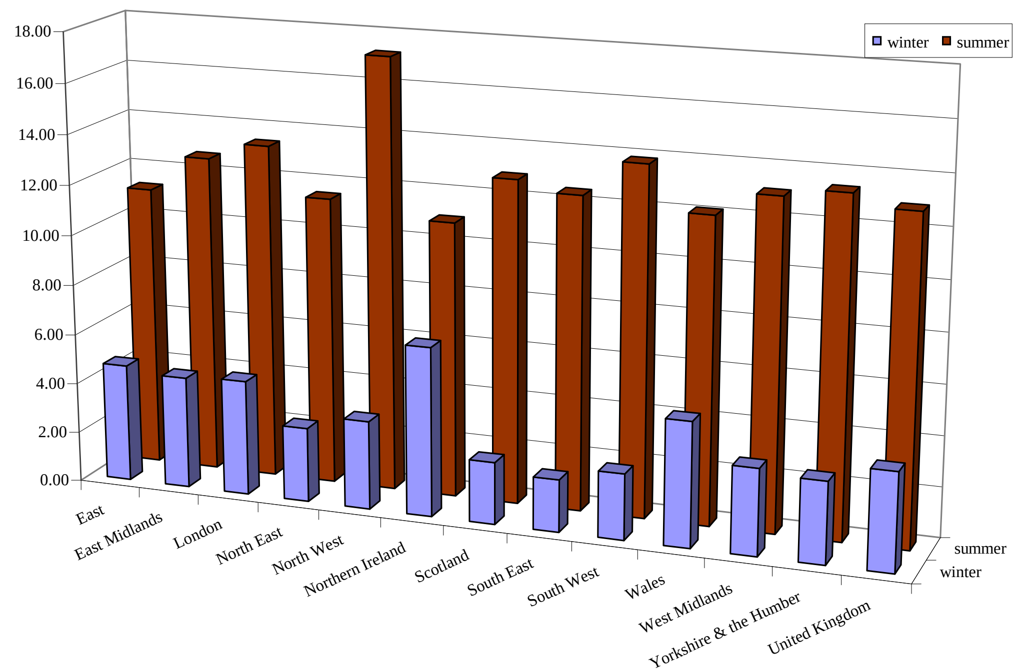
A Supporting Empirical Evidence

This Section of the Appendix first provides supporting figures and tables referred to in the text. It then provides an alternative description of the seasonality.

A.1 Supporting Material

Figure $A1$ shows similar results as Figure 1, for the period 1983-2007 using the constant-quality price index provided by the Department of Communities and Local Government (DGLG).

Figure $A1$: Average Annualized House Price Increases in Summer and Winter, 1983-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DCLG, 1983-2007.

Figure $A2$ shows the average annualized real house price increases using the Land Registry data for 1996-2012. (The difference from Figure 1 in the text is that this shows real prices.)

Figure $A2$: Average Annualized Real House Price Increases in Summer and Winter, Land Registry 1996-2012

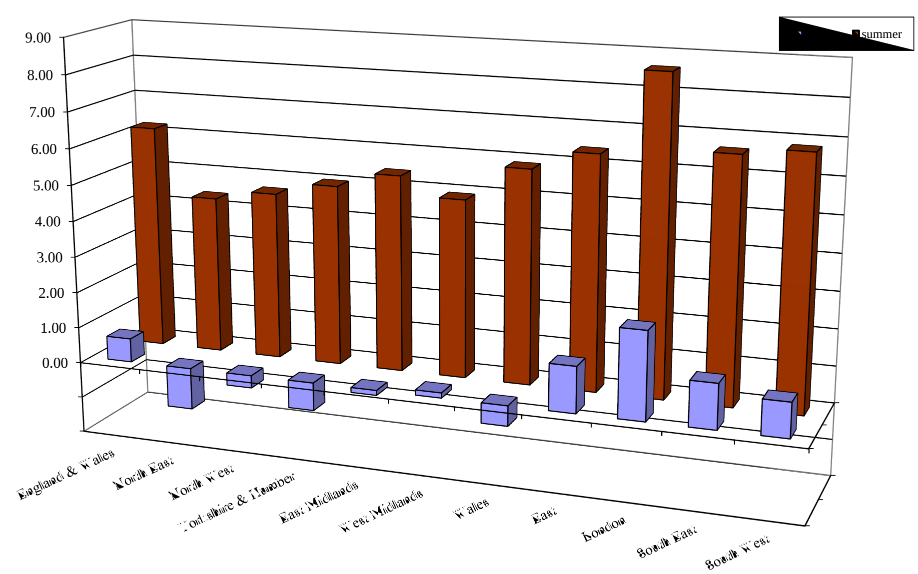


Table $A1$ shows the difference in annualized price growth rates between summers and winters in the

United Kingdom differentiating among existing houses and new houses, and buyers who were former owner occupiers, and first time buyers. The data are available from Halifax for 1983 through 2005 (note the disaggregated data are not available for later years).

Table $A1$. Differences in price growth rates between summers and winters.

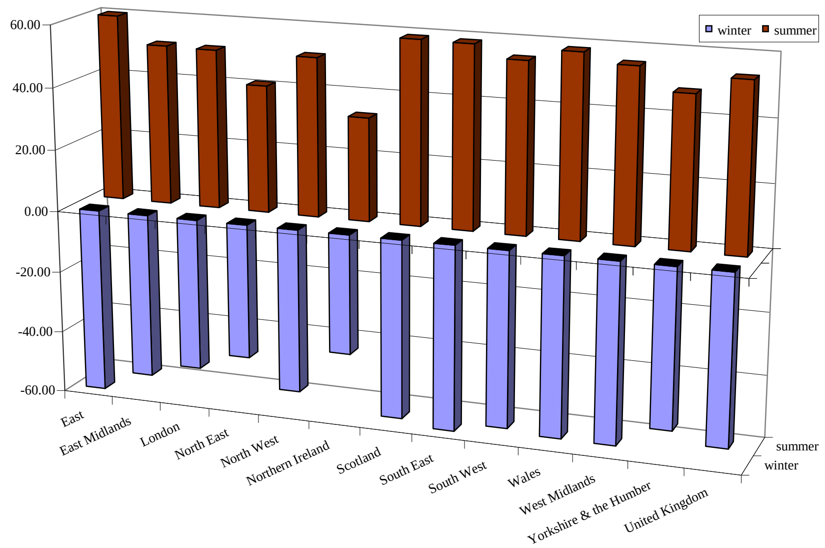
By type of house, buyer, and region. UK Halifax data. 1983-2005.

	Existing houses (All buyers)		New houses (All buyers)		Former owner occupiers (All houses)		First-time buyer (All houses)	
	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error
E. Anglia	9.092**	(3.311)	2.963	(5.735)	11.128**	(3.070)	4.759	(3.812)
E.Midlands	11.159**	(2.967)	2.814	(4.838)	11.812**	(2.997)	8.218**	(3.110)
Gr. London	5.738*	(2.643)	13.285	(7.382)	4.636	(2.451)	5.408	(2.764)
N. West	8.884**	(2.463)	-1.745	(6.076)	9.078**	(2.578)	4.917	(2.565)
North	2.292	(2.844)	2.669	(4.624)	0.954	(2.813)	3.270	(3.137)
S. East	7.419**	(2.646)	2.686	(3.614)	8.031**	(2.607)	3.815	(2.683)
S. West	9.668**	(2.981)	6.567	(4.410)	10.259**	(2.966)	5.608	(3.286)
W. Midlands	6.295*	(3.035)	10.56	(6.191)	7.127*	(3.133)	5.658	(2.961)
Yorkshire&Humb	7.543**	(2.698)	1.627	(4.959)	7.485**	(2.601)	6.289*	(3.002)
N. Ireland	10.015**	(3.415)	8.995	(5.346)	6.618	(3.882)	8.584*	(4.270)
Scotland	12.067**	(2.550)	12.414*	(5.572)	11.597**	(2.445)	4.825	(2.711)
Wales	8.886**	(3.148)	0.652	(6.082)	9.376**	(3.115)	6.755*	(3.357)
U.K.	8.069**	(2.116)	5.115*	(2.213)	8.269**	(2.080)	5.209*	(1.993)

Note: The Table shows the coefficients (and standard errors) on the dummy variable (Summer) in the regression $g_t=a+b\times Summer_t+e_t$, where g_t is the first diference in the log-house price. The equations use quarterly data from 1983 to 2005. Robust standard errors in parentheses. * Significant at the 5%; ** significant at 1%.

Figure $A3$ shows the growth rates in the number of mortgages (a proxy for the number of transactions) in the two seasons from 1983 to 2007 for different U.K. regions. The data are compiled by the Council of Mortgage Lenders (CML). As the figure shows, the number of transactions increases sharply in the summer term and accordingly declines in the winter term.

Figure $A3$: Average Annualized Increases in the Number of Transactions in Summer and Winter, 1983-200



Note: Annualized growth rates in the number of transactions in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. CML, 1983-2007.

Tables $A2a$ and $A2b$ complement Table 1 in the text, showing the differences in annualized nominal and real percentage changes in prices and transactions at more disaggregated levels of aggregation. The data come from the Land Registry and correspond to the period 1996 to 2012.

Table $A2a$. Difference in annualized percentage changes in house prices and sales volumes between semesters in the UK, by County/Unitary Authority.

	Nominal house price		Real house price		Volume of Sales	
Region	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
Bath And North East Somerset	7.801***	(2.893)	6.876**	(3.063)	168.391***	(17.271)
Bedford	4.246	(2.758)	3.321	(2.972)	135.610***	(18.064)
Blackburn With Darwen	2.605	(4.312)	1.68	(4.506)	108.125***	(17.797)
Blackpool	0.076	(3.860)	-0.849	(4.047)	109.833***	(17.743)
Blaenau Gwent	-0.278	(5.892)	-1.203	(5.943)	89.282***	(18.529)
Bournemouth	8.594***	(2.951)	7.668**	(3.090)	152.644***	(17.771)
Bracknell Forest	7.512**	(2.908)	6.587**	(3.067)	152.709***	(22.488)
Bridgend	7.352**	(3.428)	6.427*	(3.616)	116.104***	(21.728)
Brighton And Hove	9.590***	(3.096)	8.664***	(3.250)	153.371***	(15.345)
Buckinghamshire	6.333**	(2.443)	5.407**	(2.592)	171.025***	(16.633)
Caerphilly	7.852**	(3.531)	6.927*	(3.683)	109.763***	(18.751)
Cambridgeshire	6.502***	(2.410)	5.577**	(2.597)	153.133***	(15.258)
Cardiff	6.723**	(2.587)	5.798**	(2.839)	151.532***	(16.771)
Carmarthenshire	11.212***	(3.725)	10.287**	(3.945)	139.366***	(20.005)
Central Bedfordshire	6.184**	(2.624)	5.259*	(2.831)	151.578***	(20.073)
Ceredigion	10.391**	(4.371)	9.466**	(4.485)	172.681***	(18.864)
Cheshire East	5.771**	(2.441)	4.846*	(2.627)	164.372***	(16.876)
Cheshire West And Chester	2.860	(2.493)	1.935	(2.796)	151.576***	(17.382)
City Of Bristol	8.121***	(2.980)	7.196**	(3.182)	146.035***	(16.549)
City Of Derby	9.428***	(3.069)	8.503**	(3.272)	130.036***	(13.812)
City Of Kingston Upon Hull	7.192*	(3.639)	6.267*	(3.721)	108.256***	(17.974)
City Of Nottingham	8.432**	(3.176)	7.507**	(3.362)	142.728***	(14.230)
City Of Peterborough	5.717*	(3.007)	4.791	(3.190)	117.173***	(19.909)
City Of Plymouth	7.995**	(3.255)	7.070**	(3.383)	138.080***	(17.947)
Conwy	11.280***	(3.791)	10.354***	(3.885)	117.125***	(18.311)
Cornwall	6.484**	(2.755)	5.559*	(3.014)	135.242***	(16.713)
Cumbria	5.936**	(2.666)	5.011*	(2.876)	145.430***	(16.280)
Darlington	8.069**	(3.476)	7.144*	(3.619)	124.270***	(18.487)
Denbighshire	5.200	(3.388)	4.275	(3.605)	104.180***	(14.396)
Derbyshire	5.805**	(2.705)	4.879*	(2.899)	138.294***	(16.444)
Devon	6.134**	(2.539)	5.209*	(2.771)	157.589***	(17.026)
Dorset	5.657**	(2.659)	4.732	(2.841)	161.293***	(14.847)
Durham	6.048*	(3.484)	5.123	(3.695)	122.904***	(18.974)
East Riding Of Yorkshire	6.056**	(2.861)	5.130*	(3.060)	150.516***	(19.610)
East Sussex	6.315**	(2.701)	5.390*	(2.913)	146.407***	(15.545)
Essex	4.996**	(2.463)	4.071	(2.651)	141.757***	(16.147)
Flintshire	4.254	(3.207)	3.329	(3.406)	128.716***	(19.985)

Table *A2a* continued.

	Nominal house price		Real house price		Volume of Sales	
Region	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
Gloucestershire	7.202***	(2.527)	6.277**	(2.709)	141.596***	(17.546)
Greater London	7.129***	(2.467)	6.204**	(2.624)	124.953***	(14.981)
Greater Manchester	6.527**	(2.650)	5.602**	(2.796)	119.602***	(15.520)
Gwynedd	7.955**	(3.851)	7.030*	(4.093)	138.619***	(16.869)
Halton	8.894**	(4.127)	7.969*	(4.342)	127.813***	(22.070)
Hampshire	5.569**	(2.386)	4.643*	(2.554)	162.439***	(17.170)
Hartlepool	10.738**	(5.175)	9.812*	(5.362)	110.153***	(19.285)
Herefordshire	7.147**	(2.774)	6.222**	(2.959)	164.123***	(15.499)
Hertfordshire	6.770***	(2.416)	5.845**	(2.562)	146.626***	(16.305)
Isle Of Anglesey	3.746	(4.291)	2.821	(4.418)	144.922***	(19.278)
Isle Of Wight	5.220*	(2.816)	4.295	(3.063)	124.764***	(16.128)
Kent	6.024**	(2.521)	5.099*	(2.695)	140.289***	(15.823)
Lancashire	5.819**	(2.823)	4.894	(2.995)	130.537***	(17.066)
Leicester	8.988***	(3.175)	8.062**	(3.399)	120.437***	(17.282)
Leicestershire	5.217**	(2.592)	4.292	(2.768)	150.335***	(19.065)
Lincolnshire	7.414***	(2.725)	6.489**	(2.901)	137.820***	(16.957)
Luton	4.497	(3.444)	3.572	(3.552)	117.409***	(17.759)
Medway	3.899	(2.840)	2.974	(3.057)	119.805***	(17.298)
Merseyside	6.165**	(2.900)	5.240*	(3.119)	125.529***	(15.365)
Merthyr Tydfil	1.555	(8.288)	0.63	(8.366)	107.947***	(24.798)
Middlesbrough	5.205	(4.584)	4.28	(4.700)	127.499***	(19.755)
Milton Keynes	5.277*	(2.785)	4.352	(2.987)	113.533***	(17.346)
Monmouthshire	9.254***	(3.418)	8.329**	(3.693)	183.592***	(21.298)
Neath Port Talbot	5.672	(4.694)	4.747	(4.735)	97.024***	(21.134)
Newport	5.235	(3.903)	4.31	(4.119)	125.760***	(20.513)
Norfolk	6.227**	(2.545)	5.302*	(2.782)	158.746***	(17.148)
North East Lincolnshire	8.366**	(3.553)	7.441**	(3.673)	127.153***	(17.565)
North Lincolnshire	5.313	(3.577)	4.388	(3.692)	145.438***	(16.841)
North Somerset	4.258	(2.646)	3.333	(2.852)	153.704***	(20.273)
North Yorkshire	7.470***	(2.625)	6.545**	(2.818)	156.367***	(18.608)
Northamptonshire	5.213**	(2.597)	4.288	(2.816)	136.250***	(18.936)
Northumberland	8.101**	(3.191)	7.176**	(3.389)	145.089***	(17.816)
Nottinghamshire	6.384**	(2.582)	5.458*	(2.789)	140.961***	(16.668)
Oxfordshire	6.669***	(2.333)	5.744**	(2.557)	176.069***	(16.078)
Pembrokeshire	7.281*	(3.907)	6.356	(4.113)	141.997***	(17.113)
Poole	5.789**	(2.811)	4.864	(3.022)	146.006***	(17.759)
Portsmouth	7.931***	(2.967)	7.006**	(3.046)	153.175***	(18.503)

Table *A2a* continued.

