Size matters: measuring the effects of inequality and growth shocks

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Abstract

Understanding the relationship between income inequality and economic growth is of utmost importance to economists and social scientists. The empirical literature is inconclusive on the nature of their relationship and has revealed it to be highly sensitive to the estimation procedure and sample in use. The literature has, however, not focused on estimating the size of the effects of a change in economic growth on inequality, and that of inequality on growth. In this paper, for the first time, we use a novel Bayesian structural vector autoregression approach to estimate the relationship between inequality and growth via growth and inequality shocks for two large economies, China and the USA, for the years 1979 to 2018. We find that a growth shock is inequality-increasing and an inequality shock is growth-reducing. We also find, however, that the size of the effects of these shocks are very small, accounting for under 2% of the variance for both countries. Finally, we also find that the effects of the shocks dissipate within ten years, suggesting that the effects of these shocks are a short-term phenomenon. Inasmuch as the size of the effects of a growth shock on inequality and of an inequality shock on growth are so small, policy makers and researchers should focus on uncovering other macroeconomic variables that are impacted upon by these shocks and ensure that they are included in their analyses.

Keywords: Bayesian methods, structural vector autoregression, inequality, growth.

JEL codes: C32, C33, D31, D63.

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1 Introduction

One of the most important relationships examined in the economic literature is that of the relationship between income inequality and economic growth over time. Economists and philosophers have long identified causal mechanisms between inequality and economic growth (Kuznets 1955; Galor and Zeira 1992; Piketty 2014). This relationship has recently gained particular attention due to increasing interest in the long run impact of income, wealth and health inequalities on a country's growth outcomes (Bourguignon and Morrisson 2002; Anand and Segal 2008, Durlauf et al. 2009; Gabaix et al. 2016; Alvaredo et al. 2018; Clarke et al. 2018; Ho and Heindi 2018; Milanovic 2018; Kuhn et al. 2020). The recent interest in the relationship between inequality and growth has also been aided by the greater availability of international inequality statistics, especially the Gini measure[1], but also more recently with income percentile shares as an additional measure of inequality (available in the World Inequality Database 2019).

The empirical literature investigating this relationship has identified that the relationship between inequality and growth may be positive or negative, unstable or even at best non-existent (see Forbes 2000; Barro 2000; Banerjee and Duflo 2003; Knowles 2005; Castelló-Climent 2010; Halter et al. 2014; Bandyopadhyay 2018; Berg et al. 2018; Brueckner and Lederman 2018; Erman and te Kaat 2019). However, there have been no investigations that explicitly estimate the size of the effect of these entities on each other. To fill this gap in the literature, in this paper we investigate the relationship between inequality and economic growth by estimating the size of the effects as shocks on each other, using a Bayesian vector autoregression (hereafter VAR) approach.

While a growing body of empirical literature has sought to have identified causal mechanisms between inequality and economic growth (Kuznets 1955; Galor and Zeira 1992; Piketty 2014), however, there has been disagreement and considerable difficulty in pinning down this relationship empirically. Recent empirical literature has identified that the estimation of the relationship between inequality and growth is heavily dependent upon the nature of the estimation procedure involved (Herzer and Vollmer 2012; Berg et al. 2018; Juuti 2020). This literature has also suggested that the estimated relationship between economic growth and income inequality is an "artefact" of the estimation procedure used. Another strand of literature has also recently identified that the relationship is also highly sensitive to the inequality measure that is used, the time frame and country of study (Bandyopadhyay 2020). Mean-independent inequality measures reveal that there is no relationship between inequality and growth, while mean-dependent inequality measures reveal a negative relationship, albeit unstable.

Given that the relationship has proven to be difficult to empirically estimate, it is thus important for researchers to also estimate the size of the effects of inequality on growth and that of growth on inequality to ascertain the relative importance of their respective effects for policy makers. Indeed, findings revealed by Dollar and Kraay (2002) and Voitchovsky (2005), for example, have already identified that our attention should be focused on identifying the relationship at specific parts of the income distribution and that "growth is good for the poor". This literature asserts that our concern thus should be about the short term effects of a growth shock, rather than that of the entire distribution, summarised by the inequality measure.[2]

[1] The principal and most widely used source of inequality data is the World Income Inequality Database (WIID) (WIDER-UNU 2019) and the World Bank indicators.

[2] The lack of precise growth-inequality estimates stands in contrast with the standard of estimating growth-poverty elasticities.

In this paper, we use a Bayesian VAR approach to estimate the size of the effects of an inequality shock on growth, and that of a growth shock on inequality, using large two countries, China and the USA in the period 1979 to 2018. Even though the size of their GDPs are highly comparable, especially in the last twenty years, their policy approaches over the period of study are very different. In particular, Chinese economic policy has had a heavy emphasis on poverty alleviation over the period of the study, thus classifying it more as a developing country in comparison to the USA.

The macroeconomic literature has devoted a multitude of methodologies for the estimation of the effects of shocks to the economy[3]. For our analysis, we undertake our estimation using recent methods proposed by Baumeister and Hamilton (2015, 2018). Baumeister and Hamilton (2018) develops a procedure where a researcher can tailor the identifying assumptions using Bayesian prior information about the signs and also the magnitudes of the parameter values of interest. In this approach we allow our inference be guided not just by prior information about signs but also about magnitudes. The innovation offered by this method thus results in more accurate estimates of the structural VAR and the impulse responses that are generated. In addition, as a point of departure from Baumeister and Hamilton (2018), we assume the priors of the structural parameters and the covariance of the error terms to be independent.

To specify our model we include terms of trade as an additional variable that underpins the relationship, following from the empirical literature that emphasises the role of international trade in determining this relationship (Banerjee and Duflo 2003, Halter et al. 2014). Inclusion of the terms of trade also allows us to observe its relative importance in determining both growth and inequality.

There are three main findings that result from our analysis. First, we find that a growth shock is inequality-increasing. We also find that an inequality shock is growth-reducing. These results are the same for both the USA and China and accord with a lot of the earlier literature (see Forbes 2000; Barro 2000; Banerjee and Duflo 2003; Knowles 2005; Castelló-Climent 2010; Halter et al. 2014; Bandyopadhyay 2018; Berg et al. 2018; Brueckner and Lederman 2018; Erman and te Kaat 2019). Second, estimates of the variance decompositions reveal that the size of these effects are very small. This is the most salient finding of the paper. A growth shock to inequality accounts for only 2% of the variation of inequality. Similarly, an inequality shock to growth also accounts for under 2% of the variation in economic growth. In comparison, a terms of trade shock accounts for a larger amount of variation of both inequality and growth. Third, we find that the effects of these shocks are dissipated within ten to fifteen years at the most, and quite often within ten years. This result also accords with recent literature with different country examples (Bandyopadhyay 2020)[4].

For the estimation of our model we use several percentile share ratios as our prefered measure of inequality instead of using the popular Gini. There are several reasons for this. Percentile share ratios are increasingly shown in the recent literature to be better representatives of inequality over time, and indeed are being used for studies gauging long term inequality. (Gabaix et al. 2016; Smith et al. 2019). For example,

There is a large literature that measures the growth-poverty elasticities, identifying policy mechanisms which can be particulrly useful for developing economies (Bouguignon 2003, see Arndt et al. 2017 for a detailed survey of the literature). The estimations in the paper are a step forward in filling this void in the literature.

[3] For a brief introduction to the different modern approaches see Barsky and Sims (2011), Mountford and Uhlig (2009), Ramey (2011) Zeev and Pappa (2017) for a selective coverage of the methods.

[4] Bandyopadhyay (2020) uses all developed country cases, namely, Denmark, Switzerland and the UK. The effects of the shocks are estimated using a standard traditional VAR approach.

World Inequality Database (2019) with this approach also focuses entirely on the estiamation of percentile share income ratios as a relevant measure of inequality. Bandyopadhyay (2020) also reveals percentile share ratios to have favourable dynamic econometric properties as inequality measures. In addition, Cobham et al. (2013) and Cobham and Sumner (2015) recommend percentile share ratio measures and in particular the Palma measure[5], as most suitable for arriving at policy advice (Gabaix et al. 2016; Alvaredo et al. 2018; Milanovic 2018; Smith et al. 2019; Kuhn et al. 2020)[6].

The paper is organised as follows. In Section 2 we discuss the current literature on the inequality and growth relationship and the problems that characterise the estimation process identified in the literature. Section 3 presents the model that we estimate and the results estimated from the model. Section 5 discusses the results obtained in light of the current literature. Section 6 summarises the findings in the paper and concludes.

2 What does the inequality and growth literature say

There is a large and well documented literature on the estimated relationship between economic growth and inequality. The literature is inconclusive on the exact nature of the relationship. The literature reports negative, positive and no significant relationships between inequality and economic growth. (see Forbes 2000; Barro 2000; Banerjee and Duflo 2003; Knowles 2005; Castelló-Climent 2010; Herzer and Vollmer 2012; Halter et al. 2014; Niño-Zarazúa et. al 2018; Bandyopadhyay 2018; Berg et al. 2018; Brueckner and Lederman 2018; Erman and te Kaat 2019; Juuti 2020). The literature has typically shown that the relationship is highly sensitive to the sample studied, the estimation methodology and the time frame in use. In addition, given the lack of a long time series of data, the timespan of the studies are generally short, often with a small number of years but a relatively large number of cross sectional units. Recent literature also suggests that the varied results and conclusions about the estimated relationship could well be an outcome of the econometric methodology in use (Herzer and Vollmer 2012; Berg et al. 2018; Juunti 2020)

The recent empirical literature that has examined and estimated the relationship between economic growth and inequality is quite large. Using recent time dependent methods (such as panel regression or cointegation methods) and with the availability of high quality data, the literature has now rejected the inverted U relationship that derived from the seminal work of Kuznets (1955, 1956) and has uncovered several other relationships. (see Juunti 2020 for an excellent survey of the literature). Following the publication of the Deninger and Squire (1996) dataset and eventually the WIID (UNU-WIDER 2019) datasets, which have high quality data for Ginis and quintiles shares of income, the majority of further studies have used two principal approaches to estimating the relationship - cross section or panel regression approaches, which accounts for the vast majority, and time series approaches (Herzer and Vollmer 2012, Bandyopadhyay 2020). This literature typically identifies a significant positive or negative relationship between inequality and growth. Halter et al. (2014) in particular, reveals that studies using time dependent methods generate a positive relationship, while studies exploiting the cross section variation only generate a negative relationship.

[5] The Palma ratio is the ratio of the top 10 % of population's share of gross national income (GNI), divided by the poorest 40 % of the population's share of GNI. It provides a policy-relevant indicator of the extent of inequality in each country and is also considered to be particularly relevant for poverty reduction policy.

[6] However, for robustness, undertaking the estimations in this paper using available Ginis (and other mean-dependent measures) for comparative purposes is highly recommended and discussed later in the paper.

Halter et al. (2014) also find that mechanisms generating a negative relationship work over the longer term and are reflected in level-based estimators. Bandyopadhyay (2020) however, uses over 100 years data and does not obtain the same negative relationship as reported in Halter et al. (2014). Studies using non-parametric approaches (Banerjee and Duflo 2003, Bandyopadhyay 2020) on the other hand deduce that there is no significant relationship between these entities.

There is a large theoretical and empirical literature which has proposed several mechanisms that underpin the positive and negative relationships between inequality and growth. Some of the literature also concludes that the relationship is particularly dependent on the time frame (see Halter et al. 2014 for details). Inequality is growth reducing via its influence on the inter-generational transmission of inequalities in wealth (Galor and Zeira 1992 and Banerjee and Duflo 2003), via the median voter's decisions on the post-tax income distribution (Perotti 1993), via the political economy outcomes of inefficient state bureaucracies (Acemoglu et al 2003), weak legal structures (Glaeser et al., 2003) and political instability (Bénabou 1996). Due to their influence via education, evolution of wealth distribution and political economy routes, thus, the effects are slow to come into effect and pan out over the medium to long run. Inequality is growth enhancing, however, by its effect on aggregate savings (Kuznets 1955; Kaldor 1955), and via the effects on investments in research and developments (Foellmo and and Zweimueller, 2006). These effects rely upon standard economic mechanisms (such as market imperfections and convex saving functions) and thus are short term in their impact.

Another strand of literature has also identified a different set of relationships between inequality and economic growth using different types of inequality measures. This literature, has focused on the different performances of absolute, intermediate and relative inequality measures in measuring global or national level inequalities (Niño-Zarazúa et. al 2018, Bandyopadhyay 2018, 2020) Bandyopadhyay (2018) identifies using standard GMM regression methods, a stable relationship between inequality nad economic growth for mean-independent measures of inequality (i.e. the absolute Gini). However, mean-dependent measures of inequality (the relative Gini measure), reveals an unstable negative relationship. This finding is also revealed in Bandyopadhyay (2020) which undertakes a long run approach (using data for over 100 years) and finds that the relationship between economic growth and inequality is non-existent for mean-independent measures. It also finds that mean-independent measures respond to a growth shock in a different manner, compared with mean-dependent measures, owing to the different dynamic properties of the inequality measures. The relationship between inequality and economic growth is, hence, revealed to be highly dependent upon the measure of inequality in use.

