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Contract choice in agriculture with joint moral hazard in effort and risk

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Abstract

We analyze optimal contract choice in agriculture when there is joint moral hazard on the part of the farmer in the supply of effort and the riskiness of the technique of cultivation. In the presence of limited liability, high-powered incentive contracts such as fixed rental contracts will induce the farmer to adopt techniques of cultivation that are too risky from the point of view of the landlord. On the other hand, low-powered incentive contracts such as fixed wage contracts will induce the farmer to supply too little effort. We show that sharecropping contracts emerge as a natural solution to balance these two conflicting considerations. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Why are contracts often less high-powered than what purely incentive-based considerations would apparently seem to suggest? This question, which is at the heart of much of the modern theory of organizations and the principal–agent literature (Hart and Holmstrom, 1987), has been a major preoccupation of development economists for a long time. It appears in the form: why do we

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observe sharecropping tenancy in agriculture when fixed rental contracts seem superior from the point of view of incentives?[1]

The existing literature on tenancy has mainly focused on two types of explanations of why farmers can be partial rather than full residual claimants of output as part of an optimal contractual arrangement even though that hurts their incentives to supply effort.[2] The first, and the most well known one developed by Stiglitz (1974) is based on the trade-off between the landlord's need to provide incentives as well as insurance to the tenant. A fixed rental contract is optimal from the point of view of incentives, but it puts all the risk of crop failure on the tenant. A sharecropping contract is shown to achieve the right balance between risk-sharing and incentive provision. A second theory proposed by Eswaran and Kotwal (1985) argues that sharecropping enables pooling non-contractible inputs and resources of both the landlord and the tenant. For example, the landlord may be better in providing managerial effort, whereas the tenant may be better in providing supervisory effort. However, both parties need to be given incentives to provide these inputs and this is precisely what a share contract does.

This paper proposes an alternative answer to this question based on the trade-off between the landlord's desire to give incentives to the tenant encouraging the supply of effort, and discouraging undesirable risk-taking that arises in the presence of limited liability. We show that sharecropping is a contractual arrangement that optimally trades off the costs of inducing the tenant to undertake higher effort and lower risk. If there is moral hazard in effort only, the optimal contract is shown to be a fixed rent contract. If there is moral hazard in risk-taking only, then the optimal contract turns out to be a fixed wage contract. Sharecropping contracts can emerge only when there is moral hazard in both effort and risk.

Agricultural production typically involves multiple decisions which are potentially subject to moral hazard. Examples include the choice of crop-mix, of the quality and quantity of inputs such as fertilizer, pesticides and seeds, water control and soil treatment. Our work is partly motivated by the need to extend the analysis of contract choice in agriculture to allow for moral hazard in more than one action simultaneously.[3]

[1] Although the institution of sharecropping feature in the writings of Smith and Marx, Marshall was the first economist to focus on its effect on incentives.

[2] In this paper, we focus only on incentive-based theories of agricultural contracts and ignore many other well-known explanations of sharecropping. These include pure risk-sharing arguments (Cheung, 1968), or the need to screen tenants according to their unobserved ability (Hallagan, 1978). See Singh (1989) for a detailed review of the literature.

[3] The literature on interlinked contracts (see Braverman and Stiglitz, 1982) too considers the possibility of more than one input affecting the distribution of output. However, in these models, one of the inputs is contractible and the focus of the analysis is to show why the principal might want to subsidize or tax the contractible input to induce the agent to choose higher levels of the non-contractible input depending on whether they are complements or substitutes in production.

The relevance of the latter type of moral hazard, which Basu (1992) was the first to emphasize, is clearly suggested by evidence from farm-level data showing that inputs like fertilizer or pesticide significantly increases the variance of yield (e.g., Just and Pope, 1979 and Anderson and Hazell, 1989). It is often hard for the landlord to monitor the application of inputs like fertilizer just as it is costly to monitor effort.[4] Another example of riskiness of technique of production is that of crop mix which may be hard to monitor. For example, the tenant may use part of the land to grow crops (or a different variety of the same crop) which may yield the same average revenue, but whose market price or physical yields has more variability. This will increase the payoff of the farmer at the landlord's expense. Moreover, the available evidence also indicates that the existence of sharecropping cannot always be explained in terms of the risk-insurance trade off, or joint-participation by the landlord and the tenant in the production process.[5] This suggests the need for alternative theories of contract choice, and forms another motivation behind our work. Our model has testable implications that are distinguishable from existing theories of contract choice. For example, one important implication of the model is that the variance of the distribution of output in a farm is endogenous, and depends on the choice of the contract. In particular, the higher the tenant's crop share, the greater should be the fluctuation of output around the mean.[6]

