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How large is the effect of inequality on economic growth?

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## *Abstract*

Understanding the relationship between income inequality and economic growth is of great importance to social scientists. While the current empirical literature is inconclusive on the nature of their relationship, we also crucially do not know much about how large the effects of inequality and growth shocks are on each other. In this paper, we use a Bayesian structural vector autoregression approach to estimate the relationship between inequality and GDP growth for two countries, the UK and the USA. We find that the size of the effects of an inequality shock on growth, and vice versa, are very small, accounting for under 2% of the variance. We also find that the effects of the shocks dissipate within ten years, suggesting that the effects of these shocks are a short-term phenomenon. Inasmuch as the size of the effects of an inequality shock on growth, and a growth shock on inequality are so small, researchers should focus on uncovering macroeconomic mechanisms that govern the inequality-and-growth relationship using multi-equation models.

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# 1 Introduction

A great deal of attention has been devoted to understanding the relationship between income inequality and economic growth. The recent empirical literature investigating this relationship, however, is inconclusive: it has identified that the relationship between inequality and growth may be positive or negative, unstable or even at best non-existent (see Forbes 2000; Banerjee and Duflo 2003; Halter et al. 2014; Brueckner and Lederman 2018; Erman and te Kaat 2019; Bandyopadhyay 2020).

An important aspect of this literature is that it has not explicitly estimated the size of the effects of economic growth and inequality on each other. In this paper, for the first time, we thus estimate the size of the effect of inequality shocks on growth, and that of growth shocks on inequality for the UK and the USA. We find that the size of the effects are very small, mostly around 2% for both countries. In addition, we also find that the effects of inequality on growth and vice versa do not persist. The effects last for, at most, ten years. We employ a novel Bayesian structural VAR approach for our estimations using methods proposed by Baumeister and Hamilton (2018) which allows for the accurate estimation of the size of the effects and the time taken for the effects to dissipate.

The paper is organised as follows. Section 2 describes the empirical method used for the estimations. Sections 3 and 3.1 present the model and results. Section 4 concludes.

## 2 Empirical approach

The current empirical literature that examines the inequality-and-growth relationship primarily estimates the nature and direction of the relationship. Panel regression methods are most commonly used to estimate the relationship. However, panel regression approaches are not suited to estimate the magnitude of the effects of inequality and growth on each other. It is vitally important for social scientists to measure the size of the effects of inequality on growth on each other for policy purposes. In this paper, thus, to measure the size of the effects of an inequality (and growth) shock we estimate structural vector autoregressions (VARs) using the methodology of Baumeister and Hamilton (2018) (hereafter BH2018).

A core contribution of this paper and thus value-added to this literature is in using BH2018's approach for our estimations. While VAR approaches are well known for estimating structural multi-equation relationships, the Bayesian approach is now well established for its powerful abilities to incorporate prior information and generate more accurate estimates. Moon, Chorfheide and Granziera (2013) point out that in VAR models the number of parameters can be very large and that the corresponding identified set may have a very complex topology. This in turn requires researchers to provide restrictions on the parameters. However, much of the literature uses VAR models with sign restrictions following a Bayesian style method without formally acknowledging it (please see the excellent surveys of Manski 2003, Tamer 2010). In light of this literature, our paper takes a step further to formally provide a prior to the parameters of our model.

The method uses a Bayesian approach to generate prior distributions about the underlying economic structure which are then used to place some plausible restrictions on the values of the parameters estimated and generate posterior distributions. These posterior distributions are thereafter used to estimate the effects of the shocks. Variance decompositions are used to estimate the proportion of the variation in our variable interest, for

example, economic growth, due to the effect of an inequality shock.