Thus, the estimation of the inequality and growth relationship will depend upon:

1) the inequality measure that is in use - whether it is mean-dependent or mean-independent, and

2) the nature of the estimation method that is being use - namely regression analyses (for example different panel regression methods) and panel cointegration analyses.

3) the sample in use.

Thus, the literature tells of an incocnlusive story - while different inequality measures and different econometric techniques may result in different relationships between inequality and growth, these same relationships are also not robust over the long run due to the dynamic properties of the inequality measures being in flux[7]. From the policy makers' point of view though, the picture at hand is not as gloomy. Typically, policy makers of national governments work with short run effects, especially in a democratic set up, where the term in office is determined by the election cycle. Hence, our current interest in this paper, in investigating the effect of a shock is also limited to the short run as well, though the model is set up to observe the effect for up to twenty years.

Another aspect of the estimations in these studies, especially of those using GMM panel regression analyses, it is evident that the size of the regression co-efficients are quite small. This result suggests that the growth or inequality effects are quite small, something which comes through in the non-relationship obtained with the non-parametric estimates of Banerjee and Duflo (2003) and Bandyopadhyay (2020).

To our knowledge, there is currently no study which explicitly sets out to estimate the exact size of the effect of a growth shock on inequality (or an inequality shock on growth). It is important for researchers to ascertain the size of the effects of these shocks. This is because the highly sensitive and unstable nature of the inequality and growth relationship derived in earlier empirical studies could be due to the small effect each have on the other. In addition, from a policy makers' point of view, it would be useful to measure the size of the effect of a shock for GDP growth on inequality and vice versa. If the effect of a positive growth shock on inequality is estimated to be quite large, this would be of major policy significance. Also, if we find that the effect of a positive growth shock has a medium term effect, in that the impact lasts for over ten years, then this is also of great policy significance.

Thus, in this paper, to estimate the size of the effect of a shock, we undertake an estimation procedure popularised by Hamilton and Baumeister (2015, 2018) which employ a Bayesian approach. The Bayesian approach is particularly useful for empiricists to provide more informed estimates of the effects of the shocks. In their proposed method, they use the system of variables in the model (i.e. in our case inequality and growth amongst others), to generate prior "beliefs" (i.e. prior distributions) about the underlying economic structure which are then used to place some plausible restrictions on the values of the parameters estimated. The method then generates the relevant posterior distributions using the prior information. These posterior distributions are then used to estimate the effects of a growth shock on inequality (or an inequality shock on growth). In addition, as a point of departure from Baumeister and Hamilton (2018), we assume the priors of structural parameters and the covariance of the error terms to be independent.

The procedure allows us to also estimate the exact size of the effect of a shock: we are able to estimate the propotion of the variation in our variable interest, for example, economic growth, due to the effect of a inequality shock by estimating variance decompositions. Our estimated model will also estimate the proportion of variation in inequality that is due to the effect of a GDP growth shock. We describe the method and sampling algorithm in greater detail in the following empirical estimation section.

In addition to the economic growth and inequality measures as our principal entities of interest, following the empirical and theoretical literature we model the mechanism underpinning the inequality and growth relationship via one of the heavily studied macroeconomic routes - that of international trade. We thus include the terms of trade as a third variable in our three equation model. The empirical literature has given significant importance to the role of trade in underpinning this relationship (for example, see Banerjee and Duflo 2003, Barro 2000).

[7] This is due to changes in the properties of the income distribution over time from which the inequality measures are estimated. See Bandyopadhyay (2020) for more details.

3 The effect of growth shocks and inequality shocks

We now introduce the data used for our analysis. The period of study spans from 1959 to 2018. We use a three equation model for our estimations using three variables - economic growth, measured as the annual growth rate of GDP, expressed as a percentage, the terms of trade, and several measures of income inequality. We use up to five inequality measures, as percentile share ratios: percentile ratios of the 30th to 80th percentiles, (perc(30:80)) for China only), 10th to 90th percentile ratios, perc(10:90), 0th to 50th percentile ratios, perc(0:50), 50th to 90th percentile ratios, perc(50:90), and the 99th to 100th percentile ratios, perc(99:100). All of these measures have been obtained from the World Inequality Database (2019), with the exception of China, for the period 1959-1969 for 30th to 80th percentile ratio, perc(30:80). We estimate the percentile share ratio perc(30:80) from the China database CHARLS[8].

Our third variable is the terms of trade, to represent the role of trade in determining the relationship between inequality and growth. For the USA, the GDP growth and terms of trade variables have been obtained from the World Bank World Development Indicators' database. For China, these variables have been obtained from the China Stock Market & Accounting Research (CSMAR) database. We have chosen to use these two variables from the CSMAR database (and not from the World Bank database) due to continuous data for the full time period being available. For the periods over which there is an overlap in years for both data sources, there is a strong association between the variables from the two data sources.

There are several reasons why we have chosen to use the percentile share ratios as our preferred measure of inequality. There is an established literature which has identified econometric problems with the use of popularly used measures (such as the Gini) (Bandyopadhyay 2020, 2018, Niño-Zarazúa et. al 2017). Our analysis thus follows a growing body of work that uses top percentile shares and percentile ratio measures (including the Palma measure [Cobham et al. 2013; Cobham and Sumner 2015])[9] for dynamic analyses, especially for arriving at policy advice (Gabaix et al. 2016; Alvaredo et al. 2018; Milanovic 2018; Smith et al. 2019; Kuhn et al. 2020). For the sake of comparability, however, we also undertake the estimations using Ginis that are available in the UNU-WIDER (2019) database. For our purposes we require a time series of inequality measures to undertake the estimations.

We present results using two countries: China and the USA. We have chosen China and USA to represent a large developing country and a large developed country, yet being highly comparable economies. China's successful growth experience in the last thirty years places it as a highly comparable country with the United States (Zilibotti 2017, van der Wiede and Narayan 2019). In particular, the period of 2000 to 2018 in particular can be considered to be a phase over which the growth and inequality experience in China and USA is highly comparable. The institutional reforms introduced in the 1980s in China led to the rapid and stellar rise of GDP in PPP terms bringing it to comparable levels with those of OECD countries. The convergence was particularly pronounced during the first decade of the 21st century, when China grew at

[8] For China we have complete data from the WID (2019) for years 1970 to 2018. To allow for a longer time period prior to 1979, we make use of The China Health and Retirement Survey (CHARLS) dataset to generate the inequality measures for 1959 to 1969.The CHARLS database consists of a representative sample drawn from around 10,000 households and 17,500 individuals in 150 counties/districts and 450 villages/resident committees. Individuals are followed up every two years and all data are made public one year after the end of data collection.

[9] The Palma ratio is the ratio of the top 10 % of population's share of gross national income (GNI), divided by the poorest 40 % of the population's share of GNI. It provides a policy-relevant indicator of the extent of inequality in each country and is also considered to be particularly relevant for poverty reduction policy.

unprecedented annual rates close to 10%. In addition, the USA and China have also experienced comparable levels of relative intergenerational mobility for individuals born in the 1980s, particularly for income and education.

Turning to the potential nature of the inequality and growth relationship one could expect for these two countries, the case for the USA is clearly one where growth is associated with rising inequalities. One could, however, expect the situation to be the reverse for China, due to its strong equalising policies. But evidence seems to suggest rising inequalities with economic growth as well (Zilibotti 2017). Theories which describe a positive relationship between inequality and growth may work best to describe both countries' inequality-and-growth story.

To be able to measure the effects of a shock to these two entities, we estimate using a commonly studied 3-variable annual model with a system that describes the movements of economic growth, inequality and the terms of trade using the structural VAR approach. We follow the approach used in Baumeister and Hamilton (2018), with some innovations in terms of the methodology for the selection of the prior and posterior distributions, described below.

The dynamic structural specification adopted in this paper can be presented in the following form:

$$Cx_t = Fz_{t-1} + \epsilon_t \tag{1}$$

where x_t is a 3×1 vector of inequality, GDP growth and terms of trade, and $z_{t-1} = \{x'_{t-1}, x'_{t-2}, x'_{t-3}, x'_{t-4}, 1\}'$ is a 13×1 vector containing the four lags of x_t and a constant, ϵ_t is an 3×1 vector of structural innovations with the following distribution:

$$\epsilon_t \sim N(0, B) \tag{2}$$

where B is the covariance matrix of the structural innovations.[10] We assume that the 3×3 matrix C describing the contemporaneous relationship among x_t is invertible and F is a 3×13 matrix characterizing the dynamics of the system. Therefore, the equation 1 can be transformed into a reduced-form VAR specification:

$$x_t = C^{-1}Fz_{t-1} + C^{-1}\epsilon_t \tag{3}$$

We define $e_t = C^{-1}\epsilon_t$ as residuals of the reduced-form VAR. Its covariance matrix A is given by:

$$E(e_t e_t') = E(C^{-1}\epsilon_t\epsilon_t' C'^{-1}) = C^{-1}B(C')^{-1} = A \tag{4}$$

Let $G = C^{-1}F$ be the matrix capturing the dynamics of z_{t-1} in reduced-form. A can be obtained by regressing x_t on z_{t-1} and the corresponding OLS estimators are denoted as $\hat{G}$ and $\hat{A}$. In addition, the residuals of equation 3 based on OLS can be written as:

$$\hat{e}_t = x_t - \hat{G}z_{t-1} \tag{5}$$

Under usual circumstances, without any specific information about elements of A, the model would be unidentified and there would be no basis for drawing conclusions from the data about the effects of a shock

[10] In this paper, we assume that B is a diagonal matrix since different structural shocks are usually caused by different events.

to the system. The conventional approach is to place strong restrictions on the elements of A (assumed to be a "dogmatic" prior). Baumeister and Hamilton (2015, 2018) propose that prior beliefs about the underlying economic structure are used to place some plausible restrictions on the values of the parameters. For this, we use the Bayesian approach following Baumeister and Hamilton (2018) with the objective to ascertain how observations of the series of x_t revise the prior beliefs of matrices C, F, B. As stated above, the structural innovations' covariance matrix B is a diagonal matrix. The prior of C is given by $\pi(C)$. Then following Baumeister and Hamilton (2018), we assume the prior distribution of B and F conditional on C are $\pi(B|C)$ and $\pi(F|C)$, respectively.

In most cases, there is not enough information to verify that the elements in B are more likely to take specific values. Thus B is assumed to follow a uniform prior distribution. Conditional on the prior of C, we let the prior distribution of the non-zero element on the diagonal of B be a uniform distribution on the interval $[0, \lambda_i]$ for $1 \leq i \leq 3$:

$$\pi(b_{ii}|C) = f(b_{ii}; \lambda_i), b_{ii} > 0 \tag{6}$$

$$\pi(b_{ii}|C) = 0, b_{ii} \leq 0 \tag{7}$$

$$\pi(B|C) = \prod_{i=1}^{3} \pi(b_{ii}|C) \tag{8}$$

where $f(b_{ii}; \lambda_i)$ denotes the probability density of a uniform distribution on $[0, \lambda]$ at b_{ii}. For F, let $f_i^{'}$ be the i-th row of F. We assume the prior of f_i follows a normal distribution with mean a_i and covariance matrix $_i$ denoted as $N(a_i, m_i)$.

$$\pi(F|C) = \prod_{i=1}^{3} \pi(f_i|C) \tag{9}$$

where $\pi(f_i|C)$ is the pdf of f_i's prior normal distribution

$$\pi(f_i|C) = (2\pi)^{-\frac{13}{2}} |m_i|^{\frac{1}{2}} e^{-\frac{1}{2}(f_i - a_i)^{'}(m_i)^{-1}(b_i - a_i)} \tag{10}$$

Then the aggregate prior is given by the product of $\pi(C)$, $\pi(B|C)$ and $\pi(F|C)$:

$$\pi(B, C, F) = \pi(C) \prod_{i=1}^{3} \pi(b_{ii}|C)\pi(f_i|C) \tag{11}$$

where $1 \leq i \leq 3$ represents the 3 rows in C.[11] The first step of our estimation aims to track how the observations of the endogenous variables $X_N = (z_0^{'}, x_1^{'}, x_2^{'}, x_3^{'}, ...x_N^{'})^{'}$ update the prior beliefs. To be specific, the log-likelihood function conditional on Gaussian residuals can be written as:

$$L(X_N|B, C, F) = (2\pi)^{\frac{3N}{2}} |C|_+^N |B|_+^{-\frac{N}{2}} e^{-\frac{\sum_{j=1}^{N}(Cx_t - Fz_{t-1})^{'} B^{-1}(Cx_t - Fz_{t-1})}{2}} \tag{12}$$

where $|B|_+$ and $|C|_+$ denote the absolute values of their determinants. Following Baumeister and Hamil-

[11] The 3 rows correspond to the 3 endogenous variables in our system.

ton (2015, 2018), we define $\bar{X}_i$ and $\bar{Z}_i$ as follows:

$$\bar{X}_i = (x_1^{'}c_i, x_2^{'}c_i, ..., x_N^{'}c_i, A_i^{'}a_i/\sqrt{b_{ii}})^{'} \tag{13}$$

$$\bar{Z}_i = (z_0^{'}, z_1^{'}, ..., z_{T-1}^{'}, A_i^{'}/\sqrt{b_{ii}})^{'} \tag{14}$$

where A_i denotes the factor of the Cholesky decomposition of m_i, $A_i A_i^{'} = m_i^{-1}$ and $c_i^{'}$ is the i-th row of C. Then, the posterior distribution of B, C, F can be characterized by the following proposition and the proof is available in Appendix A.