Normally, when both parties are risk neutral, the best contract is a fixed rental contract which asks the tenant to pay a fixed amount to the landlord irrespective of the level of output and keep the residual for himself. This allows the tenant to capture the full marginal product of his effort and there is no departure from the full-information outcome. However, because of the presence of limited liability,

[4] During fieldwork carried out in North Indian villages by one of the authors (reported in Pandey, 1999), it was found that the landlord often did not monitor fertilizer or other input applications by their sharecropping tenants. The cost of monitoring was cited as the main reason (for example, if the distance of the leased out plot from the landlord's home was long or, if the landlord's occupation did not leave much time to spare).

[5] For example, a recent study of agricultural contracts in the American Midwest by Allen and Lueck (1992) found no positive correlation between exogenous crop variability and the likelihood of adoption of sharecropping over fixed rent contracts. Similar evidence has been obtained from Indian agriculture in an earlier study by Rao (1971) showing that high variance crops were cash rented and low-variance crops like rice were cropshared. Since insurance considerations are likely to be more important the stronger are output fluctuations, this suggests that the insurance vs. incentives trade off was unlikely to be the main driving force behind the existence of sharecropping in these studies. At the same time, many landlords in the same study by Allen and Lueck were absentee, and tenants made all the decisions. This suggests double-sided moral hazard was also unlikely to be the main explanation for the existence of sharecropping in their study.

[6] Existing empirical studies on the effect of contract choice in agriculture on efficiency, such as the well-known study of Shaban (1987), has focused exclusively on the mean of the distribution of yields. The data used in Shaban's study actually permit a test of this hypothesis.

the tenant's incentives are lower compared to the first-best because his rewards are completely flat for the range of output for which he cannot pay the rent: over this range, the landlord is the full residual claimant.[7] We show that even when the limited liability constraint is binding, the optimal linear contract is still a fixed rent contract. Any other contract that charged a lower level of fixed rent and gave a lower share to the tenant to give him the same expected payoff will elicit even lower effort.[8] By the monotone likelihood ratio property (MLRP), the higher is the output level the greater is the effect of a higher effort level in terms of increasing its likelihood. Hence, a contract that gives full incentives on high output levels and none on low output levels (like the fixed rent contract) is better than a contract that gives less than full incentives on high and medium output levels.

However, if one ignores effort and looks at the choice of riskiness of projects, limited liability would create incentives for the tenant to adopt too much risk. Basu (1992) considers this problem of 'technique moral hazard' as opposed to moral hazard in effort choice.[9] Because of limited liability the tenant is protected against downside risk, which is borne solely by the landlord. In particular, his payoff is completely flat if output falls below a certain level. However, it is strictly increasing otherwise. Hence, a contract that reduces the tenant's marginal return from high outputs is best from the point of view of discourage risk-taking. Basu shows that sharecropping contracts may emerge as an optimal contract to discourage the tenant from choosing too much risk. However, he rules out wage contracts because he focuses on absentee landlordism. We show that if the landlord is allowed to choose any linear contract, then the optimal contract is a fixed wage contract and not sharecropping in the model of Basu. Indeed, this contract achieves the first-best.

To summarize, we show that moral hazard in effort or risk alone cannot generate sharecropping contracts. However, joint moral hazard in the choice of effort and risk can. This provides an alternative explanation of the observed

[7] Shetty (1988) was the first to study the implications of limited liability in the presence of moral hazard in effort in the tenancy literature. However, he did not address the issue of the nature of optimal contracts in such an environment. Rather, he focused the effect of the wealth level of a tenant on his equilibrium supply of effort and the probability of being rationed in the land-lease market.