	Nominal house price		Real house price		Volume of Sales	
Region	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
Powys	6.062*	(3.611)	5.137	(3.846)	172.440***	(16.611)
Reading	6.170**	(2.894)	5.244*	(3.060)	124.377***	(17.524)
Redcar And Cleveland	2.963	(4.212)	2.038	(4.328)	125.396***	(19.231)
Rhondda Cynon Taff	5.388	(3.501)	4.462	(3.704)	113.962***	(18.900)
Rutland	6.252*	(3.526)	5.327	(3.681)	162.162***	(20.066)
Shropshire	7.170***	(2.649)	6.245**	(2.785)	161.467***	(17.489)
Slough	5.624*	(3.073)	4.699	(3.084)	120.085***	(19.017)
Somerset	7.097***	(2.631)	6.172**	(2.826)	154.881***	(18.975)
South Gloucestershire	6.970**	(2.930)	6.045*	(3.097)	143.025***	(19.714)
South Yorkshire	5.581**	(2.766)	4.655	(2.960)	124.611***	(16.514)
Southampton	8.101***	(2.827)	7.176**	(3.008)	141.473***	(15.223)
Southend-On-Sea	6.580**	(2.966)	5.655*	(3.135)	117.248***	(17.789)
Staffordshire	5.373**	(2.316)	4.448*	(2.497)	136.658***	(18.298)
Stockton-On-Tees	9.458**	(3.578)	8.533**	(3.709)	139.501***	(22.429)
Stoke-On-Trent	4.973	(3.584)	4.048	(3.765)	101.022***	(17.005)
Suffolk	8.058***	(2.579)	7.133**	(2.715)	149.217***	(16.729)
Surrey	7.010***	(2.473)	6.085**	(2.619)	165.804***	(15.873)
Swansea	8.138**	(3.442)	7.213**	(3.597)	138.769***	(18.312)
Swindon	6.444**	(2.747)	5.519*	(2.929)	131.208***	(20.516)
The Vale Of Glamorgan	9.078***	(2.861)	8.152***	(3.027)	149.270***	(19.246)
Thurrock	2.66	(2.841)	1.735	(3.065)	128.723***	(19.644)
Torbay	6.401**	(2.964)	5.475*	(3.142)	138.315***	(17.314)
Torfaen	6.039	(4.159)	5.114	(4.287)	142.776***	(23.876)
Tyne And Wear	6.010**	(2.864)	5.085	(3.079)	142.998***	(17.232)
Warrington	4.887	(3.022)	3.962	(3.196)	142.508***	(21.404)
Warwickshire	6.340***	(2.367)	5.414**	(2.541)	150.421***	(16.948)
West Berkshire	6.210**	(2.552)	5.285*	(2.705)	163.718***	(20.021)
West Midlands	5.283**	(2.450)	4.358	(2.686)	123.269***	(14.721)
West Sussex	7.132***	(2.646)	6.207**	(2.818)	152.414***	(16.388)
West Yorkshire	6.620**	(2.694)	5.695*	(2.931)	135.604***	(16.318)
Wiltshire	7.210***	(2.411)	6.285**	(2.631)	172.332***	(18.962)
Windsor And Maidenhead	8.398***	(2.562)	7.473***	(2.639)	169.049***	(18.020)
Wokingham	4.047	(2.694)	3.122	(2.845)	167.077***	(18.146)
Worcestershire	6.022**	(2.356)	5.097*	(2.579)	160.113***	(17.610)
Wrekin	4.225	(3.073)	3.3	(3.328)	126.385***	(19.328)
Wrexham	7.129*	(3.809)	6.204	(3.899)	133.744***	(22.804)
York	7.140***	(2.651)	6.215**	(2.807)	169.103***	(19.443)

Note: Average differences (and standard errors), by county for 1995-2012.

*Significant at 10%; ** 5%; *** 1%. Source: Land Registry.

Table $A2b$. continued. Difference in annualized percentage changes in house prices and sales volumes between semesters, by London Borough

	Nominal house price		Real house price		Volume of Sales	
Region	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
Barking And Dagenham	5.693	(3.475)	4.768	(3.653)	105.464***	(19.083)
Barnet	5.342**	(2.476)	4.417	(2.669)	131.200***	(15.716)
Bexley	3.591	(2.585)	2.666	(2.759)	123.885***	(16.208)
Brent	2.572	(2.872)	1.646	(3.030)	111.113***	(18.166)
Bromley	6.582**	(2.612)	5.657*	(2.832)	132.751***	(16.123)
Camden	6.878**	(3.274)	5.953*	(3.427)	131.223***	(16.761)
City Of Westminster	12.174***	(2.882)	11.249***	(2.962)	102.490***	(17.337)
Croydon	4.512	(2.774)	3.586	(2.938)	107.497***	(15.822)
Ealing	5.915**	(2.748)	4.990*	(2.860)	117.611***	(15.864)
Enfield	5.568**	(2.676)	4.643	(2.847)	111.523***	(16.614)
Greenwich	4.600	(2.772)	3.675	(2.906)	124.870***	(18.021)
Hackney	8.072**	(3.221)	7.147**	(3.353)	120.808***	(21.112)
Hammersmith And Fulham	10.408***	(3.134)	9.482***	(3.200)	150.335***	(19.450)
Haringey	5.987**	(2.982)	5.062	(3.104)	130.082***	(17.037)
Harrow	9.068***	(2.953)	8.142**	(3.093)	129.560***	(15.918)
Havering	5.751**	(2.590)	4.826*	(2.730)	122.226***	(17.725)
Hillingdon	5.894**	(2.622)	4.969*	(2.772)	126.889***	(16.585)
Hounslow	9.574***	(2.945)	8.649***	(3.043)	123.182***	(16.636)
Islington	9.043***	(3.024)	8.118**	(3.068)	138.837***	(17.099)
Kensington And Chelsea	10.384***	(3.440)	9.459***	(3.505)	93.290***	(17.644)
Kingston Upon Thames	7.796**	(3.168)	6.870**	(3.252)	144.271***	(16.554)
Lambeth	9.450***	(3.025)	8.525***	(3.169)	140.724***	(17.098)
Lewisham	7.475***	(2.758)	6.550**	(2.972)	140.310***	(16.931)
Merton	8.127***	(3.018)	7.202**	(3.136)	138.877***	(15.764)
Newham	3.357	(3.899)	2.432	(4.129)	50.856**	(19.325)
Redbridge	4.214	(2.851)	3.289	(3.091)	116.949***	(14.419)
Richmond Upon Thames	7.423**	(2.976)	6.497**	(3.051)	173.621***	(17.128)
Southwark	8.712***	(3.133)	7.787**	(3.230)	127.612***	(17.858)
Sutton	4.280	(2.875)	3.355	(3.024)	129.001***	(16.332)
Tower Hamlets	6.115**	(3.011)	5.189*	(3.092)	130.337***	(20.225)
Waltham Forest	4.043	(3.131)	3.118	(3.319)	108.018***	(15.064)
Wandsworth	11.483***	(3.091)	10.557***	(3.225)	151.083***	(17.161)

Note: The Table shows the average differences (and standard errors), by borough for 1995-2012. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: Land Registry Repeat Sale Index and Sales.

Table $A3$ complements Table 1 in the text, showing the differences in annualized nominal and real percentage changes in prices between summers and regions in the United Kingdom using the DCGL and Halifax datasets, as well as the corresponding figures for transactions, using CML. The data cover the period 1983-2007.

Table $A3$: Difference in Annualized Percentage Changes in U.K. House Prices (Nominal and Real) and Transactions between Summer and Winter, by Region. using DCGL, Halifax (prices) and CML (transactions). 1983-2007.

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	6.536*	(3.577)	4.870	(3.461)
East Midlands	8.231**	(3.148)	6.408**	(3.131)
Gr. London	8.788***	(3.273)	6.966**	(3.372)
North East	8.511**	(3.955)	6.845*	(3.915)
North West	13.703***	(3.323)	12.583***	(3.245)
Northern Ireland	4.237	(3.431)	2.415	(3.467)
Scotland	10.393***	(2.793)	8.571***	(2.711)
South East	10.375***	(3.496)	8.709**	(3.301)
South West	11.244***	(3.419)	9.422***	(3.459)
Wales	7.180**	(3.504)	5.358	(3.442)
West Midlands	9.623***	(3.089)	7.801**	(3.070)
Yorkshire & the Humber	10.148***	(3.114)	8.325***	(3.056)
United Kingdom	9.008***	(2.304)	7.185***	(2.314)

Note: The Table shows the average differences (and standard errors), by region for 1983-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: Department of Communities and Local Government.

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	9.885***	(3.604)	8.081**	(3.706)
East Midlands	10.247***	(3.393)	8.444**	(3.413)
Gr. London	5.696*	(3.048)	3.892	(3.221)
North East	2.197	(2.945)	0.394	(2.864)
North West	8.019***	(2.653)	6.216**	(2.548)
Northern Ireland	6.053*	(3.409)	4.25	(3.494)
Scotland	9.334***	(2.320)	7.530***	(2.272)
South East	7.104**	(3.019)	5.301*	(3.149)
South West	9.258**	(3.474)	7.454**	(3.549)
Wales	7.786**	(3.329)	5.983*	(3.288)
West Midlands	5.987*	(3.540)	4.183	(3.505)
Yorkshire & the Humber	7.253**	(2.892)	5.450*	(2.825)
United Kingdom	7.559***	(2.365)	5.756**	(2.400)

Note: The Table shows the average differences (and standard errors), by region for 1983-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: Halifax.

Region	Difference	Std. Error
East Anglia	119.420***	(11.787)
East Midlands	104.306***	(11.151)
Gr. London	99.758***	(11.577)
North East	84.069***	(9.822)
North West	103.525***	(8.963)
Northern Ireland	71.466***	(12.228)
Scotland	116.168***	(9.843)
South East	117.929***	(9.710)
South West	110.996***	(8.764)
Wales	115.900***	(13.850)
West Midlands	112.945***	(9.496)
Yorkshire & the Humber	98.904***	(8.192)
United Kingdom	107.745***	(8.432)

Note: The Table shows the average differences (and standard errors) by region for 1983-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: Council of Mortgage Lenders.

A.2 Moving patterns

A reading of the empirical literature (Goodman, 1991), suggests then that the school calendar might be a likely trigger; as noted by Goodman, however, parents of school age children are less than a third of total movers, and hence one needs an amplification effect. Our model provides such mechanism.

Figure $A4$ illustrates the fact that most people (not just parents of school-age children) move in the summer months. The data are based on the American Housing Survey (1999 and 2001).

Figure $A4$: Monthly Distribution of Moves, by Life Cycle Stage

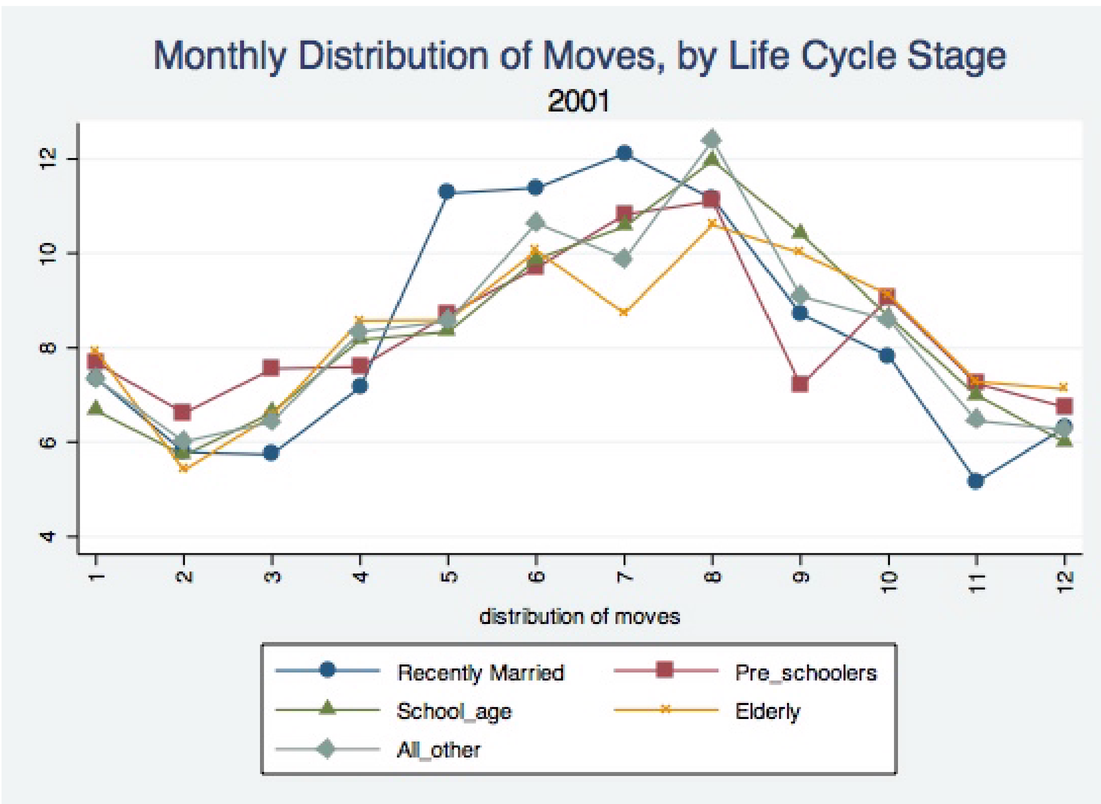
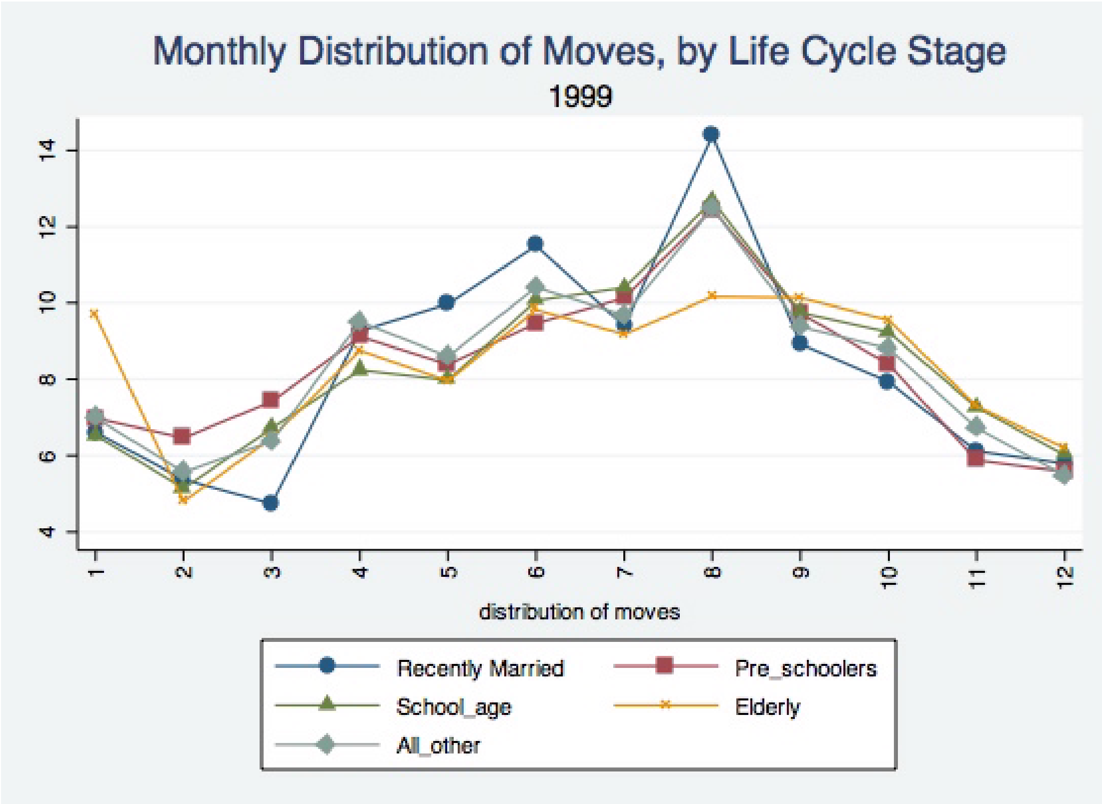