Proposition 1 *Given the aggregate prior distribution of B, C, F defined by 11 and x_t is a bounded stationary process, the posterior distribution of B, C, F conditional on observation X_N can be written as*

$$\pi(B, C, F|X_N) = \pi(C|X_N) \prod_{i=1}^{3} \pi(b_{ii}|C, X_N)\pi(f_i|C, X_N) \tag{15}$$

where

$$\pi(b_{ii}|C, X_N) = f(b_{ii}; \bar{\lambda}_i) \tag{16}$$

$$\pi(f_i|C, X_N) = N(f_i; \bar{a}_i, \bar{m}_i) \tag{17}$$

$$\bar{a}_i = (\bar{Z}_i^{'}\bar{Z}_i)^{-1}(\bar{Z}_i^{'}\bar{X}_i) \tag{18}$$

$$\bar{m}_i = (\bar{Z}_i^{'}\bar{Z}_i)^{-1} \tag{19}$$

$$\bar{\lambda}_i = \frac{2\bar{\Xi}_i}{N} \tag{20}$$

$$\bar{\Xi}_i = \bar{X}_i^{'}\bar{X}_i - \bar{X}_i^{'}\bar{Z}_i(\bar{Z}_i^{'}\bar{Z}_i)^{-1}\bar{Z}_i^{'}\bar{X}_i \tag{21}$$

And the posterior distribution of C conditional on X_N is

$$\pi(C|X_N) = \frac{\alpha_N \pi(C)(2\pi)^{-\frac{3N}{2}}|C\hat{\Sigma}_N C^{'}|_+^{-\frac{N}{2}}}{\prod_{i=1}^{3}\bar{\lambda}_i^2/6N} \prod_{i=1}^{3} \frac{2^{\frac{\bar{\Xi}_i}{N}} b_{ii}^{-\frac{N}{2}} m_i^{-\frac{1}{2}}}{\bar{m}_i^{-\frac{1}{2}}\lambda_i e^{\frac{\bar{\Xi}_i}{2}}} \tag{22}$$

where α_N is an adjustment constant to ensure that $\pi(C|X_N)$ is a probability measure.

The posterior distribution of f_i, is a normal distribution concentrating on the point $\bar{a}_i$ when the prior variance term m_i goes to infinity. In other words, a noninformative prior renders the posterior degenerate. Furthermore, when x_t's second moment is finite and m_i takes a finite value, $\bar{a}_i$ converges to the regression coefficient of $x_t^{'}c_i$ on z_{t-1} and $\bar{m}_i$ converges to 0.

$$\begin{aligned}
\bar{a}_i &= lim_{N\to\infty}(\frac{1}{N}\sum_{j=1}^{N} z_{j-1}z_{j-1}^{'} + \frac{m_i^{-1}}{N})^{-1}(\frac{1}{N}\sum_{j=1} N z_{j-1}x_j^{'}c_i + \frac{m_i^{-1}a_i}{N}) \\
&= lim_{N\to\infty}(\frac{1}{N}\sum_{j=1}^{N} z_{j-1}z_{j-1}^{'})^{-1}(\frac{1}{N}\sum_{j=1} N z_{j-1}x_j^{'}c_i) \\
&= E(z_{t-1}z_{t-1}^{'})^{-1}E(z_{t-1}x_t^{'}c_i)
\end{aligned} \tag{23}$$

$$\bar{m}_i = lim_{N\to\infty}(\frac{1}{N}\sum_{j=1}^{N} z_{j-1}z_{j-1}^{'} + \frac{m_i^{-1}}{N})^{-1} = 0 \tag{24}$$

Consistent with Baumeister and Hamilton (2015, 2018), the posterior distribution of f_i will be independent of its own prior when the sample is big enough, centering at the point $E(z_{t-1}z_{t-1}^{'})^{-1}E(z_{t-1}x_t^{'}c_i)$. On the other hand, the prior of C is informative about the posterior distribution of f_i through c_i. This result explains why much of the empircal VAR literature focuses on identifying C to undertake analyses.

Let $\hat{\Sigma}_N$ be $\frac{\sum_{j=1}^{N}\hat{e}_t\hat{e}_t^{'}}{N}$. For the mean of the posterior distribution of the diagonal element of B, $\frac{\bar{\lambda}_i}{2}$ converges to $c_i^{'}\hat{\Sigma}_N c_i$ when we have a noninformative prior for b_{ii}. Under regularity conditions, the variance of posterior b_{ii}, $\frac{(c_i^{'}\hat{\Sigma}_N c_i)^{2N}}{2}$ converges to 0 when N goes to infinity. Let Σ^* be the Σ_N estimated based on the population($N \to \infty$).

$$\begin{aligned}\Sigma^* &= lim_{N\to\infty}\Sigma_N = lim_{N\to\infty}\frac{\sum_{j=1}^{N}\hat{e}_t\hat{e}_t^{'}}{N} \\ &= E(x_t x_t^{'}) - E(x_t z_{t-1}^{'})E(z_{t-1}z_{t-1}^{'})^{-1}E(z_{t-1}x_t^{'})\end{aligned} \tag{25}$$

While the prior of b_{ii} is informative, the updated term on the prior, $\frac{\bar{\Xi}_i}{N}$ converges to $c_i^{'}\Sigma^* c_i$. The posterior distribution of b_{ii} conditional on C will also center at the point of $lim_{N\to\infty}c_i^{'}\hat{\Sigma}_N c_i$ asymptotically as long as λ_i is finite and m_i is finite. This means that the posterior of b_{ii} is also irrelevant of its own prior for large samples. The convergence property of the posterior distribution of structural parameters B, F and $\hat{\Xi}_i$ are summarized in the following Lemma, and the proof is presented in Appendix A.

Given the specification of a structural model defined by 1 and sample X_N, the posterior distribution of F, B and $\bar{\Xi}_i$ under regularity conditions about the population of X have the following convergence property

$$\begin{aligned} F|C, X_N &\to CE(x_t z_{t-1}^{'})E(z_{t-1}z_{t-1}^{'})^{-1} \\ \frac{\bar{\Xi}_i}{N}|C, X_N &\to c_i^{'}\Sigma^* c_i \\ b_{ii}|C, X_N &\to c_i^{'}\Sigma^* c_i \end{aligned} \tag{26}$$

However, the posterior distribution of C is still determined by its prior given by $\pi(C)$. According to the results of Proposition 1, the posterior distribution of C,when the prior of B and F are both noninformative, is given by:

$$\pi(C|X_N) = \frac{\alpha_N \pi(C)|C\hat{\Sigma}_N C^{'}|^{\frac{N}{2}}}{|Diag(C\hat{\Sigma}_N C^{'})|^{\frac{N}{2}}} \tag{27}$$

where α_N is a constant ensuring the sum of the probability is 1, $Diag(C\hat{\Sigma}_N C^{'})$ represents a matrix with the same diagonal elements of $C\hat{\Sigma}_N C^{'}$,but zero in all other positions. Therefore, the posterior density $\pi(C|X_N)$ is linear in its prior $\pi(C)$. Since the average of the residuals, $\hat{\Sigma}_N$ is positive definite, and C is invertible, we have the following inequality:

$$|Diag(C\hat{\Sigma}_N C^{'}| \geq |C\hat{\Sigma}_N C^{'}| \tag{28}$$

where $|Diag(C\hat{\Sigma}_N C^{'}|$ and $|C\hat{\Sigma}_N C^{'}|$ stand for the determinants of the corresponding matrix. Equality can only be achieved when the matrix $C\hat{\Sigma}_N C^{'}$ is diagonal. Let the space of the matrix C that satisfies the equality conditional on $\hat{\Sigma}_N$, be $W(\hat{\Sigma}_N)$.

Given the sample X_N, if $C \in W(\hat{\Sigma}_N)$, the posterior distribution of C can be simplified as:

$$\pi(C|X_N) = \alpha_N \pi(C) \tag{29}$$

However, $lim_{N\to\infty}\pi(C|X_N)$ goes to 0 if $C \notin W(\hat{\Sigma}_N)$. Following Baumeister and Hamilton (2015, 2018), we define the distance between C and $W(\hat{\Sigma}_N)$ as $d(C, \hat{\Sigma}_N)$:

$$d(C, \hat{\Sigma}_N) = \sum_{i=2}^{n}\sum_{j=1}^{i-1} |A_{ij}(C, \hat{\Sigma}_N)| \tag{30}$$

where $A(C, \Sigma)$ is the factor of the Cholesky decomposition of $C\Sigma C^{'} = A(C, \Sigma)A(C, \Sigma)^{'}$ and $A_{ij}(C, \Sigma_N)$ denotes the element of matrix $A(C, \Sigma)$ in the i-th row and j-th column. The asymptotic property of $\pi(C|X_N)$ can be summarized by the following proposition.

Proposition 2 *Given x_t and z_t defined in 1 and a sample denoted as X_N. Define $U_\epsilon(\hat{\Sigma}_N) = \{C : d(C, \hat{\Sigma}_N) \leq \epsilon\}$. Assume that $\frac{\bar{\lambda}_i^2}{6} \geq ln(\bar{\lambda}_i)$ For any positive ϵ, the asymptotic posterior of C under the regularity conditions about the population X satisfies*

$$lim_{N\to\infty}\pi(C \in U_\epsilon(\hat{\Sigma}_N)|X_N) = 1 \tag{31}$$

When the prior about B and F are both noinformative($\lambda_i = m_i = 0$ for $1 \leq i \leq 3$), the posterior of C conditional on an arbitrary sample X_N is

$$\pi(C|X_N) = \alpha_N \pi(C) \tag{32}$$

Furthermore, the asymptotic property of the posterior distribution of C, $\pi(C|X_N)$ does not require that x_t follows a normal distribution.[12] The main conclusion from Proposition 2 is that the asymptotic posterior of C will converge to a linear transformation of its prior when the probability that $C\Sigma^* C^{'}$ is a diagonal matrix is positive according to C's prior. In much of the empirical VAR literature, the effects of structural shocks on endogenous variables are captured by a normalized matrix, $C^{-1}B^{\frac{1}{2}}$ where:

$$\frac{dx_t}{d\epsilon_t^{'}} = C^{-1} \tag{33}$$

This specification, however, is not realistic since the results about the effects of a shock only uses information of the corresponding column of $C^{-1}B^{\frac{1}{2}}$. Therefore, we use our prior $\pi(B, C, F)$ to estimate the prior distribution of the structural matrix of parameters G, Σ in the reduced-VAR specification. In particular, $\pi(B, C, F) = \pi(C|G, \Sigma)\pi(G, \Sigma)$. This means that the posterior belief on G and Σ is completely

[12] The assumption about the VAR structure between different variables in x_t is also not necessary.

informative about its prior, implying $\pi(C|G, \Sigma, X_N) = \pi(C|G, \Sigma)$. The principal purpose of our specification is to characterize the prior belief of $\pi(C|G, \Sigma)$ based on existing evidence about inequality and growth.

4 Prior information about structural parameters

For our quantitative analysis we use a three equation system defined by equation 1 to estimate the dynamic effects of between inequality ($Inequality_t$), GDP growth rate $\hat{(}Growth_t)$, and international trade (measured as the terms of trade, $Trade_t$). In particular, the matrix C is defined as:

$$\begin{bmatrix} 1 & c_{12} & c_{13} \\ c_{21} & 1 & c_{23} \\ c_{31} & c_{32} & 1 \end{bmatrix}$$

There is a widely established literature on the determinants of economic growth, where the relationships between each of inequality, economic growth and international trade have been explored (see Temple 2021, Pavcnic 2018). This literature establishes that while there may be differences across countries and sectors, there exist significant relationships between each of these three entities. We can thus allow $c12; c13; c21; c23; c31; c32$ in C to be non-zero. The first column in the matrix C presents the effects of inequality on economic growth ($c21$) and on international trade ($c31$). For the effect of inequality on economic growth ($c21$), Section 2, discusses the mechanims via which inequality has a negative impact upon economic growth (see Bandyopadhyay 2020, Halter et al. 2014, Herzer and Vollmer 2012). We, thus, attribute this relationship as $c21 < 0$. There is also a large literature (see Pavcnic 2018 for a detailed survey) that has revealed a positive association between inequality and international trade; suggesting that greater inequality is associated with international trade[13]. We, therefore, attribute this relationship as $c31 > 0$.

The following column in the table presents the effects of economic growth on inequality ($c12$) and of economic growth on international trade ($c32$). There is a substantial empirical literature (discussed in Section 2) on the effect of economic growth on inequality which suggests that high levels of economic growth is associated with high levels inequality. On the basis of this literature we attribute this relationship in the matrix as $c12 > 0$. The literature on the relationship between economic growth and trade deduces that higher levels of economic growth are associated with higher levels of trade (Estevadeoral and Taylor 2013, Goldberg and Pavcnic 2016). We, thus, attribute this relationship as $c32 > 0$.

