Week 7: Extensions to the basic New Keynesian Model: introducing Sticky Wages (following Galí 2007)

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November 2008

1 The New Keynesian Model with Sticky Wages

- There is a continuum of differentiated labor services, all of which are used by each firm.
- Each household is specialized in one type of labor, which it supplies monopolistically. That is, workers (or their unions) are wage setters.
- But they cannot reset the wages in every period. That is, wages are sticky.
- Technology of firm i

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

uses labour of all types (j)

$$N_t(i) = \left[\int_0^1 N_t\left(i, j\right)^{\frac{\varepsilon w - 1}{\varepsilon w}} dj \right]^{\frac{\varepsilon w}{\varepsilon w - 1}}$$

- Cost minimization (given a certain expenditure in their wage bill this is the choice of firms for each type of labor)

$$N_t\left(i,j\right) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t(i)$$

where

$$W_t = \left[\int_0^1 W_t(j)^{1-\varepsilon_w} dj\right]^{\frac{1}{1-\varepsilon_w}}$$

and

$$\int_0^1 W(j) N_t\left(i,j\right) = W_t N_t(i)$$

- Optimal Price setting (as before)

$$\max \sum_k E_t \theta_p^k Q_{t.t+k} \left[P_t^* Y_{t+k,t} - \Psi(Y_{t+k,t}) \right] = 0$$

where

$$Y_{t+k,t}(j) = \left(\frac{P_t^*)}{P_{t+k}} \right)^{-\varepsilon_p} C_{t+k}$$

- Aggregation - price inflation:

$$\pi_t^p = \lambda_p \widehat{mc}_t + \beta E_t \pi_{t+1}^p$$

where $\lambda_p \equiv \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}$.

- or we can write the inflation equation as a function of the (log) deviation of the average price markup from its desired (or steady state) value, (defined as the price relative to marginal cost, so is the inverse of the real marginal cost):

$$\pi_t^p = -\lambda_p \widehat{\mu}_t^p + \beta E_t \pi_{t+1}^p$$

where

$$\widehat{\mu}^p = -\widehat{mc} \text{ and } \mu^p = \frac{\varepsilon_p}{\varepsilon_p - 1}$$

Households

- Intertemporal condition is the same as before

$$c_t = E_t c_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - \rho)$$

- But only a fraction of households/trade unions adjusting nominal wages in any given period: $1 - \theta_w$
- θ_w [0; 1] : index of wage stickiness
- Optimal wage setting
- max the expected discounted sum of utilities generated over the (uncertain) period during which the wage remains unchanged at the level W_t set in the current period. Note that the utility generated under any other wage set in the future is irrelevant from the point of view of the optimal setting of the current wage, and can thus be ignored.

$$\max U(C_{t+k/t}; N_{t+k/t})$$

- subject to (monopolistic competitive labor supplier knows the form of the demand for its labor type)

$$N_{t+k/t} = \left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}$$

(this the demand at period $t+k$ for a labor of an agent that last reset its wage at period t)

- and

$$C_{t+k/t}P_t + E_{t+k}(Q_{t+k,t+k+1}D_{t+k}) \leq D_{t+k/t} + W_t^* N_{t+k/t} - T_{t+k}$$

where $C_{t+k/t}$ is the consumption at period $t+k$ of an agent that reset its wage at period t

$$N_t \equiv \int_0^1 N_t(i)\, di$$

Optimalilty condition

$$\sum_k (\beta\theta_w)^k E_t \left[N_{t+k/t} \begin{pmatrix} U_c(C_{t+k/t}; N_{t+k/t})\frac{W_t^*}{P_t} \\ +\mathcal{M}_w U_n(C_{t+k/t}; N_{t+k/t}) \end{pmatrix} \right] = 0$$

where $\mathcal{M}_w = \frac{\varepsilon_p}{\varepsilon_p - 1}$

- Given the complete markets assumption (all agents share their labor income risk, so consumption is identical across households)

$$C_{t+k/t} = C_{t+k}$$

- Letting

$$MRS_t = -U_n(C_t; N_t)/U_c(C_t; N_t)$$

$$\sum_k (\beta\theta_w)^k E_t \left[N_{t+k/t} U_c \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k/t} \right) \right] = 0$$