Figure $A4$ continued



A.3 Aggregate Seasonality (as Reported by Publishers of House Price Indexes)

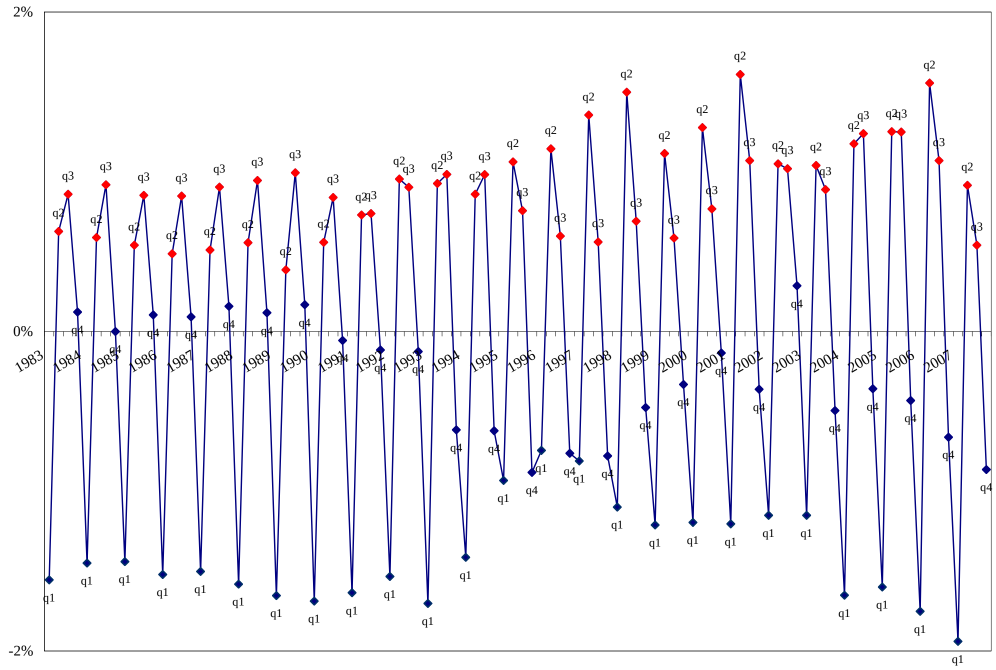
A first indication that house prices display seasonality comes from the observation that most publishers of house price indexes directly report SA data. Some publishers, however, report both SA and NSA data, and from these sources one can obtain a first measure of seasonality, as gauged by the publishers. For example, in the United Kingdom, Halifax publishes both NSA and SA house price series. Using these two series we computed the (logged) seasonal component of house prices as the ratio of the NSA house price series, P_t, relative to the SA series, P_t^*, from 1983:01 to 2007:04, $\left\{\ln \frac{P_t}{P_t^*}\right\}$. This seasonal component is plotted in Figure $A3$. (Both the NSA and the SA series correspond to the United Kingdom as a whole.)

In the United States, both the Office of Federal Housing Enterprise Oversight (OFHEO)'s house price index and the Case-Shiller index published by Standard & Poor's (S&P) are published in NSA and SA form. Figure $A4$ depicts the seasonal component of the OFHEO series for the US as a whole, measured as before as $\left\{\ln \frac{P_t}{P_t^*}\right\}$, from 1991:01 through to 2007:04. And Figure $A5$ shows the corresponding plot for the Case-Shiller index corresponding to a composite of 10 cities, with the data running from 1987:01 through to 2007:04. (The start of the sample in all cases is dictated by data availability.)

All figures seem to show a consistent pattern: House prices in the second and third quarters tend to rise above trend (captured by the SA series), and prices in the fourth, and particularly in the first quarter, tend to be in general at or below trend. The figures also make it evident that the extent of price seasonality is more pronounced in the United Kingdom than in the United States as a whole, though as shown in the text, certain cities in the United States seem to display seasonal patterns of the same magnitude as those observed in the United Kingdom. (Some readers might be puzzled by the lack of symmetry in Figure $A4$, as most expect the seasons to cancel out; this is exclusively due to the way OFHEO performs the seasonal adjustment;[47] for the sake of clarity and comparability across different datasets, we base our analysis only on the "raw", NSA series and hence the particular choice of seasonal adjustment by the publishers is inconsequential.)

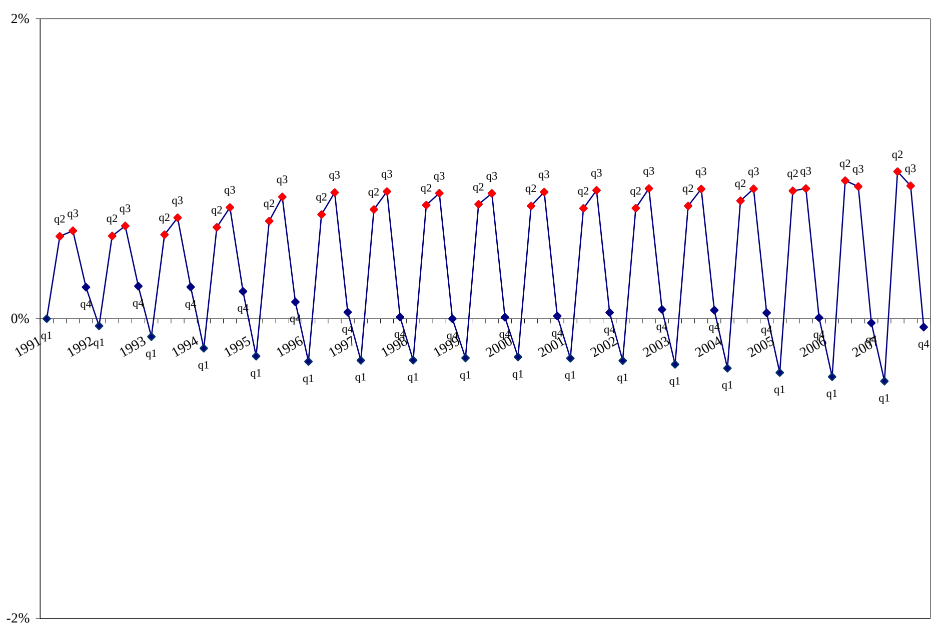
[47] OFHEO uses the Census Bureau's X-12 ARIMA procedure for SA; it is not clear, however, what the exact seasonality structure chosen is.

Figure $A3$: Seasonal Component of House Prices in the United Kingdom, 1983-2007



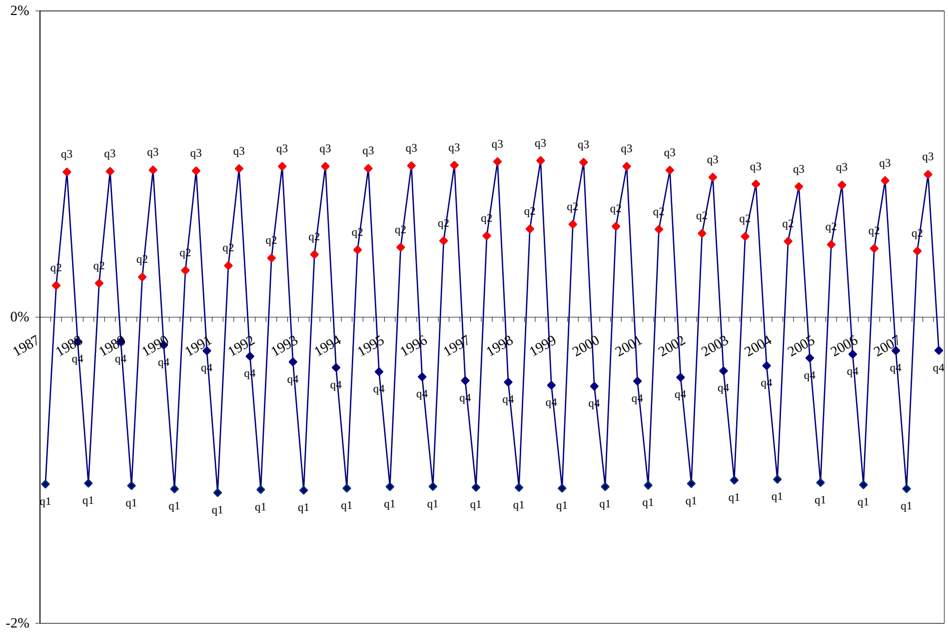
Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$. P_t is the NSA and P_t^* the SA index. Source: Halifax.

Figure $A4$: Seasonal Component of House Prices in the United States, 1991-2007



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$; P_t is the NSA and P_t^* the SA index. Source: OFHEO.

Figure $A5$: Seasonal Component of House Prices in U.S. cities, 1987-2007

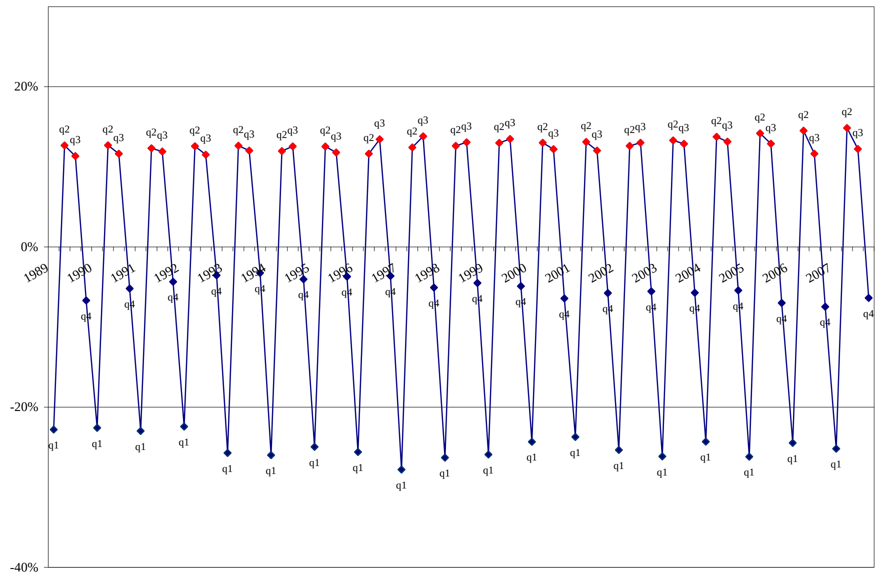


Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$. P_t is the NSA and P_t^* the SA index.

Source: Case-Shiller 10-city composite.

Last, but not least, the U.S. National Association of Realtors (NAR) publishes data on transactions both with and without seasonal adjustment. Figure $A6$ plots the seasonal component of house transactions, measured (as before) as the (logged) ratio of the NSA number of transactions Q_t, divided by the SA number of transactions Q_t^*:$\left\{\ln \frac{Q_t}{Q_t^*}\right\}$.

Figure $A6$: Seasonal Component of Housing Transactions in the United States, 1989-2007



Note: The plot shows $\left\{\ln \frac{Q_t}{Q_t^*}\right\}$; Q_t is the NSA and Q_t^* the SA number of transactions.

Source: NAR.

The seasonal pattern for transactions is similar to that for prices: Transactions surge in the second and third quarters and stagnate or fall in the fourth and first quarters. (In the United Kingdom only NSA data for transactions are available from the publishers.)

B Alternative Models of the Housing Market

We argued previously that the predictability and size of the seasonal variation in housing prices pose a challenge to existing models of the housing market. We discuss the key challenge using a simple, frictionless model and then we turn to the canonical models in the housing literature.

B.1 Frictionless Model

The equilibrium condition embedded in most dynamic general equilibrium models states that the marginal benefit of housing services should equal the marginal cost. Following Poterba (1984) the asset-market equilibrium conditions for any seasons $j = s$ (summer), w (winter) at time t is:[48]

$$d_{t+1,j'} + (p_{t+1,j'} - p_{t,j}) = c_{t,j} \cdot p_{t,j}, \tag{24}$$

where j' is the corresponding season at time $t+1$, $p_{t,j}$ and $d_{t,j}$ are the real asset price and rental price of housing services, respectively; $c_{t,j} \cdot p_{t,j}$ is the real gross (gross of capital gains) $t-$period cost of housing services of a house with real price $p_{t,j}$; and $c_{t,j}$ is the sum of after-tax depreciation, repair costs, property taxes, mortgage interest payments, and the opportunity cost of housing equity. Note that the formula assumes away risk (and hence no expectation terms are included); this is appropriate in this context because we are focusing on a "predictable" variation of prices.[49] As in Poterba (1984), we make the following simplifying assumptions so that service cost rates are a fixed proportion of the property price, though still potentially different across seasons ($c_{t,j} = c_{t+2,j} = c_j$, $j = s, w$): 1) Depreciation takes place at rate δ_j, $j = s, w$, constant for a given season, and the house requires maintenance and repair expenditures equal to a fraction κ_j, $j = s, w$, which is also constant for a given season. 2) The income tax-adjusted real interest rate and the marginal property tax rates (for given real property prices) are constant over time, though also potentially different across seasons; these rates are denoted, respectively as r_j and τ_j, $j = s, w$ (in the data, as seen, these are actually constant across seasons; we shall come back to this point below).[50] This yields $c_j = \delta_j + \kappa_j + r_j + \tau_j$, for $j = s, w$.

Subtracting (24) from the corresponding expression in the following season and using the condition

[48] See also Mankiw and Weil (1989) and Muellbauer and Murphy (1997), among others.

[49] Note that Poterba's formula also implicitly assumes linear preferences and hence perfect intertemporal substitution. This is a good assumption in the context of seasonality, given that substitution across semesters (or relatively short periods of time) should in principle be quite high.

[50] We implicitly assume the property-price brackets for given marginal rates are adjusted by inflation rate, though strictly this is not the case (Poterba, 1984): inflation can effectively reduce the cost of homeownership. This, however, should not alter the conclusions concerning seasonal patterns emphasized here. As in Poterba (1984) we also assume that the opportunity cost of funds equals the cost of borrowing.

that there is no seasonality in rents ($d_w \approx d_s$), we obtain:

$$\frac{p_{t+1,s} - p_{t,w}}{p_{t,w}} - \frac{p_{t,w} - p_{t-1,s}}{p_{t-1,s}} \frac{p_{t-1,s}}{p_{t,w}} = c_w - c_s \cdot \frac{p_{t-1,s}}{p_{t,w}}. \tag{25}$$

Using the results from the Department of Communities and Local Governments (DCLG), real differences in house price growth rates for the entire United Kingdom are $\frac{p_s - p_w}{p_w} \simeq 8.25\%$, $\frac{p_w - p_s}{p_s} \simeq 1.06\%$,[51] the left-hand side of (25) equals $7.2\% \approx 8.25\% - 1.06\% \cdot \frac{1}{1.0106}$. Therefore, $\frac{c_w}{c_s} = \frac{0.072}{c_s} + \frac{1}{1.0106}$. The value of c_s can be pinned-down from equation (24) with $j = s$, depending on the actual rent-to-price ratios in the economy. In Table $B1$, we summarize the extent of seasonality in service costs $\frac{c_w}{c_s}$ implied by the asset-market equilibrium conditions, for different values of d/p (and hence different values of $c_s = \frac{d_w}{p_s} + \frac{p_w - p_s}{p_s} = \frac{d_w}{p_s} + 0.0106$).