A similar literature demonstrates that international trade has positive impacts on economic growth (see Pavcnic 2018); thus, we attribute the relationship as $c23 > 0$. The literature on the impact of trade on inequality is quite diverse and broad, mostly studied by sector and between countries, but deduces that international trade reduces inequality (Pavcnic 2018). Thus we attribute $c13 < 0$.

Following Baumeister and Hamilton (2018), the prior distribution of all parameters follows a Student's t distribution with 3 degrees of freedom. The priors adopted for C, that have been constructed following Baumeister and Hamilton (2018), are presented in Table 1. The prior belief about C, $\pi(C)$ can be written as the product of the prior density functions of $c12; c13; c21; c23; c31; c32$ specified in Table 1. Like Baumeister

[13] While the empirical evidence reveals a positive relationship between trade and inequality, the literature however has concluded that it is not its main driver (Goldberg and Pavcnik 2016).

Parameter	Prior mode	Prior scale	Sign
c_{12}	2.3	0.6	Positive
c_{13}	1.2	0.6	Positive
c_{21}	-0.16	0.6	Negative
c_{23}	1.6	0.6	Positive
c_{31}	0.21	0.6	Positive
c_{32}	0.12	0.6	Positive

Table 1: Priors of structural parameters in C

and Hamilton (2015, 2018), the Bayesian method allows us to take both the uncertainty caused by data limitation and model specification into consideration. The priors of the dynamic effects, F, and the covariance of structural shocks, B, are defined based on an 8th order univariate autoregression. The details are outlined in the description of our algorithm in Appendix A.

4.1 Empirical results

We present our estimations below for China and USA using annual data from 1959 to 2018. For China we use distributional data from the CHARLS dataset to generate our inequality measures from 1959 to 1969 due to unavailability of these years in the World Inequality Dataset (2019). Due to the nature of the sample used in CHARLS the trends in the inequality measures estimated from CHARLS are not a perfect match with the data from the World Inequality Dataset (2019) in terms of the trend. However, these years are not directly used in the estimations because the data 20 years prior to 1979 are used for the sampling procedure described earlier. The estimations below are thus presented on the basis of data from 1979 to 2018.

The posterior impulse-response functions are plotted in Figures 1 and 2 and are calculated with respect to a one standard deviation change in the variable of interest. The red dashed lines in Figures 1 and 2 plot the median of the estimated prior distributions for 20 time periods. Although the medians of our prior distribution for structural impulse-response functions die out fairly quickly, the uncertainty we associate with this prior information grows significantly as the horizon increases. The solid blue lines in the structural impulse response functions are the medians of the posterior distribution. The shaded blue region in the impulse response panels represent the 75% posterior credibility regions and the dashed lines indicate 95% regions.

In Figure 1 we present the impulse response functions for the Chinese case, using the perc(30:80) inequality measure. The effect of an inequality shock on the three variables (inequality, GDP growth and terms of trade) are presented in the panels in the first column. An inequality shock lowers economic growth and terms of trade, in panel (2,1), but the drop is smaller for terms of trade, in panel (3,1). The inequality shock lowers GDP growth and returns to normal within 10-12 years. But it has a clear effect on the terms of trade - it intially lower trade growth but then returns to normal quickly. The second column of Figure 1 presents impulse responses for the effect of a GDP growth shock, which raises inequality as presented in the panel (1,2), and returns to normal within five to seven years. The growth shock has a negative effect on trade growth and takes a long time to return to normal. Finally, the effect of a trade shock is tabulated in column 3. The trade shock lowers inequality and then retuns to normal quickly. On the other hand, the

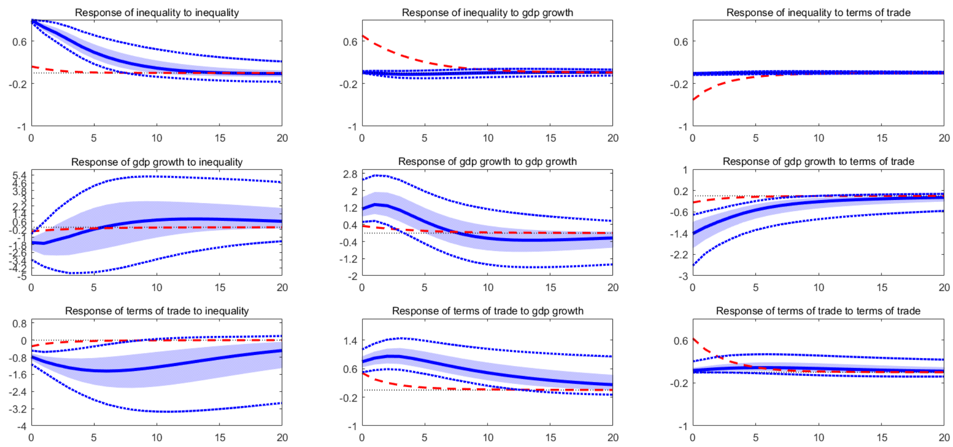

Figure 1: China's structural impulse-response functions for 3-variable VAR, using inequality measure perc(30:80). Solid blue lines: posterior median. Shaded regions: 75% posterior credibility set. Dotted blue lines: 95% posterior credibility set. Dashed red lines: prior median

trade shock also has an initial lowering of GDP growth and returns to normal quickly as well. These effects are all small and do not seem to persist.

For robustness, we estimate the models using another inequality measure perc(0:50), presented in Figure 2.[14] The inequality shock again has very similar effects on both GDP growth and terms of trade - the inequality shock lowers GDP growth and also lowers the terms of trade, with a quicker return to normal than GDP growth. We also observe the same effect that a GDP growth shock raises inequality and then returns to normal within five to ten years. All effects are again small.

We are particularly interested in the size of the contribution of each of these shocks on inequality and growth. For this, we present the historical decomposition of all three variables (GDP growth, inequality and terms of trade) in terms of the contributions of each of the separate structural shocks in Figures 3 and 4. The (red) dashed line in the panels records the actual value of our variable of interest being impacted upon by the shock (expressed as deviations from its mean). Thus for the first row, the red line presents the actual value of inequality, and for the second row and third row, the actual values of GDP growth and terms of trade. The solid blue line is the portion attributed to the indicated structural shock and the dotted blue line represents the posterior credibility sets. The shaded regions and dashed lines denote 75% and 95% posterior credibility regions, respectively. In Figure 3 the first row gives us the posterior median contribution of the inequality shock on all three entities. We can see that an inequality shock barely has any impact upon GDP growth, in panel (1,2) and terms of trade (1,3). The second row gives us the decomposition of the contributions of the GDP growth shock, and the third row panels gives us the contributions of the shock in

[14] We also generate impulse response functions for the inequality measure percentile share ratio 10th to 90th percentile shares, (Perc(10:90). The impulse response functions are very similar to those presented in the paper and are available from the authors.

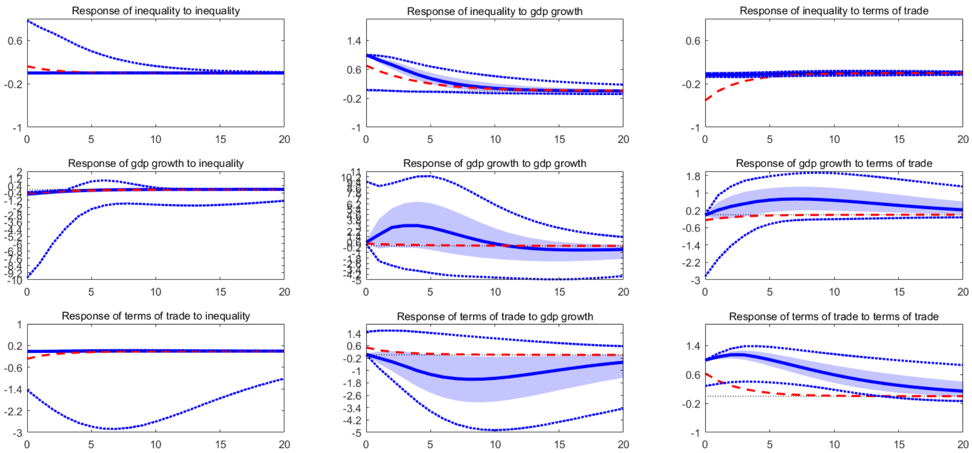

Figure 2: China's structural impulse-response functions for 3-variable VAR, using inequality measure perc(0:50). Solid blue lines: posterior median. Shaded regions: 75% posterior credibility set. Dotted blue lines: 95% posterior credibility set. Dashed red lines: prior median

terms of trade. The GDP growth shock has a small effect on inequality (in panel (2,1)). This is particularly the case in the late 1980s and early 1990s, and also later on in the late 1990s and late 2000s.

As a point of comparison, we have estimated the above estimations using a mean-dependent inequality measure, the Gini. Similar results are obtained when using the Gini as the inequality measure in Figure 10 in Appendix B.

Tables 2, 3 and 4 summarise the average contribution of the three different types of shocks using variance decompositions. We report the contribution of each of the three structural shocks to the mean-squared error of a one-year-ahead forecast of each of the three variables. Table 2 summarises the average contribution of the three different types of shocks using variance decompositions using the inequality measure perc(30:80). A GDP growth shock accounts for 1.85% of the variance of inequality and for comparison, about 6.08% of the variance of terms of trade. An inequality shock on the other hand accounts for less than 1% of variation in GDP growth (and 0.76% for terms of trade). It is not surprising that an inequality shock doesn't have much of a sizeable impact on either of the two entities (GDP growth and trade). Trade shocks account for 0.6% of the variability of GDP growth and 5.16% of the variability of inequality. It is interesting to observe that a terms of trade shock has a more perceptible impact upon inequality than the effect of a GDP shock. All said, the sizes of both of the shocks on inequality is still quite small.

As a robustness check, we repeat the above estimations using the other measures of inequality, namely, the perc(0:50) and perc(10:90) measures, in Tables 3 and 4. The perc(0:50) inequality measure represents the bottom half of the income distribution and is popularly used in the policy literature. The perc(10:90) measure is also popular for focusing on the tails of the distribution, where the metric is not influenced by the

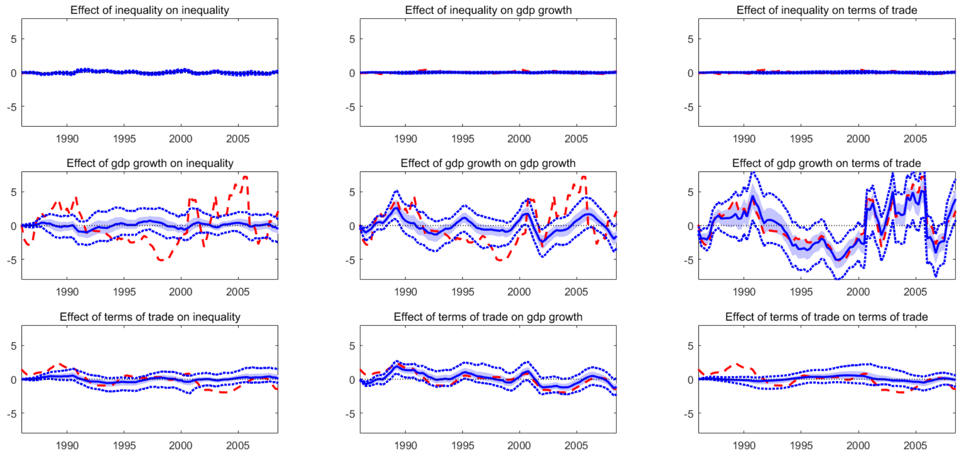

Figure 3: China's portion of historical variation in inequality (perc(30:80)), GDP growth and terms of trade attributed to each of the structural shocks. Dashed red: actual value for the deviation of the variable of interest from its mean. Solid blue: portion attributed to indicated structural shock. Shaded regions: 75% posterior credibility sets. Dotted blue: 95% posterior credibility sets.

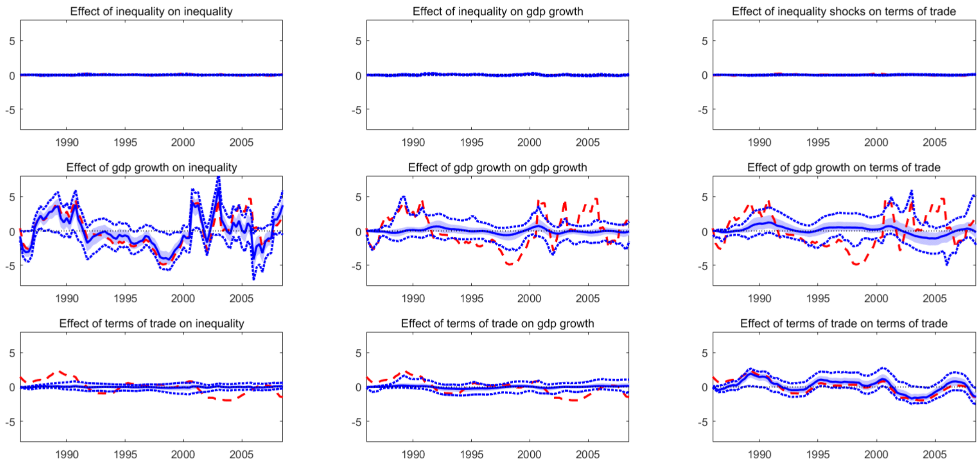

Figure 4: China's portion of historical variation in inequality (perc(0:50)), GDP growth and terms of trade attributed to each of the structural shocks. Dashed red: actual value for the deviation of the variable of interest from its mean. Solid blue: portion attributed to indicated structural shock. Shaded regions: 75% posterior credibility sets. Dotted blue: 95% posterior credibility sets.