- Full wage flexibility:

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w MRS_t$$

which holds in steady state of the sticky price model

Log linearization of the wage setting equation

$$\sum_k (\beta\theta_w)^k E_t \left(w_t^* - p_{t+k} - mrs_{t+k/t} - \log(\mathcal{M}_w \right)) = 0$$

- Noting that

$$mrs_{t+k/t} = \varphi n_{t+k/t} + \sigma c_{t+k}$$

andwe have

$$mrs_{t+k/t} = mrs_{t+k} - \varphi \varepsilon_w (w_t^* - w_{t+k})$$

- So we can write

$$(1+\varphi\varepsilon_w)w_t^* = (1-\beta\theta_w)\sum_k (\beta\theta_w)^k E_t(p_{t+k}+mrs_{t+k}+\varphi\varepsilon_w w_{t+k}+\mu^w)$$

- Defining the wage markup as $\mu_t^w = w_t - p_t - mrs_t$

$$(w_t^* - w_t) = -\gamma_w \widehat{\mu}_t^w + \beta\theta_w E_t(w_{t+1}^* - w_t)$$

where

$$\gamma_w = \frac{1 - \beta\theta_w}{1 + \varphi\varepsilon_w}$$

- Finally, using the aggregate wage index

$$W_t = \left[\theta_w \left(W_{t-1}\right)^{1-\varepsilon_w} + \left(1 - \theta_w\right) W_t^{*1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}$$

we have

$$\pi_t^w = \left(1 - \theta_w\right)\left(w_t^* - w_{t-1}\right)$$

- So the wage setting equation can be written as

$$\pi_t^w = -\lambda_w \widehat{\mu}_t^w + \beta E_t \pi_{t+1}^w$$

where

$$\lambda_w = \frac{(1 - \beta\theta_w)(1 - \theta_w)}{\theta_w(1 + \varphi\varepsilon_w)}$$

- Some manipulation

$$\pi_t^p = -\lambda_p \widehat{\mu}_t^p + \beta E_t \pi_{t+1}^p$$

$$\pi_t^w = -\lambda_w \widehat{\mu}_t^w + \beta E_t \pi_{t+1}^w$$

and with flexible prices and wages

$$\widehat{\mu}_t^p = \widehat{\mu}_t^w = 0$$

- Defining the $\widetilde{y}$ and $\widetilde{\omega}$ as the output and real wage relative to their natural level, ie the level that would prevail in the absence of nominal (price and wage rigidity), we can write

$$\begin{aligned}
\widehat{\mu}_t^p &= p_t - w_t - mpn_t - \mu^p \\
&= -\omega_t - \log(1-\alpha) - y_t + n_t - \mu^p \\
&= -\omega_t - \log(1-\alpha) - \alpha/(1-\alpha) y_t - \mu^p \\
&= -\widetilde{\omega}_t - \alpha/(1-\alpha)\widetilde{y}_t
\end{aligned}$$

and

$$
\begin{aligned}
\widehat{\mu}_t^w &= w_t - p_t - mrs_t - \mu^w \\
&= \omega_t - \varphi n_t - \sigma y_t - \mu^w \\
&= \omega_t - \varphi/(1-\alpha) y_t - \sigma y_t - \mu^w \\
&= \widetilde{\omega}_t - (\varphi + \sigma(1-\alpha))/(1-\alpha)\widetilde{y}_t
\end{aligned}
$$

The equilibrium:

We can re-write the supply side equations in terms of the gaps

$$\pi_t^p = \kappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t + \beta E_t \pi_{t+1}^p$$

$$\pi_t^w = \kappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t + \beta E_t \pi_{t+1}^w$$

where $\kappa_p = \lambda_p \frac{\alpha}{1-\alpha}$ and $\kappa_w = \lambda_w \frac{\varphi + \sigma(1-\alpha)}{1-\alpha}$

The wage identity

$$\widetilde{\omega}_t - \widetilde{\omega}_{t-1} = \pi_t^w - \pi_t^p + \Delta w_t^n$$

