Switching Monetary Policy Regimes and the Nominal Term Structure*

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Abstract

In this paper I propose a regime-switching approach to explain why the U.S. nominal yield curve on average has been steeper since the mid-1980s than during the Great Inflation of the 1970s. I show that, once the possibility of regime switches in the short-rate process is incorporated into investors' beliefs, the average slope of the yield curve generally will contain a new component called 'level risk'. Level-risk estimates, based on a Markov-Switching VAR model of the U.S. economy, are then provided. I find that the level risk was large and negative during the Great Inflation, reflecting a possible switch to lower short-rate levels in the future. Since the mid-1980s the level risk has been moderate and positive, reflecting a small but still relevant possibility of a return to the regime of the 1970s. I replicate these results in a Markov-Switching dynamic general equilibrium model, where the monetary policy rule followed by the Fed shifts between an active and a passive regime. The model also explains why in recent decades the U.S. yield curve on average has been steeper than the yield curve in countries that adopted explicit inflation targeting frameworks.

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1 Introduction

The U.S. nominal yield curve on average has been steeper since the mid-1980s than during the Great Inflation of the 1970s. This is puzzling because, in general, the *average* slope of the yield curve is thought to reflect risk premia demanded by bond investors. Therefore, in periods of high macroeconomic uncertainty such as the Great Inflation, in principle one should expect bonds to pay higher premia – that is, term premia – than they would from the mid-1980s until 2007 when macroeconomic uncertainty had reached historically low levels (the 'Great Moderation'). I refer to this apparently inconsistent relation between the yield curve's slope and macroeconomic uncertainty as the '*Slope-Volatility Puzzle*'. In this paper, I propose a theory that is based on switching macroeconomic regimes to explain this puzzle.

In the proposed framework, investors incorporate the possibility that the economy switches across different regimes into their beliefs. In particular, if the nominal short-rate fluctuates around different means across regimes, then the average slope of the yield curve in general will reflect not only the standard term premium, but also a new term attributable to the Expectations Hypothesis in the presence of regime shifts. I call this '*level risk*', and it reflects the risk that, conditional on the economy being in a low short-rate regime, long-term bonds will lose value in the case of a shift to higher short-rate regimes. The level risk will be positive in this case. Similarly, a negative level risk occurs when the short-rate is currently high and there is a possibility of switching to low short-rate regimes, which represents gains for bond holders.

I estimate the level risk during the post-World War II period using a simple Markov-Switching Vector Autoregression of the U.S. economy. This model identifies a high macroeconomic volatility regime that corresponds broadly to the Great Inflation of the 1970s. This regime is also characterized by a high average short-rate (and high inflation) and is not very persistent, with an average duration of only 7.7 years. On the other hand, the Great Moderation period appears in the model as the realization of a highly persistent regime (average duration of almost 35 years) associated with low macroeconomic volatility and a low average short-rate. If investors form beliefs according to this model, then the level risk was large and negative during the Great Inflation. Intuitively, because agents perceived this regime as relatively short-lived, there was as high probability that the short-rate would switch to lower levels in the future, causing nominal bonds to gain value. Since the mid-1980s, in contrast, the level risk has been more moderate in magnitude and positive. In this case, investors ascribed a relatively low probability to a switch back to the high short-rate regime of the 1970s, which would have caused nominal bonds to lose value. Extracting the estimated level risks from observed yield curve slope measures, I find that term premia on average were substantially *higher* during the Great Inflation of the 1970s than they have been since the mid-1980s. Therefore, my first main conclusion in this paper is that the Slope-Volatility Puzzle can be explained by differences in level risks across regimes.

One important implication of the level risk is that term structure models that do not allow for Markov-Switching regimes tend to generate biased estimates of term premia. In other words, if agents consider the possibility of regime shifts when forming expectations, then models that do not take this into account will have the term premium explain too much of the yield curve's slope. Because the level risk operates exclusively through investor's expectations, *this bias will appear even if the term structure model is fitted to particular subsamples of the data that correspond to a single economic regime.*

Next I ask: what fundamental macroeconomic changes could reproduce the Slope-Volatility Puzzle? My answer is based on a simple Markov-Switching dynamic general equilibrium model, calibrated to replicate the U.S. economy. Following Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004) and many others, I assume that monetary policy switches between a regime where the central bank accommodates inflation pressures (passive policy) and a regime where the central bank fights these pressures in a proactive manner (active policy). What sets this model apart from standard ones is that here, as in Davig and Leeper (2007), agents incorporate into their expectations the possibility of regime switches. In this model, agents acquire different levels of precautionary savings depending on the current policy regime. As a result, the average short-term interest rate differs across regimes, giving rise to potentially sizeable level risks over the yield curve.

Under my calibration, households hold more precautionary savings when the passive regime occurs. The intuition behind this is that consumption growth is more volatile in the passive than in the active regime; therefore, risk averse agents will want to hold more "insurance" when the passive regime realizes. Thus, the nominal short-rate in the passive policy regime is higher than in the active regime, giving rise to level risks. In the passive regime, level risks are large and negative as I estimated for the Great Inflation regime. In the active regime, level risks are moderate and positive, replicating my estimate for the post-1985 period. Therefore, my second main conclusion in this paper is that a general equilibrium model with a Markov-Switching monetary policy rule is capable of explaining the Slope-Volatility Puzzle.

Many economists, including Gürkaynak, Levin, and Swanson (2006, 2010), Wright (2008), and Capistrán and Ramos-Francia (2010), have highlighted gains from the adoption of an explicit inflation targeting framework. Under this policy arrangement, improved communication between the central bank and the public would reduce uncertainty about the particular way that the monetary authority will deal with inflationary pressures. This can be rationalized in the context of the model I present here. All else constant, agents would perceive a change from the active towards the passive regime as less likely under explicit inflation targeting than if the central bank did not adopt this framework. According to the model, increasing the persistence of the active regime on average flattens the yield curve in that regime, because both level risks and term premia fall. Using the Wright (2008) international dataset, I show that this prediction of the model is corroborated by the data. That is, measures of average yield curve slope in developed economies that adopted explicit inflation targeting are systematically lower than in economies that did not adopt such a framework. I interpret this as additional evidence supporting the model with a Markov-Switching monetary policy rule.

This paper is organized as follows. Section 2 includes a brief review of the literature. Section 3 documents the Slope-Volatility puzzle. Section 4 describes how allowing for a regime switching approach gives rise to the level risk. Section 5 provides level risk estimates for the United States which are shown to explain the Slope-Volatility Puzzle. In Section 6, the general equilibrium term-structure model with a Markov-Switching monetary policy rule is presented and shown to replicate the main features of this puzzle. I also show that the model replicates the yield curve evidence from inflation targeting countries. Section 7 concludes.

2 Related Literature

In this paper, I focus on how level risks on average affect the nominal U.S. yield curve. Level risks naturally emerge from the Expectations Hypothesis component of the term structure, once the short-rate is modelled as a Markov-Switching (MS) process. This insight draws on two important earlier contributions.

Hamilton (1988) pioneered in studying term structure behavior when the short rate follows a simple autoregressive MS process. He found that when regime shifts were incorporated into agents' beliefs, violations of the Expectations Hypothesis of the term structure were less severe than in single regime models. Bekaert, Hodrick, and Marshall (2001) proposed the peso problem theory which I also draw on. They noted that violations of the Expectations Hypothesis in the United States, to a large extent, are due to a 'peso problem' which is associated with the short-rate process: observed long-term yields in the United States largely can be reconciled with short-rate behavior if investors' beliefs allow for short-rate levels not observed *ex-post* in the data. This theory can be formalized by first assuming a reduced-form MS process for the short rate and then letting long-term bonds be priced by rational agents who form beliefs taking this MS process into account.

Both Hamilton (1988) and Bekaert, Hodrick, and Marshall (2001) focus on reduced-form short-rate processes, but I go one step further here and, in a no-arbitrage general equilibrium framework, relate the behavior of short- and long-term nominal yields in the United States to macro factors. In a structural micro-founded framework, I claim that changes in monetary policy regimes can explain observed changes in key U.S. macro and term structure moments over time.

There is substantial reduced-form empirical evidence in the literature showing that changes in monetary policy did affect the behavior of the U.S. term structure (and the macroeconomy) over time[1]. For example, Bikbov and Chernov (2008) have shown that the

[1]The literature on term-structure models with only yield-related factors also provides ample support for

information contained in the nominal term structure can be crucial in identifying regimes in which the Fed adopted an active or passive stance for inflation. Similarly, in the context of identified time-varying VARs with no-arbitrage bond prices, both Ang, Boivin, Dong, and Loo-Kung (2009) and Mumtaz and Surico (2009) find evidence of large movements in the Fed's response to inflation over the last six decades. Bianchi, Mumtaz, and Surico (2009) find evidence for the United Kingdom that monetary policy shocks contributed significantly more to the variability of key macro and term structure time series before than after the adoption of inflation targeting[2]. My contribution to this strand of the literature is a structural, micro-founded no-arbitrage approach to modelling the term structure in the presence of monetary policy shifts.

Interestingly, my proposed level-risk theory is related to the peso problem (or 'rare disaster') as suggested by both Rietz (1988) and Barro (2006) in the context of the equity premium puzzle. In their formulation, equity becomes very risky because there is a small probability in every period of a sudden and very sharp drop in the economy's productive capacity. In my case, a change from an active to a passive monetary policy regime, where holding nominal bonds involves significantly more risk, could be seen as a rare disaster by bond traders. As in Bansal and Yaron (2004), I also show that when shifts to a passive monetary policy regime are possible, the amount of long-run risk in the economy during the active regime is substantially higher than in a single active regime model.

The available literature on MS Dynamic Stochastic General Equilibrium (MS-DSGE) models, including Davig and Leeper (2007), Farmer, Waggoner, and Zha (2007), Davig and Doh (2008), Liu, Waggoner, and Zha (2009) and Liu and Mumtaz (2010), relies on linear approximations to the true model solution, which by construction rule out precautionary

the view that the stochastic behavior of the U.S. yield curve has varied over the past decades. See, for example, the latent-factors, regime-shifting, no-arbitrage frameworks of Bansal and Zhou (2002) and Dai, Singleton, and Yang (2007). Without relying on no-arbitrage restrictions, Ang and Bekaert (2002) show that regime shifts in the nominal term structure are present in the US, UK and Germany datasets. Evidence of regime shifts in real term structures in the US and the UK respectively also has been provided by Ang, Bekaert, and Wei (2008) and Evans (2003).

[2] The existence of shifts in the U.S. monetary policy regime are supported by a well-known macro literature that tries to identify the main reason behind the Great Moderation of the post mid-1980s. See, for example, Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004) and Boivin and Giannoni (2006). Romer and Romer (2004) also produce compelling narrative evidence of changes in the Fed's monetary policy conduct during the 20^{th} century. For other explanations for the Great Moderation, see for example, McConnell and Perez-Quiros (2000), Primiceri (2005), Sims and Zha (2006), and Justiniano and Primiceri (2008).

savings and premia in financial assets. Amisano and Tristani (2010a,b) considered non-linear solutions to MS-DSGE models with MS shocks' volatilities, but their method does not allow for changes in the Fed's response to inflationary pressures. I also contribute to this branch of the literature by offering a non-linear perturbation solution method to the standard New-Keynesian model with a Markov-Switching monetary policy rule[3].

3 The Slope-Volatility Puzzle

I now document the '*Slope-Volatility Puzzle*'. In the following subsection I derive results from the no-arbitrage theory linking the slope of the nominal yield curve to the underlying level of uncertainty in the economy. I then go on to show that these theoretical predictions are at odds with the U.S. data.

3.1 The Slope of the Yield Curve and Macroeconomic Uncertainty

Consider a long-term nominal bond that costs $B_{\tau,t}$ at time t and promises to repay the investor one dollar in $t+\tau$ (throughout this paper, I assume that bonds are zero-coupon and default-free). The continuously compounded τ-period yield to maturity is defined as $i_{\tau,t} \equiv -\frac{1}{\tau}\log B_{\tau,t}$. The economy's short-term nominal interest rate is given by $i_t \equiv i_{1,t}$. Following Dai and Singleton (2002) and letting E_t be the expectations operator conditional on date t information, I define the *τ-period nominal term premium* as $NTP_{\tau,t} \equiv i_{\tau,t} - \frac{1}{\tau}\sum_{j=0}^{\tau-1} E_t\left[i_{t+j}\right]$. This measure captures the deviations of $i_{\tau,t}$ from the pure expectations hypothesis and is positive when it is riskier to invest in the long-term bond than to invest in a sequence of short-term bonds for τ periods.

Rearranging the term premium definition, it follows that the *τ-period yield curve slope*,

[3] Chib, Kang, and Ramamurthy (2011) estimate a MS-DSGE model for the US nominal term structure. Their solution method relies on the linear/log-normal approach of Bekaert, Cho, and Moreno (2010), which assumes that the short-rate is not affected by precautionary savings effects (i.e. the average short rate is the same across regimes). As a result, their solution method by construction rules out the existence of level risks along the yield curve.

$i_{\tau,t} - i_t$, can be written as[4]

$$i_{\tau,t} - i_t = \left(\frac{1}{\tau} \sum_{j=0}^{\tau-1} E_t \left[i_{t+j} \right] - i_t \right) + NTP_{\tau,t} \ . \tag{1}$$

I call the term in parentheses in equation (1) the expectations hypothesis (EH) component of the slope. If the nominal term premium remains constant, then whenever investors revise their short-rate forecasts up (down), the EH component of the slope increases (decreases). Similarly, if the EH component remains unchanged, then an increase (decrease) in the nominal term premium increases (decreases) the slope.

The term premium definition can be used to gain some insights about the determinants of the yield curve slope in the long run. Taking unconditional expectations on both sides of equation (1), I obtain

$$E \left[i_{\tau,t} - i_t \right] = \frac{1}{\tau} \sum_{j=0}^{\tau-1} \left(E \left[i_{t+j} \right] - E \left[i_t \right] \right) + E \left[NTP_{\tau,t} \right]$$

because of the law of iterated expectations. From the assumption that the short rate follows a covariance-stationary process it follows that $\sum_{j=0}^{\tau-1} \left(E \left[i_{t+j} \right] - E \left[i_t \right] \right) = 0 \ \forall j$. Intuitively, when calculating the mean, periods where the short rate is expected to increase will cancel out those where the short rate is expected to decrease, and the EH component of the slope is equal to zero on average. As a result $E \left[i_{\tau,t} - i_t \right] = E \left[NTP_{\tau,t} \right]$, which means that if the nominal yield curve is *unconditionally* positively sloped, it is because the term premium is positive on average. Therefore, to understand why yield curves in general are positively slopped, it is important to understand the determinants of the term premium.

It can be shown that the no-arbitrage price of a τ-period bond is given by

$$B_{\tau,t} = E_t \left[M_{t,t+1} \frac{1}{\Pi_{t,t+1}} B_{\tau-1,t+1} \right] \tag{2}$$

where $M_{t,t+1}$ and $\Pi_{t,t+1}$ are the *real* stochastic discount factor (SDF) and the inflation rate between periods t and $t+1$. All else constant, an increase in the expected rate of inflation

[4] This slope definition sometimes has been refered to in the literature as 'term spread', but is not to be confused with the 'term premium'.

will reduce the price of the bond today because its expected resale value, measured in real terms, falls.

Taking a second-order expansion of the Euler condition above around the deterministic steady state, the nominal term premium is given by[5]

$$\begin{aligned} NTP_{\tau,t} \;\cong\; & RTP_{\tau,t} + Convexity^{\pi}_{\tau,t} \\ & + \left(\frac{1}{\tau} Cov_t \left[\widehat{m}_{t,t+\tau}, \widehat{\pi}_{t,t+\tau} \right] - \frac{1}{\tau} \sum_{j=0}^{\tau-1} E_t \left\{ Cov_{t+j} \left[\widehat{m}_{t+j,t+j+1}, \widehat{\pi}_{t+j,t+j+1} \right] \right\} \right) \end{aligned} \tag{3}$$

where for any variable X_t with steady state $\overline{X}$ define $\widehat{x}_t \equiv \log\left(X_t/\overline{X}\right)$, while Cov_t represents the conditional covariance operator. Therefore, the nominal term premium can be separated into three parts. The first corresponds to the *τ-period real term premium*. This is defined as $RTP_{\tau,t} \equiv r_{\tau,t} - \frac{1}{\tau}\sum_{j=0}^{\tau-1} E_t\left[r_{t+j}\right]$, where $r_{\tau,t}$ is the yield to maturity on a τ-period inflation indexed bond[6] and r_t is the real short-rate. The real term premium captures deviations of the long-term *real* yield from the EH which, as shown in Appendix A, depends only on the autocorrelation structure of the SDF[7]. The second component of $NTP_{\tau,t}$, $Convexity^{\pi}_{\tau,t}$, represents an inflation convexity term that in practice is not very relevant.

The third component appearing in parenthesis in equation (3) is what I focus on. This term corresponds to compensation for inflation risk. Its first part is positive when inflation from t until maturity co-varies positively with the investor's SDF. In this case, nominal bonds lose value exactly when wealth is most important to the investor. The second component of the inflation premium measures the (average expected) one-period-ahead inflation co-variability risk. Because inflation uncertainty is relatively low in such a short horizon, the first part of the inflation premium tends to dominate.

I use these results to relate the term premium to the underlying level of uncertainty in the economy. The dominant part of the inflation premium in equation (3) can be rewritten

[5] The following expression holds exactly if SDF and inflation are jointly log-normally distributed. Detailed derivations can be found in Appendix A.

[6] Inflation indexation is assumed to be perfect.

[7] There is evidence that the real term premium over the US term structure is close to zero or slightly negative. Compared to premia related to inflation, however, real premia are thought to be quantitatively less relevant. See Buraschi and Jiltsov (2005), Piazzesi and Schneider (2006) and Ang, Bekaert, and Wei (2008).

as

$$\frac{1}{\tau}Cov_t\left[\widehat{m}_{t,t+\tau},\widehat{\pi}_{t,t+\tau}\right]=\frac{1}{\tau}Corr_t\left[\widehat{m}_{t,t+\tau},\widehat{\pi}_{t,t+\tau}\right]\left(Var_t\left[\widehat{m}_{t,t+\tau}\right]Var_t\left[\widehat{\pi}_{t,t+\tau}\right]\right)^{1/2} \tag{4}$$

where Var_t and $Corr_t$ are the conditional variance and correlation coefficient. The square root term on the right hand side of this equation represents the product of the *ex-ante* volatilities of inflation and the SDF. If the conditional correlation between the SDF and inflation is positive (i.e. if the inflation premium is positive) and relatively constant over time, then periods associated with higher levels of inflation uncertainty will also be associated with higher inflation compensations and higher term premia. Intuitively, when inflation uncertainty is high, the future payoffs from investing in long-term nominal bonds are very uncertain if evaluated in real terms. As a result, investors will demand higher premia to hold long-term nominal bonds than when uncertainty is low.

Equation (4) also reveals that, in addition to inflation uncertainty, real uncertainty matters for term premia. To see this, note that macro models usually have the SDF be a function of real variables, such as consumption, hours worked, etc. Therefore, an increase in the level of uncertainty surrounding these real variables will increase the *ex-ante* volatility of the SDF, which in turn will have an impact on the nominal term premium through its inflation compensation component[8]. Intuitively, the investor cares about how the *real* payoff from investing in the long-term bond co-varies with his future path of consumption. If consumption becomes more difficult to predict, then investors will perceive long-term bonds as riskier and demand higher premia.