[8] Innes (1990) has shown that contracts that stipulate a fixed transfer from the agent to the principal and allows the principal to collect all output if the agent is unable to pay this fee are optimal from within the class of all *monotonic* contracts (namely, *both* the principal's and the agent's incomes are non-decreasing in output) when there is moral hazard in effort only, and the source of the agency problem is limited liability. Hence, his result shows that fixed rental contracts are actually optimal from within a larger class contracts than what we focus in this paper (namely, *linear* monotonic contracts) when there is moral hazard in effort only.

[9] Jensen and Meckling (1976) have analyzed the issue of contract choice in finance based on similar considerations.

co-existence of various types of contracts in agriculture, namely, wage contracts, sharecropping, and fixed rent tenancy based on the relative importance of moral hazard in the choice of effort and risk.[10]

A recent paper by Sengupta (1997) is the first to study the effect of moral hazard in both effort and choice of projects on contract choice. Our paper is similar in spirit to this paper in that both look at multiple sources of moral hazard simultaneously, in contrast with the existing literature on tenancy. However, the two papers are quite different in terms of the nature of the actions which are subject to moral hazard and hence the forces driving variations in optimal contractual forms. In Sengupta's model, while a modern technique is more risky than a traditional technique, it also has a different mean. Accordingly, in his analysis, contract choice crucially depends on the magnitude of the expected surplus from each type of project. In contrast, we separate the pure risk element from the pure effort element in the tenant's choice of actions. We allow the tenant to choose two actions one of which causes a mean-preserving spread in the distribution of output (i.e., increases risk in the sense of Rothschild and Stiglitz, 1970) and the other shifts the mean upward. The purpose is to focus sharply on the different kinds of conflicts of interest between the landlord and the tenant that moral hazard in effort and risk poses. In the former case, both parties gain from more effort but the tenant supplies less because he has to bear all the cost. However, risk-taking is a pure rent-seeking activity on the part of the tenant at the landlord's expense, which arises due to the presence of limited liability. Under what circumstances wage, sharecropping or fixed rent contracts will emerge in our model depends on the relative extent of the moral hazard problem in effort and risk from the landlord's point of view. A second distinguishing feature of our approach is that we take a general contracting environment where the support of output is continuous, the actions are all continuous variables, and the probability distribution of output takes a general functional form. In contrast, in the model of Sengupta, output takes two values, and there are two possible choices of project. Our approach has the advantage that it allows us to obtain general conditions

[10] Our model is distinct from the multi-task model of moral hazard developed by Holmstrom and Milgrom (1991). In the multi-task model, there are *multiple* outputs that the principal cares about and each of these outputs are functions of a single and different input all of which are provided by the agent. In our model, there is a *single* output, which is a function of *two* inputs. Both models show why incentives may more low-powered than what is predicted by the standard model of moral hazard in the supply of effort. However, in the multi-task model, it is to encourage the agent to produce reasonable amounts of all outputs the principal cares about. In our model, it is to encourage the agent to produce a single output but induce him to take actions that generate a more favorable probability distribution of this single output from the principal's point of view. Limited liability is an essential ingredient of our model in contrast with the multi-task model because due to its presence, the preference of the two parties over alternative distributions of output that have the same mean are not perfectly aligned.

determining the choice of contracts in terms of properties of the distribution function of output and the cost function of the tenant.[11] Also, by looking at the incentive constraints of effort and risk, which depend on both these variables simultaneously, we identify the importance of how these actions interact in the production technology and the farmer's cost function for contract choice.

The plan of the paper is as follows. In Section 2, we outline the basic model. In Section 3, we derive optimal linear contracts in the presence of joint moral hazard in effort and risk assuming a general probability distribution of output. In the next section, we study a specific probability distribution, namely, the normal distribution, as an illustrative example. Section 5 offers concluding remarks.

2. The model

2.1. Technology

Production requires one plot of land and one unit of labor. The landlord is endowed with one unit of land and the tenant with one unit of labor. The landlord and the tenant are both risk neutral. The tenant chooses two actions, to be referred to as effort and risk, which affect the probability distribution of output ex ante. These actions are denoted by $e \in [\underline{e}, \bar{e}]$ and $r \in [\underline{r}, \bar{r}]$ where $\bar{e} > \underline{e} \geq 0$ and $\bar{r} > \underline{r} \geq 0$.