IS

$$\widetilde{y}_t = E_t \widetilde{y}_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^n)$$

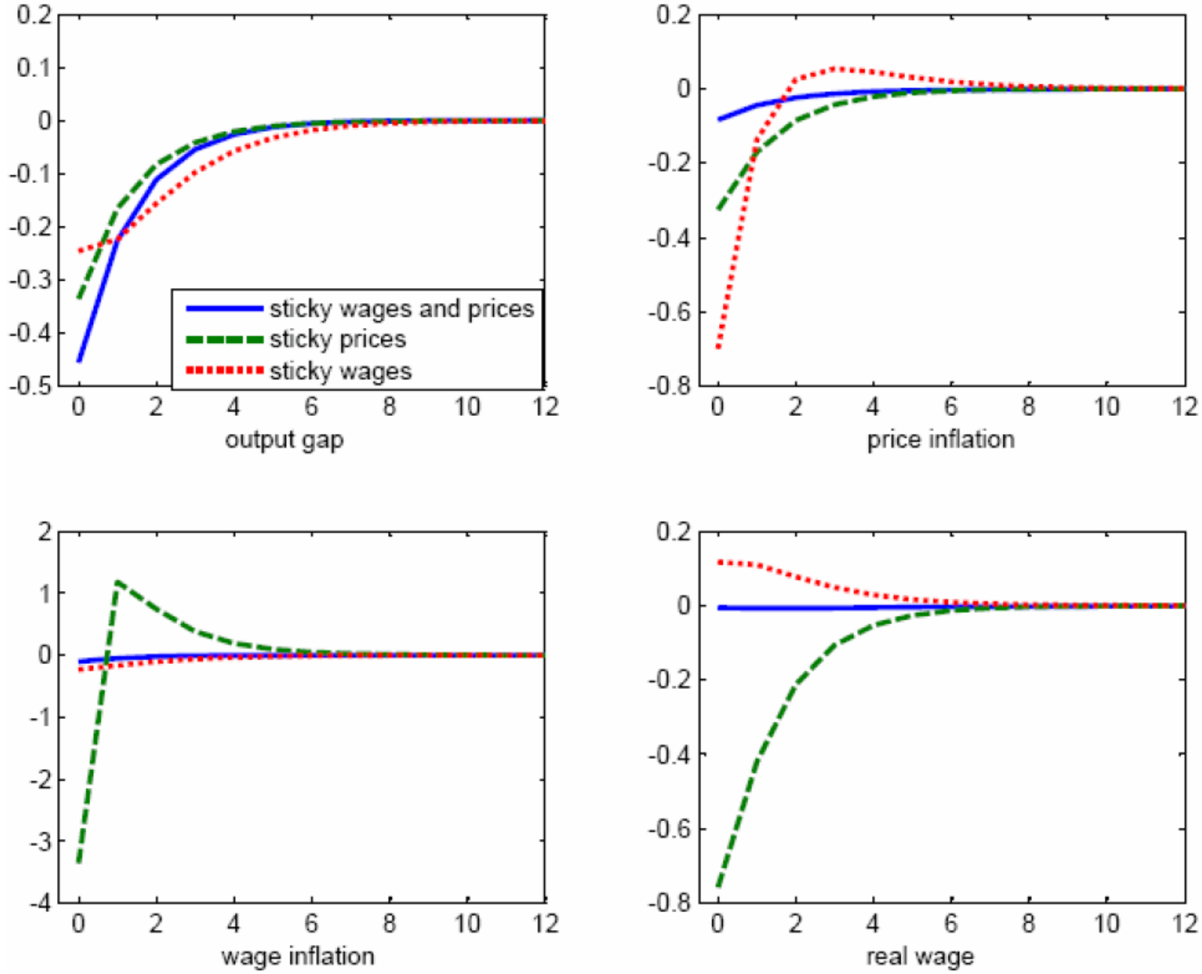
Interest rate rule: (or some other form of monetary policy)

$$i_t = \rho + \phi_y \widetilde{y}_t + \phi_p \pi_t^p + \phi_w \pi_t^w + v_t$$

Source: Galí (2007)