3.2 Evidence for the US

From the discussion above follows that periods characterized by high levels of macroeconomic uncertainty should be associated with higher term premia than periods of low uncertainty. Accordingly, differences in average term premia across periods are entirely reflected in the yield curve slope (provided that there are enough observation from each period)[9]. Do these

[8]The real term premium component of NTP_t^τ may also respond to the increase in real uncertainty. See Appendix A.

[9]In Section 3.1 I characterized the relation between the slope of the yield curve and macroeconomic volatility in terms of *unconditional* moments, resulting in $E\left[i_{\tau,t}-i_t\right]=E\left[NTP_{\tau,t}\right]$. Although applying

theoretical predictions hold for the post-World War II U.S. data?

To address that question, I construct slope measures using the 5 and 10-year zero-coupon nominal yields from the CRSP Fama-Bliss and the Gurkaynak, Sack, and Wright (2007) databases[10]. To measure the short-rate I use the 3-month T-Bill returns taken from the CRSP Fama riskfree rate file. All of the series are arranged at a quarterly frequency, and the sample goes from 1952:2 to 2008:4. Interest rates are continuously compounded, expressed in annualized terms, and observed on the last working day of each quarter.

The measures of macroeconomic uncertainty that I focus on are based on real consumption growth and inflation. The former is measured by the quarterly growth in the Real Personal Consumption Expenditures index (PCE); the latter is measured by the quarterly growth in the core PCE deflator[11]. All growth rates are continuously compounded and multiplied by 400 to be expressed in percent per annum. Consumption growth is intended as a *proxy* for the unobservable SDF.

Because bond prices are fully forward looking, inflation and consumption uncertainty must be quantified according to an *ex-ante* concept (only using information up to t to measure uncertainty from period $t+1$ onwards). Therefore I estimate univariate GARCH processes for the inflation and consumption growth series over the 1952:2-2008:4 sample, and then measure uncertainty by the GARCH-based one-quarter-ahead forecast for the conditional variance[12].

The 5- and 10-year slope measures, together with the conditional variance forecasts for

unconditional moments simplifies the exposition, it fails to agree precisely with my empirical analysis, which is interested in the co-movements between the slope and macro volatility across different *subsamples* of the U.S. data. A more appropriate characterization would then be the following. Assume that the economy switches across different regimes over time, and yet private agents believe the current regime to last indefinitely. This is the implicit assumption in a great number of papers in the macro literature, such as Lubik and Schorfheide (2004) and Smets and Wouters (2007). It then follows that $E\left[i_{\tau,t} - i_t/s_t\right] = E\left[NTP_{\tau,t}/s_t\right]$, that is the slope and term premium are on average equal conditional on each regime s_t. Section 4 shows that this equality is not true once agents incorporate the possibility of regime shifts into their beliefs, which will help to explain the Slope-Volatility Puzzle.

[10] Although the Gurkaynak, Sack, and Wright (2007) database also contains the 5-year zero-coupon yield, it only starts in 1961. The CRSP Fama-Bliss file starts in 1952, but does not contain the 10-year maturity.

[11] Consumption growth is measured by the quarterly change in the real PCE index (series code: PCECC96). For inflation I use the 'PCE deflator excluding food and energy' obtained from the St. Louis Fed webpage (series code: JCXSE). For the latter, the observations from 1952 until 1959 were estimated using the 'CPI excluding food and energy' and the 'PCE deflator all items' (series codes: CPILFESL and PCECTPI) and applying the principal components method suggested by Walczak and Massart (2001).

[12] The conditional mean of inflation and consumption growth are modelled respectively as an AR(2) and an ARMA(1,4) process. For both series the best fitting model for the conditional variance was a GARCH (1,1).

inflation and consumption growth, are shown in Figure 1. The vertical dashed lines in this figure identify subperiods of the U.S. sample where, according to Romer and Romer (2004), the Fed followed different monetary policy regimes. Each subsample is identified with the names of the Federal Reserve chairmen in office at the time[13]. I abandon these pre-specified subsamples in Section 5 and use more rigorous statistical methods to identify the U.S. regimes.

The two bottom charts in Figure 1 show a well-known stylized fact in the macro literature: from the mid-1980s until 2007, a period that corresponds roughly to my Greenspan / Bernanke subsample, the levels of macroeconomic uncertainty (in this case inflation and consumption growth uncertainty) were historically low. This is referred to in the literature as the Great Moderation[14], and usually is portrayed in terms of *ex-post* measures of uncertainty, such as the realized standard deviation of key macro time series. Figure 1 makes a similar point but uses GARCH-based volatilities which, as discussed before, better capture the level of *ex-ante* uncertainty faced by forward looking bond traders.

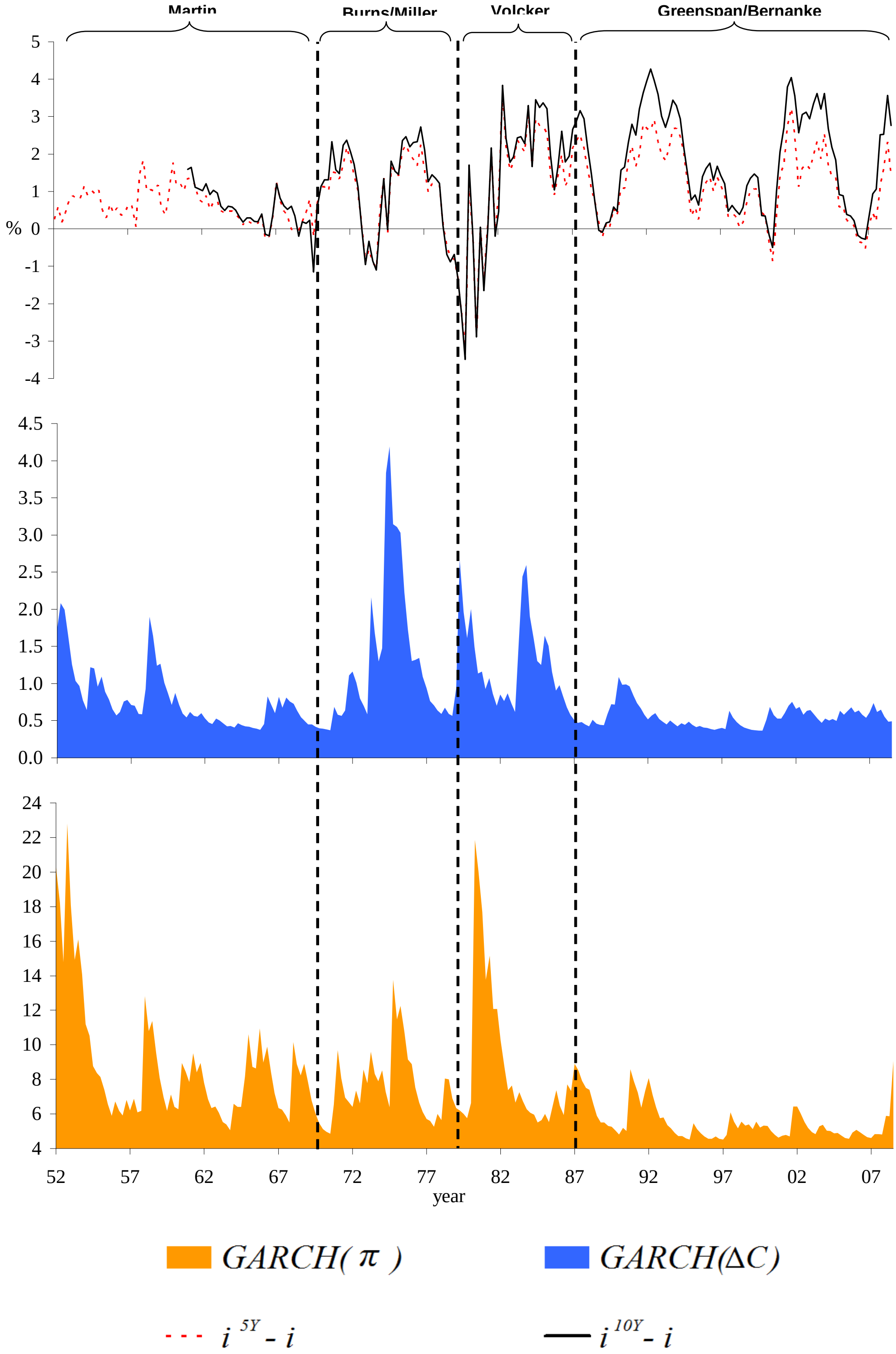
The subsamples depicted in Figure 1 can be interpreted as different regimes characterized by different levels of macroeconomic uncertainty. The Greenspan / Bernanke subsample might be viewed as a low uncertainty state, whereas the Burns / Miller and the Volcker subsamples can be seen as high uncertainty regimes. This classification is in line with the more rigorous estimates shown in Section 5. According to the theory developed in subsection 3.1, term premia and consequently yield curve slope measures in principle should reflect the underlying level of macro uncertainty. Following this logic, the yield curve slope on average should be flatter in the Greenspan / Bernanke subsample than in the Burns / Miller and Volcker subsamples.

As the top chart in Figure 1 shows, this does not hold in the data. During the Greenspan / Bernanke years, both the 5 and 10-year slope measures on average seem higher than in all three previous subsamples. If the theory above is correct, then these slope measures

[13]Romer and Romer (2004) view monetary policy in the years where Paul Volcker and Alan Greenspan were Federal Reserve chairmen as based on the same principles. Here, I separate the Volcker era from the subsequent one, because the general macroeconomic environment in the two periods was substantially different. In particular, while Volcker inherited an ambience of high/volatile inflation where credibility in monetary policy was very weak (see Goodfriend and King (2005)), the same is not true for Greenspan.

[14]See Kim and Nelson (1999) and McConnell and Perez-Quiros (2000). In addition, Stock and Watson (2002) conduct a detailed review of the literature and present some new evidence.

Figure 1: The Yield Curve Slope and Macroeconomic Uncertainty in the U.S.



Notes: $GARCH(\pi)$ and $GARCH(\Delta c)$ are the one-quarter-ahead forecasts of the conditional variance of inflation (π) and consumption growth (Δc), estimated through univariate GARCH(1,1) processes. The conditional means of inflation and consumption growth are modelled respectively as an AR(2) and an ARMA(1,4). The two yield curve slope measures are expressed in percent per annum.

tell us that investing in nominal bonds during the Greenspan / Bernanke period is riskier than in all previous subsamples. However, this is difficult to reconcile with the low levels of macroeconomic uncertainty that characterize this subsample.

This point is reinforced in Table 1, which displays in more detail certain properties of the slope, inflation, and consumption growth time series across the previously defined subsamples. The first two lines of this table show the average 5 and 10-year slope measures in each subsample, together with their associated standard errors. Two alternative measures of *ex-post* uncertainty are shown in lines three through six. These can be compared to the *ex-ante* GARCH-based measures for the sake of robustness. In particular, the third line displays the realized standard deviation of inflation, whereas lines four through six display the root mean squared forecast error (RMSFE) of inflation at three different horizons, based on a simple random-walk model[15]. The same measures of uncertainty for the case of consumption growth are shown in lines seven to ten. Finally, the last line shows the contemporaneous correlation coefficient between consumption growth and inflation.

Note that the 10-year slope on average is 179 basis points during the Greenspan / Bernanke period, the highest across all subsamples. At the same time, all ex-post measures of inflation and consumption growth uncertainty are unambiguously lower in this period than in all other subsamples.

I can now formulate the main stylized fact that motivates the remainder of this paper:

- *The Slope-Volatility Puzzle:*

  Although the inflation and consumption-based uncertainty measures suggest that the Greenspan / Bernanke subsample is the least risky for investors holding long-term nominal bonds, the nominal yield curve on average is steeper during that period than in the Martin, Burns / Miller and Volcker subsamples.

Returning to equation (4), one possible explanation for the puzzle is that $Corr_t\left[\widehat{m}_{t,t+\tau}, \widehat{\pi}_{t,t+\tau}\right]$ on average is sufficiently higher during the Greenspan / Bernanke subsample than in the

[15] The RMSFE we report are based on Atkeson and Ohanian (2001) and Stock and Watson (2007). To be more precise, let π_t be the annualized quarterly inflation rate at time t. To compute the forecast errors for inflation accumulated h periods ahead, I use the following model: $E_t\left[h^{-1}\sum_{j=1}^{h}\pi_{t+j}\right] = \frac{1}{4}\left(\pi_t + ... + \pi_{t-3}\right)$. An equivalent methodology is used for consumption growth.

Table 1: The Slope-Volatility Puzzle

		Martin (52:2-69:4)	Burns / Miller (70:1-79:2)	Volcker (79:3-87:2)	Bernanke / Greenspan (87:3-08:4)
$E(i_{5Y} - i)$		0.61	0.93	1.10	1.19
		(0.06)	(0.21)	(0.31)	(0.13)
$E(i_{10Y} - i)$		$0.51^{\dagger}$	1.09	1.32	1.79
		(0.11)	(0.25)	(0.35)	(0.18)
$SD(\pi)$		1.23	2.00	2.16	1.01
$RMSFE\,(\pi)$	$h = 4$	0.75	1.88	1.11	0.48
	$h = 8$	0.83	2.10	1.40	0.48
	$h = 12$	0.93	2.04	1.67	0.55
$SD(\Delta c)$		3.26	3.30	3.50	2.10
$RMSFE\,(\Delta c)$	$h = 4$	3.10	3.16	2.26	1.49
	$h = 8$	2.54	3.39	2.15	1.55
	$h = 12$	2.31	3.10	2.37	1.62
$Corr\,(\Delta c, \pi)$		-0.25	-0.51	-0.46	-0.17

Notes: The measures of average slope are expressed in percent per annum and the values in parenthesis are Newey-West HAC consistent standard errors, calculated using monthly slope series. SD(Δc) and inflation SD(π) are the standard deviation of consumption growth (Δc) and inflation (π), also expressed in percent per annum. *RMSFE*(π) and *RMSFE*(Δc) are the random-walk-based, root mean squared forecast errors h periods ahead for consumption growth and inflation. Corr(π,Δc) is the contemporaneous correlation coefficient between inflation and consumption. † sample starts in 1961:2.

previous ones. An increase in this correlation coefficient might offset the drop in ensuing uncertainty as the economy moves from the Volcker to the Greenspan / Bernanke periods. To rule out this possibility, the last line of Table 1 reports the realized contemporaneous correlation coefficient between inflation and consumption growth across all subsamples. This measure is a *proxy* for the unobserved conditional correlation in equation (4). Because the SDF is negatively related to consumption growth in most economic models, the correlation coefficient being negative in all subsamples is consistent with positive term premia. Note that the absolute value of the correlation coefficient changes across subsamples in the same direction as changes in uncertainty. This suggests that movements in the correlation coefficient across subsamples reinforce the puzzle, rather than helping to explain it.

4 The Level Risk

This section puts forth a theoretical approach that potentially can explain the Slope-Volatility Puzzle. It hinges on the assumption that, over time, the economy switches across different macroeconomic regimes characterized by different short-rate levels. By letting investors explicitly incorporate into their beliefs the possibility of regime shifts, I show that this assumption has important implications for the EH component of the yield curve slope.

Section 4.1 begins by assuming, for simplicity, that regimes evolve according to an exogenous Markov chain. Section 4.2 then incorporates the MS process into investors' beliefs. Conditional on a given regime, I show that in general the mean slope and term premium are not equal. The wedge between the two is what I call '*level risk*': that is, the risk of a level shift in the short-rate process in case the economy switches to a new regime. I conclude by showing that the level risk potentially can explain the Slope-Volatility Puzzle.

4.1 A Simple Markov-Switching Environment

Suppose that the short-rate, i_t, follows a regime-switching process. In particular, I assume that:

- **Assumption (1) – Markov-Switching Environment:**

  The short-rate i_t follows a Markov-Switching process with two possible states: $s_t \in \{1, 2\}$. Regimes evolve according to an exogenous Markov Chain with constant regime-switching probabilities arranged in the 2×2 matrix P. The element in the i^{th} row and j^{th} column of P represents $\Pr(s_{t+1} = j/s_t = i) \equiv p_{ij}$ for $i, j \in \{1, 2\}$. Accordingly, each line of P must sum to one. The regime-switching probabilities are known to all agents, who are also assumed to observe the realization of s_t in the beginning of period t. [16]

Assumption (1) implies that the short rate follows a potentially different dynamic, conditional on each regime. Particularly important for the results below, the short rate may fluctuate around different means across regimes. The assumptions that the Markov Chain is exogenous and features a constant transition matrix are made in order to simplify the derivations below. Additionally, I focus on the 2-regime case for expositional purposes; generalizations for an arbitrary number of regimes are simple to obtain.

I further assume that:

- **Assumption (2) – Past State Dependence:**

  The MS process for the short-rate is covariance-stationary in each regime and has the following property:

  $$E\left[i_t/\mathcal{S}_t\right] = E\left[i_t/s_t\right] \quad \text{for } t = 1, 2, 3, ...$$

  where $S_t \equiv \{s_0, s_1, ..., s_t\}$ corresponds to the history of regimes realized up to period $t - 1$.

Assumption (2) means that the average level of the short rate depends only on the current regime, not on regimes realized in previous periods. The intuition is: if, for example, regime shifts imply changes of chairman of the Fed, then the average short-rate level chosen by a new chairman does not depend on the economic dynamics during previous regimes.

[16] This is different from empirical applications of the Hamilton (1989) filter, where agents use the available information to filter out the probability of being in each regime. See Hamilton (1993).

Although this assumption is debatable, I adopt it on the grounds that it greatly simplifies the derivations that follow. Additionally, this assumption arises naturally as a property of the reduced form of the model which I analyze in Section 6. [17]

4.2 Accounting for Markov-Switching Probabilities in Agents' Beliefs

Given the proposed MS environment, I now compute the average yield curve slope, conditional on each regime. Let Ω_t represent the complete information set available to investors in period t, which summarizes all aspects of history that are relevant to the economy's future evolution, including s_t. To build the intuition, I start by considering the 2-period slope and later generalize for the slope at the τ-period horizon. In case τ equals 2, taking expectations on both sides of equation (1), conditional on $s_t = s$, yields:

$$E\left[i_{2,t} - i_t / s_t = s\right] = \frac{E\left[i_t / s_t = s\right] + E\left[E\left[i_{t+1}/\Omega_t\right]/s_t = s\right]}{2} - E\left[i_t / s_t = s\right] + E\left[NTP_{2,t} / s_t = s\right]$$

for $s = 1, 2$. Since s_t is contained in Ω_t, the law of iterated expectations implies that:

$$\begin{aligned} E\left[E\left[i_{t+1}/\Omega_t\right]/s_t = s\right] &= E\left[i_{t+1}/s_t = s\right] \\ &= p_{s1} E\left[i_{t+1}/s_t = s, s_{t+1} = 1\right] + p_{s2} E\left[i_{t+1}/s_t = s, s_{t+1} = 2\right] \end{aligned}$$

where the second equality uses the probabilities contained in $\mathbf{P}$ to parameterize expectations so that regime switches are taken into account explicitly. Combining the last two equations and using the Assumption (2) above[18] yields:

$$\begin{aligned} E\left[i_{2,t} - i_t / s_t = s\right] &= E\left[NTP_{2,t}/s_t = s\right] \\ &\quad + \left(\frac{p_{s1} E\left[i_{t+1}/s_{t+1} = 1\right] + p_{s2} E\left[i_{t+1}/s_{t+1} = 2\right] - E\left[i_t / s_t = s\right]}{2}\right) \end{aligned} \tag{5}$$

[17] See equation (17).