After these actions are chosen, nature moves and a particular value of output is realized. The distribution function of output x is given by $F(x|e, r)$ where $\underline{x} \leq x \leq \bar{x}$. We assume the supports of the distribution of output, $\underline{x}$ and $\bar{x}$, do not depend on e or r and allow them to range from $-\infty$ to $+\infty$. We assume $F(x|e, r)$ is twice continuously differentiable with respect to all arguments and denote the density function by $f(x|e, r) \equiv (\partial F(x|e, r))/(\partial x)$.

As in standard principal–agent models, it is not labor per se but the intensity with which it is applied by the tenant, namely, effort (e) that matters for

[11] This also allows us to avoid problems arising from the fact that the support of the distribution of output shifts with project choice in the models of Basu and Sengupta. As Sengupta points out, in Basu's model (where all projects have the same low output, but different high outputs) the first-best can be implemented by paying the tenant the minimum possible amount whenever output is too high. In Sengupta's model (where all projects have the same high output, but different low outputs), this cannot be done due the limited liability constraint. Still, since the value of the low output immediately reveals project choice, perturbing the landlord–tenant game slightly by expanding the set of available strategies to the landlord will enable achieving the first-best. For example, if there are two periods, the landlord can use eviction threats against the tenant if he used the wrong technique.

production. We assume that the density function satisfies the MLRP with respect to changes in e. That is, for all x:

$$\frac{\partial}{\partial x}\left[\frac{f_e(x|e,r)}{f(x|e,r)}\right] \geq 0.$$

As is well known in the principal–agent literature (see Hart and Holmstrom, 1987), this property implies that an increase in e shifts the distribution of x in a first-order stochastic dominance (FOSD) sense. That is, for all x:

$$F_e(x|e,r) \leq 0. \tag{A1}$$

Since for a higher level of effort the probability of output being less than any given value (weakly) decreases, it implies that the mean of the distribution increases with effort.[12] Because e is unobservable to the landlord, the tenant has to be rewarded on the basis of the only observable but noisy signal of e, namely, output, x.

We allow the tenant to take an additional unobservable action r, which affects the riskiness of the project. By this we mean that the tenant can deliberately cause a mean preserving spread of the distribution of output. Formally, an increase in r shifts the distribution of x in a second-order stochastic dominance (SOSD) sense.

For a given value of e, consider a family of stochastic variables indexed by r on the closed interval $[\underline{x}, \bar{x}]$. According to the definition provided by Rothschild and Stiglitz (1970), the probability distribution $F(x|e, r_2)$ is 'more risky' than $F(x|e, r_1)$ if it is obtained from the distribution for r_1 by successively displacing weight from the center towards the tails of the distribution while keeping the mean constant.[13] Formally, an increase in r is a mean-preserving increase in risk if and only if:[14]

$$\int_{\underline{x}}^{x'} [F(x|e,r_2) - F(x|e,r_1)]\,\mathrm{d}x \geq 0 \quad \forall \underline{x} \leq x' \leq \bar{x}.$$

$$\int_{\underline{x}}^{\bar{x}} [F(x|e,r_2) - F(x|e,r_1)]\,\mathrm{d}x = 0.$$

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[12] For an increase in effort, the change in the mean of the distribution of x is given by:

$$\frac{\partial E(x)}{\partial e} = \int_{\underline{x}}^{\bar{x}} x \frac{f_e(x)}{f(x)} f(x)\,\mathrm{d}x.$$

Since $(f_e(x))/(f(x))$ is increasing in x and $\int_{\underline{x}}^{\bar{x}} (f_e(x))/(f(x)) f(x)\mathrm{d}x = 0$ (which follows from totally differentiating $\int_{\underline{x}}^{\bar{x}} f(x)\mathrm{d}x = 1$, it follows that the mean of the distribution of x is increasing in e.

[13] Rothschild and Stiglitz (1970) show that if a distribution is more risky in the sense defined above, a risk-averse individual whose payoff is some concave function of x will always prefer a less risky distribution. They show that it is possible for a distribution to be more risky in terms of their definition and yet have a lower variance because the variance is a quadratic, and hence, convex function of x.