Table $B1$: Ratio of Winter-To-Summer Cost Rates

(annualized) d/p Ratio	Relative winter cost rates $\frac{c_w}{c_s}$
1.0%	448%
2.0%	334%
3.0%	276%
4.0%	241%
5.0%	218%
6.0%	201%

As the table illustrates, a remarkable amount of seasonality in service costs is needed to explain the differences in housing price inflation across seasons. Specifically, assuming annualized rent-to-price ratios in the range of 2 through 5 percent, total costs in the winter should be between 334 and 218 percent of those in the summer. Depreciation and repair costs ($\delta_j + \kappa_j$) might be seasonal, being potentially lower during the summer.[52] But income-tax-adjusted interest rates and property taxes ($r_j + \tau_j$), two major components of service costs are not seasonal. Since depreciation and repair costs

[51] In the empirical Section we computed growth rates using difference in logs; the numbers are very close using $\frac{p_{t+1,j'} - p_{t,j}}{p_{t,j}}$ instead. We use annualized rates as in the text; using semester rates of course leads to the same results.

[52] Good weather can help with external repairs and owners' vacation might reduce the opportunity cost of time—though for this to be true it would be key that leisure were not too valuable for the owners.

are only part of the total costs, given the seasonality in other components, the implied seasonality in depreciation and repair costs across seasons in the UK is even larger. Assuming, quite conservatively, that the a-seasonal component ($r_j + \tau_j = r + \tau$) accounts for only 50 percent of the service costs in the summer ($r + \tau = 0.5c_s$), then, the formula for relative costs $\frac{c_w}{c_s} = \frac{\delta_w + \kappa_w + 0.5c_s}{\delta_s + \kappa_s + 0.5c_s}$ implies that the ratio of depreciation and repair costs between summers and winters is $\frac{\delta_w + \kappa_w}{\delta_s + \kappa_s} = 2\frac{c_w}{c_s} - 1$.[53] For rent-to-price ratios in the range of 2 through 5 percent, depreciation and maintenance costs in the winter should be between 568 and 336 percent of those in the summer. (If the a-seasonal component ($r + \tau$) accounts for 80 percent of the service costs ($r + \tau = 0.8c_s$), the corresponding values are 1571 and 989 percent). By any metric, these figures seem extremely large and suggest that a deviation from the simple asset-pricing equation is called for. Similar calculations can be performed for different regions in the US; as expressed before, though the extent of price seasonality for the US as a whole is lower than in the UK, seasonality in several US cities is comparable to that in the UK and would therefore also imply large seasonality in service costs, according to condition (24).

B.2 Other search models of housing

We focus on the canonical models of Krainer (2001), Novy-Marx (2009) and Piazzesi and Schneider (2009). In these models, variations in reservation prices depend, correspondingly, on three factors: (1) variations in the value of houses common to all buyers, (2) variations in the ratio of the number of buyers to the number of sellers in the housing market, and (3) variations in the buyer's belief about the house price-to-dividend ratio. The three papers are also different in how they model search frictions. In general there are two types of search frictions: (1) finding a house or buyer, modelled as an aggregate matching function; and (2) how much a buyer likes the house, modelled as a stochastic match-specific housing utility.

[53] Call λ the aseasonal component as a fraction of the summer service cost rate: $r + \tau = \lambda c_s$, $\lambda \in (0, 1)$ (and hence $\delta_s + \kappa_s = (1 - \lambda)c_s$). Then: $\frac{c_w}{c_s} = \frac{\delta_w + \kappa_w + \lambda c_s}{\delta_s + \kappa_s + \lambda c_s} = \frac{\delta_w + \kappa_w + \lambda c_s}{c_s}$. Or $c_w = \delta_w + \kappa_w + \lambda c_s$. Hence: $\frac{c_w - \lambda v_s}{(1-\lambda)c_s} = \frac{\delta_w + \kappa_w}{(1-\lambda)c_s}$; that is $\frac{\delta_w + \kappa_w}{\delta_s + \kappa_s} = \frac{c_w}{(1-\lambda)c_s} - \frac{\lambda}{1-\lambda}$, which is increasing in λ for $\frac{c_w}{c_s} > 1$.

Krainer (2001) focuses on the second search friction and assumes housing utility is $d^i = \varepsilon^i + x$, where the stochastic value ε^i is match-specific but x is common to all buyers. He analyzes how house prices vary when x follows a Markov chain between high x_H and low x_L, with a persistent parameter λ. His model implies a negative correlation between price and time-to-sell across 'hot' ($x_t = x_H$) and 'cold' ($x_t = x_L$) markets. The case in which λ equal to zero delivers a deterministic periodic steady state with x switching between x_L and x_H. In other words, the model has a prediction for 'hot' and 'cold' seasons when $\lambda = 0$. However, Figure 3*b* and 5*b* of his paper show that there is virtually no change in price when $\lambda = 0$. In fact Krainer has noted the small change in price in his discussion of Figure 5*b*. Quoting from his paper, "in this model, prices are sticky in that they do not drop too far in down markets. Rather liquidity dries up. The reverse is true in up markets. Prices do not rise as high, but liquidity improves." The intuition in Krainer (2001) is similar to that in the frictionless model presented above. In the absence of a thick-market effect, if prices are too high in a given season, buyers prefer to wait, as the option value of waiting is high. Moreover, as agents know that in the next period the housing utility of owning a house will be low again, they are not willing to pay a high price. (One would need huge seasonality in the house dividend x_H/x_L to generate any seasonality in prices.) Like in Krainer (2001), we also focus on the second friction but unlike it, fluctuations in price in our model are driven by the thick-market-effect where the draws ε^i are stochastically higher in the season with more buyers and sellers. In order to generate seasonality, it is critical to have both i) persistence in the match quality (otherwise the increase in prices due to a temporary high house dividend will be small) and ii) a mechanism whereby the quality of transacted houses are seasonal.

Piazzesi and Schneider (2009) focuses on the first search friction and use an aggregate matching function as in Pissarides (2000, chapter 1). They analyze house prices when there is an exogenous change in buyers' beliefs about houses' price-to-dividend ratios. In other words, their mechanism can deliver any change in price levels as it is specified by the exogenous change in beliefs. For this to explain seasonality in prices, buyers' beliefs have to shift up and down regularly across seasons. We think this mechanism is thus unlikely to generate the seasonality in the data.

Novy-Marx (2009) has both types of search frictions (as in Pissarides (2000, chapter 6)), except that he uses different entry conditions for buyers and sellers. He analyzes house prices when the ratio of buyers to sellers (using his notation, θ) varies due to changes in buyers' relative propensity to enter. This mechanism could potentially generate 'hot' and 'cold' seasons if θ is higher in the hot season. We extend his model to allow for a seasonal cycle and study its quantitative implications. More specifically, we derive a periodic steady state where θ alternates between high θ^s and low θ^w deterministically. We then examine the implied seasonality in prices.

B.2.1 Seasonal Cycle in Novy-Marx (2009)

The original Novy-Marx model can be summarized as follow. There are two sources of search frictions. (1) An aggregate matching function that implies encounter rates for buyers and sellers given by:

$$\lambda_b(\theta) = \lambda\theta^\eta; \qquad \lambda_s(\theta) = \theta\lambda_b(\theta) = \lambda\theta^{\eta+1}, \eta > -1 \tag{26}$$

where $\theta = \frac{m_b}{m_s}$ is the buyer-to-seller ratio, and (2) there is a match-specific transaction value ε with cdf $\Phi(.)$. A transaction is an absorbing state. The total surplus created by a transaction is equal to $\varepsilon - V_b^* - V_s^*$ where V_b^* and V_s^* are the value for buyer and seller while searching, so the threshold for a transaction satisfies $\varepsilon^* \equiv V_b^* + V_s^*$ and the Bellman equation are

$$rV_i^* = -c_i + \lambda_i [1 - \Phi(\varepsilon^*)] E[(V_i(\varepsilon) - V_i^*) \mid \varepsilon > \varepsilon^*]$$

for $i = b, s$, where c_i is search cost and $V_i(\varepsilon)$ is the value for agent i after the transaction ε goes through, so $V_b(\varepsilon) + V_s(\varepsilon) = \varepsilon$. The total surplus is divided between the buyer and seller via Nash bargaining according to their bargaining power β_i, for $i = b, s$

$$V_i(\varepsilon) - V_i^* = \beta_i [V_b(\varepsilon) + V_s(\varepsilon) - (V_b^* + V_s^*)] = \beta_i (\varepsilon - \varepsilon^*) \tag{27}$$

which reduces the Bellman equation, $i = b, s$

$$rV_i^* = -c_i + \lambda_i \beta_i \nu_\varepsilon(\varepsilon^*) \tag{28}$$

Summing it up across buyers and sellers, and using the definition of ε^* implies an implicit function for the threshold ε^*

$$\Lambda(\theta) \nu(x) = rx + c_b + c_s \tag{29}$$

where

$$\begin{aligned} \Lambda(\theta) &= \beta_b \lambda_b(\theta) + \beta_s \lambda_s(\theta) \\ \nu(x) &\equiv [1 - \Phi(x)] E(\varepsilon - x \mid \varepsilon > x) = \int_x (z - x)\, d\Phi(z) \end{aligned} \tag{30}$$

The model is analyzed in two steps. First, given θ, he solves for equilibrium ε^* using equation (29). Then, he uses the Bellman equation (28) to derive the equilibrium values (V_b^*, V_s^*). Time-to-sell is the expected duration for seller to exit the market $E(T_s) = \frac{1}{[1-\Phi_\varepsilon(\varepsilon^*)]\lambda_s}$. The reservation/minimum price $p(\varepsilon^s) = V_s^*$ is given by (28).The transaction price $p(\varepsilon) = V_s(\varepsilon)$ is given by equation (27). The main result of the paper is Figure 4 which reports a negative correlation between $E(T_s)$ and V_s^* across markets with different θ. Finally he specifies entry conditions for buyers and sellers to solve for equilibrium θ^*.

We now introduce a seasonal cycle into the model where θ alternates between θ^s and θ^w deterministically. As in the original Novy-Marx, θ^s and θ^w are determined independently through the entry conditions. So we can proceed to study the seasonality in prices for any given (θ^s, θ^w).

Let U_i^j be the value of agent i searching in season $j = s, w$. The choice to denote this value using a different notation is very important. Here we are studying a seasonal cycle where tightness switches between θ^s and θ^w deterministically whereas in Novy-Marx the value $V_i^*(\theta^s)$ refers to the case that tightness remains at θ^s for all periods. This distinction is very important as it will become clear very soon that U_i^s / U_i^w is much smaller than $V_i^*(\theta^s) / V_i^*(\theta^w)$ for any given levels of (θ^s, θ^w).

Let ε^j be the corresponding threshold for season $j = s, w$:

$$\varepsilon^j = U_b^j + U_s^j \tag{31}$$

The Bellman equation for the value of search for agent $i = b, s$ in season s is

$$U_i^s = \delta U_i^w + \lambda_i^s \beta_i \delta \nu(\varepsilon^s) - c_i, \tag{32}$$

where δ is the discount factor between seasons (6 months). A similar Bellman equation holds for season w. So equation (32) is a set of four equations. Summing up the value for $i = b$ and s, and using definition of threshold (31), we have two equations to solve for equilibrium $(\varepsilon^s, \varepsilon^w)$ for given (θ^s, θ^w):

$$\begin{aligned} \varepsilon^s &= \delta\varepsilon^w + \Lambda(\theta^s)\,\delta\nu(\varepsilon^s) - (c_b + c_s) \qquad (33) \\ \varepsilon^w &= \delta\varepsilon^s + \Lambda(\theta^w)\,\delta\nu(\varepsilon^w) - (c_b + c_s). \end{aligned}$$

Given (θ^s, θ^w), equilibrium $(\varepsilon^s, \varepsilon^w, U_s^s, U_s^w, U_b^s, U_b^w)$ jointly satisfy the set of 6 equations given by (32) and (33).

We next derive a few analytical results that are useful in addressing the question of whether seasonal variations in θ can explain the observed seasonality in prices. We focus on the case of $\theta^s > \theta^w$, i.e. the summer season has higher buyer-to-seller ratios.

Lemma 1 *If* $\Lambda'(\theta) \geqslant 0$, *then* $\varepsilon^s > \varepsilon^w$.

Proof. Suppose not, i.e. $\varepsilon^s \leqslant \varepsilon^w$. Given $\Lambda'(\theta) \geqslant 0$, so $\Lambda(\theta^s) \geqslant \Lambda(\theta^w)$, and $\nu'(.) < 0$ implies $\nu(\varepsilon^s) > \nu(\varepsilon^w)$, but equation (33) implies

$$(1+\delta)(\varepsilon^s - \varepsilon^w) = \Lambda(\theta^s)\,\delta\nu(\varepsilon^s) - \Lambda(\theta^w)\,\delta\nu(\varepsilon^w) \tag{34}$$

hence we have $\varepsilon^s > \varepsilon^w$. Contradiction. ■

The average price of a transaction in season j is

$$P^j = E\left[p^j(\varepsilon) \mid \varepsilon > \varepsilon^j\right]$$

given the price equation (27) holds for $j = s, w$ we obtain:

$$P^j = U_s^j + \beta_s E\left[\left(\varepsilon - \varepsilon^j\right) \mid \varepsilon > \varepsilon^j\right]. \tag{35}$$

The first term is the reservation price $p^j(\varepsilon^j)$ in season j and the second term is any surplus the seller expects to receive if ε is above the threshold ε^j. This price function is similar to that of the original Novy-Marx equilibrium $\theta = \theta^j$. However, the concepts are very different. In the seasonal model, (P^s, P^w) are jointly determined in the periodic steady state whereas in Novy-Marx $P(\theta^s)$ and $P(\theta^w)$ are values for two different steady states.

It is clear from the price function that introducing thick-market effects will substantially increase P^s/P^w (by shifting up $E[(\varepsilon - \varepsilon^j) \mid \varepsilon > \varepsilon^j]$). The question is whether the model can deliver seasonality in price without the thick-market effect. First note that the second term depends on the distribution which is log-concave in Novy-Marx (both Normal and Uniform distribution are log-concave).

Lemma 2 *If $\Lambda'(\theta) \geqslant 0$ and the p.d.f. for ε is log-concave,*

$$\frac{P^s}{P^w} < \frac{U_s^s}{U_s^w},$$

i.e. average price is less seasonal than the reservation price.

Proof. It follows from Lemma 1 that $\varepsilon^s > \varepsilon^w$ and $E\left[\left(\varepsilon - \varepsilon^j\right) \mid \varepsilon > \varepsilon^j\right]$ is decreasing in ε^j if p.d.f for ε is log-concave(see Burdett (1996)). ■

Lemma 2 shows that seasonality in reservation price U_s^j is the upper bound for the seasonality in price P^j. Next we turn to seasonality in the reservation price U_s^j. Iterating forward, the Bellman equation (32) for $i = s$ implies that

$$(1+\delta)\left(U_s^s - U_s^w\right) = \delta\beta_s\left[\lambda_s^s v\left(\varepsilon^s\right) - \lambda_s^w v\left(\varepsilon^w\right)\right]. \tag{36}$$

Recall from (30), $\Lambda(\theta)$ is the weighted average of arrival rates for buyers and sellers. The arrival rate of buyers to sellers, $\lambda_s(\theta)$, is increasing in θ. So $\Lambda'(\theta) \geqslant 0$ as long as $\lambda_b(\theta)$ does not fall too much in θ. Novy-Marx assumes $\lambda_b(\theta) = \lambda$, so this condition always holds. We now proceed the analysis under the case $\Lambda'(\theta) \geqslant 0$, thus $\varepsilon^s > \varepsilon^w$. It follows from (36) that the reservation price is higher in the summer if the direct effect of higher arrival rate $\lambda_s(\theta^s) \geqslant \lambda_s(\theta^w)$ dominates the equilibrium effect of higher thresholds $\varepsilon^s \geqslant \varepsilon^w$. We next study its magnitude.