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	92.99%	1.85%	5.16%
	[0.001, 0.01]	[0.0001,0.0002]	[0.0001,0.0002]
GDP growth	0.68%	98.73%	0.60%
	[0.0001, 0.25]	[2.34,5.48]	[0.0001,0.19]
Terms of trade	0.76%	6.08%	93.16%
	[0.0001, 0.1]	[0.008,0.35]	[0.86,1.94]
Notes	Estimated contribution of each structural shock to the 4-year--ahead median squared forecast error.		

Table 2: China, decomposition of variance of 4-year-ahead forecast errors, using inequality measure percentile share ratio (30:80)

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	92.92%	2.08%	5.00%
	[0.001, 0.01]	[0.0001,0.0002]	[0.0001,0.0002]
GDP growth	0.74%	98.67%	0.59%
	[0.0001, 0.23]	[2.16,4.99]	[0.0001,0.17]
Terms of trade	0.74%	8.63%	90.63%
	[0.0001, 0.1]	[0.008,0.42]	[0.83,1.89]
Notes	Estimated contribution of each structural shock to the 4-year--ahead median squared forecast error.		

Table 3: China, decomposition of variance of 4-year-ahead forecast errors, using inequality measure percentile share ratio (0:50)

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	92.51%	1.58%	5.92%
	[0.001, 0.01]	[0,0.0001]	[0,0.0001]
GDP growth	0.71%	98.79%	0.50%
	[0.0001, 0.15]	[1.51,3.52]	[0.0001,0.11]
Terms of trade	0.76%	10.19%	89.00%
	[0.02, 0.1]	[0.00001,0.46]	[0.83,1.86]
Notes	Estimated contribution of each structural shock to the 4-year--ahead median squared forecast error.		

Table 4: China, decomposition of variance of 4-year-ahead forecast errors, using inequality measure percentile share ratio (90:10)

dynamics of the centre of the distribution. The effect of a GDP growth shock on inequality and that of an inequality shock on GDP growth is quite similar to the earlier case using the perc(30:80) inequality measure. The proportion of the variation in inequality explained by GDP growth for both inequality measures is again less than 2%. Likewise, the proportion of variation in GDP growth explained by inequality is also around 1% for both inequality measures. We also present results using the Gini measure in Appendix B in Table 8, where we obtain very similar results.

Our conclusion above, that growth shocks account for a positive fraction of inequality fluctuations accords with current literature. Variations in growth are positively associated with variations in inequality, as evinced by the recent literature using GMM and other regression approaches (Halter et al. 2014 for studies that exploit the time dimension, Barro 2000, to name a few). The negative effect of an inequality shock on GDP growth is also documented in the panel regression applications' literature (for example Forbes 2000, Knowles 2005; Halter et al. 2014 for studies exploiting cross sectional variation and Bandyopadhyay 2020 for mean-dependent inequality measures). It is also clear from these studies that, indeed, the effects must be small due to the very small size of the regression coefficients of inequality and growth in the regressions estimated in these studies. These studies, however, do not emphasise or further analyse the cause of the small magnitude of these effects.

To examine a different country's growth and inequality experience, we now undertake our estimations for the USA. The growth and inequality experience has been vastly different in the USA due to a historically different policy framework compared with China. To illustrate the different inequality experiences for the two countries, Figure 9 in Appendix B plots some selected inequality measures estimated for the USA and China.

In Figures 5 and 6 we present the structural impulse response functions for the USA, using the perc(0:50) and the perc (50:90) inequality measures[15]. A clear impact is observed for an inequality shock on GDP growth in panel (2,1): an inequality shock leads to a drop in GDP growth, for both sets of results with perc(0:50) and perc (50:90). The red dashed lines plot the median of our prior distribution for impulse-response functions for the 20 time periods. Again, the medians of our prior distribution for structural impulse-response functions die out fairly quickly. The effects are small but do not seem to persist for long; for the effect of an inequality shock on growth, it lasts for between five to ten years. In panel (1,2) we record effect of a growth shock on inequality. For both measures of inequality, the growth shock leads to a drop in inequality. For robustness, we also estimate the impulse response functions using the perc(10:90) and perc(99:100) measures where we obtain similar results, results available with authors. We thus observe for both countries, and also for other countries we have worked with (namely UK and France) that using different inequality percentile measures do not necessarily yield very different results. Bandyopadhyay (2020) shows that when using a large range of inequality measures, some inequality measures do result in different effects of a GDP shock on inequality[16].

[15] We present results with perc(99:100) in place of perc(10:90) which was used for the China example simply for variety. Results with perc(10:90) for the US are available with the authors; they are very similar to the perc(99:100) results presented.

[16] Bandyopadhyay (2020) shows that mean-independent inequality measures perform slightly differently from mean-dependent inequality measures in being impacted upon by a GDP growth shock.

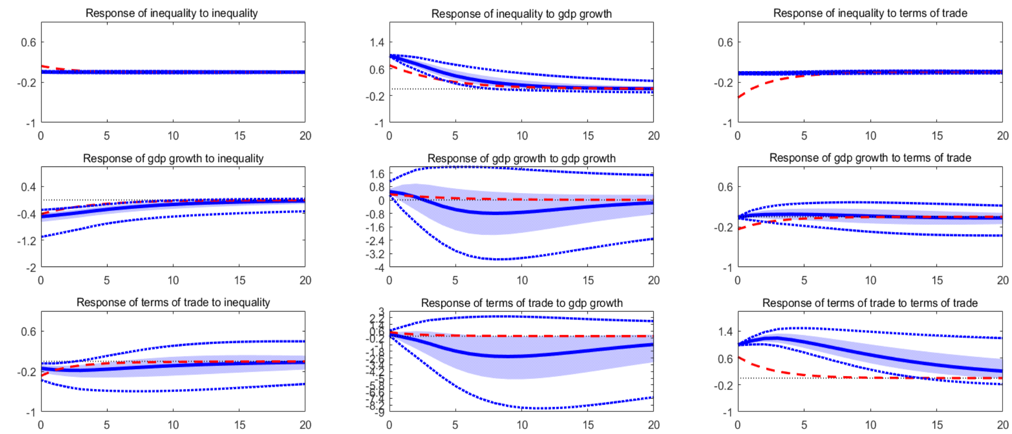

Figure 5: United States' structural impulse-response functions for 3-variable VAR, using inequality measure perc(0:50). Solid blue lines: posterior median. Shaded regions: 75% posterior credibility set. Dotted blue lines: 95% posterior credibility set. Dashed red lines: prior median

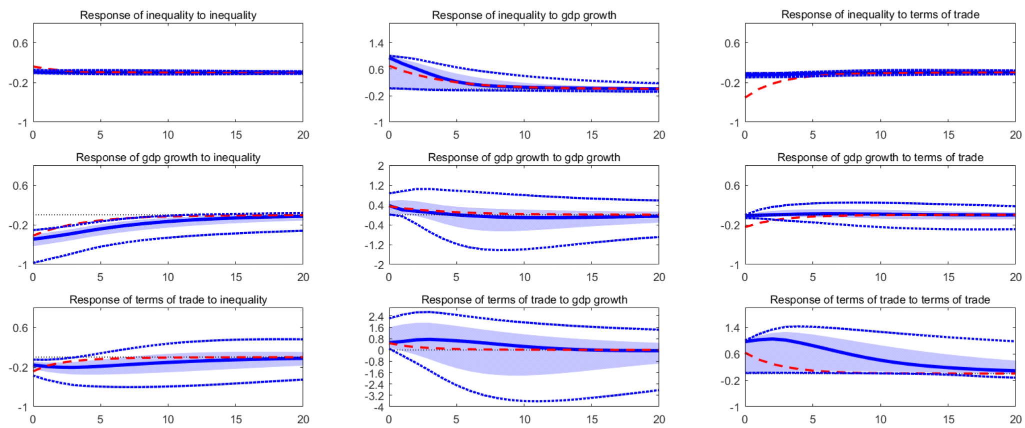

Figure 6: United States' structural impulse-response functions for 3-variable VAR, using inequality measure perc(50:90). Solid blue lines: posterior median. Shaded regions: 75% posterior credibility set. Dotted blue lines: 95% posterior credibility set. Dashed red lines: prior median

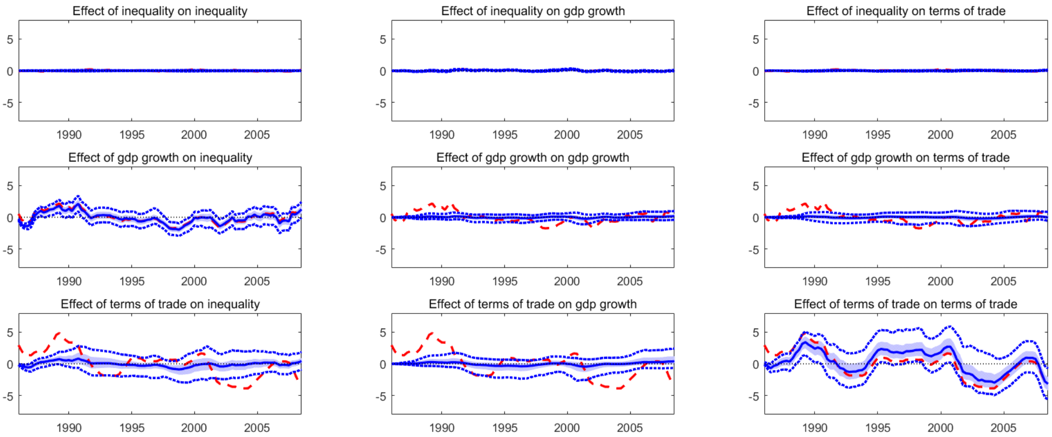

Figure 7: United States' portion of historical variation in inequality (perc(0:50)), GDP growth and terms of trade attributed to each of the structural shocks. Dashed red: actual value for the deviation of the variable of interest from its mean. Solid blue: portion attributed to indicated structural shock. Shaded regions: 75% posterior credibility sets. Dotted blue: 95% posterior credibility sets.

In order to identify the relative contribution of these structural shocks, we also present the historical decomposition of all three variables (GDP growth, inequality and terms of trade) in terms of the contributions of the three different sources. Figures 7 and 8 present the variance decompositions in the nine panels, where the (red) dashed line records the actual value of our variable of interest being impacted upon by the shock (expressed as deviations from its mean). The solid blue line is the portion attributed to the indicated structural shock, the dotted blue line represents the posterior credibility sets and the shaded regions and dashed lines denote 75% and 95% posterior credibility regions, respectively.

In the figures the first row gives us the posterior median contribution of inequality shocks on all three entities. We observe a negligible effect of an inequality shock on GDP growth and trade, compared to the China estimations.

The effect of a GDP growth shock on inequality in panel (2,1), is a bit more observable, with an initial increase in inequality and gradually tapers out in the 1990s. The third row gives us the decomposition of the contributions of a trade shock: the effect of a trade shock on inequality (3,1) is quite small (given by the blue solid line), as is also the case for its effect on GDP growth. Similar results are obtained when using the Gini as the inequality measure in Figure 11 in Appendix B.

These results are also revealed in Tables 5 to 7 which summarise the average contribution of the three different types of shocks using variance decompositions. The tables report the contribution of each of the three structural shocks to the mean-squared error of a one-year-ahead forecast of each of the three variables for three different types of inequality measures. We present estimates using three different percentile share inequality measures that reveal the similarities in their effects. We also present results using the Gini

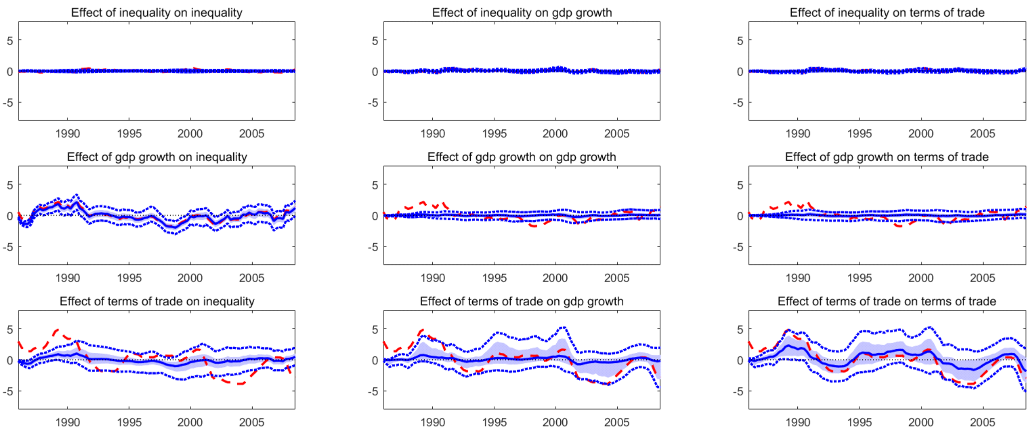

Figure 8: United States' portion of historical variation in inequality (perc(50:90)), GDP growth and terms of trade attributed to each of the structural shocks. Dashed red: actual value for the deviation of the variable of interest from its mean. Solid blue: portion attributed to indicated structural shock. Shaded regions: 75% posterior credibility sets. Dotted blue: 95% posterior credibility sets.

measure (as a mean-dependent inequality measure) in Appendix B, in Table 9. Results obtained with the Gini measure are very similar to those obtained with the percentile share ratios.