[14] The second condition follows from integration by parts: $\int_{\underline{x}}^{\bar{x}} [F(x,r_2) - F(x,r_1)]\mathrm{d}x = [xF(x|e,r_2) - xF(x|e,r_1)]_{\underline{x}}^{\bar{x}} - \int_{\underline{x}}^{\bar{x}} xf(x|e,r_2)\mathrm{d}x + \int_{\underline{x}}^{\bar{x}} x\mathrm{d}f(x|e,r_1)\mathrm{d}x = 0$.

We use a differential version of these conditions as in Diamond and Stiglitz (1974):

$$\int_{\underline{x}}^{\bar{x}} F_r(x|e,r)\,\mathrm{d}x = 0 \tag{A2}$$

$$\int_{\underline{x}}^{x'} F_r(x|e,r)\,\mathrm{d}x \geq 0 \quad \forall \underline{x} \leq x' \leq \bar{x}.$$

The tenant incurs a private cost for his actions $C(e, r)$. We assume that this function satisfies the following properties: it is twice continuously differentiable, monotonically increasing and convex in e and r. Also, $C(0, 0) = 0$. We assume that e and r are either substitutes or separable in the farmer's cost function, i.e., $C_{er}(e, r) \geq 0$.[15] A possible interpretation of this condition is that the tenant's cost depends on the total labor time he spends on the farm and it can be allocated among these two alternative activities.

2.2. Information and contracts

There is moral hazard in the provision of both effort and risk. That is, both e and r are unobservable to the landlord and hence, cannot be contracted on. Contracts must be based on the only signal of these inputs, namely, output x. We assume that the tenant has no wealth so that the only way the landlord can receive any transfers from the tenant is from realized output. We consider reward functions for the tenant that are linear in output. In particular, the tenant receives $sx - R$ when x is realized: s stands for the share of output of the tenant and R is a fixed transfer from the tenant to the landlord. If $R < 0$ that can be interpreted as fixed wage component, and if $R > 0$, a fixed rent component. In the tenancy literature, the linearity of the reward rule is usually motivated by the commonly observed form of contracts in agriculture, namely, fixed wage, sharecropping, or fixed rent contracts (Singh, 1989). Also, as Hart and Holmstrom (1987) have pointed out, if landlords cannot monitor trade of output among different farmers, then nonlinear contracts will be effectively replaced by a linear contract with a constant marginal share.[16]

Given that both parties are risk neutral, the first-best contract in this environment is a fixed rental contract that requires the tenant to pay a fixed amount to the landlord irrespective of the level of output and keep the residual for himself. In a one-shot model, this is equivalent to selling off the farm to tenant, which is a standard solution to agency problems. However, if the tenant has limited wealth

[15] If e and r are strongly complementary in the tenant's cost function, then they will tend to move together as the contract changes. This will eliminate the basic trade off our model is based on.

[16] This is exactly the same reason why price discrimination cannot be practiced if buyers can trade among themselves. See Hart and Holmstrom (1987) and Singh (1989) for more detailed discussions of various arguments advanced to justify linear reward functions in the principal–agent literature.

and credit markets are imperfect, then the size of fixed ex ante transfers from the tenant to the landlord would be restricted. For the same reason, it puts an upper bound to the ex post punishment that the tenant receives in any state of the world. Since the tenant has zero wealth in our model, contracts in our model satisfy a limited-liability constraint of the following form: the tenant's income has to be non-negative for all realizations of output. This is the main source of incentive problems in this model. Because the tenant's payoff is completely flat for output less than the critical value at which the limited liability constraint binds, on the one hand, this causes him to undersupply effort. On the other hand, this causes the tenant to choose more risky actions because he does not care about downside risk, but shares the benefit of high values of output.