We estimate our model using data for two countries, UK and the USA, for the years 1959 to 2018. Both countries are comparable in their macroeconomic structures, and have similar policy mechanisms. We use a three equation model for our estimations, with three variables: economic growth, measured as the annual growth rate of GDP, a variety of percentile share ratio measures of income inequality, and the terms of trade as a third variable that underpins the inequality-and-growth relationship<sup>1</sup>. We use percentile shares ratios as our preferred measure of inequality due to the recent literature identifying econometric problems with the Gini for time dependent analyses (Bandyopadhyay 2020). We present results with several measures for robustness, namely the 0th to 50th percentile ratio, perc(0:50), 90th to 10th percentile ratio, perc (90:10), and 90th to 100th percentile ratio, perc (90:100)<sup>2</sup>. Inequality measures have been obtained from the World Inequality Database (2019). GDP growth and terms of trade have been obtained from the World Bank's World Development Indicators' database.

### 3 Modelling the growth and inequality relationship

To estimate our three-variable model we follow BH2018, with some additional innovations for the selection of the prior and posterior distributions.

The VAR specification can be presented as:

$$Cx_t = Fz_{t-1} + \epsilon_t \quad (1)$$

here  $x_t$  is a  $3 \times 1$  vector of inequality, GDP growth and terms of trade, and  $z_{t-1}$  is a  $12 \times 1$  vector containing the four lags of  $x_t$  and a constant,  $\epsilon_t$  is a  $3 \times 1$  vector of structural innovations following the distribution:

$$\epsilon_t \sim N(0, D) \quad (2)$$

We assume matrix  $C$  can be inverted; equation (1) can thus be transformed into:

$$x_t = C^{-1}Fz_{t-1} + C^{-1}\epsilon_t \quad (3)$$

$$E(C^{-1}\epsilon_t\epsilon_t'C'^{-1}) = C^{-1}D(C')^{-1} = A \quad (4)$$

where  $C^{-1}D$ , and  $\hat{A}$  can be obtained by OLS regression of  $x_t$  on  $z_{t-1}$ . In addition, the residuals of equation (3) can be written as:

$$C^{-1}\epsilon_t = x_t - C^{-1}Dz_{t-1} \quad (5)$$

Following BH2018, we ascertain how the observations of the series of  $x_t$  revise the prior beliefs of matrices  $C, F, D$ .

Let  $D$  be a diagonal matrix, the prior of  $C$ ,  $p(C)$ . The conditional distribution of  $D$  and  $F$  are  $p(D|C)$  and  $p(F|C, D)$  respectively.

The prior distribution of  $D$  is:

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<sup>1</sup>For example, Forbes (2000), Banerjee and Duflo (2003) amongst many others model international trade as a mechanism via which the inequality and growth relationship is governed.

<sup>2</sup>The perc(0:50) inequality measure represents the bottom half of the income distribution, perc(90:10) represents the two ends of the income distribution, while perc(90:100) focuses on the top end.

$$p(d_{ii}|C) = f(d_{ii}, u_i, \sigma_i) \text{ for } d_{ii} > 0, \text{ and } 0 \text{ o.w} \quad (6)$$

where  $u_i$  and  $\sigma_i$  are parameters corresponding to the prior of  $C$ ,  $p(C)$ . For  $F$ , we assume that each row of  $F$  follows a normal distribution,  $N(a_i, d_{ii}m_i)$  as a prior.

$$p(F|D, C) = \prod_{i=1}^{12} p(f_i|D, C) \quad (7)$$

Then the aggregate prior is given by:

$$p(C, F, D) = p(C) \prod_{i=1}^{12} p(d_{ii}|C) p(f_i|C, D) \quad (8)$$

where  $i = 1$  to 12 represents the 12 diagonals in  $F^3$ . With this prior, we can express the log likelihood function of the observations as:

$$p(x_0, \dots, x_{60}|C, D, F) = \prod_{t=1}^{60} f(\epsilon_t|C, F, D) \quad (9)$$

where  $\epsilon_t$  is the error term in equation (1), following  $N(0, D)$  and  $t = 1, \dots, 60$  is the number of years. To implement, we collect the unknown elements of  $C$  in a vector  $c$  and define the distribution of  $c$  as:

$$c \sim N(0, u, \sigma) \quad (10)$$

which is the prior for  $C, p(C)$ . Maximising the likelihood function generates the first posterior of  $p(C, F, D|x_0, \dots, x_{60})$ . We hereafter employ the Metropolis-Hastings algorithm to draw the parameters  $\hat{C}, \hat{F}, \hat{D}$  from the posterior distribution to generate 20,000 impulse response functions (IRFs) and their confidence intervals. The impulse responses are estimated as follows:

$$\begin{aligned} \frac{dx_{t+s}}{d\epsilon'_t} &= \frac{\Delta x_{t+s}}{d\epsilon'_t} + \frac{\Delta x_{t+s-1}}{d\epsilon'_t} + \dots + \frac{\Delta x_t}{d\epsilon'_t} \\ &= (I_3 - C^{-1}F_1 - C^{-1}F_2 - \dots - C^{-1}F_4)^{-1}C^{-1} \end{aligned} \quad (11)$$

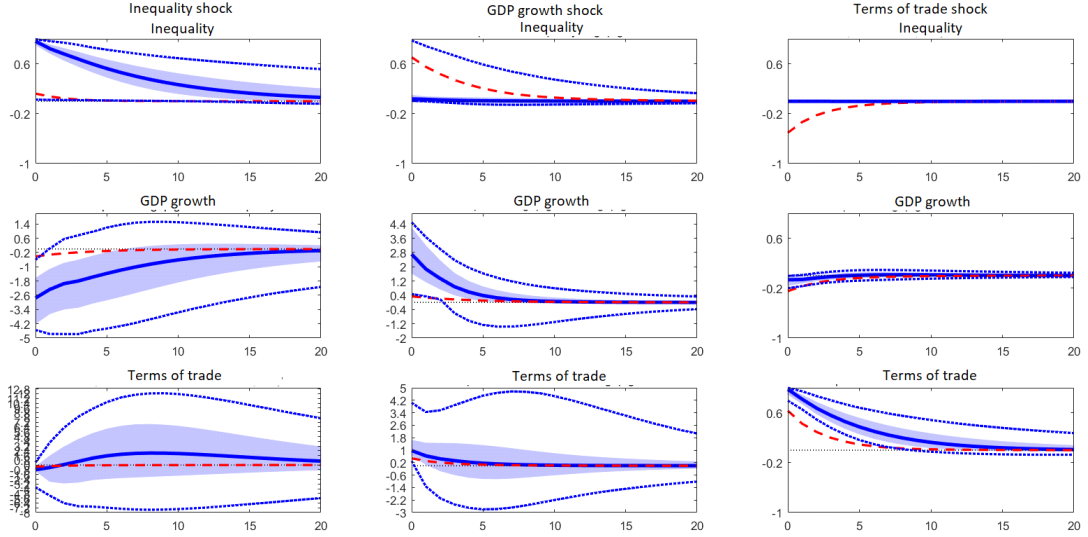
where  $F_j$  corresponds to the coefficients of lag  $j$  terms of  $x_t$ .

To generate the draws from the posterior distribution  $p(C, F, D|x_0, x_1, \dots, x_{60})$ , we undertake the following method. Let  $C_1 = C_*$ , where  $C_*$  is derived by maximizing the log-likelihood function. We generate  $C_n = C_n + f(\hat{P}^{-1})v_{n+1}$ , where  $v_{n+1}$  is a 3 by 1 vector of independent normal distributions with mean 0 and variance 1. Let  $L(C)$  be the log-likelihood function when  $C = C$ ,  $\hat{P}\hat{P}' = \frac{d^2 L(C)}{dC dC'}|_{C=C_*}$  and  $f(a) = (X'(a)X(a) - (X'(a)Z)(Z'Z)^{-1}(Z'X(a)))$  for any  $a$ . If the log-likelihood on  $C_{n+1}$  is greater than log-likelihood on  $C_n$ , then  $C_{n+1} = C_{n+1}$  o.w  $C_{n+1} = C_n$ . We repeat this procedure for  $n = 20000$  which thus gives us  $C$ . Thereafter we repeat the same procedure to generate draws for  $F$  and  $D$ .