We find that the effect of a GDP growth shock in all three tables accounts for similar amounts of the variation in inequality at under 2% and by comparison, slightly larger of the variance of terms of trade. An inequality shock also accounts for a very small amounts of variation in GDP growth (under 2%), and 3% for the terms of trade. By contrast, the terms of trade shock seems to have a more perceptile effect on inequality and with some variation in its contribution for the different measures of inequality. The effects of all the shocks are thus quite comparable for China and the USA.

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	86.97%	1.94%	11.1%
	[0.00, 0.06]	[0.00,0.02]	[0.01,0.03]
GDP growth	1.78%	97.36%	0.86%
	[0.03, 0.31]	[0.12,0.69]	[0.0001,0.11]
Terms of trade	1.3%	8.63%	90.07%
	[0.0001, 0.41]	[0.00,1.32]	[0.07,1.81]
Notes	Estimated contribution of each structural shock to the 4-year--ahead median squared forecast error.		

Table 5: USA, decomposition of variance of 4-year-ahead forecast errors, using inequality measure percentile share ratio (50:90)

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	74.63%	1.78%	23.59%
	[0.001, 0.04]	[0.00001,0.0002]	[0.00001,0.03]
GDP growth	1.24%	97.72%	1.04%
	[0.00001, 0.05]	[0.34,0.82]	[0.0001,0.04]
Terms of trade	4.82%	10.50%	84.68%
	[0.0001, 0.23]	[0.01,0.43]	[0.04,1.74]
Notes	Estimated contribution of each structural shock to the 4-year--ahead median squared forecast error.		

Table 6: USA, decomposition of variance of 4-year-ahead forecast errors, using inequality measure percentile share ratio (10:90)

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	92.00%	1.49%	6.51%
	[0.001, 0.1]	[0.0001,0.002]	[0.0001,0.003]
GDP growth	0.83%	98.53%	0.65%
	[0.00001, 0.04]	[0.34,0.82]	[0.0001,0.03]
Terms of trade	6.41%	0.83%	92.76%
	[0.01, 0.35]	[0.0001,0.11]	[0.81,1.88]
Notes	Estimated contribution of each structural shock to the 4-year--ahead median squared forecast error.		

Table 7: USA, decomposition of variance of 4-year-ahead forecast errors, using inequality measure percentile share ratio (99:100)

5 Discussion

There are several findings from our estimations which matter for both the policy maker and the empirical researcher.

- First, we find that a growth shock is inequality-increasing and an inequality shock is growth-reducing. We obtain this effect for both the USA and China. The two countries having very different policy structures and in particular, China's drive towards poverty alleviation, does not seem to impact upon this salient relationship. This finding conforms with much of the earlier cross country literature, as discussed in Section 2.
- Second, perhaps the most striking finding is that the size of the effect of a GDP growth shock on inequality is very small. We also observe a similar small effect of an inequality shock on GDP growth. This is the case for both China and the USA.[17] The effect of a terms of trade shock on inequality is larger in magnitude for all cases studied, for some cases by a factor of ten.

For China, the size of the effect of a growth shock on inequality explains under 2% of the variance in inequality for all the inequality measures tested. Likewise, for the USA, a growth shock also explains a similarly small amount of the variance of inequality; namely, under 2%. The effect of an inequality shock is

[17] This result is also borne out with our empirical results with other countries that we have tested (namely, the UK and France, not presented)

also very small for both countries. It is quite remarkable that the size of these effects for both countries are very similar, in spite of the two countries having different socio-economic structures. The differences in the results for different inequality measures are also negligible.

This particular result of the size of the effect of GDP growth on inequality being small may be due to the fact of the lack of social mobility that has been highlighted for both the US and China. The effects of a GDP growth spurt may take a very long time to channel into having an impact upon inequality. At worst, the required channels via which a growth spurt were to reduce inequality may not be adequately available for these countries.

- Finally, we observe that the impact for all of the shocks (namely, growth, inequality and terms of trade) do not persist for very long. For both countries, and for all variables in the model, the shocks taper off in their effects within ten to fifteen years at the most. This result accords with that observed in Bandyopadhyay (2020) for a number of developed countries such as Denmark, Switzerland and the UK.

For the policy maker, the third finding, that the effects of the shocks are short, is good news. However, it also means that the socio-economic effects of the shock needs to be dealt with in the immediate aftermath of the shock, which is bad news. For all the impulse responses generated, there is an immediate increase in inequality after a growth shock (and an immediate drop in GDP growth after an inequality shock). Further modelling is thus required to identify which aspects of social wellbeing or which macroeconomic variables are most immediately impacted upon due to a growth or inequality shock.

That the size of the effects of inequality and growth shocks upon each other is only 2% calls for a discussion. In terms of the simple arithmetic of the variance decomposition, it is easy to see from our empirical results that a terms of trade shock has a much larger impact upon inequality and growth compared to that of growth and inequality on each other for both countries. For the US, in Tables 5 and 6, 11% and 23% of the variance in inequality is explained by the terms of trade shock, respectively. For the China results, around 5% of variation in inequality is explained by a terms of trade shock. Compared to that a GDP growth shock explains only 1.78% of the variation in inequality for both countries, and around 1% of growth is explained by an inequality shock. Other studies using this approach (for example Baumeister and Hamilton 2018) in measuring the effects on monetary policy, domestic demand and supply shocks on inflation, reveal that 69% of variation in inflation is explained by a supply shock and 28% is determined by a demand shock. In contrast, a monetary policy shock only explains 5%. Comparing our findings to these statistics for fiscal and monetary shocks, it is thus easy to conclude that the effects of inequality and growth shocks on each other are thus quite small.

That a growth shock or an inequality shock explains so little variation of inequality or growth, respectively, however raises a worrying concern. The small size implies that these shocks are therefore impacting upon other macroeconomic variables that are not included in the model; variables which have a clearer and direct impact upon social wellbeing or other macroeconomic aspects. In our estimations, for example, we find that the terms of trade shock explains the variation in inequality to a greater extent than a growth shock. Thus, "inequality and growth" empirical studies employing models that are solely devoted to the effect of changes in inequality upon growth, (or changes in growth on inequality), should model a more elaborate system of equations, identifying pathways of the impact of growth and/or inequality shocks on a large number of variables.

In addition, the assessments undertaken in the paper would ideally be complemented with more countries' examples; for example, some fast growing small economies at different stages of development, with high or low inequalities. Smaller countries and middle income/rank developing countries may have a different experience, and the size of a growth shock on inequality and vice versa may be larger. Some possible examples that may be interesting to study could be Vietnam, Korea, Botswana, South Africa, Argentina, Chile or Brazil to name a few.

6 Conclusion

In this paper we have examined the relationship between inequality and growth by estimating the impact of their individual shocks for two large countries, China and the USA. We use a Bayesian vector autoregression approach following Baumeister and Hamilton (2015, 2018) to estimate the size and direction of the effects of the shocks. As a point of departure from the literature, we assume the priors of the structural parameters and the covariance of the error terms to be independent. We conclude three salient findings. First, we find that a growth shock on inequality is inequality-increasing. We also find that an inequality shock is growth-reducing. These two results conform with much of the empirical literature.

The second salient finding is that the size of these effects are strikingly small. Variance decompositions reveal that at the most 2% of the variations in growth are explained by an inequality shock. Likewise, we find that under 2% of variation in inequality are explained by a growth shock.

The third finding is that the effect of these shocks dissipate within ten years. The results are remarkably similar for both countries. This result is also borne out in Bandyopadhyay (2020), where the effects of a growth shock on inequality also dissipates in around ten years for three major developed countries (namely, Denmark, Switzerland and UK). To ensure the robustness of our results, we use up to five inequality measures for the analysis, namely several percentile share ratios, each representing different parts of the income distribution. It is interesting that for both China and the USA the results are very similar for all inequality measures.

The results obtained in the paper, for the first time using these novel methods, suggest that the inequality and growth relationship is a highly individual experience for each country and that it would be ideal for researchers and policy makers to analyse countries on an individual basis, rather than relying upon a generalised result for all countries using a cross country approach. While the findings in this paper accord with several conclusions in the empirical literature, one of the striking findings in this paper is that these effects have a short to medium term effect only. The effect of both growth and inequality shocks last for under ten years. It is possible developed and developing countries will have different response times to these shocks.

7 Supplementary material

Appendices A and B are available below as supplementary materials provided with the paper.

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Appendix A

Proof of Proposition 1. The log-likelihood function conditional on Gaussian residuals is given by:

$$L(X_N|B,C,F) = (2\pi)^{\frac{3N}{2}}|C|_+^N|B|_+^{-\frac{N}{2}}\prod_{i=1}^{N} e^{-\sum_{t=1}^{N}(x_t^{'}c_i - z_{t-1}^{'}f_i)^2/2b_{ii}}$$

Given the Cholesky decomposition of $m_i^{-1} = A_i A_i^{'}$, the prior of f_i conditional on C is given by

$$\pi(f_i|C) = (2\pi)^{-\frac{13}{2}}|m_i|^{-\frac{1}{2}}e^{-\frac{(\zeta_i-\theta_i f_i)^{'}(\zeta_i-\theta_i f_i))}{2}} \tag{34}$$

where $\zeta_i = A_i^{'}a_i\sqrt{b_{ii}}$ and $\theta_i = A_i^{'}\sqrt{b_{ii}}$. The prior distribution of f_i conditional on C can be integrated with $\bar{a}_i$ defined in 18. Moreover, $\bar{X}_i - \bar{Z}_i\bar{a}_i$ is not correlated with $\bar{Z}_i$ and the quadratic term in the prior of F can be calculated as

$$\begin{aligned}
(\bar{X}_i - \bar{Z}_i f_i)^{'}(\bar{X}_i - \bar{Z}_i f_i) &= (\bar{X}_i - \bar{X}_i\bar{a}_i + \bar{Z}_i\bar{a}_i - \bar{Z}_i f_i)^{'}(\bar{X}_i - \bar{X}_i\bar{a}_i + \bar{Z}_i\bar{a}_i - \bar{Z}_i f_i) \\
&= (\bar{X}_i - \bar{Z}_i\bar{a}_i)^{'}(\bar{X}_i - \bar{Z}_i\bar{a}_i) + (f_i - \bar{a}_i)^{'}\bar{Z}_i^{'}\bar{Z}_i(f_i - \bar{a}_i) \\
&= \bar{X}_i^{'}\bar{X}_i - \bar{X}_i^{'}\bar{Z}_i(\bar{Z}_i^{'}\bar{Z}_i)^{-1}\bar{Z}_i^{'}\bar{X}_i + (f_i - \bar{a}_i)^{'}\bar{Z}_i^{'}\bar{Z}_i(f_i - \bar{a}_i) \\
&= \bar{\Xi}_i + (f_i - \bar{a}_i)^{'}\bar{m}_i^{-1}(f_i - \bar{a}_i)
\end{aligned} \tag{35}$$

where $\bar{\Xi}_i$ is defined as:

$$\bar{\Xi}_i = \bar{X}_i^{'}\bar{X}_i - \bar{X}_i^{'}\bar{Z}_i(\bar{Z}_i^{'}\bar{Z}_i)^{-1}\bar{Z}_i^{'}\bar{X}_i \tag{36}$$

and $(\bar{Z}_i^{'}\bar{Z}_i)$ is replaced by $\bar{m}_i$ because of 19. Since $\pi(F|C) = \prod_{i=1}^{3}\pi(f_i|C)$, the product of $\pi(F|C)$ and the log-likelihood function of X_N can be defined as:

$$L(X_N|B,C,F)\pi(F|C) = (2\pi)^{\frac{3N}{2}}|C|_+^N|B|_+^{-\frac{N}{2}}\prod_{i=1}^{3}(2\pi)^{-\frac{13}{2}}|m_i|^{-\frac{1}{2}}e^{-\frac{\bar{\Xi}_i+(f_i-\bar{a}_i)^{'}\bar{m}_i^{-1}(f_i-\bar{a}_i)}{2}} \tag{37}$$