Figure 6.3: Sticky Wages and the Effects of a Monetary Policy Shock

the more sticky are prices (ie, wages and the price level) the more quantities have to adjust



- The more sticky are prices (wages and prices) the more quantities have to move
- Price inflation and real wage dynamics different depending on rigidities - blue line more plausible

Monetary Policy Design with Sticky Wages and Prices

- Distortions: sticky prices, sticky wages, monopolistic competition in goods and labour markets
- Distortions present even in the flexible price/wage version of the model (or in the steady state)
- 1) Price is a markup over marginal cost

$$P = \mathcal{M}_P \frac{W_t}{MPN_t}$$

- 2) Wage is a markup over marginal rate os substitution between labour and leisure

$$\frac{W}{P} = \mathcal{M}_w MRS$$

- Efficiency

$$\frac{W}{P} = MRS = MPN$$

- If you assume an employment subsidy τ

$$P = \mathcal{M}_P \frac{(1-\tau)W}{MPN}$$

would imply

$$MPN = (1-\tau)\mathcal{M}_P\mathcal{M}_w MRS$$

- So, steady state efficiency could be achieved by setting

$$\tau = 1 - \frac{1}{\mathcal{M}_P\mathcal{M}_w}$$

- What is government imposes income taxes? What is the optimal tax level?

Monetary Policy Design with Sticky Wages and Prices

- Distortions: sticky prices, sticky wages, monopolistic competition in goods and labour markets
- Instrument: subsidy and interest rate policy
- Number of instruments<number of distortions
- Replicating the natural equilibrium allocation is generally unfeasible.

Second Order Approximation to Welfare Losses

- (assuming an efficient steady state or an optimal subsidy)

$$\mathcal{W} \simeq \frac{1}{2} E_0 \sum_{k=0}^{\infty} \beta^k \left[\frac{\varepsilon_p}{\lambda_p} \pi_t^{p2} + \frac{\varepsilon_w (1-\alpha)}{\lambda_w} \pi_t^{w2} + \frac{\varphi + \alpha + \sigma(1-\alpha)}{1-\alpha} \tilde{y}_t^2 \right]$$

- Optimal monetary policy

- Minimize the loss function subject to

$$\pi_t^p = \kappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t + \beta E_t \pi_{t+1}^p$$

$$\pi_t^w = \kappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t + \beta E_t \pi_{t+1}^w$$

$$\widetilde{\omega}_t - \widetilde{\omega}_{t-1} = \pi_t^w - \pi_t^p + \Delta w_t^n$$

- When only prices (wages) are sticky - optimal policy fully stabilizes price (wage) inflation

- The behavior of the real variables are identical, because optimal policy stabilizes the wedge between the marginal rate of transformation and the marginal rate of substitution.