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<sup>3</sup>The 12 diagonals correspond to the 3 variables and 4 lags.

Figure 1: UK's structural IRFs for 3-variable VAR, using inequality measure perc(90:100).



### 3.1 Empirical results

We now present our estimates for the UK and the USA. In Figure 1 the posterior IRFs for the UK are calculated with respect to a one standard deviation change in the variable of interest. We use perc(90:100) as our inequality measure<sup>4</sup>. The red dashed lines plot the median of the estimated prior distributions for 20 time periods. The solid blue lines are the median of the posterior distribution. The shaded blue region represents the 75% posterior credibility regions and the dashed lines indicate 95% regions.

The effect of an inequality shock on the three variables are presented in the first column. An inequality shock lowers economic growth, in panel (2,1), and terms of trade, in panel (3,1). The effect of the inequality shock on GDP growth dissipates within 10-12 years.

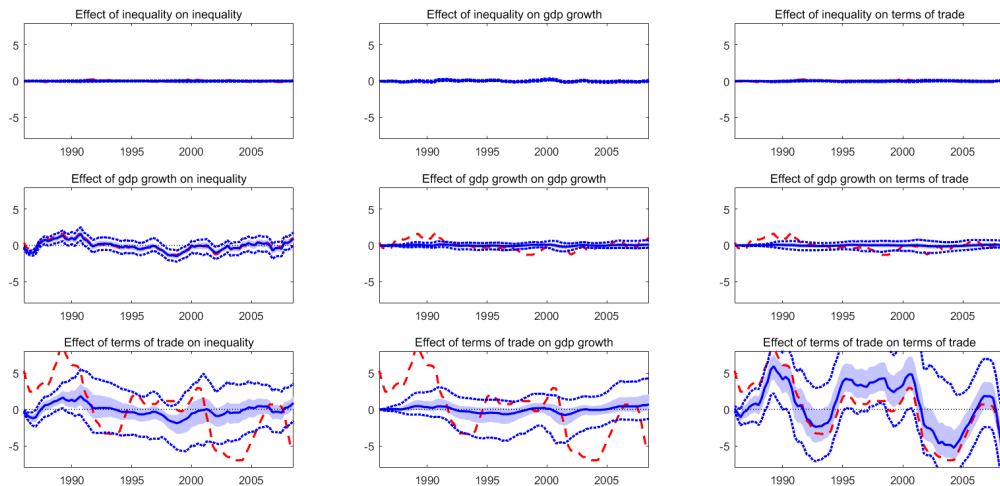
A GDP growth shock raises inequality (panel (1,2)), and returns to normal within five to seven years. The growth shock has a negative effect on the terms of trade, (panel(2,2)). A terms of trade shock lowers inequality (panel (1,3)) and returns to normal quickly, and also lowers GDP growth. These effects are all small and do not persist.

To estimate the size of the contribution of these shocks, we estimate the historical decomposition of all three variables, presented in Figure 2, using inequality measure perc(0:50)<sup>5</sup>. The (red) dashed line records the actual value of our variable of interest (as deviations from its mean). The solid blue line is the portion attributed to the indicated

<sup>4</sup>Estimates using other inequality measures are available further in the paper and also from the authors. The results for all inequality measures used are very similar, hence we present results for a variety of inequality measures in the paper for robustness.

<sup>5</sup>Results using other inequality measures used are not presented for brevity and are all available from the authors. The values of the estimates are very close to each other.