Lemma 3 *If $c_b = c_s = 0$,*

$$\frac{U_s^s}{U_s^w} - 1 = \left(\frac{\lambda_s^s v\left(\varepsilon^s\right)}{\lambda_s^w v\left(\varepsilon^w\right)} - 1\right)\left(\frac{1-\delta}{\frac{\lambda_s^s \delta \nu\left(\varepsilon^s\right)}{\lambda_s^w v\left(\varepsilon^w\right)} + 1}\right). \tag{37}$$

Proof. Iterating the Bellman equation forward to obtain

$$\left(1-\delta^2\right) U_s^s = \lambda_s^w \beta_s \delta^2 \nu\left(\varepsilon^w\right) + \lambda_s^s \beta_s \delta v\left(\varepsilon^s\right) - (1+\delta) c_i$$

together with (36), the result follows. ■

Note that given that δ is the discount factor between the two seasons (6 months), it is very close to 1, so the ratio $\frac{U_s^s}{U_s^w}$ is substantially smaller than the $\frac{V_s^*(\theta^s)}{V_s^*(\theta^w)}$ in Figure 2&4 of Novy-Marx which from equation (28) under $c_b = c_s = 0$ is

$$\frac{V_s^*\left(\theta^s\right)}{V_s^*\left(\theta^w\right)} = \frac{\lambda_s^s v\left(\varepsilon^*\left(\theta^s\right)\right)}{\lambda_s^w v\left(\varepsilon^*\left(\theta^w\right)\right)}.$$

The result is intuitive as $\frac{V_s^*(\theta^s)}{V_s^*(\theta^w)}$ is across steady states whereas $\frac{U_s^s}{U_s^w}$ is across seasons along the periodic steady state.

Time-to-sell in season j is

$$E\left(T_s^j\right) = \frac{1}{\lambda_s^j\left[1-\Phi\left(\varepsilon^j\right)\right]} \tag{38}$$

Similar to that of U_s^j, there are two effects: (1) higher θ^s increases the arrival rate $\lambda_s\left(\theta^s\right)$, which decreases time-to-sell; and (2) a higher threshold lowers the probability of a transaction conditional on meeting, thus increases time-to-sell.

To summarize, under Novy-Marx assumptions of $\eta = 0$, $c_b = c_s = 0$ and given a logconcave distribution for ε, conditions for Lemma 1-3 are satisfied. We have higher price, shorter time-to-sell and higher transactions in the hot season relative to the cold. Novy-Marx consider the case $\varepsilon \sim N\left(\mu, \sigma^2\right)$, so

$$\nu(x) = (\mu - x) N\left(\frac{\mu - x}{\sigma}\right) + \sigma n\left(\frac{\mu - x}{\sigma}\right)$$

where $N(.)$ is the c.d.f. and $n(.)$ is the p.d.f for the Normal distribution. Define $y^j = \frac{\varepsilon^j - \mu}{\sigma}$, using the implicit equations (33), equilibrium (y^s, y^w) jointly satisfy

$$\begin{aligned}\delta y^w &= y^s - \frac{(1-\delta)\mu}{\sigma} + \Lambda(\theta^s)\delta\left[y^s N(y^s) + n(y^s)\right] \\ \delta y^s &= y^w - \frac{(1-\delta)\mu}{\sigma} + \Lambda(\theta^w)\delta\left[y^w N(y^w) + n(y^w)\right].\end{aligned}$$

Given values (y^s, y^w), $\varepsilon^j = \mu + \sigma y^j$, $i = s, w$, thus $\left[U_s^s, U_s^w, P^s, P^w, E(T_s^s), E(T_s^w)\right]$ are obtain from (27), (32) and (38).

We derive the quantitative results using Novy-Marx parameters except we use the 6% annual interest, consistent with Blake (2011) and our own calibration.

Novy-Marx reports quantitative results for various levels of θ as he is interested in comparing across steady states. For our interest of studying seasonality, we need to specify the average level $\bar{\theta}$ and then look at the seasonal cycle around it. We set $\bar{\theta} = 10$ so that the average time-to-sell is around 6.5 months. Note that this number is larger than that is required in Figure 2 of Novy-Marx due to the lower interest rate used here. We then compute the periodic steady state with $\theta^s = (1+a)\bar{\theta}$ and $\theta^w = (1-a)\bar{\theta}$.

We compute the extent in seasonality of a variable X as in our paper: $4 * \ln\left(\frac{X^s}{X^w}\right)$. Given time-to-sell is counter-seasonal, its seasonality is a negative number. It is not surprising that both the extent of seasonality in price and time-to-sell is increasing in the driver of the seasonality $\frac{\theta^s}{\theta^w}$. To have a sense of how large $\frac{\theta^s}{\theta^w}$ should be, we turn to the implied seasonality in transaction. Transaction in season j is related to time-to-sell in season j by

$$Q^j = m_s^j * \{\underbrace{\lambda_s^j\left[1 - \Phi\left(\varepsilon^j\right)\right]}_{\text{prob. of sale}}\} = \frac{m_s^j}{E\left(T_s^j\right)}$$

where m_s^j is measure of sellers/houses in season j. Thus seasonality in transaction is approximately equal to seasonality in m_s^j minus the seasonality in $E\left(T_s^j\right)$, where the later is a negative number given time-to-sell is counter-seasonal. Novy-Marx's model can predict any level of seasonality in m_s^j depending on the entry conditions (similarly, it can predict any level of θ^s/θ^w). Since our objective is to understand whether the model can predict the level of price seasonality in the data, we focus on the case where the model matches the seasonality in m_s^j in the data which is about 28%. The predicted seasonality in transaction is simply equal to (28% minus predicted seasonality in time-to-sell)

The results are reported in Table $B2$ where the bargaining power for seller is $\beta_s = 0.8$ as in Novy-Marx. The results demonstrate that increasing $\frac{\theta^s}{\theta^w}$ has a much larger effect on the time-to-sell ratio than price ratio across seasons. In other words, Figure 4 of Novy-Marx is essentially a flat line when it comes to comparing across 'hot season' and 'cold season'. More specifically, when $\theta^s$is 20 percent above and θ^w is 20 percent below the annual average, predicted seasonality in transactions is 150 percent (close to the U.S. level) but seasonality in price is only 1.6 percent (a third of the U.S. level at 4.8). Making θ^s 30 percent above average (and θ^w 30 percent below) increases the seasonality in price to 2.4% but it sharply increases the seasonality in transaction to 215 percent. To summarize, we find that the implied seasonality in price is too small for reasonable levels of seasonality in transactions when the buyer-to-seller's ratio is the driving force: buyer-to-sell ratios affect time-to-sell directly through the arrival rate of buyers while its effect on transaction prices is through seller's reservation prices. Thus, we conclude that seasonal variation in reservation price that is based on variations in buyer-to-seller's ratio alone cannot generate enough seasonality in price.

Table $B2$. Seasonality in Novy-Marx model with $\frac{\theta^s}{\theta^w} = \frac{1+a}{1-a}$						
	Seasonal Ratio			Seasonality $= 4 * \ln\left(\frac{X^s}{X^w}\right)$		
a	$\frac{\theta^s}{\theta^w}$	$\frac{E(T_s^w)}{E(T_s^s)}$	$\frac{P^s}{P^w}$	Time-to-sell	Price	Transaction
0.1	1.2	$\frac{7.0 \text{ months}}{6.0 \text{ months}} = 1.16$	1.002	-60%	0.8%	88%
0.2	1.5	$\frac{7.7 \text{ months}}{5.6 \text{ months}} = 1.36$	1.004	-122%	1.6%	150%
0.3	1.9	$\frac{8.5 \text{ months}}{5.3 \text{ months}} = 1.60$	1.006	-187%	2.4%	215%

C Microfoundations for First-order Stochastic Dominance

In this Section we provide microfoundations for the key assumption in our model, namely:

$$F(., v') \leq F(., v) \Leftrightarrow v' > v, \tag{39}$$

where v denotes the stock of houses for sale (or vacancies). The derivation makes explicit the relation between v and the quality of the (best) match.

Suppose the quality of the match between any given person and any given house follows a distribution $G(x)$. Suppose further the actual number of houses viewed by a buyer, denoted by n, is a stochastic Poisson process with arrival rate λ. The arrival rate (per buyer) is the outcome of a homogeneous matching function $m(b, v)$, which depends on the number of buyers and sellers in the market:

$$\lambda = \frac{m(b, v)}{b} = b^{\alpha-1} m(1, v/b)$$

In equilibrium in our model, $b = v$, so

$$\lambda = v^{\alpha - 1} m$$

where $m = m(1,1)$ is constant. We assume that $\alpha > 1$, i.e., the arrival rate (which governs the number of viewings) is increasing in the number of vacancies (or houses that can be viewed).

The distribution of quality when n houses are viewed, using the order statistics is given by:

$$F_n(x) = G(x)^n$$

so the distribution of quality for a buyer in a market with v vacancies is:

$$\begin{aligned} F(x) &= \sum_n G(x)^n \left(e^{-\lambda} \frac{\lambda^n}{n!} \right) \\ &= e^{-\lambda} \sum_n \left(\frac{[\lambda G(x)]^n}{n!} \right) \\ &= e^{-\lambda} e^{\lambda G(x)} = e^{-\lambda(1 - G(x))} \\ &= e^{-v^{\alpha-1} m (1 - G(x))} \end{aligned}$$

Thus $F(x)$ satisfies assumption (39) for any given $G(x)$ and it can be interpreted as the distribution of the maximum match quality from a finite sample, whose size follows a Poisson distribution with arrival rate increasing in v.

This derivation is helpful to understand the foundations for the assumption. For calibration purposes, however, this is of little help, as the underlying distribution G is not known and hence we do not know the shape of F. Therefore, to avoid a deeper level of assumptions, in the paper we just use a “generic” F stochastically increasing in v and take stance only at the calibration stage.

D Efficiency Properties of the Model and Robustness

D.1 Efficiency Properties of the model

This section discusses the efficiency of equilibrium in the decentralized economy. For a complete derivation, see Section E.3 of this Appendix. The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The key difference between the planner's solution and the decentralized solution is that the former internalizes the thick-market effect. It is evident that the equilibrium level of transactions in the decentralized economy is not socially efficient because the optimal decision rules of buyers and sellers takes the stock of vacancies in each period as given, thereby ignoring the effects of their decisions on the stock of vacancies in the following periods. The thick-market effect generates a negative externality that makes the number of transactions in the decentralized economy inefficiently high for any given stock of vacancies (transacting agents do not take into account that, by waiting, they can thicken the market in the following period and hence increase the overall quality of matches).[54]

The efficient level of seasonality in housing markets, however, will depend on the exact distribution of match quality $F(\varepsilon, v)$. Under likely scenarios, the solution of the planner will involve a positive level of seasonality; that is, seasonality can be an efficient outcome. Indeed, in some circumstances, a planner may be willing to completely shut down the market in the cold season, to fully seize the benefits of a thick market.[55] This outcome is not as unlikely as one may a priori think. For example, the academic market for junior economists is extremely seasonal.[56] Extreme seasonality of course relies on the specification of utility—here we simply assume linear preferences; if agents have sufficiently concave utility functions (and intertemporal substitution across seasons is extremely low), then the planner

[54]This result is similar to that in the stochastic job matching model of Pissarides (2000, chapter 8), where the reservation productivity is too low compared to the efficient outcome in the presence of search externalities.

[55]The same will happen in the decentralized economy when the ratio $(1 - \phi^s) / (1 - \phi^w)$ is extremely high, e.g. the required ratio is larger than 10 for the calibrated parametrs we use.

[56]And it is perhaps highly efficient, given that it has been designed by our well-trained senior economists.

may want to smooth seasonal fluctuations. For housing services, however, the concern of smoothing consumption across two seasons in principle should not be too strong relative to the benefit of having a better match that is on average long lasting (9 to 13 years in the two countries we analyze).

D.2 Model Assumptions

It is of interest to discuss four assumptions implicit in the model. First, we assume that each buyer only visits one house and each seller meets only one buyer in a given season. We do this for simplicity so that we can focus on the comparison across seasons. One concern is whether allowing the buyer to visit other houses may alter the results.[57] This is, however, not the case here. Note first that the seller's outside option is also to sell to another buyer. More formally, the surplus to the buyer if the transaction for her first house goes through is:

$$\tilde{S}_b^s(\varepsilon) \equiv H^s(\varepsilon) - \tilde{p}^s(\varepsilon) - \{E^s[S_b^s(\eta)] + \beta B^w\}, \tag{40}$$

where $E^s[S_b^s(\eta)]$ is the equilibrium expected surplus (as defined in (13)) for the buyer if she goes for another house with random quality η. By definition $S_b^s(\eta) \geqslant 0$ (it equals zero when the draw for the second house η is too low). Compared to (13), the outside option for the buyer is higher because of the possibility of buying another house. Similarly, the surplus to the seller if the transaction goes through is:

$$\tilde{S}_v^s(\varepsilon) \equiv \tilde{p}^s(\varepsilon) - \{\beta V^w + u + E^s[S_v^s(\eta)]\}. \tag{41}$$

The key is that both buyer and seller take their outside options as given when bargaining. The price $\tilde{p}^s(\varepsilon)$ maximizes the Nash product with the surplus terms $\tilde{S}_b^s(\varepsilon)$ and $\tilde{S}_v^s(\varepsilon)$. The solution implies

[57] Concretely, one might argue that the seller of the first house can now only capture part of the surplus of the buyer in excess of the buyer's second house. In this case, for the surplus (and hence prices) to be higher in the summer one would need higher dispersion of match quality in the summer. This intuition is, however, incomplete. Indeed, one can show that higher prices are obtained independently of the level of dispersion.

$(1-\theta)\,\tilde{S}_v^s(\varepsilon) = \theta \tilde{S}_b^s(\varepsilon)$, but the Nash bargaining for the second house implies that $(1-\theta)\,E^s\left[S_v^s(\eta)\right] = \theta E^s\left[S_b^s(\eta)\right]$, so:

$$(1-\theta)\left[\tilde{p}^s(\varepsilon) - (\beta V^w + u)\right] = \theta\left[H^s(\varepsilon) - \tilde{p}^s(\varepsilon) - \beta B^w\right],$$

which has the same form as (16); thus it follows that the equilibrium price equation for $\tilde{p}^s(\varepsilon)$ is identical to (17)—though the actual level of prices is different, as the cutoff match-quality is different. Our qualitative results on seasonality in prices continue to hold as before, and quantitatively they can be even stronger. Recall that in the baseline model we find that seasonality in the sum of buyer's and seller's values tends to reduce the quality of marginal transactions in the summer relative to winter because the outside option in the hot season is linked to the sum of values in the winter season: $B^w + V^w$. Intuitively, allowing the possibility of meeting another party in the same season as an outside option could mitigate this effect and hence strengthen seasonality in prices. To see this, the cutoff quality $\tilde{\varepsilon}^s$ is now defined by: $H^s(\tilde{\varepsilon}^s) = \beta(B^w + V^w) + u + E^s\left[S^s(\eta)\right]$. Compared to (4), the option of meeting another party as outside option shows up as an additional term, $E^s\left[S^s(\eta)\right]$, which is higher in the hot season.