The realized incomes of the tenant (T) and the landlord (L) under this contract are:

$$y_T(x) = \max\{sx - R, 0\}$$
$$y_L(x) = \min\{x - (sx - R), x\}.$$

Let the point where the limited liability binds be given by $\hat{x}$.

$$\hat{x} \equiv \frac{R}{s}.$$

Notice that our restriction to linear contracts of this form also rules out non-monotonic contracts.[17]

The tenant's expected payoff is given by:

$$U_T = \int_{\hat{x}}^{\bar{x}} (sx - R) f(x|e,r) \mathrm{d}x - C(e,r). \quad (1)$$

Since $\int_{\hat{x}}^{\bar{x}}(sx - R)f(x|e,r)\mathrm{d}x = (1 - F(\hat{x}))[sE(x|x \geq \hat{x}) - R]$, where $E(x|x \geq \hat{x})$ is the conditional expectation of x given that $x \geq \hat{x}$, the tenant's reward has a simple interpretation. With probability $1 - F(\hat{x})$ it is positive, in which case the tenant's expected payoff is his share (s) of the conditional mean of the distribution of output less the fixed rent payment (R). With probability $F(\hat{x})$, the tenant's reward is zero.

On the other hand, the landlord's expected payoff is

$$U_L = E(x) - \int_{\hat{x}}^{\bar{x}} (sx - R) f(x|e,r) \mathrm{d}x \quad (2)$$

where $E(x) \equiv \int_{\underline{x}}^{\bar{x}} xf(x|e,r)\mathrm{d}x$.

Notice that if $s = 0$, the tenant gets no share of output and to ensure a positive expected payoff to the tenant he must receive a transfer from the landlord, i.e.,

[17] Innes (1990) and Zou (1995) show that if such contracts are allowed then the first-best can be achieved whether or not there is moral hazard in effort, or risk, or both. As Innes (1990) points out, non-monotonic contracts create incentives for the agent to revise his output reports upward (for example, by borrowing) or for the principal to destroy output. Moreover, most observed forms of contracts such as debt, equity, or stock options are monotonic.

$R<0$. This is therefore a fixed wage contract. Next, if $s=1$, the landlord gets no share of output and to ensure him non-zero expected returns, he must receive a transfer from the tenant, i.e., $R>0$ (otherwise, $\hat{x}<0$ and hence, both terms in the landlord's expected payoff, $\int_{\underline{x}}^{\hat{x}} x f(x|e,r)\mathrm{d}x + R\int_{\underline{x}}^{\bar{x}} f(x|e,r)\mathrm{d}x$, are negative). This is a fixed rent contract. For $0<s<1$, we have a sharecropping contract.

The tenant's exogenously given outside reservation utility is $\bar{u}$. We assume the landlord has all the bargaining power and can make take-it-or-leave-it offers to the tenant subject to providing him with an expected payoff of at least $\bar{u}$. This will be referred to as the participation constraint (PC):

$$U^{\mathrm{T}} \geq \bar{u}. \tag{3}$$

The tenant's choice of e and r faced with a given contract (s, R) are given by the incentive constraints:

$$-s\int_{\hat{x}}^{\bar{x}} F_e(x|e,r)\mathrm{d}x = C_e(e,r) \tag{4}$$

$$-s\int_{\hat{x}}^{\bar{x}} F_r(x|e,r)\mathrm{d}x = C_r(e,r). \tag{5}$$

(See the Appendix for details.)

Let $a_e^{\mathrm{T}}(s,R) \equiv -s\int_{\hat{x}}^{\bar{x}} F_e(x)\mathrm{d}x > 0$ and $a_r(s,R) \equiv -s\int_{\hat{x}}^{\bar{x}} F_r(x)\mathrm{d}x > 0$. These are the expected marginal products of e and r accruing to the tenant. Notice that by Eq. (A2), $\int_{\hat{x}}^{\bar{x}} F_r(x)\mathrm{d}x + \int_{\underline{x}}^{\hat{x}} F_r(x)\mathrm{d}x = 0$. Hence, $a_r(s,R) = s\int_{\underline{x}}^{\hat{x}} F_r(x)\mathrm{d}x$.

We assume that the technology displays diminishing returns to effort to ensure that an interior maximum exists, namely, $-\int_{\hat{x}}^{\bar{x}} F_{ee}(x)\mathrm{d}x \leq 0$. A sufficient condition for this is $F_{ee}(x) \geq 0 \forall \underline{x} \leq x \leq \bar{x}$. This assumption is referred to as convexity of distribution function condition (CDFC) in the principal–agent literature (Hart and Holmstrom, 1987) and is used to justify the first-order approach. We make a similar assumption regarding the choice of r to ensure the existence of an interior maximum: $F_{rr}(x) \geq 0 \forall \underline{x} \leq x \leq \bar{x}$.