Figure 2: UK's portion of historical variation in inequality (perc(0:50)), GDP growth and terms of trade attributed to each of the structural shocks.



structural shock and the dotted blue line represents the posterior credibility sets. The shaded regions and dashed lines denote 75% and 95% posterior credibility regions, respectively. In Figure 2 we can see that an inequality shock barely has any impact upon GDP growth, in panel (1,2) and on the terms of trade (panel (1,3)). The GDP growth shock has a small effect on inequality (panel (2,1)). This is particularly the case in the late 1980s and early 1990s.

Table I summarises the average contribution of the three types of shocks via variance decompositions, using the inequality measure perc(0:50). We report the contribution of each of the three shocks to the mean-squared error of a four-year-ahead forecast of the variables. A growth shock accounts for 2.48% of the variance of inequality and a terms of trade shock accounts for 3.01% of inequality. An inequality shock on the other hand accounts for 1.83% of variation in GDP growth. The sizes of the effects of both shocks on growth and inequality is strikingly small. We repeat the above estimations using another inequality measure, perc(90:10), in Table II, resulting in similar conclusions.

For robustness, we examine the USA, which has a different growth and inequality experience compared to the UK due to historically different policy frameworks. Figure 3 presents the IRFs for the USA, using perc(99:100). Like the UK case, an inequality shock leads to a drop in growth, and a growth shock leads to an increase in inequality. The effects are again small and do not persist for long<sup>6</sup>.

Figure 4 presents the decompositions of the effects of shocks, with perc(0:50) as the inequality measure. We observe a small effect of an inequality shock on GDP growth and

<sup>6</sup>For robustness, we estimate the impulse response functions using several other inequality measures, such as the perc(90:10) measure where we obtain similar results, all available with authors.

Table I: UK, decomposition of variance of 4-year-ahead forecast errors, using perc(0:50)

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	94.51%	2.48%	3.01%
	[0.001, 0.01]	[0.0001,0.0002]	[0.0001,0.0003]
GDP growth	1.83%	97.03%	1.15%
	[0.0001, 0.003]	[0.31,0.79]	[0.0001,0.02]
Terms of trade	2.42%	1.42%	96.36%
	[0.002, 0.39]	[0.001,0.13]	[0.83,1.87]
Parentheses indicate 95% credibility intervals			

Table II: UK, decomposition of variance of 4-year-ahead forecast errors, using perc(90:10)

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	90.50%	2.48%	7.42%
	[0.001, 0.1]	[0.0001,0.003]	[0.0001,0.005]
GDP growth	1.33%	95.03%	3.65%
	[0.0001, 0.06]	[0.31,0.73]	[0.0001,0.12]
Terms of trade	7.11%	3.83%	89.06%
	[0.01, 0.39]	[0.001,0.14]	[0.73,1.76]
Parentheses indicate 95% credibility intervals			

Figure 3: USA's structural IRFs for 3-variable VAR, using inequality measure perc(99:100).

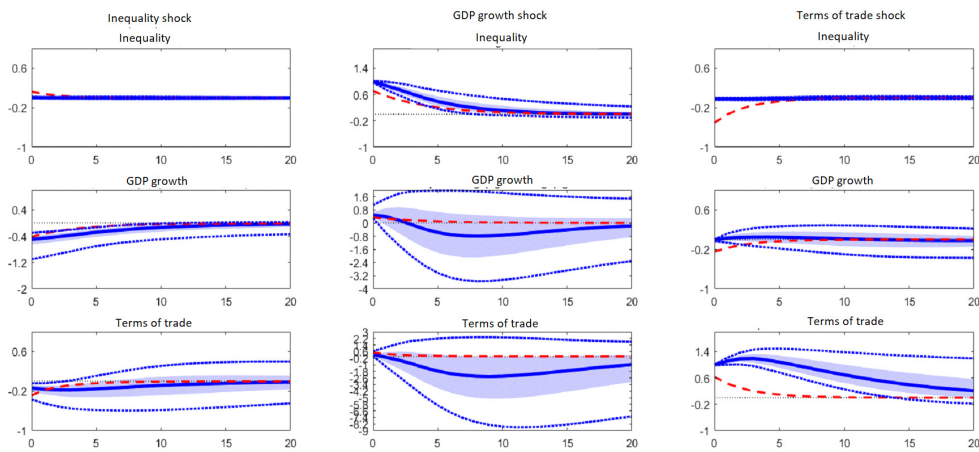


Figure 4: USA's portion of historical variation in inequality (perc(0:50)), GDP growth and terms of trade attributed to each of the structural shocks.