[18] See Appendix C for the case where Assumption 2 is dropped.

for $s \in \{1, 2\}$. Therefore, unlike the case analyzed in Section 3.1, once investors' beliefs incorporate the regime-switching probabilities, the mean slope conditional on regime s in general is not equal to the mean term premium in that particular regime. Instead, it is equal to the mean term premium *plus* the expression in parentheses in equation (5). This new term arises from the EH component of the slope definition in equation (1). Unlike the case analyzed in Section 3.1, conditional on a given regime, the EH component now does not cancel out in expectations. Intuitively, the new term takes into account the risk that the average level of the short-rate process switches due to a regime change. I refer to the expression in parentheses in equation (5) as the *'level risk'*.

For a short-rate process that is covariance-stationary in each regime, it is easy to show that equation (5) can be written more compactly as:

$$E\left[i_{2,t} - i_t/s_t = 1\right] = \frac{\left(1 - p_{11}\right)\mathcal{D}_s E\left[i_t/s_t\right]}{2} + E\left[NTP_{2,t}/s_t = 1\right] \tag{6}$$

$$E\left[i_{2,t} - i_t/s_t = 2\right] = \frac{-\left(1 - p_{22}\right)\mathcal{D}_s E\left[i_t/s_t\right]}{2} + E\left[NTP_{2,t}/s_t = 2\right] \tag{7}$$

where $\mathcal{D}_s E\left[i_t/s_t\right]$ is defined as the average short-rate differential across regimes, i.e. $\mathcal{D}_s E\left[i_t/s_t\right] \equiv E\left[i_t/s_t = 2\right] - E\left[i_t/s_t = 1\right]$. Without loss of generality, let $E\left[i_t/s_t = 2\right] \geq E\left[i_t/s_t = 1\right]$, which implies that $\mathcal{D}_s E\left[i_t/s_t\right]$ is non-negative. It follows that, because $p_{11}, p_{22} \in [0, 1]$, the level risk is non-negative in the low-average short-rate regime 1 and non-positive in the high-average short-rate regime 2. Intuitively, conditional on the economy being in regime 1, the possibility of future regime changes introduces a risk of increase in the average short-rate level, which would in turn reduce bond prices. Conditional on regime 2, the opposite would be true: regime switches would represent a risk of reduction in the average level of the short rate, which would increase bond prices. If p_{11} increases, everything else being constant, then the level risk associated with regime 1 falls, because switching from regime 1 becomes less likely. Similarly, the level risk associated with regime 2 falls in absolute value as p_{22} increases.

Note that only in two particular cases does the level risk equal zero. First, if $E\left[i_t/s_t = 2\right] = E\left[i_t/s_t = 1\right]$, then switching regimes implies no change in the average level of the short rate.

As a result level risks are zero in each regime. Also, if investors believe regime $s \in \{1, 2\}$ to be an absorbing state, that is if $p_{ss} = 1$, then the level risk conditional on this regime is zero: once regime s is reached, investors perceive it as lasting indefinitely; consequently, further regime shifts are not considered when investors form beliefs about the future.

Generalizing equations (6) and (7) for $\tau > 2$ is simple. Following Hamilton (1994), the probability that an observation of regime i will be followed k periods ahead by an observation of regime j, that is $\Pr(s_{t+k} = j/s_t = i)$, is the element in the i^{th} row and j^{th} column of $\mathbf{P}^k$. I denote this probability by $\left[\mathbf{P}^k\right]_{ij}$. The generalizations of equations (6) and (7) for the case $\tau \geq 2$ are then:

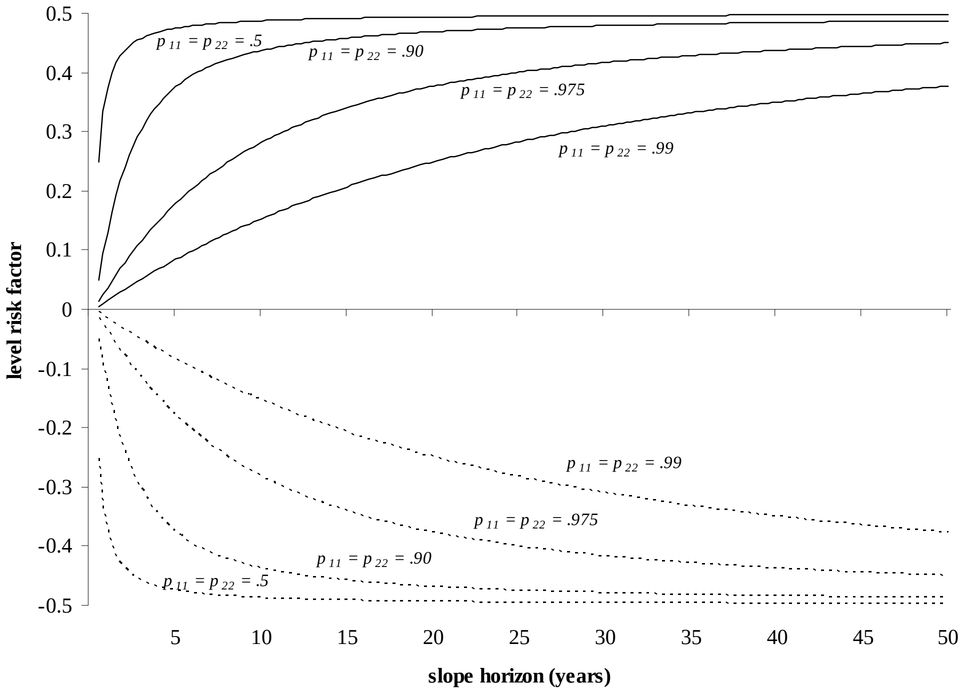
$$E\left[i_{\tau,t} - i_t/s_t = 1\right] = \left(\frac{(\tau-1)}{\tau} - \frac{1}{\tau}\sum_{k=1}^{\tau-1}\left[\mathbf{P}^k\right]_{11}\right)\mathcal{D}_s E\left[i_t/s_t\right] + E\left[NTP_t^\tau/s_t = 1\right] \tag{8}$$

$$E\left[i_{\tau,t} - i_t/s_t = 2\right] = -\left(\frac{(\tau-1)}{\tau} - \frac{1}{\tau}\sum_{k=1}^{\tau-1}\left[\mathbf{P}^k\right]_{22}\right)\mathcal{D}_s E\left[i_t/s_t\right] + E\left[NTP_t^\tau/s_t = 2\right]\ . \tag{9}$$

It can be shown that as $\tau \to \infty$, the factors multiplying the interest rate differential $\mathcal{D}_s E\left[i_t/s_t\right]$ in equations (8) and (9) converge to $1 - p^{erg}$ and $-p^{erg}$ respectively, where p^{erg} is the ergodic probability associated with regime 1 (and the one associated with regime 2 is $1 - p^{erg}$). In case $p_{11} = p_{22} < 1$, the ergodic probability of the Markov-Chain is $p^{erg} = 0.5$, which implies that in the limit, where $\tau \to \infty$, half of the interest rate gap $\mathcal{D}_s E\left[i_t/s_t\right]$ is reflected in the average slope in each regime. For example, for an interest rate differential of 4% across the two regimes, in the limit as $\tau \to \infty$ the level risk in regimes 1 and 2 will be sizeable – namely 2% and -2% respectively.

Is the level risk quantitatively relevant for values of τ far form the limit? Figure 2 plots on the vertical axis the factors multiplying the interest rate differential $\mathcal{D}_s E\left[i_t/s_t\right]$ in equations (8) and (9) against the slope horizon τ on the horizontal axis. Each full line corresponds to factors associated with equation (8) for a particular choice of the regime-switching probabilities. The dashed lines show the same, but in case of equation (9). Figure 2 displays only the case where $p_{11} = p_{22}$, which implies that the level risk associated with regime 2 is the mirror image of that associated with regime 1. For example, the line labeled

Figure 2: The Level Risk for Different Slope Horizons



Notes: Each full line corresponds to the factors multiplying the mean interest rate gap from equation (8) as a function of τ for a different choice of regime-switching probabilities. The dashed lines represent the same for equation (9).

$p_{11} = p_{22} = 0.90$ says that, at the 5-year horizon (i.e. $\tau = 20$), slightly less than 40% of the interest rate differential $\mathcal{D}_s E\left[i_t/s_t\right]$ is going to be reflected in the average slope conditional on regime 1. The dashed line shows that the same quantity, but with a negative sign, appears in the slope conditional on regime 2.

Note that the level risk increases in absolute value as the slope horizon increases. Intuitively, the probability of a regime switch happening during the life of the long-term bond considered in the slope measure increases in τ. Additionally, as the probability of remaining in the same regime gets closer and closer to one, the level risk takes longer to reach its limiting value as $\tau \to \infty$. As a result, it becomes quantitatively less relevant for values of τ far from the limit. However, even for the case where $p_{11} = p_{22} = 0.99$, the magnitude of the level risk at the 10-year horizon corresponds to a sizeable 15% of the mean interest rate differential across regimes.

One important implication of the level risk is that, when the true data-generating process displays regime switches, term structure models that do not allow for MS regimes probably will generate biased term premia estimates. In other words, if in reality agents consider the possibility of regime shifts when forming expectations, then models that do not take this into account will tend to force the term premium to explain too much of the cross section of yields. I emphasize that, because the level risk operates exclusively through investor's expectations, *this bias appears even if the term structure model is fitted to particular subsamples of the data that correspond to a single economic regime.* Consider, for example, the case where a researcher tries to fit a model that does not allow for regime switches to a particular subsample of the data encompassing a regime characterized by a relatively low short rate. If this regime is not an absorbing state, then the existence of a positive level risk implies that term premia estimates based on this model could be biased upwards.

To conclude, let me return to Figure 1. Can the level risk help to explain the Slope-Volatility Puzzle? In principle the answer is yes. When agents incorporate the MS short-rate process into their beliefs, the average slope in each subsample depicted in this figure may represent a combination of term premium and level risk. If, for example, there was a negative level risk during the Burns / Miller and Volcker subsamples and a positive level risk in the Greenspan / Bernanke years, then the puzzle potentially could be resolved. For this to be true, the mean short-rate in the regime represented by the Burns / Miller and Volcker subsamples must be higher than in the Greenspan / Bernanke regime. Additionally, both regimes must not be perceived by investors as absorbing states.

5 Level Risks and the Slope-Volatility Puzzle

This section provides level-risk estimates for different regimes of the U.S. economy. Subsection 5.1 models the dynamics of the U.S. economy according to a two-states Markov-Switching Vector Autoregression (MS-VAR). Subsection 5.2 demonstrates that, if investors form expectations based on the estimated MS-VAR, the level risks were moderate and positive in the Greenspan / Bernanke years and large and negative in the Burns / Miller and Volcker subsamples. After controlling for the level risk, the term premium in the Greenspan / Bernanke years were substantially smaller than in the Burns / Miller and Volcker periods.

Accounting for level risks thus solves the Slope-Volatility Puzzle.

5.1 A Simple MS-VAR of the U.S. Economy

I model the dynamics of the U.S. economy during the post-World War II period according to a quarterly MS-VAR which includes three variables: the inflation rate π_t; the growth rate of consumption Δc_t; and the short-term nominal interest rate i_t. The MS-VAR can be written as

$$\begin{aligned} \mathbf{Y}_t &= \Phi_{0(s_t)} + \Phi_{1(s_t)}\mathbf{Y}_{t-1} + ... + \Phi_{q(s_t)}\mathbf{Y}_{t-q} + \mathbf{u}_t \\ \mathbf{u}_t &\sim N\left(\mathbf{0}, \mathbf{\Sigma}_{(s_t)}\right) \end{aligned} \tag{10}$$

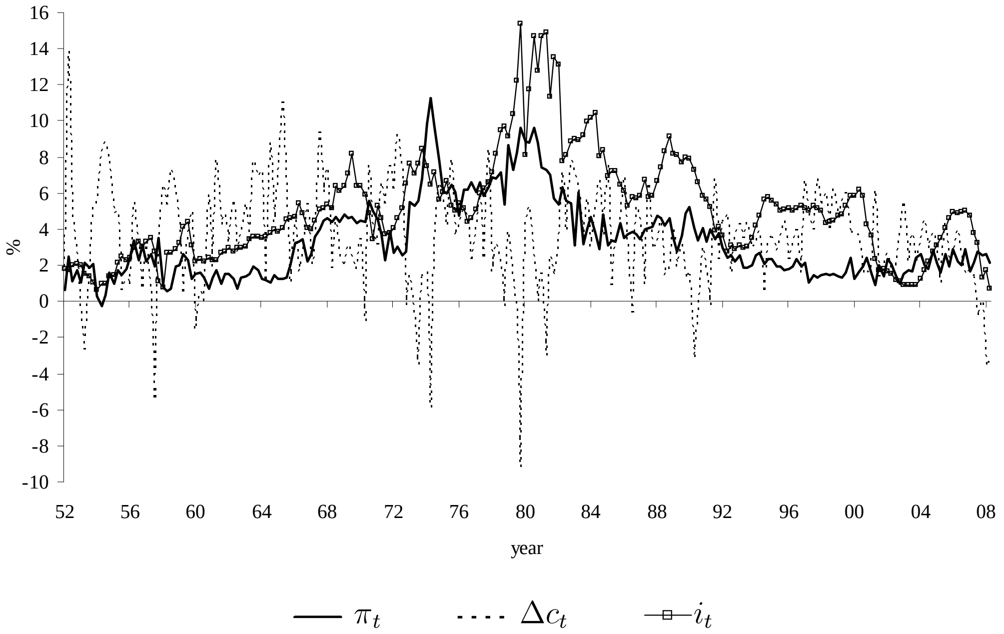
where $\mathbf{Y}_t \equiv (\pi_t, \Delta c_t, i_t)'$ and $\mathbf{u}_t$ is a 3×1 vector of *iid* reduced-form innovations. The unknown regime-switching parameters are organized in the 3×1 vector of intercepts $\Phi_{0(s_t)}$, in the 3×3 matrices of autoregressive coefficients $\Phi_{k(s_t)}$ for $k = 1, ..., q$ and finally in the 3×3 covariance matrix of error terms $\mathbf{\Sigma}_{(s_t)}$. In this empirical application I consider the more general case where there are potentially more than two regimes indexed by $s_t \in \{1, 2, ..., K\}$. As before, regimes switch over time according to an exogenous and ergodic Markov-Chain with a $K \times K$ transition matrix given by $\mathbf{P}$.

This MS-VAR can be viewed as a reduced form of the general equilibrium model that I develop in Section 6, where regime switches trigger changes in the coefficients of the monetary policy rule followed by the monetary authority. As noted by Benati and Surico (2009), changes in the coefficients of the monetary policy rule can affect both the autoregressive coefficients and the covariance matrix of the innovations in the model's reduced-form[19].

The time series included in the MS-VAR were described in Section 3.2 and are shown in Figure 3. I use data from 1952:2 until 2008:4 in order to be consistent with the available yield curve slope data shown previously. I also include in the dataset q observations before 1952:2 as an initial condition for the MS-VAR.

[19] In reality, the reduced form associated with the DSGE model from Section 6 has both linear and quadratic terms (the model is solved to a 2^{nd} order approximation), while for simplicity the MS-VAR has only linear terms. This does not represent a serious problem, because in the context of the DSGE model the variables included in the MS-VAR are well approximated by a linear solution.

Figure 3: Time Series Included in the MS-VAR



Notes: Inflation (π_t), consumption growth (Δc_t) and the short-rate (i_t) are measured in percent per annum.

Assuming that agents form expectations based on the knowledge of the current regime realization and all of the parameters in equation (10), the level risk can be computed easily using the method discussed in Section 4. In the two-regimes case, the estimated transition matrix and the expected value of the variables included in the MS-VAR, $E\left[\mathbf{Y}_t/s_t\right]$, can be used to evaluate the level risk terms in equations (8) and (9).

5.2 Estimation Results

I estimated equation (10) via Maximum Likelihood by applying the Hamilton (1989)[20] filter. To choose the number of regimes K and the MS-VAR lag length q, I used standard information criteria. Table 2 shows some important model selection information for different choices of K and q. The first column reports the number of estimated parameters in each specification. The second through fourth columns respectively report the maximized value

[20] It was assumed in Section 4.2 that the agents in the economy observe the current and all past regime realizations. The econometrician, on the other hand, needs to filter out probabilities for the regime realizations conditional on the available information. That is, since at any point in the time series the researcher does not know *ex-ante* the state of the Markov-Chain, the best she can do is to use an optimal filter to ascribe probabilities for each state. This filter is described in Hamilton (1989). Here, the estimation algorithm was implemented via Krolzig's MSVAR package for OX that uses the EM methods discussed in Krolzig (1997).

Table 2: MS-AR Model Selection Criteria

		number of parameters	Log-likelihood	AIC	BIC
$K=1$	$q=1$	15	-1136.8	10.18	10.45
	$q=2$	21	-1121.8	10.12	10.53
	$q=3$	27	-1112.6	10.12	10.66
	$q=4$	33	-1092.9	10.03	10.70
$K=2$	$q=1$	38	-1030.1	9.41	$\boxed{9.98}$
	$q=2$	56	-1007.6	9.37	10.22
	$q=3$	74	-992.9	9.40	10.52
	$q=4$	92	-965.7	9.32	10.71
$K=3$	$q=1$	60	-989.8	9.25	10.15
	$q=2$	87	-939.0	$\boxed{9.04}$	10.35
	$q=3$	114	-939.4	9.28	11.00
	$q=4$	141	-917.6	9.33	11.45

Notes: In the lines, K is the number of regimes and q is the MS-VAR's lag-length. In the columns, AIC is the Akaike Information Criterion and BIC is the (Schwartz) Bayesian Information Criterion.

of the log-likelihood, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). According to the AIC and BIC criteria, the best fitting models are highlighted in the table.

Both the AIC and BIC select models with $K > 1$. Note that the number of estimated parameters in case $K > 1$ increases dramatically with the MS-VAR's lag-length. As a result, both information criteria point to models with low lag-lengths. It is important to note that as the number of estimated parameters increase, it is more likely that the optimization algorithm used to estimate the model will get stuck in local maxima. In Table 2, for the model with $K = 3$, when I increase the lag length from $q = 2$ to $q = 3$, the log-likelihood decreases. This is a sign that the algorithm is stuck in a local maximum. To avoid this problem, I choose the best fitting model according to the BIC criterion ($K = 2$ and $q = 1$). Apart from having desirable large sample properties, this criterion penalizes models with an excessive number of parameters more heavily than does the AIC.