A second simplification in the model is that buying and selling houses involve no transaction costs. This assumption is easy to dispense with. Let $\bar{\tau}_b^j$ and $\bar{\tau}_v^j$ be the transaction costs associated with the purchase $(\bar{\tau}_b^j)$ and sale $(\bar{\tau}_v^j)$ of a house in season j. Costs can be seasonal because moving costs and repairing costs may vary across seasons.[58] The previous definitions of surpluses are modified by replacing price p^j with $p^j - \bar{\tau}_v^j$ in (12) and with $p^j + \bar{\tau}_b^j$ in (13). The value functions (15) and (14), and the Nash solution (16) continue to hold as before. So, the price equation becomes:

$$p^s(\varepsilon) - \bar{\tau}_v^s = \theta\left[H^s(\varepsilon) - \bar{\tau}_v^s - \bar{\tau}_b^s\right] + (1-\theta)\,\frac{u}{1-\beta}, \tag{42}$$

[58] Repair costs (both for the seller who's trying to make the house more attractive and for the buyer who wants to adapt it before moving in) may be smaller in the summer because good weather and the opportunity cost of time (assuming vacation is taken in the summer) are important inputs in construction). Moving costs, similarly, might be lower during vacation (because of both job and school holidays).

which states that the net price received by a seller is a weighted average of housing value net of total transaction costs and the present discounted value of the flow value u. And the reservation equation becomes:

$$\varepsilon^s =: H^s(\varepsilon^s) - \left(\bar{\tau}_b^j + \bar{\tau}_v^j\right) = \beta\left(B^w + V^w\right) + u. \tag{43}$$

The extent of seasonality in transactions depends only on total costs $(\bar{\tau}_b^j + \bar{\tau}_v^j)$ while the extent of seasonality in prices depends on the distribution of costs between buyers and sellers. One interesting result is that higher transactional costs in the winter do not always result in lower winter house prices. Indeed, if most of the transaction costs fall on the seller ($\bar{\tau}_v^j$ is high relative to $\bar{\tau}_b^j$), prices could actually be higher in the winter for θ sufficiently high. On the other hand, if most of the transaction costs are borne by the buyer, then seasonal transaction costs could potentially be the driver of seasonality in vacancies (and hence transactions and prices). As said, our theoretical results on seasonality in prices and transactions follow from $v^s > v^w$, independently of the particular trigger (that is, independently of whether it is seasonal transaction costs for the buyer or seasonal moving shocks; empirically, they are observationally equivalent, as they both lead to seasonality in vacancies, which we match in the quantitative exercise[59]).

Third, the model presented so far assumed observable match-quality. In Section F of this Appendix we derive the case in which the seller cannot observe the match quality ε. We model the seller's power θ in this case as the probability that the seller makes a take-it-or-leave-it offer; $1-\theta$ is then the probability that the buyer makes a take-it-or-leave-it offer upon meeting.[60] In that setting, $\theta = 1$ corresponds to the case in which sellers always post prices. When ε is observable, a transaction goes through whenever the total surplus is positive. However, when the seller does not observe ε, a transaction goes through only when the surplus to the buyer is positive. Since the seller does not

[59] Furthermore, empirically, we are unaware of data on direct measures of moving costs or propensities to move, much less so at higher frequency.

[60] Samuelson (1984) shows that in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all.

observe ε, the seller offers a price that is independent of the level of ε, which will be too high for some buyers whose $\varepsilon's$ are not sufficiently high (but whose ε would have resulted in a transaction if ε were observable to the seller). Therefore, because of the asymmetric information, the match is privately efficient only when the buyer is making a price offer. We show that our results continue to hold; the only qualitative difference is that the extent of seasonality in transactions is now decreasing in θ. This is because when ε is unobservable there is a second channel affecting a seller's surplus and hence the seasonality of reservation quality, which is opposite to the effects from the seasonality of outside option described above: When the seller is making a price offer, the surplus of the seller is higher in the hot season and hence sellers are more demanding and less willing to transact, which reduces the seasonality of transactions; the higher the seller's power, θ, the more demanding they are and the lower is the seasonality in transaction.

E Derivation for the model with observable value

E.1 Solving for prices

To derive $p^s(\varepsilon)$ in (17), use the Nash solution (16),

$$[p^s(\varepsilon) - \beta V^w - u](1-\theta) = [H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w]\theta,$$

so

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u. \tag{44}$$

Using the value functions (14) and (15),

$$(1-\theta)V^s - \theta B^s = \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u$$

solving out explicitly,

$$(1-\theta)V^s - \theta B^s = \frac{(1-\theta)u}{1-\beta}$$

substitute into (44) to obtain (17).

E.2 The model without seasons

The value functions for the model without seasons are identical to those in the model with seasonality without the superscripts s and w. It can be shown that the equilibrium equations are also identical by simply setting $\phi^s = \phi^w$. Using (20), (7) and (18) to express the average price as:

$$P^s = \frac{u}{1-\beta} + \theta \left[\frac{\beta(1+\beta\phi^s)h^w(\varepsilon^w) + \left(1-\beta^2 F^s(\varepsilon^s)\right)(1+\beta\phi^w)E\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right]}{\left(1-\beta^2\right)\left(1-\beta^2\phi^w\phi^s\right)} \right], \tag{45}$$

Using (5),

$$\frac{\varepsilon}{1-\beta\phi} = u + \frac{\beta\phi}{1-\beta\phi}(1-\beta)(V+B)$$

and $B+V$ from (7),

$$B+V = \frac{u}{1-\beta} + \frac{1}{1-\beta^2}\left\{ \frac{1-F}{1-\beta\phi} E\left[\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right] + \beta \frac{1-F}{1-\beta\phi} E\left[\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right] \right\}$$

which reduces to:

$$B+V = \frac{u}{1-\beta} + \frac{1-F(\varepsilon)}{(1-\beta)(1-\beta\phi)} E\left(\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right).$$

It follows that

$$\varepsilon = u + \frac{\beta\phi}{1-\beta\phi}\left[1 - F(\varepsilon)\right] E\left(\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right),$$

and the law of motion for vacancy implies:

$$v = \frac{1-\phi}{1-\phi F(\varepsilon)}.$$

E.3 Analytical derivations of the planner's solution

The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The interesting comparison is the level of reservation quality achieved by the planner with the corresponding level in the decentralized economy. To spell out the planner's problem, we follow Pissarides (2000) and assume that in any period t the planner takes as given the expected value of the housing utility service per person in period t (before he optimizes), which we denote by q_{t-1}, as well as the beginning of period's stock of vacancies, v_t. Thus, taking as given the initial levels q_{-1} and v_0, and the sequence $\{\phi_t\}_{t=0\ldots}$, which alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem is to choose a sequence of $\{\varepsilon_t\}_{t=0,..}$ to maximize

$$U\left(\{\varepsilon_t, q_t, v_t\}_{t=0\ldots}\right) \equiv \sum_{t=0}^{\infty} \beta^t \left[q_t + uv_t F\left(\varepsilon_t; v_t\right)\right] \tag{46}$$

subject to the law of motion for q_t :

$$q_t = \phi_t q_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF\left(x; v_t\right), \tag{47}$$

the law of motion for v_t (which is similar to the one in the decentralized economy):

$$v_{t+1} = v_t \phi_{t+1} F\left(\varepsilon_t; v_t\right) + 1 - \phi_{t+1}, \tag{48}$$

and the inequality constraint:

$$0 \leqslant \varepsilon_t \leqslant \bar{\varepsilon}(v_t), \tag{49}$$

where the upper bound $\bar{\varepsilon}$ can potentially be infinite.

The planner faces two types of trade-offs when deciding the optimal reservation quality ε_t: A static one and a dynamic one. The static trade-off stems from the comparison of utility values generated by occupied houses and vacancies in period t in the objective function (46). The utility per person generated from vacancies is the rental income per person, captured by $uv_tF(\varepsilon_t)$. The utility generated by occupied houses in period t is captured by q_t, the expected housing utility service per person conditional on the reservation value ε_t set by the planner in period t. The utility q_t, which follows the law of motion (47), is the sum of the pre-existing expected housing utility q_{t-1} that survives the moving shock and the expected housing utility from the new matches. By increasing ε_t, the expected housing value q_t decreases, while the utility generated by vacancies increases (since $F(\varepsilon_t)$ increases). The dynamic trade-off operates through the law of motion for the stock of vacancies in (48). By increasing ε_t (which in turn decreases q_t), the number of transactions in the current period decreases; this leads to more vacancies in the following period, v_{t+1}, and consequently to a thicker market in the next period. We first derive the case where the inequality constraints are not binding, i.e. markets are open in both the cold and hot seasons.

The Planner's solution when the housing market is open in all seasons

Because the sequence $\{\phi_t\}_{t=0,\ldots}$ alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem can be written recursively. Taking (q_{t-1}, v_t), and $\{\phi_t\}_{t=0,..}$ as given, and provided that the solution is interior, that is, $\varepsilon_t < v_t$, the Bellman equation for the planner is given by:

$$
\begin{aligned}
W\left(q_{t-1}, v_t, \phi_t\right) &= \max_{\varepsilon_t}\left[q_t + u v_t F\left(\varepsilon_t; v_t\right) + \beta W\left(q_t, v_{t+1}, \phi_{t+1}\right)\right] \\
s.t. \qquad & q_t = \phi_t q_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF\left(x; v_t\right), \\
v_{t+1} &= v_t \phi_{t+1} F\left(\varepsilon_t; v_t\right) + 1 - \phi_{t+1}.
\end{aligned} \tag{50}
$$

The first-order condition implies

$$
\left(1 + \beta \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial q_t}\right) v_t\left(-\varepsilon_t f\left(\varepsilon_t; v_t\right)\right) + \left(\beta \phi_{t+1} \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial v_{t+1}} + u\right) v_t f\left(\varepsilon_t; v_t\right) = 0,
$$

which simplifies to

$$
\varepsilon_t\left(1 + \beta \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial q_t}\right) = u + \beta \phi_{t+1} \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial v_{t+1}}. \tag{51}
$$

Using the envelope-theorem conditions, we obtain:

$$
\frac{\partial W\left(q_{t-1}, v_t, \phi_t\right)}{\partial q_{t-1}} = \phi_t\left(1 + \beta \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial q_t}\right) \tag{52}
$$

and

$$
\begin{aligned}
\frac{\partial W\left(q_{t-1}, v_t, \phi_t\right)}{\partial v_t} &= \left(u + \beta \phi_{t+1} \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial v_{t+1}}\right)\left(F\left(\varepsilon_t; v_t\right) - v_t T_{1t}\right) \\
&\quad + \left(1 + \beta \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial q_t}\right)\left(\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF\left(x; v_t\right) + v_t T_{2t}\right)
\end{aligned} \tag{53}
$$

where $T_{1t} \equiv \frac{\partial}{\partial v_t}\left[1 - F\left(\varepsilon_t; v_t\right)\right] > 0$ and $T_{2t} \equiv \frac{\partial}{\partial v_t}\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF\left(x; v_t\right) > 0$. In the periodic steady state, the first order condition (51) becomes

$$\varepsilon^j \left(1 + \beta \frac{\partial W^{j'}\left(q^j, v^{j'}\right)}{\partial q^j}\right) = u + \beta \phi^{j'} \frac{\partial W^{j'}\left(q^j, v^{j'}\right)}{\partial v^{j'}} \tag{54}$$

The envelope condition (52) implies

$$\frac{\partial W^j\left(q^{j'}, v^j\right)}{\partial q^{j'}} = \phi^j \left[1 + \beta\left(\phi^{j'} + \beta \phi^{j'} \frac{\partial W^j\left(q^{j'}, v^j\right)}{\partial q^{j'}}\right)\right]$$

which yields:

$$\frac{\partial W^j\left(q^{j'}, v^j\right)}{\partial q^{j'}} = \frac{\phi^j\left(1 + \beta \phi^{j'}\right)}{1 - \beta^2 \phi^j \phi^{j'}} \tag{55}$$

Substituting this last expression into (53), we obtain:

$$\frac{\partial W^j\left(q^{j'}, v^j\right)}{\partial v^j} = \left(u + \beta \phi^{j'} \frac{\partial W^{j'}\left(q^j, v^{j'}\right)}{\partial v^{j'}}\right) A^j + D^j,$$

where

$$A^j \equiv F^j\left(\varepsilon^j\right) - v^j T_1^j; \qquad D^j \equiv \frac{1 + \beta \phi^{j'}}{1 - \beta^2 \phi^j \phi^{j'}} \left(\int_{\varepsilon^j}^{\bar{\varepsilon}^j} x dF^j(x) + v^j T_2^j\right), \tag{56}$$

Hence, we have

$$\frac{\partial W^j\left(q^{j'}, v^j\right)}{\partial v^j} = \left\{u + \beta \phi^{j'}\left[\left(u + \beta \phi^j \frac{\partial W^j\left(q^{j'}, v^j\right)}{\partial v^j}\right) A^{j'} + D^{j'}\right]\right\} A^j + D^j,$$

which implies

$$\frac{\partial W^j\left(q^{j'}, v^j\right)}{\partial v^j} = \frac{u A^j\left(1 + \beta \phi^{j'} A^{j'}\right) + \beta \phi^{j'} D^{j'} A^j + D^j}{1 - \beta^2 \phi^j \phi^{j'} A^j A^{j'}}. \tag{57}$$

Substituting (55) and (57) into the first-order condition (54),

$$\varepsilon^j\left(1+\beta\frac{\phi^{j'}\left(1+\beta\phi^j\right)}{1-\beta^2\phi^j\phi^{j'}}\right)=u+\beta\phi^{j'}\frac{uA^{j'}\left(1+\beta\phi^jA^j\right)+\beta\phi^jD^jA^{j'}+D^{j'}}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}}$$

simplify to:

$$\varepsilon^j\left(\frac{1+\beta\phi^{j'}}{1-\beta^2\phi^j\phi^{j'}}\right)=\frac{\left(1+\beta\phi^{j'}A^{j'}\right)u+\beta^2\phi^j\phi^{j'}A^{j'}D^j+\beta\phi^{j'}D^{j'}}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}},\tag{58}$$

and the stock of vacancies, v^j, $j=s,w$, satisfies (8) as in the decentralized economy.

The thick-market effect enters through two terms: $T_1^j\equiv\frac{\partial}{\partial v^j}\left[1-F^j\left(\varepsilon^j\right)\right]>0$ and $T_2^j\equiv\frac{\partial}{\partial v^j}\int_{\varepsilon^j}^{\bar{\varepsilon}^j}xdF^j\left(x\right)>0$. The first term, T_1^j, indicates that the thick-market effect shifts up the acceptance schedule $\left[1-F^j\left(\varepsilon\right)\right]$. The second term, T_2^j, indicates that the thick-market effect increases the conditional quality of transactions. The interior solution (58) is an implicit function of ε^j that depends on $\varepsilon^{j'}$, v^j, and $v^{j'}$. It is not straightforward to derive an explicit condition for $\varepsilon^j<v^j$, $j=s,w$. Abstracting from seasonality for the moment, i.e. when $\phi^s=\phi^w$, it follows immediately from (8) that the solution is interior, $\varepsilon<v$. Moreover (58) implies the planner's optimal reservation quality ε^p satisfies:

$$\frac{\varepsilon^p}{1-\beta\phi}=\frac{u+\frac{\beta\phi}{1-\beta\phi}\left(\int_{\varepsilon^p}^{\bar{\varepsilon}}xdF\left(x\right)+vT_2\right)}{1-\beta\phi F\left(\varepsilon^p\right)+\beta\phi vT_1}.\tag{59}$$

Comparing (59) with (23), the thick-market effect, captured by T_1 and T_2, generates two opposite forces. The term T_1 decreases ε^p, while the term T_2 increases ε^p in the planner's solution. Thus, the positive thick-market effect on the acceptance rate T_1 implies that the number of transactions is too low in the decentralized economy, while the positive effect on quality T_2 implies that the number of transactions is too high. Since $1-\beta\phi$ is close to zero, however, the term T_2 dominates. Therefore, the overall effect of the thick-market externality is to increase the number of transactions in the decentralized economy relative to the efficient outcome. As discussed in the text, comparing the extent in seasonality in the decentralized equilibrium to the planner's solution depends on the exact distribution $F(\varepsilon, v)$. We next derive the case in which the Planner finds it optimal to close down the market in the cold season.