Table III: USA, decomposition of variance of 4-year-ahead forecast errors, using perc(0:50)

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	92.92%	2.08%	5%
	[0.001, 0.01]	[0.0001,0.0002]	[0.0001,0.0002]
GDP growth	0.74%	98.67%	0.59%
	[0.0001, 0.23]	[2.16,4.99]	[0.0001,0.17]
Terms of trade	0.74%	8.63%	90.63%
	[0.0001, 0.01]	[0.008,0.42]	[0.83,1.89]
Parentheses indicate 95% credibility intervals			

trade. The effect of a growth shock on inequality in panel (2,1), reveals an initial increase in inequality gradually tapering out in the 1990s.

Table III reports the contribution of each of the three shocks, using the inequality measure, perc(0:50)<sup>7</sup>. A growth shock accounts for under 2% of the variation in inequality, and by comparison, a larger amount of the variance of terms of trade. An inequality shock also accounts for less than 1% of variation in growth and the terms of trade.

To summarize:

- First, we find that a growth shock is inequality-increasing and an inequality shock is growth-reducing for both countries.
- Second, the most striking finding is that the size of the effects of the GDP growth shocks and inequality shocks are very small (around 2%).

<sup>7</sup>Further results using other percentile measures of inequality for robustness are also available from the authors, not presented here for brevity. All results using the different measures of inequality are very similar.



- Finally, we observe that the effects of the shocks do not persist, dissipating within ten to fifteen years.

That the effects of GDP growth shocks on inequality and inequality shocks on growth, respectively, are very small is a new addition to the empirical literature of inequality and growth. Its small magnitude suggests that social scientists should research alternative socio-economic mechanisms which determine inequality and growth independently, rather than one on each other. Likewise, that these shocks do not persist for long is also a new finding in the empirical literature (other than evidence uncovered in Bandyopadhyay (2020) using standard VAR methods). The lack of persistence of their effects and the small size of the effects, jointly suggests that both theoretical and empirical researchers should focus on alternative mechanisms when researching the determinants of inequality and growth.

## 4 Conclusion

In this paper we use a Bayesian VAR approach to estimate the direction and size of the effects of inequality and growth shocks on each other and conclude three salient findings. First, we find that a growth shock is inequality-increasing and an inequality shock is growth-reducing. This result conforms with much of the empirical literature. Second, the most important finding is that the size of these effects are remarkably small. Variance decompositions reveal that at the most 2% of the variation in growth is explained by an inequality shock. Likewise, we find that under 2% of variation in inequality is explained by a growth shock. This is a striking result, in light of the long literature on inequality and growth, where a huge amount of importance has been attributed to the effects of economic growth on inequality, and that of inequality on growth.

The third and most important finding is that these shocks dissipate within ten years. While this is good news for the policy maker, it also implies that both empirical and theoretical researchers should focus on other mechanisms that have a more persistent impact on inequality and growth, independently. In addition, further research is needed on whether developing countries and transition economies have different experiences.

The second finding, that the size of these effects is small, is of concern to researchers and policy makers. That a growth shock or an inequality shock explains so little variation of inequality and growth, respectively, implies that these shocks impact upon other macroeconomic mechanisms that are not included in the model. Future research on the relationship between inequality and growth, thus, should explicitly model macroeconomic mechanisms that govern this relationship using multi-equation models instead of single equation models.

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