All of the parameter estimates for the best fitting MS-VAR with $K = 2$ and $q = 1$

Table 3: MS-VAR Conditional Moments

	Regime 1	Regime 2
Average regime duration in years	34.6	7.7
Ergodic Probabilities	0.818	0.182
$E[\pi_t/s_t]$	2.38	4.93
$E[\Delta c_t/s_t]$	3.30	4.33
$E[i_t/s_t]$	3.96	6.80
$SD[\pi_t/s_t]$	1.39	2.90
$SD[\Delta c_t/s_t]$	2.65	3.73
$SD[i_t/s_t]$	2.23	3.29

Notes: The conditional moments were computed using numerical simulation of the MS-VAR with *K=2* and *q=1*. The average regime duration and ergodic probabilities were computed according to Hamilton (1994).

are reported in Appendix B. Some selected conditional moments for the MS-VAR also are shown in Table 3, while the filtered and smoothed regime probabilities are shown in Figure 4.

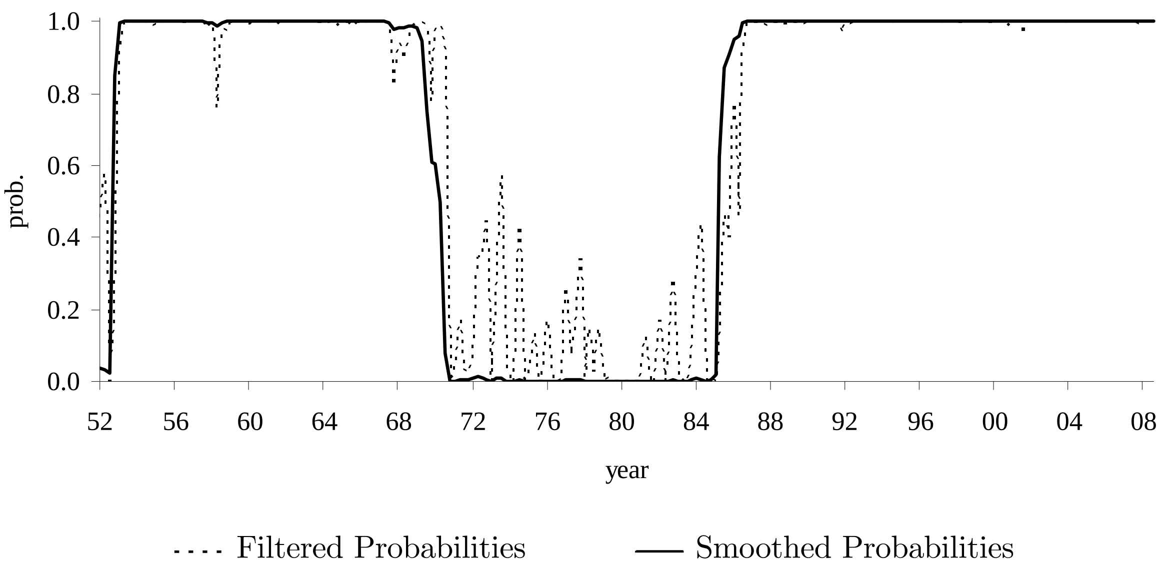
For convenience I reproduce here the estimated MS-VAR transition matrix

$$\begin{pmatrix} \widehat{p}_{11} & \widehat{p}_{12} \\ \widehat{p}_{21} & \widehat{p}_{22} \end{pmatrix} = \begin{pmatrix} \underset{(0.007)}{0.993} & 0.007 \\ 0.033 & \underset{(0.023)}{0.967} \end{pmatrix}$$

where the numbers in parentheses are standard errors. Note that, since p_{11} is close to unity, regime 1 is very persistent and close to being an absorbing state. According to Table 3, this regime has an average duration of about 35 years. On the other hand, the probability of remaining in regime 2, p_{22}, is considerably smaller, implying an average duration of only 7.7 years for regime 2. Note, however, that the estimate for p_{22} is not very precise.

Based on the smoothed regime probabilities, regime 1 almost exactly encompasses the Martin and the Greenspan / Bernanke subsamples from Section 3. In particular, the MS-VAR interprets the Great Moderation years as realizations of this regime. On the other

Figure 4: MS-VAR Filtered and Smoothed Probatilities of Regime 1



Notes: Probabilities computed using the filters described in Hamilton (1994) for the MS-VAR with *K=2* and *q=1*.

hand, regime 2 roughly covers the Burns / Miller and Volcker subsamples from Section 3 (this regime also appears with a high probability very briefly from 1952:2 to 1953:1) - it therefore encompasses the Great Inflation years. The conditional moments in Table 3 reveal that when regime 1 occurs, the inflation rate and the short-term interest rate fluctuate around significantly lower levels than when regime 2 is realized. Moreover, confirming the results obtained in Section 3.2, the economy conditional on regime 1 is substantially more stable than when regime 2 realizes[21].

I now use the mean short-rate gap across regimes and the regime-switching probabilities estimated above to parameterize investors' beliefs. Using equations (8) and (9), it is straightforward to compute the level risk conditional on each regime.

Using the information reported in Table 3, we note that the mean interest rate differential across regimes 1 and 2, $\mathcal{D}_s E\left[i_t/s_t\right]$, is equal to 2.84%. Using the estimated ergodic probabilities reported in the same table, it is easy to show that in the limit as $\tau \to \infty$ the level risk conditional on regime 1 is given by $(1 - 0.818) \times 2.84\% = 0.52\%$. A similar calculation for regime 2 yields a limiting level risk of $-0.818 \times 2.84\% = -2.32\%$. Therefore,

[21] Interestingly, the regime classification that emerges from Figure 4 is very similar to the one estimated by Ang and Bekaert (2002).

Table 4: Yield Curve Slope Decomposition

			Regime 1	Regime 2	*Great Moderation* (85:3-08:4)	*Great Inflation* (70:3-85:2)
(1)	Average Slope	τ = 1 year	0.38	0.46	0.40	0.48
		τ = 3 years	0.75	0.78	0.92	0.80
		τ = 5 years	0.99	0.91	1.25	0.94
		τ = 10 years	1.47	1.08	1.84	1.08
(2)	Level Risk	τ = 1 year	0.03	-0.14	0.03	-0.14
		τ = 3 years	0.10	-0.45	0.10	-0.45
		τ = 5 years	0.16	-0.70	0.16	-0.70
		τ = 10 years	0.26	-1.15	0.26	-1.15
(3)	Residual [(1)-(2)]	τ = 1 year	0.35	0.60	0.37	0.61
		τ = 3 years	0.65	1.22	0.82	1.25
		τ = 5 years	0.84	1.61	1.09	1.64
		τ = 10 years	1.22	2.23	1.58	2.23

Notes: Average slope measures conditional on regimes 1 and 2 were computed for the sample 1985:3-2008:4 and 1970:3-1985:2 respectively. Level risk measures are based on the estimated MS-VAR.

in the limit the estimated level risk becomes sizeable, especially for the less persistent regime 2. Observe the sign of the level risk in each regime. In regime 2 investors anticipate that a future switch to regime 1 is possible. This switch would imply that the short rate fluctuates around a significantly lower level than the current one. As a result, there is a *negative* level risk. In regime 1, on the other hand, investors believe that there is a possibility of switching to the highly volatile regime 2, which implies a shift towards higher levels of the short rate. In this case a *positive* level risk arises. Because regime 1 is more persistent than regime 2, the level risk is larger in absolute value in the latter than in the former regime.

For values of τ far away from the limit, the estimated level risks at different maturities are reported in the second (regime 1) and third (regime 2) columns of Table 4. In both regimes, the level risk is relevant in terms of magnitude: in regime 2, the level risk grows quickly to -115 basis points at the 10-year horizon, while in regime 1 it is still capable of increasing the average 10-year slope by 26 basis points.

By rearranging equations (8) and (9), we can see that subtracting the estimated level risk from the average slope in each regime makes it possible to recover the term premium. However, because my MS-VAR does not include the yield curve slope, I cannot directly obtain the mean value of the slope conditional on each regime. I therefore use the sample average of observed slope measures as a *proxy*.

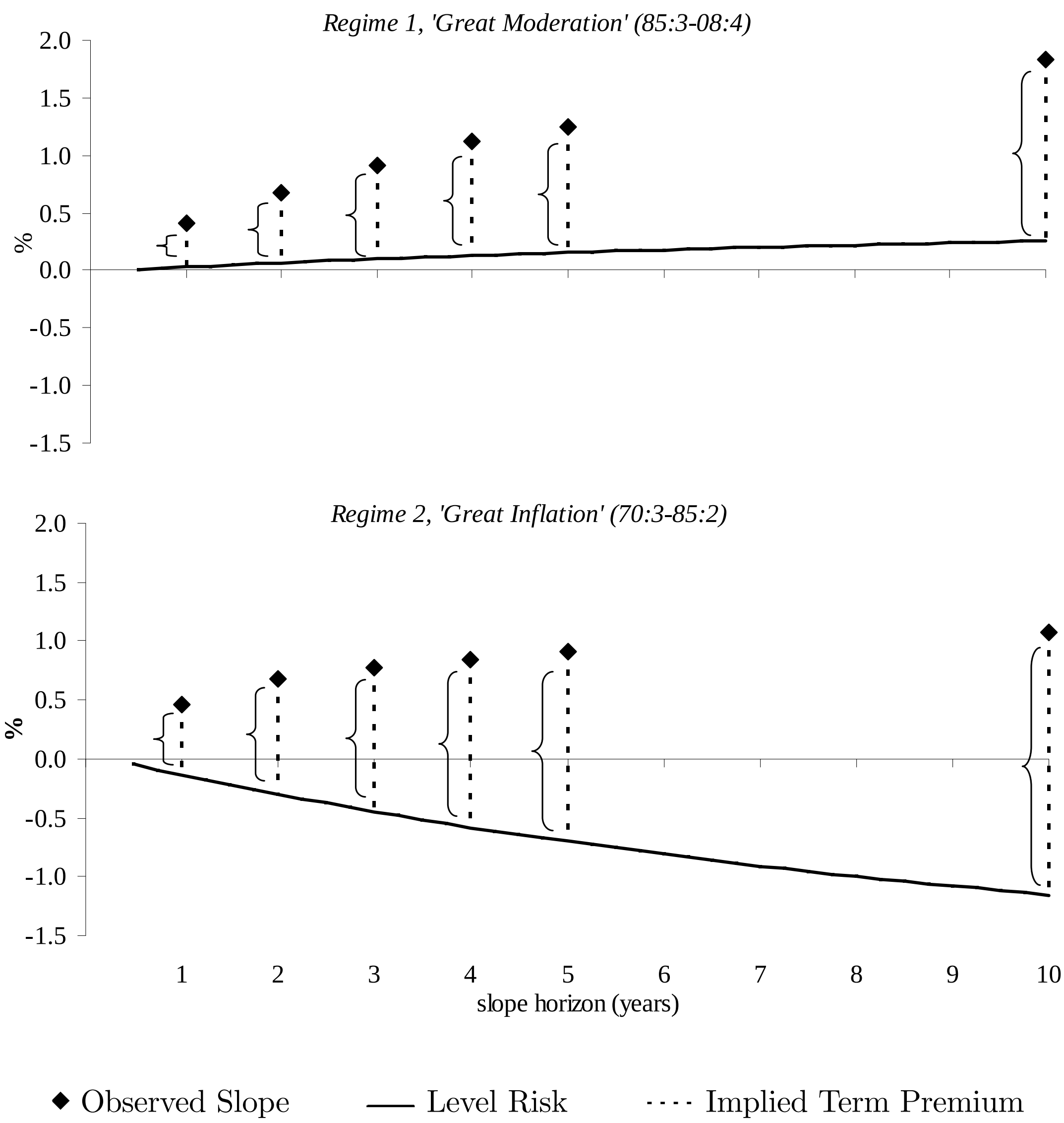
Sample averages of slope measures at different maturities[22] are reported in panel (1) of Table 4. In the second column I report average yield curve slopes for the quarters when regime 1 was associated with a smoothed probability greater that 50%. The third column reports the same measures for quarters where regime 2 was the predominant regime according to the smoothed probabilities. The fourth and fifth columns of the table report average slope measures for two particular subsamples of the data of interest: first, the 1985:3-2008:4 subsample, which is the portion of regime 1 that corresponds to the 'Great Moderation'; second, the 1970:3-1985:2 subsample, which corresponds to the 'Great Inflation' years of regime 2. Panel (3) of Table 4 shows the residuals obtained after subtracting the level risk from these average slope measures.

The slope averages across regimes restate the puzzle from Section 3: conditional on the high volatility regime 2, the slope on average is *lower* than in regime 1. This is true for the 5- and 10-year maturities reported in the second and third columns of Table 4, but if we restrict our attention to the Great Inflation and Great Moderation subsamples (fourth and fifth columns of Table 4), this is true for almost all reported maturities. Importantly, once I control for the level risk, the puzzle disappears completely. That is, once level risks are subtracted from the slope measures, regime 2 displays substantially higher premia than regime 1 at all maturities analyzed[23]. The same is true when I restrict my attention to the Great Inflation and Great Moderation subsamples. In other words, based on slope measures nominal bonds seem less risky in the Great Inflation than in the Great Moderation not because of a seemingly abnormal pattern of term premia across regimes, but rather simply because of an outcome of beliefs that incorporate the possibility of regime switches.

[22] In addition to the 5 and 10-year slope measures discussed before, I consider in this section the 1, 2, 3, and 4-year slope measures calculated again using the CRSP database.

[23] This is similar to Bekaert, Hodrick, and Marshall (2001). They find that some term structure 'anomalies' can be explained by a combination of term premia and a 'peso problem' in the short-rate process.

Figure 5: Decomposing the Observed Mean Slope



An alternative way to illustrate this last result is shown in Figure 5. In the top chart, the full line plots the estimated level risks in regime 1 against increasing maturities in the horizontal axis. The diamond markers correspond to the average slope measures from Table 4 for the 'Great Moderation' subsample. The bottom chart shows the same analysis but for the level risks conditional on regime 2 and the average slope measures that correspond to the 'Great Inflation' subsample.

Although the yield curve in both regimes on average is positively sloped at all available horizons, the decomposition of the slope is quite different. In regime 1 the slope measures are a combination of positive term premia and positive level risks, but in regime 2 the yield curve is positively sloped *in spite of substantially negative level risks*. Conditional on regime 2, it takes very high term premia to offset the negative level risks and to generate the observed slope. The residual term premia, indicated in Figure 5 by the curly brackets, are in fact significantly larger in regime 2 than in regime 1.

6 Level Risks in a Structural Model with MS Monetary Policy Regimes

In the MS-VAR estimated in the previous section, what makes the short-rate fluctuate around different levels across regimes, giving rise to level risks? Similarly, what makes the economy less volatile in regime 1 than in regime 2? The reduced-form MS-VAR is silent about the possible structural changes experienced by the macroeconomy once it switches to a new regime.

In this section, I investigate whether differences in how the Fed conducts monetary policy jointly can explain the different macro dynamics and the yield curve shape across regimes. I find that the main features of the data explored in the previous sections can be replicated in a simple MS-DSGE that allows the economy to switch over time between active and passive monetary policy regimes. Private agents incorporate the MS possibility into their beliefs, which in turn has important implications for both the macroeconomic equilibrium and the yield curve.

In Section 6.1 I start by describing the MS-DSGE model and proposing an approximate non-linear solution method. The proposed solution method allows agents to accumulate precautionary savings, thus giving rise to non-trivial bond premia. In particular, the model endogenously generates a short-rate differential across regimes as private agents acquire different levels of precautionary savings depending on the monetary policy stance. As a result, level risks appear along the yield curve. In Section 6.2 I show that under a plausible choice of parameters the model is able to replicate the Slope-Volatility Puzzle. In other words, when there is a passive policy regime, the amount of macro uncertainty is substantially higher than in the active regime; as a result, term premia are higher. However, because of level risks generated endogenously through precautionary savings, during the passive regime the yield curve is less steep than in the active one, thus reproducing the Slope-Volatility Puzzle.

6.1 Model Description and Solution

6.1.1 Monetary Policy Regimes

Following Davig and Leeper (2007) and many others[24], I assume that the Fed sets the short-term nominal interest rate i_t according to the following MS feedback rule

$$i_t = \bar{i} + \phi_{\pi(s_t)}\widehat{\pi}_t + \phi_{y(s_t)}\widehat{y}_t$$

where $\widehat{\pi}_t$ and $\widehat{y}_t$ are the log deviations of aggregate inflation and output from the deterministic steady state[25]. The crucial difference between this formulation and more standard Taylor rules is that here the policy reaction coefficients $\phi_{\pi(s_t)}$ and $\phi_{y(s_t)}$ at time t depend on the regime realization $s_t \in \{1, 2\}$. [26] As a result, a regime switch can trigger changes in how the Fed sets the short rate in order to fight deviations of inflation and output from their steady-state levels.

Without loss of generality, I set $\phi_{\pi(1)} \geq \phi_{\pi(2)}$, meaning that monetary policy in regime 1 is at least as 'active' with respect to inflation deviations as in regime 2. In particular, when $\phi_{\pi(1)} > \phi_{\pi(2)}$, regime 2 is considered 'less active' than regime 1. When $\phi_{\pi(2)} < 1$, policy in regime 2 is said to be 'passive'.

In a model with fixed parameters, a passive monetary policy rule implies indeterminacy of the equilibrium solution[27]. In that case, the policy rate increases less than one-to-one with an increase in inflation; consequently, the ex-post short-term real interest rate falls. However, in the case of a regime-switching Taylor rule, this is not necessarily true. As

[24] See, for example, Farmer, Waggoner, and Zha (2007), Davig and Doh (2008), Liu, Waggoner, and Zha (2009) and Liu and Mumtaz (2010).

[25] Adding an autocorrelated monetary policy shock to the monetary policy rule does not significantly change any of my results.

[26] A potentially interesting extension would be to analyze models with a regime-dependent inflation target. However, in a recent study, Liu, Waggoner, and Zha (2010) found no empirical support for this specification in the U.S. data.

[27] When $\phi_y \neq 0$, the threshold above which ϕ_π respects the Taylor principle is slightly below but still very close to unity for reasonable Taylor-rule parameterizations. See Bullard and Mitra (2002).

Davig and Leeper (2007) show, although monetary policy is passive in one regime, if the probability of switching to an active enough regime is sufficiently high, then the model has a unique stable solution.

For consistency with my previous results, I assume an MS environment in line with Assumption (1) from Section 5. That is, I again let the economy switch between two different regimes over time. These regimes evolve according to an exogenous Markov Chain indexed by $s_t \in \{1, 2\}$, with transition matrix $\mathbf{P}$. Finally, private agents again are assumed to observe the current regime realization s_t before making decisions; accordingly, the complete information set available to private agents at date t will be denoted by $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$.

6.1.2 Private Agents

The macro model contains four agents: households, final and intermediate good producers and a monetary authority. The latter was described in the previous section. I now analyze the behavior of each remaining agent in turn. Detailed model derivations can be found in Appendix D.