The Planner's solution when the housing market is closed in the cold season

Setting $\varepsilon_t^w = \bar{\varepsilon}_t^w$, the Bellman equation (50) can be rewritten as:

$$
\begin{aligned}
W^s\left(q_{t-1}^w, v_t^s\right) &= \max_{\varepsilon_t^s} \left[\begin{array}{c} \phi^s q_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + u v_t^s F_t^s\left(\varepsilon_t^s\right) \\ +\beta\left(q_{t+1}^w + u\left[v_t^s \phi^w F_t^s\left(\varepsilon_t^s\right) + 1 - \phi^w\right]\right) \\ +\beta^2 W^s\left(q_{t+1}^w, v_{t+2}^s\right) \end{array} \right] \\
& s.t. \\
q_{t+1}^w &= \phi^w \left[\phi^s q_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x)\right], \\
v_{t+2}^s &= \phi^s\left[v_t^s \phi^w F_t^s\left(\varepsilon_t^s\right) + 1 - \phi^w\right] + 1 - \phi^s.
\end{aligned} \tag{60}
$$

Intuitively, "a period" for the decision of ε_t^s is equal to $2t$. The state variables for the current period are given by the vector $\left(q_{t-1}^w, v_t^s\right)$, the state variables for next period are $\left(q_{t+1}^w, v_{t+2}^s\right)$, and the control variable is ε_t^s. The first order condition is:

$$
\begin{aligned}
0 &= v_t^s\left(-\varepsilon_t^s f_t^s\left(\varepsilon_t^s\right)\right) + u v_t^s f_t^s\left(\varepsilon_t^s\right) \\
&\quad +\beta\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s\left(\varepsilon_t^s\right)\right) + u v_t^s \phi^w f_t^s\left(\varepsilon_t^s\right)\right) \\
&\quad +\beta^2\left[\frac{\partial W^s}{\partial q_{t+1}^w}\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s\left(\varepsilon_t^s\right)\right)\right) + \frac{\partial W^s}{\partial v_{t+2}^s}\left(\phi^s v_t^s \phi^w f_t^s\left(\varepsilon_t^s\right)\right)\right],
\end{aligned}
$$

which simplifies to:

$$
\begin{aligned}
0 &= -\varepsilon_t^s + u + \beta\left(-\phi^w \varepsilon_t^s + u\phi^w\right) \\
&\quad +\beta^2\left[\frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\left(-\phi^w \varepsilon_t^s\right) + \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\phi^s \phi^w\right]
\end{aligned}
$$

and can be written as:

$$\varepsilon_t^s \left[1 + \beta\phi^w + \beta^2 \phi^w \frac{\partial W^s \left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\right] = (1 + \beta\phi^w) u + \beta^2 \phi^w \phi^s \frac{\partial W^s \left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s} \tag{61}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W^s \left(q_{t-1}^w, v_t^s\right)}{\partial q_{t-1}^w} = \phi^s + \beta\phi^w\phi^s + \beta^2 \phi^w \phi^s \frac{\partial W^s \left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}, \tag{62}$$

and

$$\begin{aligned} & \frac{\partial W^s \left(q_{t-1}^w, v_t^s\right)}{\partial v_t^s} \\ = \; & (1 + \beta\phi^w) \left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s (x) + v_t^s T_{2t}^s \right) + (1 + \beta\phi^w) u \left[F_t^s (\varepsilon_t^s) - v_t^s T_{1t}^s\right] \\ & + \beta^2 \frac{\partial W^s \left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w} \phi^w \left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s (x) + v_t^s T_{2t}^s \right) \\ & + \beta^2 \frac{\partial W^s \left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s} \phi^s \phi^w \left[F_t^s (\varepsilon_t^s) - v_t^s T_{1t}^s\right], \end{aligned}$$

where $T_{1t}^s \equiv \frac{\partial}{\partial v_t^s} \left[1 - F_t^s (\varepsilon^s)\right] > 0$ and $T_{2t}^s \equiv \frac{\partial}{\partial v_t^s} \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s (x) > 0$. Rewrite the last expression as:

$$\begin{aligned} & \frac{\partial W^s \left(q_{t-1}^w, v_t^s\right)}{\partial v_t^s} \\ = \; & \left(1 + \beta\phi^w + \beta^2 \phi^w \frac{\partial W^s \left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\right) \left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s (x) + v_t^s T_{2t}^s \right) \\ & + \left((1 + \beta\phi^w) u + \beta^2 \phi^s \phi^w \frac{\partial W^s \left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s} \right) \left[F_t^s (\varepsilon_t^s) - v_t^s T_{1t}^s\right] \end{aligned} \tag{63}$$

In steady state, (62) and (63) become

$$\frac{\partial W^s (q^w, v^s)}{\partial q^w} = \frac{\phi^s (1 + \beta\phi^w)}{1 - \beta^2 \phi^w \phi^s}, \tag{64}$$

and

$$
\begin{aligned}
& \frac{\partial W^s(q^w, v^s)}{\partial v^s}\left(1-\beta^2\phi^s\phi^w\left[F^s(\varepsilon^s)-v^sT_1^s\right]\right) \\
= & \left(1+\beta\phi^w+\beta^2\phi^w\frac{\phi^s(1+\beta\phi^w)}{1-\beta^2\phi^w\phi^s}\right)\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s(x)+v^sT_2^s\right) \\
& +(1+\beta\phi^w)\,u\left[F^s(\varepsilon^s)-v^sT_1^s\right].
\end{aligned}
\tag{65}
$$

Substituting into the FOC (61),

$$
\begin{aligned}
& \varepsilon^s\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s} \\
= & (1+\beta\phi^w)\,u+\beta^2\phi^w\phi^s\frac{(1+\beta\phi^w)\,u\left[F^s(\varepsilon^s)-v^sT_1^s\right]+\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s(x)+v^sT_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s(\varepsilon^s)-v^sT_1^s\right]}
\end{aligned}
$$

which simplifies to

$$
\frac{\varepsilon^s}{1-\beta^2\phi^w\phi^s}=\frac{u+\frac{\beta^2\phi^w\phi^s}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s(x)+v^sT_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s(\varepsilon^s)-v^sT_1^s\right]}, \tag{66}
$$

which is similar to the Planner's solution with no seasons in (59), with $\beta^2\phi^w\phi^s$ replacing $\beta\phi$.

F Model with unobservable match quality

Assume that the seller does not observe ε. As shown by Samuelson (1984), in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all. Hence, we adopt a simple price-setting mechanism: The seller makes a take-it-or-leave-it offer p^{jv} with probability $\theta \in [0,1]$ and the buyer makes a take-it-or-leave-it offer p^{jb} with probability $1-\theta$. ($\theta = 1$ corresponds to the case in which sellers post prices.) Broadly speaking, we can interpret θ as the "bargaining power" of the seller. The setup of the model implies that the buyer accepts any offer

p^{sv} if $H^s(\varepsilon) - p^{sv} \geqslant \beta B^w$; and the seller accepts any price $p^{sb} \geqslant \beta V^w + u$. Let S_v^{si} and $S_b^{si}(\varepsilon)$ be the surplus of a transaction to the seller and the buyer when the match quality is ε and the price is p^{si}, for $i = b, v$:

$$\begin{aligned} S_v^{si} &\equiv p^{si} - (u + \beta V^w), && (67) \\ S_b^{si}(\varepsilon) &\equiv H^s(\varepsilon) - p^{si} - \beta B^w. && (68) \end{aligned}$$

Note that the definition of S_v^{si} implies that

$$p^{sv} = S_v^{sv} + p^{sb} \tag{69}$$

i.e. the price is higher when the seller is making an offer. Since only the buyer observes ε, a transaction goes through only if $S_b^{si}(\varepsilon) \geqslant 0$, $i = b, v$, i.e. a transaction goes through only if the surplus to the buyer is non-negative regardless of who is making an offer. Given $H^s(\varepsilon)$ is increasing in ε, for any price p^{si}, $i = b, v$, a transaction goes through if $\varepsilon \geqslant \varepsilon^{si}$, where

$$H^s\left(\varepsilon^{si}\right) - p^{si} = \beta B^w. \tag{70}$$

$1 - F^s(\varepsilon^{si})$ is thus the probability that a transaction is carried out. From (2), the response of the reservation quality ε^{si} to a change in price is given by:

$$\frac{\partial \varepsilon^{si}}{\partial p^{si}} = \frac{1 - \beta^2 \phi^w \phi^s}{1 + \beta \phi^w}. \tag{71}$$

Moreover, by the definition of $S_b^{si}(\varepsilon)$ and ε^{si}, in equilibrium, the surplus to the buyer is:

$$S_b^{si}(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1 + \beta\phi^w}{1 - \beta^2\phi^w\phi^s}\left(\varepsilon - \varepsilon^{si}\right). \tag{72}$$

F.1 The Seller's offer

Taking the reservation policy ε^{sv} of the buyer as given, the seller chooses a price to maximize the expected surplus value of a sale:

$$\max_{p} \{[1 - F^{s}(\varepsilon^{sv})][p - \beta V^{w} - u]\}$$

The optimal price p^{sv} solves

$$[1 - F^{s}(\varepsilon^{sv})] - [p - \beta V^{w} - u] f^{s}(\varepsilon^{sv}) \frac{\partial \varepsilon^{sv}}{\partial p^{s}} = 0. \tag{73}$$

Rearranging terms we obtain:

$$\underbrace{\frac{p^{sv} - \beta V^{w} - u}{p^{sv}}}_{\text{mark-up}} = \underbrace{\left[\frac{p^{sv} f^{s}(\varepsilon^{sv}) \frac{\partial \varepsilon^{s}}{\partial p^{s}}}{1 - F^{s}(\varepsilon^{sv})}\right]^{-1}}_{\text{inverse-elasticity}},$$

which makes clear that the price-setting problem of the seller is similar to that of a monopolist who sets a markup equal to the inverse of the elasticity of demand (where demand in this case is given by the probability of a sale, $1 - F^{s}(\varepsilon^{s})$). The optimal decisions of the buyer (71) and the seller (73) together imply:

$$S_{v}^{sv} = \frac{1 - F^{s}(\varepsilon^{sv})}{f^{s}(\varepsilon^{sv})} \frac{1 + \beta \phi^{w}}{1 - \beta^{2} \phi^{w} \phi^{s}}. \tag{74}$$

Equation (74) says that the surplus to a seller generated by the transaction is higher when $\frac{1-F^{s}(\varepsilon^{sv})}{f^{s}(\varepsilon^{sv})}$ is higher, i.e. when the conditional probability that a successful transaction is of match quality ε^{sv} is lower. Intuitively, the surplus of a transaction to a seller is higher when the house is transacted with a stochastically higher match quality, or loosely speaking, when the distribution of match quality has a "thicker" tail.

Given the price-setting mechanism, in equilibrium, the value of a vacancies to its seller is:

$$V^s = u + \beta V^w + \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}. \tag{75}$$

Solving out V^s explicitly,

$$V^s = \frac{u}{1-\beta} + \theta \frac{\left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}, \tag{76}$$

which is the sum of the present discounted value of the flow value u and the surplus terms when its seller is making the take-it-or-leave-it offer, which happens with probability θ. Using the definition of the surplus terms, the equilibrium p^{sv} is:

$$p^{sv} = \frac{u}{1-\beta} + \theta \frac{\left[1 - \beta^2 F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}. \tag{77}$$

F.2 The Buyer's Offer

The buyer offers a price that extracts all the surplus from the seller, i.e.

$$S_v^{sb} = 0 \Leftrightarrow p^{sb} = u + \beta V^w$$

Using the value function V^w from (76), the price offered by the buyer is:

$$p^{sb} = \frac{u}{1-\beta} + \theta \frac{\beta^2 \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}. \tag{78}$$

The buyer's value function is:

$$
\begin{aligned}
B^s &= \beta B^w + \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S_b^{sv}\left(\varepsilon\right) \mid \varepsilon \geqslant \varepsilon^{sv}\right] \\
&\quad + \left(1-\theta\right)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S_b^{sb}\left(\varepsilon\right) \mid \varepsilon \geqslant \varepsilon^{sb}\right],
\end{aligned} \tag{79}
$$

where $E^s[.]$ indicates the expectation taken with respect to the distribution $F^s(.)$. Since the seller does not observe ε, the expected surplus to the buyer is positive even when the seller is making the offer (which happens with probability θ). As said, buyers receive zero housing service flow until they find a successful match. Solving out B^s explicitly,

$$
\begin{aligned}
B^s &= \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S_b^{sv}\left(\varepsilon\right) \mid \varepsilon \geqslant \varepsilon^{sv}\right] + \left(1-\theta\right)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S_b^{sb}\left(\varepsilon\right) \mid \varepsilon \geqslant \varepsilon^{sb}\right] \\
&\quad + \beta\left\{\theta\left(1 - F^w\left(\varepsilon^{sv}\right)\right) E^w\left[S_b^{wv}\left(\varepsilon\right) \mid \varepsilon \geqslant \varepsilon^{wv}\right] + \left(1-\theta\right)\left[1 - F^w\left(\varepsilon^{sb}\right)\right] E^w\left[S_b^{wb}\left(\varepsilon\right) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\}.
\end{aligned} \tag{80}
$$

F.3 Reservation quality

In any season s, the reservation quality ε^{si}, for $i = v, b$, satisfies

$$
H^s\left(\varepsilon^{si}\right) = S_v^{si} + u + V^w + \beta B^w, \tag{81}
$$

which equates the housing value of a marginal owner in season s, $H^s\left(\varepsilon^s\right)$, to the sum of the surplus generated to the seller (S_v^{si}), plus the sum of outside options for the buyer (βB^w) and the seller ($\beta V^w + u$). Using (2), ε^{si} solves:

$$
\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^{si} = S_v^{si} + u + \frac{\beta\phi^w\left(1-\beta^2\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(B^w + V^w\right) - \frac{\beta^2\phi^w\left(1-\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^s + B^s\right). \tag{82}
$$

The reservation quality ε^s depends on the sum of the outside options for buyers and sellers in both seasons, which can be derived from (76) and (80):

$$\begin{aligned} & B^s + V^s && (83) \\ = \; & \frac{u}{1-\beta} + \\ & \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right] + \\ & \beta\left\{\theta\left(1 - F^w\left(\varepsilon^{sv}\right)\right) E^w\left[S^{wv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wv}\right] + (1-\theta)\left[1 - F^w\left(\varepsilon^{sb}\right)\right] E^w\left[S^{wb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\}, \end{aligned}$$

where $S^{si}(\varepsilon) \equiv S_b^{si}(\varepsilon) + S_v^{si}$ is the total surplus from a transaction with match quality ε. Note from (82) that the reservation quality is lower when the buyer is making a price offer: $\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\varepsilon^{sv} - \varepsilon^{sb}\right) = S_v^{sv}$. Also, because of the asymmetric information, the match is privately efficient when the buyer is making a price offer.

The thick-and-thin market equilibrium through the distribution F^j affects the equilibrium prices and reservation qualities $\left(p^{jv}, p^{jb}, \varepsilon^{jv}, \varepsilon^{jb}\right)$ in season $j = s, w$ through two channels, as shown in (77), (78), and(82)): the conditional density of the distribution at reservation ε^{jv}, i.e. $\frac{f^j\left(\varepsilon^{jv}\right)}{1-F^j\left(\varepsilon^{jv}\right)}$, and the expected surplus quality above reservation ε^{jv}, i.e. $\left(1 - F^j\left(\varepsilon^{ji}\right)\right) E^j\left[\varepsilon - \varepsilon^{ji} \mid \varepsilon \geqslant \varepsilon^{ji}\right]$, $i = b, v$. As shown in (74), a lower conditional probability that a transaction is of marginal quality ε^{jv} implies higher expected surplus to the seller S_v^{jv}, which increases the equilibrium prices p^{jv} and p^{jb} in (77) and (78). Similarly as shown in (72) and the assumption of first order stochastic dominance, using integration by parts, expected surplus to the buyer $\left(1 - F^j\left(\varepsilon^{ji}\right)\right) E^s\left[S_b^{si}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{si}\right]$, $i = b, v$ is higher in the hot season with higher vacancies. These two channels affect V^j and B^j in (76) and (80), and as a result affect the reservation qualities ε^{jv} and ε^{jb} in (5).