3. Contract choice with moral hazard in effort and risk

The expected social surplus is the sum of the landlord's and the tenant's payoffs:

$$S = E(x) - C(e,r).$$

Hence, if e and r were fully contractible, the first-order conditions to maximize social surplus will be:

$$-\int_{\underline{x}}^{\bar{x}} F_e(x|e,r)\mathrm{d}x - C_e(e,r) = 0.$$

$$-\int_{\underline{x}}^{\bar{x}} F_r(x|e,r)\mathrm{d}x - C_r(e,r) = 0.$$

However, $\int_{\underline{x}}^{\bar{x}} F_r(x|e,r)\mathrm{d}x = 0$ by Eq. (A2) and hence, $r^* = 0$. Therefore, e^* is characterized by

$$-\int_{\underline{x}}^{\bar{x}} F_e(x|e^*,0)\mathrm{d}x - C_e(e^*,0) = 0.$$

The full-information benchmark provides the following characterization of agency costs under joint moral hazard in effort and risk in the presence of limited liability. The proof of the following and all other propositions in the paper are in the Appendix.

Proposition 1. *Consider any linear contract (s, R) and suppose that the limited liability constraint is binding. Then in the presence of joint moral hazard in effort and risk,* $e < e^*$ *and* $r \geq r^*$. *For* $s > 0$, $r > r^*$.

This result shows that both types of agency costs are, in general, present under any linear contract so long as the limited liability constraint in binding. The only exception is a wage contract (i.e., $s = 0$) where the tenant does not supply *any* non-contractible input at all. In this case, while the agency cost from the undersupply of effort is the greatest, there is no loss of surplus from inefficient risk-taking. For all other contracts, there is too little supply of effort and too much risk-taking compared to the first-best.

While Proposition 1 establishes that agency costs arising from low effort and high risk-taking will exist under any linear contract, the landlord obviously will want to design optimal contracts to minimize these agency costs. We now turn to the characterization of optimal contracts. When we derived the tenant's optimal responses of effort and risk (e, r) in Section 2.2, we took as given the contract offered by the landlord, (s, R). Now, we characterize the landlord's choice of the optimal contract (s, R) anticipating the tenant's ICs, Eqs. (4) and (5). In addition, the landlord has to respect the PC of the tenant, Eq. (3), when choosing s and R. We can reduce the landlord's choice of instruments to that of choosing s only by always adjusting R to provide the tenant with the same level of expected utility whenever s is changed.[18] Totally differentiating the tenant's PC, we get (see the section on deriving the landlord's first-order condition in the Appendix for details):

$$\frac{\partial R}{\partial s} = E[x|x \geq \hat{x}].$$

[18] It should be noted here that the PC of the tenant may not bind due to limited liability if the reservation payoff is very low. See Banerjee et al. (2000) for an analysis of this possibility in a model where only effort is subject to moral hazard. For our present purpose, this is not relevant as we consider changes in s and R that give the tenant the same expected payoff. This could be the outside option or some higher value, and henceforth, we take $\bar{u}$ to represent the larger of these two numbers.

This yields the following first-order condition for the landlord with respect to s:

$$\frac{\partial U_L}{\partial s} = -\left[\int_{\underline{x}}^{\bar{x}} F_e(x|e,r)\,\mathrm{d}x\right]\frac{\partial e}{\partial s} + \left(s\int_{\hat{x}}^{\bar{x}} F_e(x|e,r)\,\mathrm{d}x\right)\frac{\partial e}{\partial s}$$
$$+ s\left(\int_{\hat{x}}^{\bar{x}} F_r(x|e,r)\,\mathrm{d}x\right)\frac{\partial r}{\partial s}. \tag{6}$$

See the Appendix for the steps involved in deriving the landlord's first-order condition. Let $a_e^L(s,R) \equiv -\int_{\underline{x}}^{\bar{x}} F_e(x)\mathrm{d}x + s\int_{R/s}^{\bar{x}} F_e(x|e,r)\mathrm{d}x > 0$ denote the marginal product of e accruing to the landlord. Since by Eq. (A2), the social marginal product of r is zero, the marginal product of r accruing to the landlord is the negative of the marginal product of r accruing to the tenant, $a_r(s, R)$.