Following Rudebusch and Swanson (2008), the household sector has a representative infinitely-lived agent endowed with Epstein and Zin (1989, 1991) and Weil (1990) preferences. Letting C_t and N_t represent the household's consumption and labor supply, those preferences are described by:

$$V_t = \begin{cases} u\left(C_t, N_t\right) + \beta \left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if} \quad u\left(C_t, N_t\right) \geq 0 \text{ everywhere} \\ \\ u\left(C_t, N_t\right) - \beta \left[E_t \left(-V_{t+1}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if} \quad u\left(C_t, N_t\right) \leq 0 \text{ everywhere} \end{cases} \tag{11}$$

where V_{t+1} denotes the utility continuation value to the household. The period utility is given by $u\left(C_t, N_t\right) = e^{b_t}\left(\frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta}\right)$, where b_t represents a time-preference shock. As Epstein and Zin (1989) show, these preferences disentangle the coefficient of risk aversion from the elasticity of intertemporal substitution (EIS), which are constrained in standard expected utility preferences to be the reciprocal of one another. In the particular parametrization above, the degree of risk aversion is associated with (but not equal to) $\alpha \in \mathbb{R}$, whereas the

EIS is given by $1/\gamma$. When $\alpha = 0$, the standard expected utility case is recovered.

The representative household maximizes (11) subject to the budget constraint

$$P_t C_t + E_t \widetilde{M}_{t+1} X_{t+1} \leq X_t + P_t W_t N_t + T_t$$

where P_t is the aggregate price level and $E_t \widetilde{M}_{t+1} X_{t+1}$ is the value of a complete portfolio of state-contingent assets with $\widetilde{M}_{t+1}$ representing the *nominal* stochastic discount factor and X_{t+1} the portfolio holdings from period t to $t+1$. Additionally, W_t represents the real wage rate and T_t summarizes all lump-sum transfers to the household. It follows from the household's optimization problem that the one-period *real* SDF and the labor supply are respectively given by

$$\begin{aligned} M_{t,t+1} &= \beta \left[\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} e^{b_{t+1} - b_t} \\ W_t &= \chi \frac{N_t^{\eta}}{C_t^{-\gamma}} \end{aligned} \tag{12}$$

where, under complete markets, the SDF can be used to price nominal bonds of different maturities recursively. More specifically, letting $\Pi_{t+1} \equiv P_{t+1}/P_t$, the price of a τ-period nominal bond is given by $B_{\tau,t} = E_t \left[M_{t,t+1} B_{\tau-1,t+1} \Pi_{t+1}^{-1} \right]$. In the specific case of a one-period bond, the pricing condition becomes $B_{1,t} = E_t \left[M_{t,t+1} \Pi_{t+1}^{-1} \right]$. Note that the term in parenthesis in the SDF expression, which also appears in expected utility models, captures the current consumption risk. The term in square brackets containing the continuation utility value introduces aversion to long-run consumption and labor risks.

Final good firms operate under perfect competition, and the representative producer is endowed with the following technology

$$Y_t = \left(\int_0^1 Y_{t(f)}^{\frac{1}{1+\lambda_t}} di \right)^{1+\lambda_t}$$

where Y_t is the quantity of final goods produced through a combination of $Y_{t(f)}$ of each intermediate good $f \in [0, 1]$. Following Steinsson (2003) and Smets and Wouters (2003), I allow the degree of substitutability across differentiated intermediate goods λ_t to vary over time. A decrease in λ_t reduces the monopoly power of intermediate producers, which in turn reduces their price markup. Profit maximization in the final-good sector yields a demand curve for each intermediate good

$$Y_{t(f)} = \left(\frac{P_{t(f)}}{P_t}\right)^{-\frac{1+\lambda_t}{\lambda_t}} Y_t \tag{13}$$

where $P_{t(f)}$ is the price of intermediate good f. The aggregate price level is then given by $P_t = \left(\int_0^1 P_{t(f)}^{-\frac{1}{\lambda_t}} di\right)^{-\lambda_t}$.

In the intermediate-good sector, all firms have identical Cobb-Douglas production functions given by

$$Y_{t(f)} = A_t \overline{K}^{\theta} N_{t(f)}^{1-\theta} \tag{14}$$

where the level of capital $\overline{K}$ is assumed for simplicity to be fixed, $N_{t(f)}$ is the amount of labor employed by firm f, and $Y_{t(f)}$ is its level of output. The aggregate level of technology is denoted by A_t. As in Rotemberg (1982), firms can reset the prices of each differentiated good in every period, but incur intangible quadratic adjustment costs in doing so

$$\frac{\xi}{2}\left(\frac{P_{t(f)}}{P_{t-1(f)}}\frac{1}{\overline{\Pi}} - 1\right)^2 P_t Y_t$$

where $\overline{\Pi}$ is the steady state rate of inflation. These costs do not affect the firm's cash-flows, but must be considered in the optimization problem. Therefore, each intermediate firm f chooses $P_{t(f)}$ so as to maximize the expected discounted sum of future profits corrected by the adjustment costs

$$E_t\left\{\sum_{j=0}^{\infty} M_{t,t+j}\frac{P_t}{P_{t+j}}\left[D_{t+j(f)} - \frac{\xi}{2}\left(\frac{P_{t+j(f)}}{P_{t+j-1(f)}}\frac{1}{\overline{\Pi}} - 1\right)^2 P_{t+j}Y_{t+j}\right]\right\}$$

where $D_{t(f)}$ represents the period t profit which is given by $P_{t+j(f)}Y_{t+j(f)} - P_{t+j}W_{t+j}N_{t+j(f)}$. Since firms are owned by households, $M_{t,t+j} \equiv \prod_{k=1}^{j} M_{t+k}$ is used to discount future profits. The optimization problem is constrained by sequences of equations (13) and (14) starting from period t onwards.

The market clearing condition in the final good market is given by

$$Y_t = C_t + G_t + \delta\overline{K}$$

where G_t represents a shock to the market clearing condition and, following Rudebusch and Swanson (2008), a constant amount $\delta\overline{K}$ of the final good is used to repair the depreciated capital. If G_t is interpreted as government purchases, then it is assumed that the government runs a balanced budget financed through lump-sum taxes obtained from the household sector.

To complete the model description, assume that the four exogenous shocks follow AR(1) processes:

$$\begin{aligned} b_t &= \rho_b b_{t-1} + \sigma_b \varepsilon_t^b \\ \log A_t &= \rho_A \log A_{t-1} + \sigma_A \varepsilon_t^A \\ \log(1+\lambda_t) &= (1-\rho_\lambda)\log\left(1+\overline{\lambda}\right) + \rho_\lambda \log(1+\lambda_{t-1}) + \sigma_\lambda \varepsilon_t^\lambda \\ \log G_t &= (1-\rho_G)\log\overline{G} + \rho_G \log G_{t-1} + \sigma_G \varepsilon_t^G \end{aligned}$$

with independently and identically distributed innovations $\varepsilon_t^k \sim N(0,1)$ for $k \in \{b, A, \lambda, G\}$.

6.1.3 Model Solution

The macro model has five (exogenous) predetermined variables: b_t, A_t, λ_t, G_t and s_t. The first four are standard continuously differentiable variables, whereas the last is a discrete variable that follows a Markov chain with transition matrix $\mathbf{P}$. In order to write the Markov chain compactly, I define a 2×1 vector $\boldsymbol{\xi}_t \equiv (1[s_t = 1], 1[s_t = 2])'$ where $1[s_t = j]$ is an indicator function which is equal to one if $s_t = j$. Hamilton (1994) shows that the Markov chain can be represented in terms of $\boldsymbol{\xi}_t$ by the autoregressive process $\boldsymbol{\xi}_{t+1} = \mathbf{P}\boldsymbol{\xi}_t + \boldsymbol{\nu}_t$, where $\boldsymbol{\nu}_t$ is a heteroskedastic zero mean vector of innovations that can only assume discrete values. Collecting the differentiable predetermined variables in a $n_x \times 1$ vector $\mathbf{x}_t \equiv (b_t, \log A_t, \log(1+\lambda_t), \log G_t)'$ and letting $\mathbf{y}_t$ represent the $n_y \times 1$ vector of (natural logarithms of the) non-predetermined variables, the macroeconomic system together with the bond pricing equations can be written as

$$
\begin{aligned}
&E_t\left[f\left(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \boldsymbol{\xi}_t\right)\right]=0 \\
&\mathbf{x}_{t+1}=\left(I_4-\boldsymbol{\Lambda}\right) \overline{\mathbf{x}}+\boldsymbol{\Lambda} \mathbf{x}_t+\Sigma \sigma \boldsymbol{\varepsilon}_{t+1} \\
&\boldsymbol{\xi}_{t+1}=\mathbf{P} \boldsymbol{\xi}_t+\boldsymbol{\nu}_t
\end{aligned} \tag{15}
$$

where $\boldsymbol{\varepsilon}_t \equiv \left(\varepsilon_t^b, \varepsilon_t^A, \varepsilon_t^\lambda, \varepsilon_t^G\right)'$ is the vector of *i.i.d.* standard normal shocks. The coefficient matrices are given by $\boldsymbol{\Lambda} = diag(\rho_b, \rho_A, \rho_\lambda, \rho_G)$, $\Sigma = diag\left(\sigma_b, \sigma_A, \sigma_\lambda, \sigma_G\right)$ and $\overline{\mathbf{x}} = \left(0, 0, \log\left(1+\overline{\lambda}\right), \log \overline{G}\right)'$, while σ is a perturbation parameter that scales uncertainty in the model.

The model above forms a system of Markov-Switching non-linear rational expectations equations for which an analytical solution is not known. Solution methods based on standard linear approximations have been proposed in the literature – e.g. Davig and Leeper (2007) and Farmer, Waggoner, and Zha (2010b)–, but they would give rise to zero risk premia implied in the prices of financial assets and therefore are not useful in my context. I will therefore look for a second-order approximation to the true model solution using the perturbation techniques suggested by Schmitt-Grohe and Uribe (2004)[28].

In order to be able to apply perturbation techniques to the system (15), one needs to be able to differentiate it with respect to all state variables. But because $\boldsymbol{\xi}_t$ is discrete, it is impossible to apply these techniques directly to the system above. Therefore I define an *extended system*[29] of equations in which the dependence of the control variables on regimes is made explicit. It is then straightforward to implement perturbation methods to the extended system, because it is fully differentiable in the state variables.

In order to write the extended system, I introduce a state-contingent notation. That is,

[28] In a model where only the shock volatilities follow exogenous MS processes, Amisano and Tristani (2010a,b) show that the perturbation solution is particularly easy to obtain. However, their solution method does not apply to the general case where other model parameters, such as the ones in the policy rule, are allowed to follow MS processes.

[29] Note that I rewrite the *non-linear* Markov Switching model represented by system (15) into the extended form. Davig and Leeper (2009) apply a similar transformation to a Markov-Switching model *after* log-linearizing the equilibrium conditions and refer to this as the "linear representation". Since here the model remains non-linear after being transformed into the extended form, I refer to it as the "extended system" to avoid confusion.

I denote the value of the vector of endogenous variables $\mathbf{y}_t$ contingent on $s_t = s \in \{1, 2\}$ by $\mathbf{y}_{t(s)}$. Using this notation, the expected value of $\mathbf{y}_{t+1}$ conditional on Ω_t can be parameterized as follows: $E\left[\mathbf{y}_{t+1}/\Omega_t^{-s}, s_t = s\right] = p_{s1}E\left[\mathbf{y}_{t+1(1)}/\Omega_t^{-s}\right] + p_{s2}E\left[\mathbf{y}_{t+1(2)}/\Omega_t^{-s}\right]$. The extended non-linear system therefore can be written as:[30]

$$
\begin{gathered}
F\left(\mathbf{y}_{t+1(1)}, \mathbf{y}_{t+1(2)}, \mathbf{y}_{t(1)}, \mathbf{y}_{t(2)}, \mathbf{x}_t\right) \equiv E\left[\begin{array}{c} f_1\left(\mathbf{y}_{t+1(1)}, \mathbf{y}_{t+1(2)}, \mathbf{y}_{t(1)}, \mathbf{x}_t\right) \\ f_2\left(\mathbf{y}_{t+1(1)}, \mathbf{y}_{t+1(2)}, \mathbf{y}_{t(2)}, \mathbf{x}_t\right) \end{array} /\Omega_t^{-s}\right] = 0 \\
\mathbf{x}_{t+1} = \left(I_4 - \mathbf{\Lambda}\right)\overline{\mathbf{x}} + \mathbf{\Lambda}\mathbf{x}_t + \Sigma\sigma\boldsymbol{\varepsilon}_{t+1} \\
\boldsymbol{\xi}_{t+1} = \mathbf{P}\boldsymbol{\xi}_t + \boldsymbol{\nu}_t
\end{gathered} \tag{16}
$$

where each equilibrium condition in the original system (15) is represented by two entries in $F(\cdot)$, each contingent on one possible realization of $\boldsymbol{\xi}_t$. Note that the expectations operator does not condition on s_t because the MS probabilities are already dealt with by the state-contingent notation. Observe too that $f_1(\bullet)$, for example, depends on the $t+1$ vector of control variables contingent on regime 1 and 2. Writing the system this way makes explicit the fact that, in general, the solution in each regime will depend crucially on the behavior of the economy in the alternative regime. Expectations that policy may switch in the future will affect households' and firms' decisions today and will lead to a different equilibrium relative to a model without switching regimes. Only when both regimes are absorbing states, that is $p_{11} = p_{22} = 1$, will the solution in each regime be independent of the behavior of the economy in the alternative regime. Unlike in (15), the extended system only has differentiable predetermined variables. Therefore a perturbation solution can be obtained easily[31].

[30] See Appendix E for more details on the extended non-linear system.

[31] To write the extended system, it is crucial that there be no regime-dependent state variables in the model. Imagine, for example, that time-varying capital is included in the model. Then, in order to rewrite the system in extended form, we would need to keep track of all the history of realized regimes. It then would be impossible to solve the model using the method proposed here.

The model solution I seek takes the form

$$\begin{aligned}
\begin{pmatrix} \mathbf{y}_{t(1)} \\ \mathbf{y}_{t(2)} \end{pmatrix} &= \begin{pmatrix} g^1(\mathbf{x}_t, \sigma) \\ g^2(\mathbf{x}_t, \sigma) \end{pmatrix} \\
\mathbf{x}_{t+1} &= (I_4 - \mathbf{\Lambda})\,\overline{\mathbf{x}} + \mathbf{\Lambda}\mathbf{x}_t + \Sigma\sigma\boldsymbol{\varepsilon}_{t+1} \\
\boldsymbol{\xi}_{t+1} &= \mathbf{P}\boldsymbol{\xi}_t + \boldsymbol{\nu}_t \ .
\end{aligned}$$

My aim is to approximate $g^1(\bullet)$ and $g^2(\bullet)$ around the deterministic steady state defined by $\mathbf{x}_{t+1} = \mathbf{x}_t = \overline{\mathbf{x}}$ and $\sigma = 0$, which implies that $\overline{g}^j = g^j(\overline{\mathbf{x}}, 0)$ for $j \in \{1, 2\}$. This is a convenient approximation point because an analytical solution to the non-linear system can be found easily. Note that the regime-switching parameters $\phi_{\pi(s_t)}$ and $\phi_{y(s_t)}$ do not affect the economy in the deterministic steady state, which implies that $\overline{g}^1 = \overline{g}^2 = \overline{g}$.

Using insights from Schmitt-Grohe and Uribe (2004), a second-order approximate solution to the vector of control variables conditional on $s_t = s$ is given by

$$\mathbf{y}_{t(s)} \cong \overline{g} + \overline{g}_x^s(\mathbf{x}_t - \overline{\mathbf{x}}) + \frac{1}{2}\begin{bmatrix} (\mathbf{x}_t - \overline{\mathbf{x}})' \,\overline{g}_{xx[1]}^s (\mathbf{x}_t - \overline{\mathbf{x}}) \\ \cdots \\ (\mathbf{x}_t - \overline{\mathbf{x}})' \,\overline{g}_{xx[n_y]}^s (\mathbf{x}_t - \overline{\mathbf{x}}) \end{bmatrix} + \frac{1}{2}\overline{g}_{\sigma\sigma}^s \sigma^2 \quad \text{for } s = 1, 2 \tag{17}$$

where $\overline{g}_x^s$ is a $n_y \times n_x$ matrix of first derivatives of $g^s(\bullet)$ with respect to the state variables and $\overline{g}_{xx[k]}^s$ for $k = 1, ..., n_y$ are symmetric $n_x \times n_x$ matrices of second derivatives of $g^s(\bullet)$ again with respect to the state variables. The $n_y \times 1$ vector $\overline{g}_{\sigma\sigma}^s$ denotes the second derivatives of $g^s(\bullet)$ with respect to the scalar σ. All matrices of first and second derivatives are evaluated at the deterministic steady state. Finally, note that the $\overline{g}_\sigma^s$ and $\overline{g}_{x\sigma}^s$ terms omitted from equation (17) are proven to be equal to zero by Schmitt-Grohe and Uribe (2004).

From equation (17), some important properties of the model solution emerge. First, uncertainty as measured by σ only shifts the constant term of the policy function by $\overline{g}_{\sigma\sigma}^s$. This shift causes the control variables to fluctuate around a *stochastic steady state*, which corrects for the precautionary savings motive and gives rise to risk premia in financial assets. Because the cross derivatives between the state variables and σ are all zero up to a second-order, the risk premia *within any given regime* are constant. Note, however, that the second derivative term with respect to σ will in general depend on the current regime realization which in turn will give rise to discrete changes in risk premia as regimes alternate.

Standard perturbation methods solve for the unknown coefficients of the above Taylor expansion by taking derivatives of (16) with respect to $\mathbf{x}_t$ and σ, which are equal to zero and can be evaluated easily at the deterministic steady state. All unknown coefficients of equation (17) are then determined by solving relatively simple systems of equations[32].

6.2 Model Analysis

Here I start by calibrating the parameters of the MS-DSGE model. I then analyze how well the model that is based on switching monetary policy regimes is able to replicate the key empirical macro and yield curve moments analyzed in Section 5.

For convenience, the empirical moments I focus on in this section are reproduced in Table 5. In particular, I am interested in simultaneously replicating two sets of empirical moments. First, the MS-DSGE model should be able to reproduce the variances of inflation, consumption growth, and the short rate, conditional on each regime of the MS-VAR. These are displayed in the top panel of Table 5. Second, I require that the MS-DSGE replicates the yield curve slope decomposition based on the MS-VAR, which I reproduce in the bottom panel of Table 5 (for simplicity, I focus here only on the 10-year maturity). For the latter set of moments, I focus in this section on the empirical moments shown in the fourth and fifth columns of Table 4, which correspond to the Great Inflation and Great Moderation subsamples.

The objective of this calibration exercise is to verify whether the model implied moments, conditional on the monetary policy regimes 1 (more active) and 2 respectively, replicate the empirical moments for the Great Moderation and Great Inflation subsamples shown in Table 5. Put more formally:

[32] Conditions for uniqueness of a bounded solution in Markov-Switching DSGE models have been established by Farmer, Waggoner, and Zha (2010a), but apply to the case of linearized models. More general conditions that apply to our original non-linear system (15) are not yet available in the literature. In what follows, I consider only bounded equilibria that are unique for a linearized version of the model using the Farmer, Waggoner, and Zha (2010a) method.