Moreover, we can obtain the prior of B, C, F and observation X_N by calculating the product of $\pi(C)$, $\pi(B|C)$, and $L(X_N|B,C,F)\pi(F|C)$

$$\begin{aligned}
\pi(B,C,F,X_N) &= \pi(C)\pi(B|C)\pi(F|C)\pi(X_N|B,C,F) \\
&= \pi(C)(2\pi)^{-\frac{3N}{2}}|C|_+^N\prod_{i=1}^{3}\frac{b_{ii}^{-\frac{N}{2}}}{\lambda_i}(2\pi)^{-\frac{13}{2}}m_i^{-\frac{1}{2}}e^{-\frac{\bar{\Xi}_i+(f_i-\bar{a}_i)^{'}\bar{m}_i^{-1}(f_i-\bar{a}_i)}{2}} \\
&= \pi(C)(2\pi)^{-\frac{3N}{2}}|C|_+^N\prod_{i=1}^{3}\frac{b_{ii}^{-\frac{N}{2}}}{\lambda_i e^{\frac{\bar{\Xi}_i}{2}}}(2\pi)^{-\frac{13}{2}}\frac{m_i^{-\frac{1}{2}}}{\bar{m}_i^{-\frac{1}{2}}}e^{-\frac{(f_i-\bar{a}_i)^{'}\bar{m}_i^{-1}(f_i-\bar{a}_i)}{2}} \\
&= \pi(C)(2\pi)^{-\frac{3N}{2}}|C|_+^N\prod_{i=1}^{3}\frac{\frac{2\bar{\Xi}_i}{N}b_{ii}^{-\frac{N}{2}}m_i^{-\frac{1}{2}}}{\bar{m}_i^{-\frac{1}{2}}\lambda_i e^{\frac{\bar{\Xi}_i}{2}}}f(b_{ii};\bar{\lambda}_i)N(f_i;\bar{a}_i,\bar{m}_i)
\end{aligned} \tag{38}$$

Equation 38 can be transformed into:

$$\pi(B,C,F,X_N) = \pi(X_N)\pi(C|X_N)\pi(B|C,X_N)\pi(F|C,X_N) \tag{39}$$

Therefore, the posterior distribution of f_i and b_{ii} are given by:

$$\begin{aligned} \pi(F|C,X_N) &= \prod_{i=1}^{3} N(f_i;\bar{a}_i,\bar{m}_i) \\ \pi(B|C,X_N) &= \prod_{i=1}^{3} f(b_{ii},\bar{\lambda}_i) \end{aligned} \tag{40}$$

The remaining term $\pi(X_N)\pi(C|X_N)$ is defined as:

$$\pi(X_N)\pi(C|X_N) = \pi(C)(2\pi)^{-\frac{3N}{2}}|C|_+^N \prod_{i=1}^{3} \frac{2^{\frac{\bar{\Xi}_i}{N}} b_{ii}^{-\frac{N}{2}} m_i^{-\frac{1}{2}}}{\bar{m}_i^{-\frac{1}{2}}\lambda_i e^{\frac{\bar{\Xi}_i}{2}}} \tag{41}$$

From 41, C's posterior distribution conditional on X_N, $\pi(C|X_N)$ is a linear function in the expression on the right hand side of 41.

$$\pi(C|X_N) = \pi(X_N)^{-1}\pi(C)(2\pi)^{-\frac{3N}{2}}|C|_+^N \prod_{i=1}^{3} \frac{\frac{2\bar{\Xi}_i}{N} b_{ii}^{-\frac{N}{2}} m_i^{-\frac{1}{2}}}{\bar{m}_i^{-\frac{1}{2}}\lambda_i e^{\frac{\bar{\Xi}_i}{2}}} \tag{42}$$

Because the residuals are not related to the prior of C, we can normalize 42 by $\hat{\Sigma}_N$ and let α_N be the adjustment constant to ensure that the integration of the posterior distribution $\pi(C|X_N)$ is 1. Thus we obtain the expression of $\pi(C|X_N)$ stated in Proposition 1

$$\pi(C|X_N) = \frac{\alpha_N \pi(C)(2\pi)^{-\frac{3N}{2}}|C\hat{\Sigma}_N C'|_+^{-\frac{N}{2}}}{\prod_{i=1}^{3}\bar{\lambda}_i^2/6N} \prod_{i=1}^{3} \frac{2^{\frac{\bar{\Xi}_i}{N}} b_{ii}^{-\frac{N}{2}} m_i^{-\frac{1}{2}}}{\bar{m}_i^{-\frac{1}{2}}\lambda_i e^{\frac{\bar{\Xi}_i}{2}}} \tag{43}$$

Q.E.D

Proof of Lemma 1. Given a structural VAR defined by equation 1, the covariance matrix of the posterior f_i is:

$$E[(f_i-\bar{a}_i)(f_i-\bar{a}_i)'|C,X_N] = \frac{1}{N(N^{-1}\sum_{t=1}^{N} z_{t-1}z_{t-1}' + \frac{m_i^{-1}}{N})} \tag{44}$$

since x_t has a bounded second moment and m_i is finite, the expression above converges to 0 as N goes to infinity. Moreover, $\bar{a}_i$ is the regression coefficient of $c_i^{'}x_t$ on z_{t-1}

$$\begin{aligned}
\bar{a}_i &= lim_{N\to\infty}(\frac{1}{N}\sum_{j=1}^{N} z_{j-1}z_{j-1}^{'} + \frac{m_i^{-1}}{N})^{-1}(\frac{1}{N}\sum_{j=1} Nz_{j-1}x_j^{'}c_i + \frac{m_i^{-1}a_i}{N}) \\
&= lim_{N\to\infty}(\frac{1}{N}\sum_{j=1}^{N} z_{j-1}z_{j-1}^{'})^{-1}(\frac{1}{N}\sum_{j=1} Nz_{j-1}x_j^{'}c_i) \\
&= E(z_{t-1}z_{t-1}^{'})^{-1}E(z_{t-1}x_t^{'}c_i)
\end{aligned} \tag{45}$$

Therefore the convergence of F is obtained as:

$$F = (f_1, f_2, f_3)^{'} \to CE(x_t z_{t-1}^{'})E(z_{t-1}z_{t-1}^{'})^{-1} \tag{46}$$

According to Proposition 1, $\frac{\bar{\Xi}_i}{N}$ is:

$$\begin{aligned}
\frac{\bar{\Xi}_i}{N} &= \frac{\bar{X}_i^{'}\bar{X}_i - \bar{X}_i^{'}\bar{Z}_i(\bar{Z}_i^{'}\bar{Z}_i)^{-1}\bar{Z}_i^{'}\bar{X}_i}{N} = \frac{N^{-1}\sum_{t=1}^{N} c_i^{'}x_t x_t^{'}c_i + N^{-1}a_i^{'}m_i^{-1}a_i/b_{ii}}{N} \\
&- (N^{-1}\sum_{t=1}^{N} c_i^{'}x_t z_{t-1}^{'} + N^{-1}a_i^{'}m_i^{-1}/b_{ii})(N^{-1}\sum_{t=1}^{N} z_{t-1}z_{t-1}^{'} + N^{-1}m_i^{-1}/b_{ii})^{-1}\times \\
&(N^{-1}\sum_{t=1}^{N} z_{t-1}x_t^{'}c_i + N^{-1}m_i^{-1}a_i/b_{ii})
\end{aligned} \tag{47}$$

since m_i, a_i and b_{ii} are finite, the expression above converges to $c_i^{'}\Sigma^* c_i$ as N goes to infinity. The posterior second moment of b_{ii} is calculated as:

$$E[(b_{ii} - \frac{\bar{\lambda}_i}{2})^2|X_N] = \frac{\bar{\lambda}_i^2}{12} = \frac{\bar{\Xi}_i^2}{3N} \to 0 \tag{48}$$

Because $\bar{\Xi}_i/N$ converges to a non-zero value as proven above, 48 goes to 0 when N goes to infinity. Moreover, the convergence of the posterior expectation of b_{ii} is given by:

$$\frac{2\bar{\Xi}_i}{2N} \to c_i^{'}\Sigma^* c_i \tag{49}$$

The convergence is achieved because of 55. Thus all the claims in Lemma 1 are proven. **Q.E.D**

Proof of Proposition 2. According to the Cholesky decomposition of $C\Sigma C^{'} = A(C,\Sigma)A(C,\Sigma)^{'}$, the determinant of $C\Sigma C^{'}$ can be expressed as:

$$|C\Sigma C^{'}| = \prod_{i=1}^{3} A(C,\Sigma)_{ii}^2 \tag{50}$$

And the diagonal element of $C\Sigma C'$ is given by:

$$c_i'\Sigma c_i = \sum_{j=1}^{i} A(C,\Sigma)_{ij}^2 \tag{51}$$

Based on Proposition 1, the posterior $\bar{\lambda}_i$ can be expressed as a linear combination of prior λ_i and $\bar{\Xi}_i$.

$$\bar{\lambda}_i = \lambda_i + e^{\bar{\Xi}} \tag{52}$$

Because $\bar{\Xi}_i$ is the sum of residuals obtained by regressing $\bar{X}_i$ on $\bar{Z}_i$, therefore it is greater than $Nc_i'\hat{\Sigma}_N c_i$. Combining 52 with 51, the following inequality is true:

$$\begin{aligned} \bar{\lambda}_i \geq e^{Nc_i'\hat{\Sigma}_N c_i} &\rightarrow ln(\bar{\lambda}_i) \geq Nc_i'\hat{\Sigma}_N c_i \\ &\rightarrow ln(\bar{\lambda}_i) \geq N\sum_{j=1}^{i} A(C,\hat{\Sigma}_N)_{ij}^2 \end{aligned} \tag{53}$$

If $C \notin U_\epsilon(\hat{\Sigma}_N|X_N)$, then $A(C,\hat{\Sigma}_N)_{ij}^2 \geq \frac{\epsilon}{3}$ for any $i > j$. Therefore, $c_i'\hat{\Sigma}_N c_i > A(C,\Sigma_N)_{ii}^2 + \frac{\epsilon}{3}$. Combining with 53 and the assumption about $\bar{\lambda}_i$, the following inequality is true:

$$\prod_{i=1}^{3} \frac{\bar{\lambda}_i^2}{6N} \geq \prod_{i=1}^{3} ln(\bar{\lambda}_i)/N \geq \prod_{i=1}^{3} c_i'\hat{\Sigma}_N c_i > \prod_{i=1}^{3} A(C,\hat{\Sigma}_N)_{ii}^2 \tag{54}$$

According to the posterior distribution derived by Proposition 1, the probability of $C \notin U_\epsilon(\hat{\Sigma}_N|X_N)$ can be written as:

$$\begin{aligned} &\int_{C\notin U_\epsilon(\hat{\Sigma}_N|X_N)} \frac{\alpha_N \pi(C)(2\pi)^{-\frac{3N}{2}}(C\hat{\Sigma}_N C')}{\prod_{i=1}^{3}\bar{\lambda}_i^2/6N} \prod_{i=1}^{3} \frac{\frac{2\bar{\Xi}_i}{N} b_{ii}^{-\frac{N}{2}} m_i^{-\frac{1}{2}}}{\bar{m}_i^{-\frac{1}{2}} \lambda_i e^{\frac{\bar{\Xi}_i}{2}}} dC \\ &\leq \int_{C\notin U_\epsilon(\hat{\Sigma}_N|X_N)} \frac{\alpha_N \pi(C)(2\pi)^{-\frac{3N}{2}}}{\prod_{i=1}^{3}\bar{\lambda}_i^2/6N} \left(\frac{\prod_{i=1}^{3} A(C,\hat{\Sigma}_N)_{ii}^2}{\prod_{i=1}^{3}\frac{\epsilon}{3} + A(C,\hat{\Sigma}_N)_{ij}^2}\right)^{\frac{N}{2}} \prod_{i=1}^{3}\prod_{i=1}^{3} \frac{\frac{2\bar{\Xi}_i}{N} b_{ii}^{-\frac{N}{2}} m_i^{-\frac{1}{2}}}{\bar{m}_i^{-\frac{1}{2}} \lambda_i e^{\frac{\bar{\Xi}_i}{2}}} dC \rightarrow 0 \end{aligned} \tag{55}$$

Since b_{ii} λ_i and m_i are finite, $\bar{m}_i$ and $\frac{2\bar{\Xi}_i}{N}$ are bounded according to Proposition 1, the expression in the second line of 55 goes to 0 as N goes to infinity. When $\hat{\Sigma}_N$ converges to Σ^*, the probability that C is not in $U_\epsilon(\Sigma^*)$ follows:

$$\begin{aligned} \pi(C \notin U_\epsilon(\Sigma^*)) &= \pi(d(C,\Sigma^*) > \epsilon)) \\ &\leq \pi([2d(C,\hat{\Sigma}_N) + 2\sum_{i=2}^{3}\sum_{j=1}^{i-1}(A(C,\Sigma^*)_{ij} - A(C,\hat{\Sigma}_N)_{ij})^2] > \epsilon|X_N) \\ &\leq \pi(2d(C,\hat{\Sigma}_N) > \frac{\epsilon}{2}|X_N) + \pi(2\sum_{i=2}^{3}\sum_{j=1}^{i-1}(A(C,\Sigma^*)_{ij} - A(C,\hat{\Sigma}_N)_{ij})^2 > \frac{\epsilon}{2}|X_N) \end{aligned} \tag{56}$$

$\pi(2d(C,\hat{\Sigma}_N) > \frac{\epsilon}{2}|X_N)$ converges to 0 due to 55. $\pi(2\sum_{i=2}^{3}\sum_{j=1}^{i-1}(A(C,\Sigma^*)_{ij} - A(C,\hat{\Sigma}_N)_{ij})^2 > \frac{\epsilon}{2}|X_N)$

converges to 0 because of the $\hat{\Sigma}_N$ converges to Σ^* by construction. Therefore, $\pi(2d(C,\hat{\Sigma}_N) > \frac{\epsilon}{2}|X_N) + \pi(2\sum_{i=2}^{3}\sum_{j=1}^{i-1}(A(C,\Sigma^*)_{ij} - A(C,\hat{\Sigma}_N)_{ij})^2 > \frac{\epsilon}{2}|X_N)$ goes to 0 as N goes to infinity and $\pi(C \notin U_\epsilon(\Sigma^*))$ is zero as stated. **Q.E.D**

Metropolis Hastings algorithm for posterior draws The choices of priors of B, C, F are based on the features of our dataset. We first use an 8-th order univariate auto-regression on endogenous variables to obtain an initial covariance matrix of residuals denoted as R. Let $\hat{s}_{it}$ be the residual of variable i in period t according to the auto-regression above:

$$\hat{s}_{it} = x_{i,t} - \sum_{j=1}^{8}\hat{\beta}_j x_{i,t-j} \tag{57}$$

Then the ij-th element of R is denoted as $R_{ij} = \frac{\sum_{j=1}^{N}\hat{s}_{it}\hat{s}_{jt}}{N}$. Thus the prior of mean of b_{ii} is equal to the i-th diagonal element of CRC', implying $\lambda_i = 2CRC'_{ii}$.