F.4 Stock of vacancies

In any season s, the average probability that a transaction goes through is $\left\{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+(1-\theta)\left[1-F^s\left(\varepsilon^{sb}\right)\right]\right\}$, and the average probability that a transaction does not through is $\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}$. Hence, the law of motion for the stock of vacancies (and for the stock of buyers) is

$$\begin{aligned} v^s &= (1-\phi^s)\left\{v^w\left[\theta\left(1-F^w\left(\varepsilon^{wv}\right)\right)+(1-\theta)\left(1-F^w\left(\varepsilon^{wb}\right)\right)\right]+1-v^w\right\} \\ &\quad +v^w\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}, \end{aligned}$$

where the first term includes houses that received a moving shock this season and the second term comprises vacancies from last period that did not find a buyer. The expression simplifies to

$$v^s = v^w\phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}+1-\phi^s, \tag{84}$$

that is, in equilibrium v^s depends on the equilibrium reservation quality $\left(\varepsilon^{wv},\varepsilon^{wb}\right)$ and on the distribution $F^w(.)$.

An equilibrium is a vector $\left(p^{sv},p^{sb},p^{wv},p^{wb},B^s+V^s,B^w+V^w,\varepsilon^{sv},\varepsilon^{sb},\varepsilon^{wv},\varepsilon^{wb},v^s,v^w\right)$ that jointly satisfies equations (77),(80),(82), (83) and (84), with the surpluses S_v^j and $S_b^j(\varepsilon)$ for $j=s,w$, derived as in (74), and (72). Using (84), the stock of vacancies in season s is given by:

$$v^s = \frac{(1-\phi^w)\phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}+1-\phi^s}{1-\phi^w\phi^s\left\{\theta F^s\left(\varepsilon^{sv}\right)+(1-\theta)F^s\left(\varepsilon^{sb}\right)\right\}\left\{\theta F^w\left(\varepsilon^{wv}\right)+(1-\theta)F^w\left(\varepsilon^{wb}\right)\right\}}. \tag{85}$$

Given $1-\phi^s > 1-\phi^w$, as in the observable case, it follows that, in equilibrium $v^s > v^w$.

F.5 Seasonality in Prices

Let

$$p^{s} \equiv \frac{\theta\left[1-F^{s}\left(\varepsilon^{sv}\right)\right] p^{sv}+(1-\theta) p^{sb}}{\theta\left[1-F^{s}\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

be the average price observed in season s. Given $p^{sv}=S_{v}^{sv}+p^{sb}$, we can rewrite it as

$$p^{s}=p^{sb}+\frac{\theta\left[1-F^{s}\left(\varepsilon^{sv}\right)\right] S_{v}^{sv}}{\theta\left[1-F^{s}\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

using (78)

$$\begin{aligned} p^{s} &= \frac{u}{1-\beta}+\theta \frac{\beta^{2}\left[1-F^{s}\left(\varepsilon^{sv}\right)\right] S_{v}^{sv}+\beta\left[1-F^{w}\left(\varepsilon^{wv}\right)\right] S_{v}^{wv}}{1-\beta^{2}}+\frac{\theta\left[1-F^{s}\left(\varepsilon^{sv}\right)\right] S_{v}^{sv}}{1-\theta F^{s}\left(\varepsilon^{sv}\right)} \\ &= \frac{u}{1-\beta}+\theta\left(\frac{\left[1-\theta F^{s}\left(\varepsilon^{sv}\right)\right] \beta^{2}+1-\beta^{2}}{\left[1-\theta F^{s}\left(\varepsilon^{sv}\right)\right]\left(1-\beta^{2}\right)}\right)\left[1-F^{s}\left(\varepsilon^{sv}\right)\right] S_{v}^{sv}+\frac{\theta \beta\left[1-F^{w}\left(\varepsilon^{wv}\right)\right] S_{v}^{wv}}{1-\beta^{2}} \end{aligned}$$

we obtain,

$$p^{s}=\frac{u}{1-\beta}+\theta\left\{\frac{\left[1-\theta \beta^{2} F^{s}\left(\varepsilon^{sv}\right)\right]\left[1-F^{s}\left(\varepsilon^{sv}\right)\right] S_{v}^{sv}}{\left[1-\theta F^{s}\left(\varepsilon^{sv}\right)\right]\left(1-\beta^{2}\right)}+\frac{\beta\left[1-F^{w}\left(\varepsilon^{wv}\right)\right] S_{v}^{wv}}{1-\beta^{2}}\right\}. \tag{86}$$

Since the flow u is a-seasonal, house prices are seasonal if $\theta > 0$ and the surplus to the seller is seasonal. As in the case with observable match quality, when sellers have some "market power" ($\theta > 0$), prices are seasonal. The extent of seasonality is increasing in the seller's market power θ. To see this, note that the equilibrium price is the discounted sum of the flow value (u) plus a positive surplus from the sale. The surplus S_v^{sv}, as shown in (74), is seasonal. Given $v^s > v^w$, Assumption 2 implies hazard rate ordering, i.e. $\frac{f^w(x)}{1-F^w(x)} > \frac{f^s(x)}{1-F^s(x)}$ for any cutoff x, i.e. the thick-market effect lowers the conditional probability that a successful transaction is of the marginal quality ε^{sv} in the hot season, that is, it implies a "thicker" tail in quality in the hot season. In words, the quality of matches goes up in the summer and hence buyers' willingness to pay increases; sellers can then extract a higher surplus in the summer; thus, $S_v^{sv} > S_v^{wv}$. As in the case with observable ε, there is an equilibrium effect through the seasonality of cutoffs. As shown in (82), the equilibrium cutoff ε^{sv} depends on the surplus to the seller (S_v^{sv}) and on the sum of the seller's and the buyer's outside options, while the equilibrium cutoff ε^{sb} depends only on the sum of the outside options. The seasonality in outside options tends to reduce $\varepsilon^{si}/\varepsilon^{wi}$ for $i = b, v$. This is because the outside option in the hot season s is determined by the sum of values in the winter season: $B^w + V^w$, which is lower than in the summer. However, the seasonality in the surplus term, $S_v^{sv} > S_v^{wv}$ (shown before), tends to increase $\varepsilon^{sv}/\varepsilon^{wv}$ (the marginal house has to be of higher quality in order to generate a bigger surplus to the seller). Because of these two opposing forces, the equilibrium effect is likely to be small (even smaller than in the observable case.)

Given that θ affects S_v^{sv} only through the equilibrium vacancies and reservation qualities, it follows that the extent of seasonality in price is increasing in θ.

F.6 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s \left[\theta \left(1 - F^w \left(\varepsilon^{wv}\right)\right) + (1 - \theta) \left(1 - F^w \left(\varepsilon^{wb}\right)\right)\right]. \tag{87}$$

(An isomorphic expression holds for Q^w). As in the observables case, seasonality in transactions stems from three sources. First, the direct effect from a larger stock of vacancies in the summer, $v^s > v^w$. Second the amplification through the thick-market effects that shifts up the probability of a transaction. Third, there is an equilibrium effect through cutoffs. As pointed out before, this last effect is small. As in the case with observable ε, most of the amplification stems from the thick-market effect. What is new when ε is unobservable is that the extent of seasonality in transactions is decreasing in the seller's market market power θ. This is because higher θ leads to higher surplus in the summer relative to winter, S_v^{sv}/S_v^{wv}, which in turn increases $\varepsilon^{sv}/\varepsilon^{wv}$ and hence decreases Q^s/Q^w); the higher is θ, the stronger is this effect (it disappear when $\theta = 0$).

G Model's Additional Statistics

G.1 Time-on-market and Transaction Probabilities

For the baseline seasonal model with $\frac{1-\phi^s}{1-\phi^w} = 1.25$, for the U.S., the steady state transaction probabilities are $1 - F^s(\varepsilon^s) = 0.31$ in the summer and $1 - F^w(\varepsilon^w) = 0.25$ in the winter. The transaction probabilities are seasonal, and indeed the source of the amplification mechanism that makes the volume of transaction more seasonal than the number of houses for sale. Under these probabilities, we can compute the steady state median time-on-market for each season. Let x^s be the number of semester that a house stays on the market if it is put on sale in season s. The distribution for x^s can be computed using Table $G1$.

Table $G1$. Distribution of time-to-sell

Stays exactly x^s semester	pdf of x^s
0	$(1 - F^s)$
1	$F^s(1 - F^w)$
2	$F^s F^w(1 - F^s)$
3	$(F^s)^2 F^w(1 - F^w)$
∞	

Thus given the steady state probabilities, we can derive the distribution of time-to-sell x_s for houses that are put on the market in season $s = s, w$. The numbers are reported in Table $G2$

Table $G2$. Distribution of time-to-sell					
x^s	pdf of x^s	cdf of x^s	x^w	pdf of x^w	cdf of x^w
0	0.31	0.31	0	0.25	0.25
1	0.17	0.48	1	0.23	0.48
2	0.16	0.64	2	0.13	0.61

Thus the median TOM is around 6 months for both seasons, being slightly higher in the winter than summer, this is also the time-to-sell used in Piazzesi and Schneider (2009). Our predicted median time-to-sell is consistent with the median number of months reported in Ungerer (2012) and Diaz and Jerez (2012), where they use the median number of months for newly built and report numbers of 5.2 months for 1974-2011, and 5.7 months for 1960-2012.

(Note that the average TOM in the market in our model is given, correspondingly by $F^s \frac{1+F^w}{1-F^s F^w}$ and $F^w \frac{1+F^s}{1-F^s F^w}$. Given the well known problem with the average TOM reported in the data, we prefer to focus on the median, which is less sensitive to some of these concerns.[61])

G.2 Likelihood of an agent having η houses for sale

A any point in time, we can derive the distribution of houses for sale for a given agent. The support for the distribution is from 0 to infinite. However, given that both v and $(1-\phi)$ are small, the distribution concentrates around 0 or 1 house for sale. To see this in brief, consider the model without season with steady state $v = 0.17$ and transaction probabilities $[1 - F(\varepsilon)] = 0.28$. There are two reasons why the probability that an agent has more than one house for sale is close zero. First, the steady state v is small so very few houses are for sale. Second and more importantly, the probability of a moving

[61] An example illustrating the problems with the average is described in http://www.manausa.com/how-long-does-it-take-to-sell-a-home/#ixzz2MmFiytY9

Suppose the total time on the market for a house is the sum of 1) 30 days "For Sale By Owner"; 2) 180 days with Broker A; 3) 10 days with Broker B. This is a total of 220 days, yet the MLS would report it as "10 days." The average only informs on the average of the final listing periods for those homes. Most problematic, the average does not include the days on the market of houses that failed to sell.

shock is very small because it is set to match the average duration of staying in a house which is 9 years for the U.S., $(1-\phi)=0.056$. More specifically, conditional on having $\eta \geqslant 1$ houses for sale it is highly unlikely that an agent can transit to having $\eta+1$ house for sale. This requires three events: the agent fails to sell, buys a new house but receives a moving shock immediately after; which happens with probability $(1-F(\varepsilon))\,F(\varepsilon)\,(1-\phi)=0.01$. Thus, it is unlikely that agents will have more than one house for sale. The answer for the baseline seasonal model (with $\frac{1-\phi^s}{1-\phi^w}=1.25$) is very similar because the moving probability in both summer and winter are also very small, $(1-\phi^s)=0.062$ and $(1-\phi^w)=0.049$; and steady state $v^s=0.180$ and $v^w=0.167$.

We next provide more details on how one could derive the full distribution for the number of houses for sale. As in the paper, v_t is the measure of houses for sale, $(1-F_t)$ is the transaction probability and $(1-\phi)$ is the probability of a moving shock.

To compute the likelihood of an agent having η houses for sale at any period t, it is useful to divide the population into two broad types: the matched agents (m) and the non-matched agents (n). Within each broad group, agents are also different with regards to the number of houses they have for sale. Thus it is useful to denote the type of an agent at time t as $s_t=(k,i)$ for $k=m,n$ denoting matched or unmatched and $i=0,1,...$ denoting the number of houses owned by the agent.

We next tables describe the probability of the number of houses for sale at the beginning of period $t+1$ for all types of agents $s_t=(k,i)$ in period t. The probability is different across $k=m,n$ and between $i=0$ and any $i>0$. Therefore, there are four tables to report.

Table $G3$ is the distribution of those who are matched to a house and have no house to sell at time t. Tables $G4$, $G5$, and $G6$ show the distributions for the remaining cases. The first column in each table shows the potential number of houses in the next period. The second column shows the corresponding probabilities. The third column explains how the probability is derived, and the last column shows the state in which it transitions to.

Table $G3$: $s_t = (m, 0)$

η_{t+1}	$\Pr(\eta_{t+1} \mid s_t)$	events	s_{t+1}
i	ϕ	stay at $t+1$	$(m, 0)$
$i+1$	$1-\phi$	move at $t+1$	$(n, 1)$

Table $G4$: $s_t = (m, i)$, $i > 0$

η_{t+1}	$\Pr(\eta_{t+1} \mid s_t)$	events	s_{t+1}
$i-1$	$(1-F_t)\phi$	sold at t, stay at $t+1$	$(m, i-1)$
i	$(1-F_t)(1-\phi)$	sold at t, move at $t+1$	(n, i)
i	$F_t\phi$	didn't sell at t, stay at $t+1$	(m, i)
$i+1$	$F_t(1-\phi)$	didn't sell at t, move at $t+1$	$(n, i+1)$

Table $G5$: $s_t = (n, 0)$

η_{t+1}	$\Pr(\eta_{t+1} \mid s_t)$	events	s_{t+1}
i	F_t	didn't buy at t	$(n, 0)$
i	$(1-F_t)\phi$	bought at t, stay at $t+1$	$(m, 0)$
$i+1$	$(1-F_t)(1-\phi)$	bought at t, move at $t+1$	$(n, 1)$

Table $G6$: $s_t = (n, i)\,,\ i > 0$			
η_{t+1}	$\Pr\left(\eta_{t+1} \mid s_t\right)$	events	s_{t+1}
$i-1$	$(1-F_t)\,F_t$	sold and didn't buy at t	$(m, i-1)$
$i-1$	$(1-F_t)^2\,\phi$	sold and bought at t, stay at $t+1$	$(m, i-1)$
i	$(1-F_t)^2\,(1-\phi)$	sold and bought at t, move at $t+1$	(n, i)
i	F_t^2	didn't sell and didn't buy at t	(n, i)
i	$F_t\,(1-F_t)\,\phi$	didn't sell and bought at t, stay at $t+1$	(m, i)
$i+1$	$F_t\,(1-F_t)\,(1-\phi)$	didn't sell and bought at t, move at $t+1$	$(n, i+1)$

We aggregate across all types of agents to derive $\Pr\left(\eta_{t+1} = i\right)$.

$$\Pr\left(\eta_{t+1} = i\right) = \sum_{s_t} \Pr\left(\eta_{t+1} = i \mid s_t\right) \Pr\left(s_t\right)$$

where given the initial distribution of types, $\Pr(s_0)$, we can compute $\Pr(s_t) = \Pr(s_t \mid s_{t-1}) \Pr(s_{t-1})$ with $\Pr(s_t \mid s_{t-1})$ given in the above four tables. Given that agents can only buy and sell one house in a period, the relevant s_t in the summation includes only those with $(i-1, i, i+1)$ houses for sale. The total number of houses for sale in period $t+1$ is

$$v_{t+1} = \sum_i i \Pr\left(\eta_t = i\right).$$

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