Table 5: Summary of the Empirical Moments

	Great Moderation	*Great Inflation*
(1) Macro Moments:		
$SD\left[\pi_t/s_t\right]$	1.39	2.90
$SD\left[\Delta c_t/s_t\right]$	2.65	3.73
$SD\left[i_t/s_t\right]$	2.23	3.29
(2) Yield Curve Moments:		
$E\left[i_{10Y,t} - i_t/s_t\right]$	1.84	1.08
of which:		
$\quad E\left[NTP_{10Y,t}/s_t\right]$	1.58	2.23
$\quad LevelRisk_{10Y}$	0.26	-1.15

Notes: The macro moments in panel (1) are reproduced from table 3, whereas the decomposition of the 10-year average slope in panel (2) is reproduced from the fourth and fifth columns of table 4.

- **The Proposed Calibration Exercise:**

  Consider the MS-DSGE model described above where $\phi_{\pi(1)} > \phi_{\pi(2)}$. Do the model implied moments conditional on regime 1 (regime 2) replicate the empirical Great Moderation (Great Inflation) moments shown in Table 5?

The spirit of this exercise is to study how well the model, relying *only* on shifts in monetary policy, replicates the macro and yield curve moments in Table 5. To keep the results easy to interpret I refrain from analyzing models with MS volatilities, even though this extra feature potentially could improve the model's fit.

6.2.1 Choice of Parameters

I calibrate the model such that, conditional on regime 1, it fits the Great Moderation moments in Table 5. The parameter choices shown in Table 6 follow estimated DSGE models such as Lubik and Schorfheide (2004), Smets and Wouters (2007) and Justiniano and Primiceri (2008).

The parameters in the monetary policy rule conditional on the more active regime are set according to the post-1982 estimates in Lubik and Schorfheide (2004), i.e. $\phi_{\pi(1)} = 2.19$ and $\phi_{y(1)} = 0.075$. In regime 2, the Fed's response to inflation is set to the lowest value that guarantees the existence of a unique stable model equilibrium, i.e. $\phi_{\pi(2)} = 0.948$. For simplicity, I also set $\phi_{y(2)} = 0.075$. [33] As in Section 5, the transition probabilities were set to $p_{11} = 0.993$ and $p_{22} = 0.967$.

For the household's preferences, I choose $\beta = 0.99$ that implies an annualized real discount rate of 4% in the deterministic steady state. The utility consumption curvature γ is set to 2: this implies an EIS of one half, in line with micro data estimates such as Vissing-Jorgensen (2002). Following Smets and Wouters (2007), I set the inverse Frisch elasticity of the labor supply η to 0.4. I set α according to the 'best fit' specification in Rudebusch and Swanson (2008). A traditional measure of risk aversion suggested by Epstein and Zin (1989),

[33] In a reduced form macro-term structure model, Ang, Boivin, Dong, and Loo-Kung (2009) find that the Fed's policy response to the output gap was roughly stable over the sample period I analyze.

Table 6: The Benchmark Calibration

Monetary Policy Rule:				Exogenous Processes:			
$\phi_{\pi(1)}$	2.19	$\phi_{\pi(2)}$	0.948	ρ_b	0.83	σ_b	0.020
$\phi_{y(1)}$	0.075	$\phi_{y(2)}$	0.075	ρ_A	0.98	σ_A	0.005
p_{11}	0.993	p_{22}	0.967	ρ_λ	0.18	σ_λ	0.051
				ρ_G	0.94	σ_G	0.008
Structural Parameters:				**The Steady State:**			
β	0.99	θ	0.33	$\overline{\Pi}$	1.004		
γ	2	$\overline{\lambda}$	0.2	$\overline{K}/\left(4\overline{Y}\right)$	2.5		
η	0.40	ξ	233	$\overline{G}/\overline{Y}$	0.2		
α	-108	δ	0.02				

$\gamma + \alpha(1-\gamma)$, would then imply a coefficient of relative risk aversion of 110. Although high, this risk aversion measure applies only to endowment economies and, in models with a flexible labor supply, suffers from a substantial upward bias (see Swanson (2009)). Moreover, estimated DSGE models with recursive preferences fitted to U.S. bond prices usually feature a high level of risk aversion (e.g. Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2011)).

The capital share in the production function θ is set to 0.33. Also, I choose $\overline{\lambda} = 0.2$, implying a steady-state price markup of 20%. The price adjustment costs parameter ξ is set to 233 which, for a linearized version of my model, corresponds to a Calvo coefficient of 0.75. [34]

The four exogenous shock processes were calibrated as follows: the parameters associated with the shocks to the household's preferences and to the price markup follow the estimates in Justiniano and Primiceri (2008) and Smets and Wouters (2007) respectively for the post-Great Inflation subsamples[35]. For the technology and government expenditure shocks, I

[34] Setting $\xi = \frac{\varphi\left(1-\theta+\theta\frac{1+\overline{\lambda}}{\overline{\lambda}}\right)\frac{1}{\overline{\lambda}}}{(1-\varphi)(1-\varphi\beta)(1-\theta)}$ in the model considered here yields the same linearized Phillips curve slope as in a Calvo (1983) version of this model where a fraction φ of the intermediate good producers are not able to reset prices in each period. See Keen and Wang (2007).

[35] Although the markup shock in Smets and Wouters (2007) follows an ARMA(1,1) process, here I consider a more standard AR(1) process. The parameters ρ_λ and σ_λ are calibrated so that the dynamics of the AR(1) are as close as possible to the ARMA(1,1) in Smets and Wouters (2007). None of the results change significantly if I had used instead the ARMA(1,1) process.

fit an AR(1) model to a constructed Solow-residuals series and to the Real Government Consumption series over the 1985:3-2008:4 sample.

Finally, the non-stochastic steady state of the model is set as: $\overline{K}/\left(4\overline{Y}\right) = 2.5$, $\overline{G}/\overline{Y} = 0.20$ and $\delta = 0.02$. The value of χ makes labor in the deterministic steady state equal to one. In the stochastic steady state, $\overline{\Pi}$ makes the model fit the average short-rate in the Great Moderation regime.

6.2.2 Results

Result I: *The nominal term premium is higher in the passive than in the active monetary policy regime.*

Table 7 shows the empirical moments discussed above alongside comparable moments implied by the calibrated MS-DSGE model. Panel (1) focuses on the macro moments, while panels (2) and (3) focus on yield curve moments. In particular, panel (2) explores the decomposition of the average slope into term premium and level risk, whereas panel (3) decomposes the nominal term premium into a real term premium component and a compensation for inflation risk (see equation (3)). [36]

As expected, conditional on the active policy regime, the model replicates the relatively low macro volatilities observed during the Great Moderation period fairly well (the calibration above was tailored to fit these moments). When the model economy switches to the passive policy regime, however, both real and nominal volatilities become substantially higher. Although the model conditional on regime 2 overshoots the level of nominal volatility observed in the Great Inflation period, it qualitatively replicates the empirical macro moments in each subsample of the data adequately.

Note that the model-implied 10-year nominal term premium in regime 2 is 35 basis points higher than in regime 1. Even though the nominal term premium conditional on regime 2

[36] The model-implied nominal term premium decomposition was obtained as follows: let $\widetilde{B}_{\tau,t} \equiv exp(-\tau r_{\tau,t})$ represent the period t price of a bond that pays one unit of the final good at $t+\tau$. I include in the MS-DSGE model the following recursive pricing conditions: $\widetilde{B}_{\tau,t} = E_t\left[M_{t,t+1}\widetilde{B}_{\tau-1,t+1}\right]$ for $\tau \geq 1$ with initial condition $\widetilde{B}_{0,t} = 1\ \forall t$. The real term premium is then computed using the definition in Section 3.1.

is not as high as my estimate for the Great Inflation regime, the model generates a higher nominal term premium in the regime associated with higher levels of macro uncertainty. Thus it replicates an important result obtained in Section 5.

To better understand this result, I explore the decomposition shown in panel (3) of Table 7. Consider, for example, an economy that switches from a passive to an active regime. Two forces pressure the 10-year nominal term premium in opposing directions. On the one hand, as the level of nominal uncertainty falls, the inflation risk portion of the nominal term premium – the inflation risk shown in table 7 includes an inflation convexity term, as shown in equation (3) – drops by 62 basis points. On the other hand, the 10-year real term premium increases by 27 basis points. Intuitively, as the Fed becomes more aggressive in fighting inflationary pressures, the volatility of the short-term *real* interest rate increases, making long-term real bonds riskier. Given my choice of parameters, the first effect dominates and, as a result, the nominal term premium *decreases* as the economy goes from an active to a passive regime.

Result II: *The model endogenously generates realistic level risks along the yield curve.*

Perhaps the most striking feature of Table 7 is that the MS-DSGE model is able to *endogenously* generate level risks very much in line with the ones estimated in Section 5. As discussed above, for this to be true the nominal short-rate process that results from the model must fluctuate around a different mean conditional on each regime. That is, $\mathcal{D}_s E\left[i_t/s_t\right]$ must be different from zero. Because the short-rate in both regimes is equal to $\log\left(\overline{\Pi}/\beta\right)$ in the deterministic steady state (in fact, the deterministic steady state is the same across regimes for all variables in the model), the existence of level risks implies that the model generates a short-rate differential $\mathcal{D}_s E\left[i_t/s_t\right]$ endogenously in the *stochastic* steady state.

Returning to equation (17), note that uncertainty causes the constant term in the approximate policy function to shift from the deterministic steady state $\overline{g}$ to the stochastic steady state $\overline{g}+\frac{1}{2}\overline{g}^s_{\sigma\sigma}$ for $s=1,2$. The size of this shift for any given variable may depend upon the monetary policy regime. That is, each regime may be characterized by differ-

Table 7: Actual vs. Model-Based Moments

	US Data		MS-DSGE Model	
	Great Moderation	*Great Inflation*	Regime 1 $\phi_{\pi(1)} = 2.19$	Regime 2 $\phi_{\pi(2)} = 0.95$
(1) Macro volatility:				
$SD\left[\pi_t/s_t\right]$	1.39	2.90	1.27	4.92
$SD\left[\Delta c_t/s_t\right]$	2.65	3.73	2.57	3.41
$SD\left[i_t/s_t\right]$	2.23	3.29	2.72	4.62
(2) Slope decomposition:				
$E\left[i_{10Y,t} - i_t/s_t\right]$	1.84	1.08	1.61	0.95
$=$				
$E\left[NTP_{10Y,t}/s_t\right]$	1.58	2.23	1.43	1.78
$+$				
$Level\ Risk_{10Y}$	0.26	-1.15	0.18	-0.83
(3) Term Premium decomposition:				
$E\left[NTP_{10Y,t}/s_t\right]$	1.58	2.23	1.43	1.78
$=$				
$E\left[RTP_{10Y,t}/s_t\right]$	–	–	0.66	0.39
$+$				
$Inflation\ Risk_{10Y}$	–	–	0.77	1.39

Notes: E[x] and SD(x) respectively represent the mean and standard deviation of x. The empirical moments from Table 5 are reproduced here. The two last columns display model implied theoretical moments, except for the standard deviation of consumption growth, which was simulated. All variables are expressed in percent per annum. The inflation risk shown in the last row includes an inflation convexity term (see equation (3)).

ent levels of precautionary savings. Accordingly, the conditional mean of the short rate in general will be different across regimes[37].

To better understand this mechanism, Table 8 displays first moments of key macro variables conditional on each regime. The model generates a nominal short-rate differential of 2.04%, close to the 2.84% estimate based on the MS-VAR from Section 5. This differential is a result of three mechanisms in the model:

1. The real short-rate mechanism:

   In Table 8, the real short-rate r_t on average is 0.79% higher in the passive than in the active regime, producing an upward pressure on i_t in the former regime. To understand this mechanism, remember that consumption uncertainty is higher in the passive than the active regime. The risk-averse household thus responds by forming more precautionary savings in the passive than in the active regime (in Table 8, the average level of consumption is lower in the passive regime), implying that the expected growth rate of consumption is positive in the passive, and negative in the active, regime. The short-term real rate therefore is higher in the passive regime in order to counter the household's desire to smooth consumption over time.

2. The inflation level mechanism:

   The short-term nominal rate is also higher in the passive than in the active regime because inflation on average is 2.04% higher in the former regime. Due to the real short-rate mechanism explained above, intermediate good producers face a higher discount on their future profits in the passive than in the active regime. Because current profit has a higher weight in their objective function, firms choose higher optimal prices in the passive than in the active regime. In other words, intermediate firms are more concerned with the future implications of their pricing decision today in the active regime.

[37] Even though level risks are associated with the EH component of the slope in the presence of a MS short-rate process, they only appear in the MS-DSGE model because of the second-order term $\overline{g}^s_{\sigma\sigma}$ in equation (17). It follows that level risks are zero if one considers a standard first order approximation to the model solution, or if private agents are risk-neutral. Therefore, in the MS-DSGE model, level risks behave very much like standard premia.

Table 8: Understanding the Nominal Short-Rate Differential Across Regimes

	(a) Regime 1 $\phi_{\pi(1)} = 2.19$	(b) Regime 2 $\phi_{\pi(2)} = 0.95$	(b) - (a)
$E\left[i_t/s_t\right]$	3.90%	5.94%	2.04%
$E\left[r_t/s_t\right]$	2.93%	3.72%	0.79%
$E\left[\pi_t/s_t\right]$	0.87%	2.01%	1.14%
$E\left[C_t/s_t\right]$	1.90	1.89	-0.01
$E\left[\frac{C_{t+1}}{C_t}/s_t\right]$	-0.01%	0.06%	0.07%

Notes: Model-implied theoretical moments are expressed in percent per annum, except for consumption which is expressed in level.

3. The inflation risk mechanism:

   As discussed above, the difference between i_t and r_t represents compensation for inflation risk (again including an inflation convexity term). Because inflation uncertainty is higher in the passive than in the active regime, short-term nominal bonds pay higher premia in the former than in the latter regime. Using the moments reported in Table 8, note that the compensation for inflation risk falls from 0.21% to 0.10% as policy switches from passive to active.

What happens to the 10-year average yield curve slope when the economy switches from a passive to an active regime? First, as a result of the three channels explained above, the level risk switches from being highly negative to moderately positive, imposing an upward pressure on the 10-year slope. At the same time, because the short-term real rate is more volatile in the active than in the passive regime, the real term premium increases, also putting upward pressure on the 10-year slope. Finally, going in the opposite direction, the drop in inflation uncertainty as the economy switches to the active regime results in a sharp drop in the compensation for inflation risk that is paid by 10-year nominal bonds. Under the proposed calibration, the first two effects dominate and the 10-year slope actually increases as the economy switches from a passive to an active policy regime. Therefore I conclude that the MS-DSGE model in which monetary policy switches between active and passive regimes is able to replicate the Slope-Volatility Puzzle.

Result III: *The MS economy in the active regime is riskier than a corresponding economy with a fixed active policy.*

Table 9 compares the active regime of the MS-DSGE model under two different assumption for p_{11}: in the second column, p_{11} is set as in the benchmark calibration to 0.993, while in the third column the active regime is assumed to be an absorbing state, that is $p_{11} = 1$. All other model parameters are kept at the values showed in Table 6. In the case of $p_{11} = 1$, once the economy reaches regime 1 the MS-DSGE model behaves exactly as a simpler model with a fixed active monetary policy rule.

Note that the 10-year nominal term premium is higher in the active regime with $p_{11} = 0.993$ than with $p_{11} = 1$. This can be explained by a combination of two mechanisms. The first, in line with the Barro-Rietz rare disasters theory, is as follows. When $p_{11} = 0.993$ there is a risk that during the life of the bond a (relatively) rare bad event will occur and the economy switch to the passive regime in which nominal bonds are very risky. The risk of a sudden change in policy is priced into nominal bonds. As a result, term premia in the active regime are higher when $p_{11} = 0.993$ than when $p_{11} = 1$.

A second mechanism, which resembles the long-run risk theory of Bansal and Yaron (2004), operates in parallel to the one just described. In order to explain this new mechanism, Figure 6 plots impulse response functions to a negative technological shock[38]. The solid lines represent the active regime when $p_{11} = 1$, whereas the dashed lines correspond to the active regime when $p_{11} = 0.993$. In both cases, the technological shock reduces the price of the 10-year nominal bond (inflation expectations increase) exactly when the level of consumption falls, making this bond a risky asset. Comparing models under different p_{11} values, observe that both consumption and the bond price suffer more pronounced drops

[38] As Rudebusch and Swanson (2008) show, technological shocks are the most important determinants of the term premium in general equilibrium models like the one considered here.

Table 9: Absorbing vs. Non-Absorbing Active Monetary Policy Regimes

	MS-DSGE conditional on $s_t = 1$	
	$p_{11} = 0.993$	$p_{11} = 1.000$
(1) Macro volatility:		
$SD\left[\pi_t / s_t\right]$	1.27	1.20
$SD\left[\Delta c_t / s_t\right]$	2.57	2.46
$SD\left[i_t / s_t\right]$	2.72	2.60
(2) Slope decomposition:		
$E\left[i_{10Y,t} - i_t / s_t\right]$	1.61	1.19
$=$		
$E\left[NTP_{10Y,t} / s_t\right]$	1.43	1.19
$+$		
$Level\ Risk_{10Y}$	0.18	0.00
(3) Term Premium decomposition:		
$E\left[NTP_{10Y,t} / s_t\right]$	1.43	1.19
$=$		
$E\left[RTP_{10Y,t} / s_t\right]$	0.66	0.67
$+$		
$Inflation\ Risk_{10Y}$	0.77	0.52

Notes: $E[\mathrm{x}]$ and $SD(\mathrm{x})$ respectively represent the mean and standard deviation of x. The empirical moments are reproduced here from table 5. The two last columns display model implied theoretical moments, except for the standard deviation of consumption growth which was simulated. All variables are expressed in percent per annum. The inflation risk shown in the last row includes an inflation convexity term (see equation (3)).

when this parameter is set to 0.993 than to 1, implying that investing is this bond is riskier in the former case[39]. Intuitively, when $p_{11} = 0.993$ the model equilibrium in the active regime is affected through private agents' expectations by the possibility of switching to the passive regime, generating more pronounced inflation responses than when $p_{11} = 1$. [40] Therefore, following the shock, an equally active central bank has to fight higher inflationary pressures when $p_{11} = 0.993$ than when $p_{11} = 1$, resulting in sharper drops in consumption in the former case. As in the case analyzed by Rudebusch and Swanson (2008), the possibility of regime switches can be seen as increasing the amount of long-run risk in the economy relative to a situation in which the active policy regime is perceived as an absorbing state[41].

Result IV: *The average yield curve slope is higher in IT than in non-IT countries.*

On Table 9, note that the 10-year yield curve slope is 42 basis points higher in an economy with $p_{11} = 0.993$ than in a similar economy with $p_{11} = 1$. Why? First, in light of Result III described above, the nominal term premium is higher when p_{11} is set to 0.993 than when it is set to 1. This channel alone is responsible for 24 out of the 42 basis-points difference in the slope across models. The remaining 18 basis points are explained by the level risk, which is positive when $p_{11} = 0.993$, but is equal to zero in an absorbing state (see Section 4.2).