$$(a)\ \widehat{\mu}_t^p = 0 \Rightarrow p_t - w_t + mpn_t = mpn_t - mrs_t = 0$$

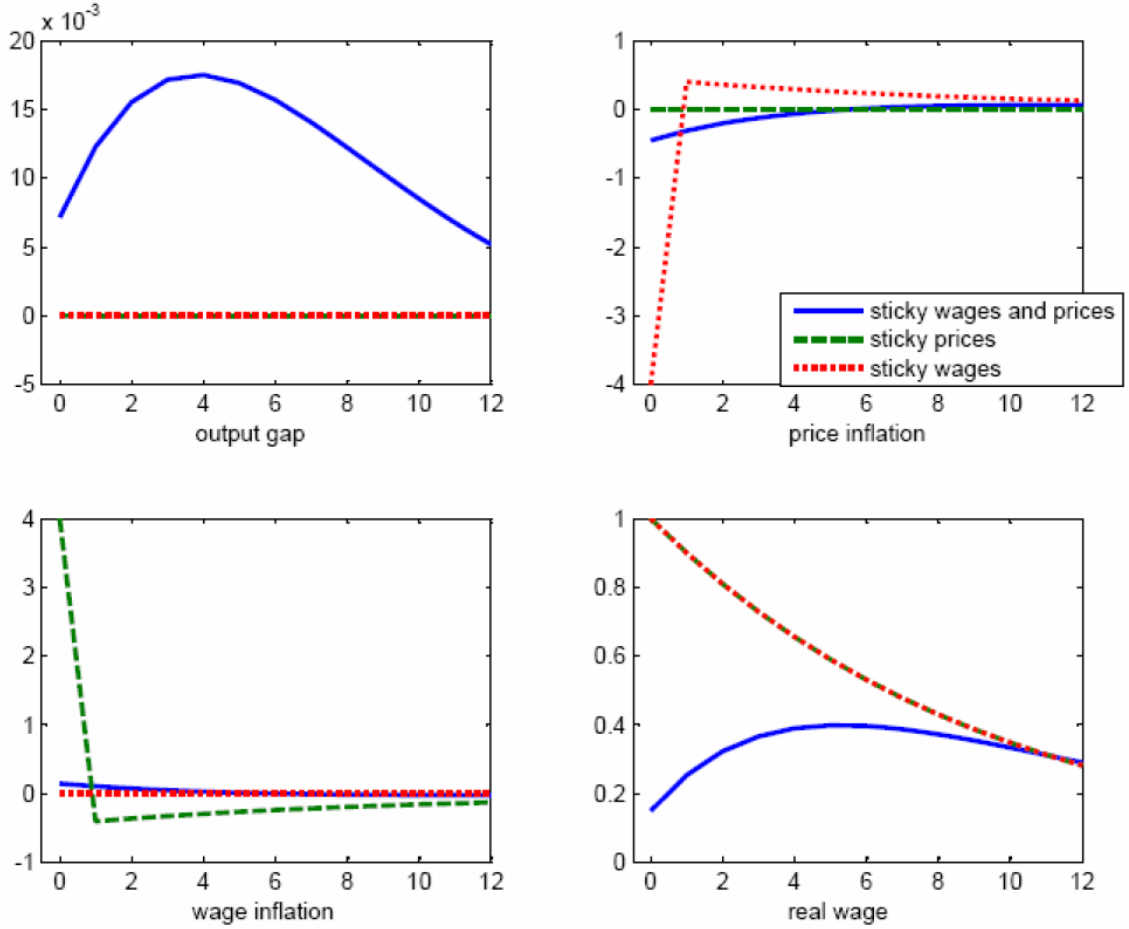
$$(b)\ \widehat{\mu}_t^w = 0 \Rightarrow w_t - p_t - mrs_t = mpn_t - mrs_t = 0$$

- When both prices and wages are sticky the natural allocation can no longer be attained.

- In that case the optimal policy strikes a balance between attaining the output and real wage adjustments warranted by the rise in productivity and, on the other hand, keeping wage and price inflation close to zero to avoid the distortions associated with nominal instability.

Source: Galí (2007)

Figure 6.4: The Effects of a Technology Shock under the Optimal Policy



- It is optimal to raise the real wage smoothly, through a mix of negative price inflation and positive wage inflation.

- The implied sluggishness of the real wage, combined with the improvement in technology, accounts for the observed overshooting of output, which rises above its natural level.

Special case: Exercise

- Show that when $\kappa_p = \kappa_w = \kappa$ and $\varepsilon_p = \varepsilon_w = \varepsilon$ the optimal policy can be written as

$$\lambda_w \pi_t^p + \lambda_p \pi_t^w + \frac{\lambda_p}{\varepsilon} \Delta \widetilde{y}_t = 0$$

or

$$\pi_t + \frac{\vartheta}{\varepsilon} \Delta \widetilde{y}_t = 0$$

where $\pi_t = (1 - \vartheta)\pi_t^p + \vartheta \pi_t^w$ and $\vartheta = \frac{\lambda_p}{\lambda_p + \lambda_w}$

- What are the policy trade-offs of the policymaker? To which extent is the output gap stabilized under the optimal policy?

- Does the output gap need to be observable to implement optimal policy?

- Can you propose a simple interest rate rule that approximate this optimal policy?