As typically obtained in the empirical VAR literature, magnitudes of auto-regression coefficients are declining in order. This suggests that the diagonal elements of m_i corresponding to higher lags tend to be close to 0. Following Doan et.al (1984), we define the following structure:

$$\begin{aligned} t_1 &= (\frac{1}{1}, \frac{1}{2^{2\phi}}, ..\frac{1}{8^{\phi}})' \\ t_2 &= (R_{11}^{-1}, R_{22}^{-1}, R_{33}^{-1})' \\ t_3 &= \phi_1(t_1 \otimes t_2, \phi_2)' \end{aligned} \tag{58}$$

where the j-th diagonal element of m_i is equal to the j-th element in vector t_3: $m_{i,kk} = t_{3,k}$. The no-negative power index ϕ captures the confidence in the prior distribution, $\phi_1 \geq 0$ corresponds to our prior belief about zero higher order coefficients, and $\phi_2 \geq 0$ governs our prior belief about fixed term. In our analysis of economic growth and inequality, we let $\phi = 1.5$, $\phi_1 = 150$ and $\phi_2 = 0.5$.

Using the priors generated by auto-regression on endogenous variables, we create draws of C and B and draw B and F according to the posterior distribution given by Proposition 1. The objective function of the Metropolis Hastings algorithm for C is:

$$\begin{aligned} T(C) = log(\pi(C)) + \frac{N}{2}log(|C\hat{\Sigma}_N C'|) - \sum_{i=1}^{3} 3log(\bar{\Xi}(C)_i/N) \\ + \sum_{i=1}^{3}\frac{1}{2}log(\bar{m}(C)_i/m(C)_i) - \prod_{i=1}^{3}\lambda(C)_i + \sum_{i=1}^{3}\frac{\bar{\Xi}(C)_i}{2} \end{aligned} \tag{59}$$

According to the priors generated by auto-regression, $\lambda_i(C) = c_i' R c_i$ and $\bar{\Xi}(C)_i = c_i' R c_i$. And R is given by

$$R = \sum_{t=1}^{N} x_t x_t' + \iota m^{-1}\iota' - (\sum_{t=1}^{N} x_t z_{t-1}' + \iota m^{-1})(\sum_{t=1}^{N} z_{t-1}z_{t-1}' + m^{-1})^{-1}(\sum_{t=1}^{N} z_{t-1}x_t' + m^{-1}\iota') \tag{60}$$

where m is the matrix obtained by auto-regression with $m_{i,kk} = t_{3,k}$ and ι is a 3×13 matrix defined as:

$$\iota = [B, 0] \tag{61}$$

in which B is a diagonal matrix with $B_{ii} = b_{ii}$ for $1 \leq i \leq 3$.

We first construct a 9×1 vector, ω containing all the structural parameters that need to be estimated $(c_{12}, c_{13}, c_{21}, c_{23}, c_{31}, c_{32})$. The initial value of ω in our quantitative analysis is the value maximizing 59 denoted as ω_0. Regarding the Student's t distribution that we adopt for our priors, the scale can be denoted as:

$$\hat{\zeta} = \frac{d^2 T(C(\omega_0))}{d\omega d\omega'} \tag{62}$$

Based on this initial value, the Metropolis Hastings algorithm can help us draw ω according to the posterior of C derived in Table 1. Let the value of ω in the initial loop be $\omega_0 = \hat{\omega}$. The algorithm updates the draw of ω in the following manner:

$$\omega_s = \omega_{s-1} + q(A(\hat{\zeta}))' p_t \tag{63}$$

where $A(\hat{\zeta})A(\hat{\zeta})' = \hat{\zeta}$ is the Cholesky factor of $\hat{\zeta}$ and p_t denotes a 9 variables draw from a student distribution with 3 degrees of freedom. q is a tuning parameter to control the updating process. When $T(C(\omega_s)) < T(C(\omega)_{s-1})$, $\omega_s = \omega_{s-1}$ with probability $\frac{T(C(\omega_s))}{T(C(\omega)_{s-1})}$. Otherwise, $\omega_s = \omega_s$. q in our analysis is set to ensure the probability that a new draw will be kept is 0.5. Finally, the candidates of ω_s after $K_0 = 10000$ burn-in draws is the size $K_1 = 50000$ draw based on the posterior distribution given by Table 1. After generating the draw of ω which represents C, the candidate draw of B and F can be drawn based on their posterior distribution derived in Proposition 1

Appendix B

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	59.22%	8.89%	31.89%
	[0.23, 2.23]	[0.03,0.67]	[0.19,0.91]
GDP growth	1.03%	96.72%	2.25%
	[0.00001, 0.003]	[0.01,0.02]	[0.0001,0.01]
Terms of trade	3.48%	6.04%	90.48%
	[0.01, 0.21]	[0.41,1.98]	[2.79,6.13]
Notes	Estimated contribution of each structural shock to the 4-year--ahead median squared forecast error.		

Table 8: USA, decomposition of variance of 4-year-ahead forecast errors, using inequality measure percentile share ratio (99:100)

	Inequality shock	GDP growth shock	Terms of trade shock
Inequality	95.27%	4.12%	0.61%
	[0.01, 0.6]	[0.00001,0.0001]	[0.00001,0.0001]
GDP growth	2.02%	96.79%	1.20%
	[0.00003, 0.00003]	[0.01,0.01]	[0.0001,0.0001]
Terms of trade	4.83%	3.85%	91.32%
	[0.03, 0.71]	[0.01,0.58]	[2.32,5.36]
Notes	Estimated contribution of each structural shock to the 4-year--ahead median squared forecast error.		

Table 9: USA, decomposition of variance of 4-year-ahead forecast errors, using inequality measure percentile share ratio (99:100)

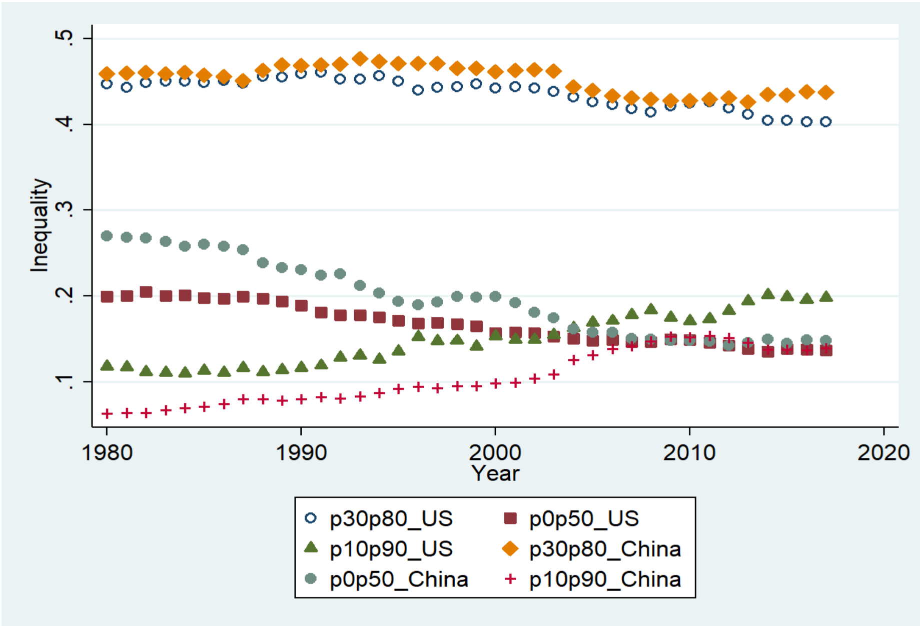

Figure 9: Trends of selected inequality measures (percentile share ratios) for China and USA for 1979 to 2018.

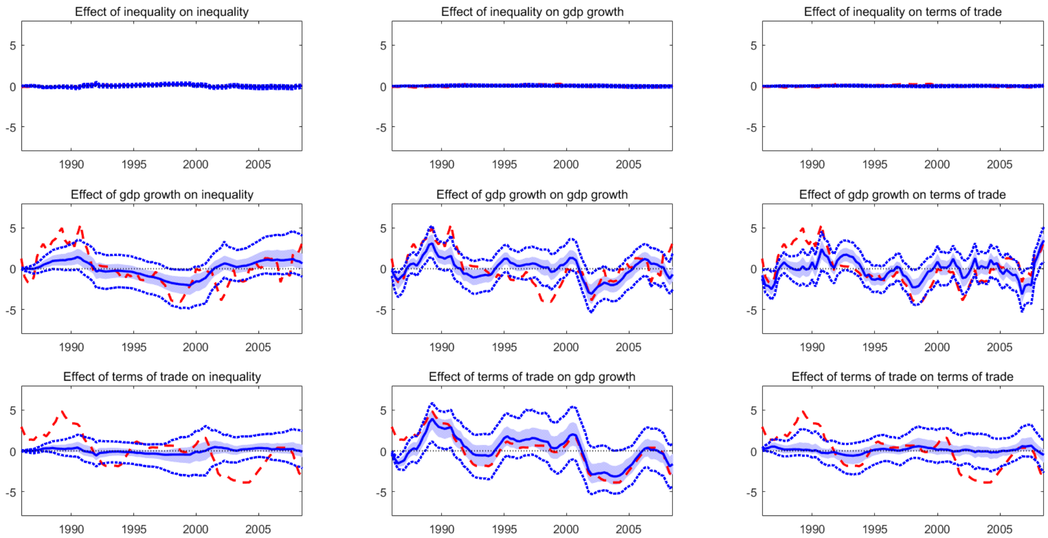

Figure 10: China' portion of historical variation in inequality (using the Gini), GDP growth and terms of trade attributed to each of the structural shocks. Dashed red: actual value for the deviation of the variable of interest from its mean. Solid blue: portion attributed to indicated structural shock. Shaded regions: 75% posterior credibility sets. Dotted blue: 95% posterior credibility sets.

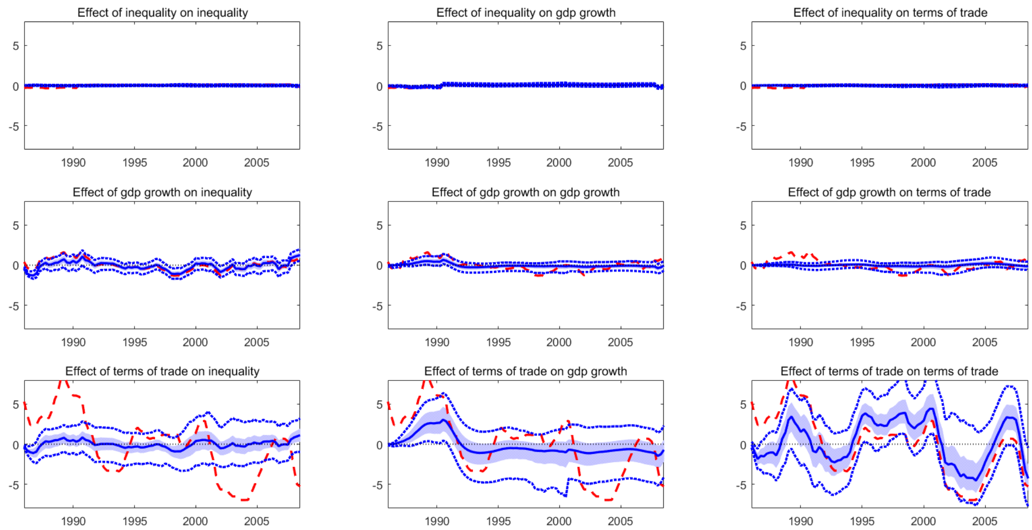

Figure 11: United States' portion of historical variation in inequality (using the Gini), GDP growth and terms of trade attributed to each of the structural shocks. Dashed red: actual value for the deviation of the variable of interest from its mean. Solid blue: portion attributed to indicated structural shock. Shaded regions: 75% posterior credibility sets. Dotted blue: 95% posterior credibility sets.