To verify this model's prediction in the data, I compare the observed average yield curve slope across IT and non-IT countries. I focus only on developed economies after the mid-1980s, a period characterized by particularly benign inflation developments in both groups of countries. The general idea here is that, relative to the case where IT is not adopted, private agents in IT countries may believe that future switches to passive monetary policy regimes are less likely to occur[42]. In terms of the MS-DSGE model, the probability that the economy remains in the active regime, p_{11}, therefore would be higher when IT is adopted

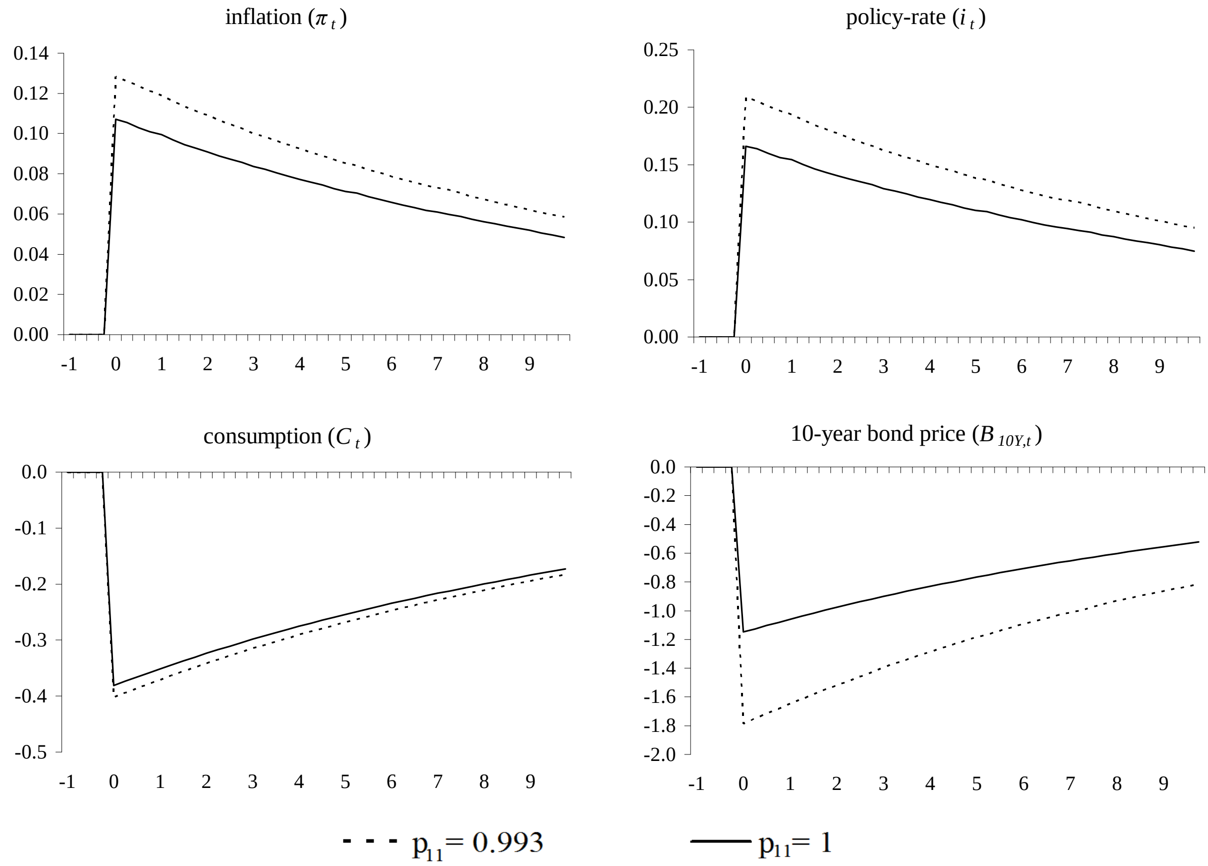
[39] This also can be observed in Table 9, which shows that the MS-DSGE model generates more macroeconomic volatility in the active regime with $p_{11} = 0.993$ than with $p_{11} = 1$.

[40] See Davig and Leeper (2007).

[41] It is interesting to note that the extra premium charged by investors when p_{11} is lower comes in the form of an extra compensation for bearing inflation risk, whereas the real term premium remains almost the same; see panel (3) of Table 9.

[42] This exercise implicitly assumes that, within an IT regime, the monetary authority responds to inflationary pressures actively.

Figure 6: Impulse Responses to a Negative Technological Shock in the Active Regime



Notes: Impulse response functions to a negative one standard deviation shock to technology. Full and dashed lines correspond respectively to the MS-DSGE model under $p_{11}= 1$ and $p_{11}= 0.993$. The vertical axes represent percentage deviations from the stochastic steady state, where the deviations of the inflation rate and the policy rate are expressed in percent per annum. The numbers in the horizontal axes represent years following the shock.

than otherwise. So, assuming all else constant, the yield curve in IT countries should be flatter on average than in non-IT countries.

There are many ways to justify why p_{11} would tend to be higher in active regimes with than without IT[43]. First, a higher p_{11} could represent improvements in communication between the Fed and the general public once IT is in place. According to this view, IT would make clearer to the public that shifts in inflation away from the objective are going to be dealt with actively. Second, in IT countries emphasis is shifted away from the particular individuals conducting monetary policy in a given point in time to the monetary policy framework itself. A higher p_{11} therefore could signal that, with IT, changes in the individuals in charge of the central bank are less likely to significantly modify the way that policy is conducted. Finally, increases in p_{11} could represent the gain in central bank accountability prompted by IT. Under IT, a central bank that decides to adopt a passive stance with respect to inflation would have to explain to the public the short and long-term implications of its decision; as a result, the probability of switching from the active regime decreases.

Using the Wright (2008) database of international monthly zero-coupon yields of up to ten years maturity, I compute slope measures for ten different countries: three non-IT countries (Germany[44], Japan, and US) and seven countries that adopted IT (New Zealand, Canada, UK, Sweden, Australia, Switzerland, and Norway). For the non-IT countries, I compute the average 10-year slope for the sample which corresponds to the Great Moderation subsample from Section 5, specifically Sep/1985 - Dec/2008. For each IT country, I instead consider the average 10-year slope from the month of IT adoption until Dec/2008.

Table 10 shows the results. The top panel reports the 10-year average slope for the non-IT countries; the bottom panel shows the measure for the IT countries. Also, to facilitate comparisons between IT and non-IT countries, I show in parentheses the average slope in each of the non-IT countries taken over the same sample as for the IT country. For example, the values in parentheses for New Zealand correspond to the average slope in the

[43] See, for example, Bernanke, Laubach, Mishkin, and Posen (1999).

[44] Bernanke, Laubach, Mishkin, and Posen (1999) consider Germany as an early case of inflation targeting. I decided to include Germany in the non-IT group because there an explicit inflation objective only has been set for the long run whereas short- to medium-term inflation targets have not been announced (this is also true after the ECB started to operate in 1999).

Table 10: Yield Curve Slope in non-IT vs. IT Countries

Non-IT Countries:

	Sample	10-Year Slope
US	Sep/85 - Dec/08	1.90
Germany	Sep/85 - Dec/08	1.25
Japan	Sep/85 - Dec/08	1.23
Avg. non-IT		**1.46**

IT Countries:

	Sample	10-Year Slope	
		IT Country	(US, Germany, Japan)
New-Zealand	Feb/90 - Dec/08	-0.01	(1.91^*, 1.19^*, 1.37^*)
Canada	Mar/91 - Dec/08	1.60	(1.95^*, 1.24 , 1.49)
United Kingdom	Oct/92 - Dec/08	0.26	(1.81^*, 1.48^*, 1.61^*)
Sweden	Jan/93 - Dec/08	1.35	(1.78^*, 1.51^*, 1.61^*)
Australia	Sep/94 - Dec/08	0.64	(1.62^*, 1.65^*, 1.58^*)
Switzerland	Jan/00 - Dec/08	1.28	(1.83^*, 1.23 , 1.26)
Norway	Apr/01 - Dec/08	0.77	(2.07^*,1.30^* , 1.22^*)
Average IT		**0.89**	

Notes: The average measures of yield curve slope were computed using the Wright (2008) database of international zero-coupon yields. As a proxy for the short-rate, the 3-month zero-coupon yield was used in the slope computation.

US, Germany and Japan taken over the Feb/90 - Dec/08 sample.

If the model's predictions are correct, then the non-IT countries should be associated with steeper yield curves than the IT countries. Indeed, Table 10 reveals that the average 10-year slope across the non-IT countries was 1.46%, more than 60% higher than in the IT countries. Using the numbers in parentheses shown in Table 10, I can compare pairs of IT and non-IT countries while fixing the same sample. Each time the slope in the non-IT country is higher than that of the IT country, I indicate it with a $*$ in the table. I find that in the great majority of the pair-wise comparisons, the IT countries had flatter yield curves than the non-IT countries.

When making cross-country comparisons, it should be noted that country-specific factors potentially could influence the results. For example, there could be specific features of some IT countries that make the yield curve flatter in those countries, even though perceptions about monetary policy are not significantly different there from in the non-IT countries. However, it is reassuring that in *almost all* pair-wise comparisons the model prediction was verified. That is, the results shown in Table 10 appear to be robust enough across different pairs of IT and non-IT countries, thus providing evidence in favor of the explanation that I propose which is based on monetary policy regimes.

7 Conclusions

In this paper I studied how shifts in the monetary policy regime might have affected the average slope of the U.S. nominal term structure in the past decades. My first contribution was to show that, in the presence of a Markov-Switching short-rate process, measures of the average yield curve slope reflect not only term premia but also level risks. I provide level risk estimates based on a simple reduced form Markov-Switching Vector Autoregression: they are large and negative during the Great Inflation and moderate and positive after 1985. Controlling for level risks, the average slope measures imply that term premia in the Great Inflation were substantially higher than after 1985.

My second contribution was to show that a calibrated dynamic general equilibrium model, where the Taylor rule shifts between an active and a passive stance for inflation, replicates my U.S. level risks and term premia estimates. Because the model was solved using a second-order rather than a standard first-order approximation method, I can analyze the different levels of precautionary savings that characterize each policy regime. The model-implied differences in term premia and level risks across regimes are a result of the optimal behavior of private agents and therefore are entirely endogenous.

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A No-Arbitrage Bond Prices

In this appendix I will show how to derive the approximate no-arbitrage bond pricing formulas discussed in section 3.1. I will start by pricing inflation protected bonds and then will move on to nominal bonds.

(i) Pricing Inflation Protected Bonds:

Let $\widetilde{B}_{\tau,t}$ represent the period t price of a bond that pays one unit of the consumption good at $t+\tau$. Because their payoffs are already expressed in terms of consumption, these bonds are not subject to inflation risk. Under no-arbitrage $\widetilde{B}_{\tau,t}$ is determined by $\widetilde{B}_{\tau,t} = E_t\left[M_{t+1}\widetilde{B}_{\tau-1,t+1}\right]$. Denote the τ-period real yield to maturity by $r_{\tau,t} \equiv -\frac{1}{\tau}\log \widetilde{B}_{\tau,t}$. Then, taking a second order approximation to the Euler equation above we get:

$$\widehat{r}_{\tau,t} = -\frac{1}{\tau}\sum_{j=1}^{\tau} E_t\left[\widehat{m}_{t+j}\right] - \frac{1}{2\tau}Var_t\left[\sum_{j=1}^{\tau}\widehat{m}_{t+j}\right] + O\left(\epsilon^3\right) \tag{18}$$

where $O\left(\epsilon^3\right)$ contains the terms of order higher than two that are ignored.
Using the equation above, it can be shown that the second order approximate real term premium is given by:

$$RTP_{\tau,t} \equiv \widehat{r}_{\tau,t} - \tfrac{1}{\tau}\textstyle\sum_{j=1}^{\tau} E_t\left[r_{t+j-1}\right] \cong -\tfrac{1}{2\tau}Var_t\left[\textstyle\sum_{j=1}^{\tau}\widehat{m}_{t+j}\right] + \tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau} E_t\left\{Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right\}$$

where I made use of the law of iterated expectations to eliminate the first moments of the SDF. It is important to note that $E_t\left\{Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right\} \neq Var_t\left[\widehat{m}_{t+j}\right]$ for $j \geq 1$. I can also rewrite the real term premium as:

$$\begin{aligned}
RTP_{\tau,t} &\cong -\tfrac{1}{2\tau}Var_t\left[\textstyle\sum_{j=1}^{\tau}\widehat{m}_{t+j}\right] + \tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau} E_t\left\{Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right\} \\
&= -\tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau} Var_t\left[\widehat{m}_{t+j}\right] - \tfrac{2}{2\tau}\textstyle\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau} Cov_t\left[\widehat{m}_{t+j},\widehat{m}_{t+k}\right] \\
&\quad + \tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau} E_t\left\{Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right\}
\end{aligned}$$

But note that:

$$\begin{aligned}
&(1)\ Var_{t+j-1}\left[\widehat{m}_{t+j}\right] = E_{t+j-1}\left[\widehat{m}^2_{t+j}\right] - \left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2 \\
&\quad\Downarrow \\
&\quad E_t\left[Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] = E_t\left[\widehat{m}^2_{t+j}\right] - E_t\left[\left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2\right]
\end{aligned}$$

$$\begin{aligned}
(2)\ Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] &= E_t\left[\left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2\right] - \left(E_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right]\right)^2 \\
&= E_t\left[\left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2\right] - \left(E_t\left[\widehat{m}_{t+j}\right]\right)^2 \\
&\Downarrow \\
Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] + \left(E_t\left[\widehat{m}_{t+j}\right]\right)^2 &= E_t\left[\left(E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right)^2\right]
\end{aligned}$$

Combining the two results above I get

$$\begin{aligned}
E_t\left[Var_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] &= E_t\left[\widehat{m}^2_{t+j}\right] - Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right] - \left(E_t\left[\widehat{m}_{t+j}\right]\right)^2 \\
&= Var_t\left[\widehat{m}_{t+j}\right] - Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right]
\end{aligned}$$

Then the real term premium can be written as:

$$\begin{aligned}
RTP_{\tau,t} &\cong -\tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau} Var_t\left[\widehat{m}_{t+j}\right] - \tfrac{1}{\tau}\textstyle\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau} Cov_t\left[\widehat{m}_{t+j},\widehat{m}_{t+k}\right] \\
&\quad + \tfrac{1}{2\tau}\textstyle\sum_{j=1}^{\tau}\left\{Var_t\left[\widehat{m}_{t+j}\right] - Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right]\right\} \\
&= -\tfrac{1}{\tau}\textstyle\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau} Cov_t\left[\widehat{m}_{t+j},\widehat{m}_{t+k}\right] - \underbrace{\frac{1}{2\tau}\sum_{j=1}^{\tau} Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right]}_{\text{SDF convexity term}}
\end{aligned}$$

Which implies that:

$$RTP_{\tau,t} \cong -\frac{1}{\tau}\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau} Cov_t\left[\widehat{m}_{t+j},\widehat{m}_{t+k}\right] - \frac{1}{2\tau}\sum_{j=1}^{\tau} Var_t\left[E_{t+j-1}\left[\widehat{m}_{t+j}\right]\right]$$

Therefore the real term premium depends on the autocorrelation structure of the SDF and a convexity term.

(ii) Pricing Nominal Bonds:

Taking a second order approximation to equation (2) I obtain:

$$\begin{aligned}\left(i_{\tau,t}-\overline{i_\tau}\right) = & -\tfrac{1}{\tau}\sum_{j=1}^{\tau} E_t\left[\widehat{m}_{t+j}\right] + \tfrac{1}{\tau}\sum_{i=1}^{\tau} E_t\left[\widehat{\pi}_{t+j}\right] \\ & -\tfrac{1}{2\tau} Var_t\left[\sum_{j=1}^{\tau}\widehat{m}_{t+j}\right] - \tfrac{1}{2\tau} Var_t\left[\sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right] \\ & +\tfrac{1}{\tau} Cov_t\left[\sum_{j=1}^{\tau}\widehat{m}_{t+j},\sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right] + O\left(\epsilon^3\right)\end{aligned}$$

Combining this equation with equation (18) yields:

$$\begin{aligned}\left(i_{\tau,t}-\overline{i_\tau}\right) = & \left(r_{\tau,t}-\overline{r_\tau}\right) + \tfrac{1}{\tau}\sum_{i=1}^{\tau} E_t\left[\widehat{\pi}_{t+j}\right] \\ & -\tfrac{1}{2\tau} Var_t\left[\sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right] + \tfrac{1}{\tau} Cov_t\left[\sum_{j=1}^{\tau}\widehat{m}_{t+j},\sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right] + O\left(\epsilon^3\right)\end{aligned}$$

which is a Fisher-type equation extended to take into account the risk premia implied in long-term bond prices. The nominal term premium according to the definition in the text can therefore be written as:

$$\begin{aligned}NTP_{\tau,t} \cong & RTP_{\tau,t} + Convexity_{\tau,t}^{\pi} \\ & +\tfrac{1}{\tau} Cov_t\left[\sum_{j=1}^{\tau}\widehat{m}_{t+j},\sum_{j=1}^{\tau}\widehat{\pi}_{t+j}\right] - \tfrac{1}{\tau}\sum_{j=1}^{\tau} E_t\left\{Cov_{t+j-1}\left[\widehat{m}_{t+j},\widehat{\pi}_{t+j}\right]\right\}\end{aligned}$$

where the first moments of inflation drop out due to the law of iterated expectations. The inflation convexity term is given by

$$Convexity_{\tau,t}^{\pi} \equiv -\frac{1}{\tau}\sum_{j=1}^{\tau-1}\sum_{k=j+1}^{\tau} Cov_t\left[\widehat{\pi}_{t+j},\widehat{\pi}_{t+k}\right] - \frac{1}{2\tau}\sum_{j=1}^{\tau} Var_t\left[E_{t+j-1}\left[\widehat{\pi}_{t+j}\right]\right]$$

B MS-VAR Parameter Estimates

The parameter estimates for the best fitting MS-VAR model from Section 5 together with their respective standard errors (in parenthesis) are reported below:

$$\widehat{\mathbf{P}} = \begin{pmatrix} \underset{(0.007)}{0.993} & 0.007 \\ 0.033 & \underset{(0.023)}{0.967} \end{pmatrix} \quad \widehat{\Phi}_{0(1)} = \begin{pmatrix} \underset{(0.14)}{0.01} \\ \underset{(0.59)}{3.09} \\ \underset{(0.13)}{-0.17} \end{pmatrix} \quad \widehat{\Phi}_{0(2)} = \begin{pmatrix} \underset{(0.65)}{0.80} \\ \underset{(1.61)}{9.15} \\ \underset{(0.81)}{0.44} \end{pmatrix}$$

$$\widehat{\Phi}_{1(1)} = \begin{pmatrix} \underset{(0.06)}{0.76} & \underset{(0.02)}{0.04} & \underset{(0.03)}{0.10} \\ \underset{(0.24)}{-0.53} & \underset{(0.08)}{0.27} & \underset{(0.14)}{0.14} \\ \underset{(0.05)}{0.08} & \underset{(0.02)}{0.07} & \underset{(0.03)}{0.93} \end{pmatrix} \quad \widehat{\Phi}_{1(2)} = \begin{pmatrix} \underset{(0.09)}{0.77} & \underset{(0.05)}{-0.02} & \underset{(0.06)}{0.07} \\ \underset{(0.23)}{-0.47} & \underset{(0.13)}{0.01} & \underset{(0.14)}{-0.39} \\ \underset{(0.12)}{0.15} & \underset{(0.07)}{0.03} & \underset{(0.07)}{0.82} \end{pmatrix}$$

$$\widehat{\mathbf{\Sigma}}_{(1)} = \begin{pmatrix} 0.36 & -0.18 & 0.04 \\ -0.18 & 5.75 & 0.39 \\ 0.04 & 0.39 & 0.28 \end{pmatrix} \quad \widehat{\mathbf{\Sigma}}_{(2)} = \begin{pmatrix} 1.51 & -1.22 & 0.21 \\ -1.22 & 9.32 & 0.99 \\ 0.21 & 0.99 & 2.55 \end{pmatrix}$$

C Level Risks Without Assumption 2

I show in this appendix that without imposing assumption 2 from Section 4.1 level risks become substantially less tractable. For the case of $\tau = 2$, I showed in Section 4.2 it is easy to see that the level risk is given by:

$$LR^{(s)}_{\tau=2} = \frac{p_{s1}E\left[i_{t+1}/\begin{smallmatrix}s_t=s\\ s_{t+1}=1\end{smallmatrix}\right]+p_{s2}E\left[i_{t+1}/\begin{smallmatrix}s_t=s\\ s_{t+1}=2\end{smallmatrix}\right]-E[i_t/s_t=s]}{2}$$

where $LR^{(s)}_{\tau} \equiv E\left[i_{\tau,t} - i_t/s_t = s\right] - E\left[NTP_{\tau,t}/s_t = s\right]$ is the level risk at the 2-period horizon conditional on regime s. For the case $\tau = 3$ the expressions get significantly more complicated. Start from the 3-period slope definition:

$$i_{3,t} - i_t = \frac{i_t+E[i_{t+1}/\Omega_t]+E[i_{t+2}/\Omega_t]}{3} - i_t + NTP_{3,t}$$

Taking conditional expectations on both side I get:

$$\begin{aligned} E\left[i_{3,t} - i_t/s_t\right] &= \frac{-2E[i_t/s_t]+E[E[i_{t+1}/\Omega_t]/s_t]+E[E[i_{t+2}/\Omega_t]/s_t]}{3} + E\left[NTP_{3,t}/s_t\right] \\ &= \frac{-2E[i_t/s_t]+E[i_{t+1}/s_t]+E[i_{t+2}/s_t]}{3} + E\left[NTP_{3,t}/s_t\right] \end{aligned}$$

where the second equality follow from the law of iterated expectations. But the conditionally expected short-rates can be written as:

$$E\left[i_{t+1}/s_t = s\right] = p_{s1}E\left[i_{t+1}/\begin{smallmatrix}(s_t,s_{t+1})\\ =(s,1)\end{smallmatrix}\right] + p_{s2}E\left[i_{t+1}/\begin{smallmatrix}(s_t,s_{t+1})\\ =(s,2)\end{smallmatrix}\right]$$

and

$$\begin{aligned} E\left[i_{t+2}/s_t = s\right] = {} & p_{s1}p_{11}E\left[i_{t+2}/\begin{smallmatrix}(s_t,s_{t+1},s_{t+2})\\ =(s,1,1)\end{smallmatrix}\right] + p_{s2}p_{21}E\left[i_{t+2}/\begin{smallmatrix}(s_t,s_{t+1},s_{t+2})\\ =(s,2,1)\end{smallmatrix}\right] \\ & +p_{s1}p_{12}E\left[i_{t+2}/\begin{smallmatrix}(s_t,s_{t+1},s_{t+2})\\ =(s,1,2)\end{smallmatrix}\right] + p_{s2}p_{22}E\left[i_{t+2}/\begin{smallmatrix}(s_t,s_{t+1},s_{t+2})\\ =(s,2,2)\end{smallmatrix}\right] \end{aligned}$$

As a result the 3-period level risk becomes:

$$LR^{(s)}_{\tau=3} = \frac{1}{3}\left[\begin{array}{c} -2E\left[i_t/s_t\right] + p_{s1}E\left[i_{t+1}/\begin{smallmatrix}s_t=s\\ s_{t+1}=1\end{smallmatrix}\right] + p_{s2}E\left[i_{t+1}/\begin{smallmatrix}s_t=s\\ s_{t+1}=2\end{smallmatrix}\right] \\ +p_{s1}p_{11}E\left[i_{t+2}/\begin{smallmatrix}(s_t,s_{t+1},s_{t+2})\\ =(s,1,1)\end{smallmatrix}\right] + p_{s2}p_{21}E\left[i_{t+2}/\begin{smallmatrix}(s_t,s_{t+1},s_{t+2})\\ =(s,2,1)\end{smallmatrix}\right] \\ +p_{s1}p_{12}E\left[i_{t+2}/\begin{smallmatrix}(s_t,s_{t+1},s_{t+2})\\ =(s,1,2)\end{smallmatrix}\right] + p_{s2}p_{22}E\left[i_{t+2}/\begin{smallmatrix}(s_t,s_{t+1},s_{t+2})\\ =(s,2,2)\end{smallmatrix}\right] \end{array}\right]$$

I will stop at $\tau = 3$, but for the usual long-term maturities of interest such as $\tau = 3$ or $\tau = 40$ it is easy to see that the level risk expressions become prohibitively large. In other words, without assumption 2 the number of different conditional expectations terms one needs to keep track of in order to compute an expression for the level risk increases very fast with maturity.

D The New-Keynesian Model – Detailed Derivations

This appendix reports some more detailed derivations for the MS-DSGE model described in Section 6.1.

D.1 Households

Assuming $u\left(C_{t(h)}, N_{t(h)}\right) \geq 0$ everywhere, the representative household solves the following problem:

$$V\left(X_t\right) = \max_{C_t,N_t,X_{t+1}} \Big\{ u\left(C_t, N_t\right) + \beta \left[E_t V\left(X_{t+1}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}} \\ -\Lambda_t \left[P_t C_t + E_t \widetilde{M}_{t,t+1} X_{t+1} - X_t - P_t W_t N_t - D_t\right]\Big\}$$

Letting $V\left(X_t\right) \equiv V_t$, I take first order conditions (FOCs) to get:

$$\Lambda_t P_t = \frac{\partial u(C_t,N_t)}{\partial C_t} \qquad -\Lambda_t P_t W_t = \frac{\partial u(C_t,N_t)}{\partial N_t} \qquad \Lambda_t \widetilde{M}_{t,t+1} = \beta \left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{\alpha}{1-\alpha}} V_{t+1}^{-\alpha} \frac{\partial V_{t+1}}{\partial X_{t+1}}$$

where the envelope condition is given by $\frac{\partial V(X_t)}{\partial X_t} = \Lambda_t$. Note that the optimized value function is given by $V_t = u\left(C_t, N_t\right) + \beta \left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$. Therefore, substituting the Envelope Condition into the FOCs I get:

$$\Lambda_t P_t = \frac{\partial u(C_t,N_t)}{\partial C_t} \qquad -\Lambda_t P_t W_t = \frac{\partial u(C_t,N_t)}{\partial N_t} \qquad \Lambda_t \widetilde{M}_{t,t+1} = \beta \left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{\alpha}{1-\alpha}} V_{t+1}^{-\alpha} \Lambda_{t+1}$$

Combining the first and second FOCs above yields the labor supply equation $\chi \frac{N_t^{\eta}}{C_t^{-\gamma}} = W_t$, whereas combining the first and third FOCs yields the nominal stochastic discount factor:

$$\widetilde{M}_{t,t+1} = \beta \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}}}\right)^{-\alpha} \frac{\frac{\partial u(C_{t+1},N_{t+1})}{\partial C_{t+1}}}{\frac{\partial u(C_t,N_t)}{\partial C_t}} \frac{1}{\Pi_{t+1}}$$

Imposing $u\left(C_t, N_t\right) = e^{b_t}\left(\frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta}\right)$ one obtains equations (12) in the text, where $\widetilde{M}_{t,t+1} \equiv M_{t,t+1}/\Pi_{t+1}$.

D.2 Firms

(a) Final Good Producers:
The representative final good producer chooses $Y_{t(f)}$ for $f \in [0, 1]$ to solve:

$$\max_{Y_{t(f)}} P_t Y_t - \int_0^1 P_{t(f)} Y_{t(f)} df \qquad s.t. \qquad Y_t = \left(\int_0^1 Y_{t(f)}^{\frac{1}{1+\lambda_t}} di\right)^{1+\lambda_t}$$

The first order condition can be seen as the demand curve for the differentiated good f, i.e. $Y_{t(f)} = \left(\frac{P_{t(f)}}{P_t}\right)^{-\frac{1+\lambda_t}{\lambda_t}} Y_t$. Zero-profit in the final good sector implies that $P_t = \left(\int_0^1 P_{t(i)}^{-\frac{1}{\lambda_t}} di\right)^{-\lambda_t}$.

(b) Intermediate Good Producer:
Since capital is fixed, the real marginal cost of each firm f is given by W_t divided by the marginal product of labor, i.e. $MC_{t(f)} = \frac{W_t}{(1-\theta) A_t \overline{K}^{\theta} N_{t(f)}^{-\theta}}$. Using the demand for $Y_{t(f)}$, the real marginal cost of firm f can be written as:

$$MC_{t(f)} = \left(\frac{P_{t(f)}}{P_t}\right)^{-\frac{\theta}{1-\theta}\frac{1+\lambda_t}{\lambda_t}} \underbrace{\left[\frac{1}{(1-\theta)\overline{K}^{\frac{\theta}{1-\theta}}} \frac{W_t}{A_t} \left(\frac{Y_t}{A_t}\right)^{\frac{\theta}{1-\theta}}\right]}_{\equiv MC_t}$$

where MC_t represents the average real marginal in the intermediate good sector. In period t each firm f faces the following price-setting problem:

$$\max_{P_{t(f)}} E_t \left\{ \sum_{j=0}^{\infty} M_{t,t+j} \frac{P_t}{P_{t+j}} \left[P_{t+j(f)} Y_{t+j(f)} - P_{t+j} W_{t+j} N_{t+j(f)} - \frac{\xi}{2} \left(\frac{P_{t+j(f)}}{P_{t+j-1(f)}} \frac{1}{\overline{\Pi}} - 1 \right)^2 P_{t+j} Y_{t+j} \right] \right\}$$

subject to $Y_{t+j(f)} = A_{t+j} \overline{K}^{\theta} N_{t+j(f)}^{1-\theta}$ and $Y_{t+j(f)} = \left(\frac{P_{t+j(f)}}{P_{t+j}} \right)^{-\frac{1+\lambda_{t+j}}{\lambda_{t+j}}} Y_{t+j}$, where $M_{t,t+j}$ is the real SDF between periods t and $t+j$. The FOC with respect to $P_{t(f)}$ is:

$$\left[\left(1 - \frac{1+\lambda_t}{\lambda_t} \right) \left(\frac{P_{t(f)}}{P_t} \right)^{-\frac{1+\lambda_t}{\lambda_t}} \frac{Y_t}{P_t} + \frac{1+\lambda_t}{\lambda_t} \left(\frac{P_{t(f)}}{P_t} \right)^{-\frac{1+\lambda_t}{\lambda_t} \frac{1}{1-\theta} - 1} MC_t \frac{Y_t}{P_t} - \xi \left(\frac{P_{t(f)}}{P_{t-1(f)}} \frac{1}{\overline{\Pi}} - 1 \right) \frac{1}{P_{t-1(f)}} \frac{1}{\overline{\Pi}} Y_t \right]$$
$$+ E_t \left\{ M_{t+1} \left[\xi \left(\frac{P_{t+1(f)}}{P_{t(f)}} \frac{1}{\overline{\Pi}} - 1 \right) \frac{P_{t+1(f)}}{P_{t(f)}^2} \frac{1}{\overline{\Pi}} Y_{t+1} \right] \right\} = 0$$

where to simplify notation I let $M_{t+1} \equiv M_{t,t+1}$.

D.3 Market Clearing Conditions

(a) Symmetric Equilibrium:
Since firms in the intermediate good sector are identical in every aspects, I can now impose the condition for a symmetric equilibrium $P_{t(i)} = P_t \forall i$ to get:

$$MC_t = \frac{1}{1+\lambda_t} + \frac{\lambda_t}{1+\lambda_t} \xi \left(\frac{\Pi_t}{\overline{\Pi}} - 1 \right) \frac{\Pi_t}{\overline{\Pi}} - \frac{\lambda_t}{1+\lambda_t} E_t \left\{ M_{t+1} \left[\xi \left(\frac{\Pi_{t+1}}{\overline{\Pi}} - 1 \right) \frac{\Pi_{t+1}}{\overline{\Pi}} \frac{Y_{t+1}}{Y_t} \right] \right\}$$

(b) Labor Market Clearing Condition:
The labor market clears when:

$$N_t \equiv \int_0^1 N_{t(f)} df = \left(\frac{Y_t}{A_t \overline{K}^{\theta}} \right)^{\frac{1}{1-\theta}} \int_0^1 \left(\frac{P_{t(f)}}{P_t} \right)^{-\frac{1+\lambda_t}{\lambda_t(1-\theta)}} df$$

Solving the last expression for Y_t one gets $Y_t = \left[\int_0^1 \left(\frac{P_{t(f)}}{P_t} \right)^{-\frac{1+\lambda_t}{\lambda_t(1-\theta)}} df \right]^{-(1-\theta)} A_t \overline{K}^{\theta} N_t^{1-\theta}$, where the term in square brackets is the index of price dispersion across firms. Under Rotemberg (1982) adjustment costs, all firms charge the same price and therefore this index is equal to one so that aggregate output follows $Y_t = A_t \overline{K}^{\theta} N_t^{1-\theta}$.

(c) Final Goods Market:
The goods market clearing condition is simply $Y_t = C_t + G_t + \delta \overline{K}$.

E The Extended Non-Linear System

This appendix describes in detail the equations that form the extended system as in equation (16) from Section 6.1.3. For $s \in \{1, 2\}$:

1: Household's Preferences
$$V_{t(s)} = \frac{C_{t(s)}^{1-\gamma}}{1-\gamma} - \chi \frac{N_{t(s)}^{1+\eta}}{1+\eta} + \beta \left\{ p_{s1} E \left[V_{t+1(1)}^{1-\alpha} / \Omega^{-s} \right] + p_{s2} E \left[V_{t+1(2)}^{1-\alpha} / \Omega^{-s} \right] \right\}^{\frac{1}{1-\alpha}}$$

2: Labor Supply
$$W_{t(s)} = \chi \frac{N_{t(s)}^{\eta}}{C_{t(s)}^{-\gamma}}$$

3: Short-term Nominal Bond Euler Equation
$$e^{-i_{t(s)}} = p_{s1} E \left[\beta \left[\frac{V_{t+1(1)}}{\left(p_{s1} E \left[V_{t+1(1)}^{1-\alpha} / \Omega^{-s} \right] + p_{s2} E \left[J_{t+1(2)}^{1-\alpha} / \Omega^{-s} \right] \right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(1)}}{C_{t(s)}} \right)^{-\gamma} \frac{1}{\Pi_{t+1(1)}} / \Omega^{-s} \right]$$
$$+ p_{s2} E \left[\beta \left[\frac{V_{t+1(2)}}{\left(p_{s1} E \left[V_{t+1(1)}^{1-\alpha} / \Omega^{-s} \right] + p_{s2} E \left[V_{t+1(2)}^{1-\alpha} / \Omega^{-s} \right] \right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(2)}}{C_{t(s)}} \right)^{-\gamma} \frac{1}{\Pi_{t+1(2)}} / \Omega^{-s} \right]$$

4: Optimal Pricing Equation in the Intermediate Sector

$$MC_{t(s)} = \frac{1}{1+\lambda_t} + \xi \frac{\lambda_t}{1+\lambda_t} \left(\frac{\Pi_{t(s)}}{\overline{\Pi}} - 1 \right) \frac{\Pi_{t(s)}}{\overline{\Pi}}$$

$$-p_{s1} \frac{\lambda_t}{1+\lambda_t} E \left[\beta \left[\frac{V_{t+1(1)}}{\left(p_{s1} E\left[V_{t+1(1)}^{1-\alpha} \right] + p_{s2} E\left[V_{t+1(2)}^{1-\alpha} \right] \right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(1)}}{C_{t(s)}} \right)^{-\gamma} \left[\xi \left(\frac{\Pi_{t+1(1)}}{\overline{\Pi}} - 1 \right) \frac{\Pi_{t+1(1)}}{\overline{\Pi}} \frac{Y_{t+1(1)}}{Y_{t(s)}} \right] / \Omega^{-s} \right]$$

$$-p_{s2} \frac{\lambda_t}{1+\lambda_t} E \left[\beta \left[\frac{V_{t+1(2)}}{\left(p_{s1} E\left[V_{t+1(1)}^{1-\alpha} \right] + p_{s2} E\left[V_{t+1(2)}^{1-\alpha} \right] \right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(2)}}{C_{t(s)}} \right)^{-\gamma} \left[\xi \left(\frac{\Pi_{t+1(2)}}{\overline{\Pi}} - 1 \right) \frac{\Pi_{t+1(2)}}{\overline{\Pi}} \frac{Y_{t+1(2)}}{Y_{t(s)}} \right] / \Omega^{-s} \right]$$

5: Real Marginal Cost

$$MC_{t(s)} = \frac{1}{(1-\theta)\overline{K}^{\frac{\theta}{1-\theta}}} \frac{W_{t(s)}}{A_t} \left(\frac{Y_{t(s)}}{A_t} \right)^{\frac{\theta}{1-\theta}}$$

6: Monetary Policy Rule

$$i_{t(s)} = \log \frac{\overline{\Pi}}{\beta} + \phi_{\pi(s)} \left(\pi_{t(s)} - \overline{\pi} \right) + \phi_{y(s)} \left(y_{t(s)} - \overline{y} \right)$$

7: Market Clearing Condition in the Final Good Sector

$$Y_{t(s)} = C_{t(s)} + G_t + \delta \overline{K}$$

8: Market Clearing Condition in the Labor Market

$$Y_{t(s)} = A_t \overline{K}^{\alpha} N_{t(s)}^{1-\alpha}$$

9: No-Arbitrage Bond Prices

$$B_{t(s)}^{\tau} = p_{s1} E \left[\beta \left[\frac{V_{t+1(1)}}{\left(p_{s1} E\left[V_{t+1(1)}^{1-\alpha} \right] + p_{s2} E\left[V_{t+1(2)}^{1-\alpha} \right] \right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(1)}}{C_{t(s)}} \right)^{-\gamma} \frac{1}{\Pi_{t+1(1)}} B_{t+1(1)}^{\tau-1} / \Omega^{-s} \right]$$

$$+p_{s2} E \left[\beta \left[\frac{V_{t+1(2)}}{\left(p_{s1} E\left[V_{t+1(1)}^{1-\alpha} \right] + p_{s2} E\left[V_{t+1(1)}^{1-\alpha} \right] \right)^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \left(\frac{C_{t+1(2)}}{C_{t(s)}} \right)^{-\gamma} \frac{1}{\Pi_{t+1(2)}} B_{t+1(2)}^{\tau-1} / \Omega^{-s} \right]$$

for $\tau = 1, ..., T$