To find out the effect of s on e and r, we totally differentiate the tenant's first-order conditions with respect to s and obtain:

$$\frac{\partial e}{\partial s} = \frac{\Delta_{rr} A_e - \Delta_{er} A_r}{\Delta} \tag{7}$$

$$\frac{\partial r}{\partial s} = \frac{\Delta_{ee} A_r - \Delta_{er} A_e}{\Delta} \tag{8}$$

where we have used the following definitions to simplify notation:

$$\Delta_{ee} = \left(C_{ee} + s\int_{\frac{R}{s}}^{\bar{x}} F_{ee}(x)\,\mathrm{d}x\right),\ \Delta_{rr} = \left(C_{rr} + s\int_{\frac{R}{s}}^{\bar{x}} F_{rr}(x)\,\mathrm{d}x\right),$$

$$\Delta_{er} = \left(C_{er} + s\int_{\frac{R}{s}}^{\bar{x}} F_{er}(x)\,\mathrm{d}x\right),\ \Delta = \Delta_{ee}\Delta_{rr} - \Delta_{er}^2,$$

$$A_e = -\int_{\frac{R}{s}}^{\bar{x}} F_e(x)\,\mathrm{d}x + F_e\left(\frac{R}{s}\right)\left[E\left(x|x \geq \frac{R}{s}\right) - \frac{R}{s}\right] \text{ and}$$

$$A_r = -\int_{\frac{R}{s}}^{\bar{x}} F_r(x)\,\mathrm{d}x + F_r\left(\frac{R}{s}\right)\left[E\left(x|x \geq \frac{R}{s}\right) - \frac{R}{s}\right].$$

For the Hessian of the tenant's maximization problem with respect to e and r to be negative definite we need the following conditions to be satisfied:

$$\Delta_{ee} > 0$$
$$\Delta_{rr} > 0$$
$$\Delta_{ee}\Delta_{rr} - \Delta_{er}^2 > 0.$$

The first term in the expression for $\partial e/\partial s$ captures the direct effect of increasing the crop-share of the tenant on e from the tenant's IC with respect to e, Eq. (4). However, r changes with s as well, and this affects the incentive constraint for e. This indirect effect is captured by the second term. The interpretation of the expression for $\partial r/\partial s$ is similar. We show in the Proof of Proposition 2 that $A_e \geq 0$ so long as the distribution of output satisfies the MLRP. Hence, the direct effect of an increase in s on e is always positive. Next, consider the expression for the direct effect of an increase in s on r, A_r:

$$A_r = -\int_{\frac{R}{s}}^{\bar{x}} F_r(x)\mathrm{d}x + F_r\left(\frac{R}{s}\right)\left[E\left(x|x \geq \frac{R}{s}\right) - \frac{R}{s}\right].$$

The first term of this expression is positive by Eq. (A2). Unlike in the case of e, here, the second term may or may not be negative because the sign of $F_r(R)$ depends on the specific value of R. A sufficient condition for A_r to be positive is that the probability of defaulting under a fixed rent contract, $F(R)$, does not decrease with an increase in r. This is likely to be the case so long as the value of R is not too high, which means, given the PC, the reservation payoff of the tenant, $\bar{u}$, is not too low. For example, if the distribution of x is symmetric with respect to the mean, then this condition translates to R being not greater than the unconditional mean $E(x)$ of output.

The following proposition shows that if risk is not subject to moral hazard, then the tenant should be given as high-powered incentives as possible to maximize the supply of effort. However, due to limited liability, even with a fixed rent contract, the supply of effort will be less than the first-best because the tenant does not internalize the effect of his choice of e over a range of output (namely, $x \leq R$). That is, he cares only about the truncated distribution of output for $x \geq R$ as opposed to the entire distribution.

Proposition 2. *If r is contractible, then the optimal contract is a fixed rental contract which stipulates* $r^* = \underline{r}$. *Effort is less than the first-best level under this contract.*

However, if one looks at r, the only reason it is supplied is because there are states of the world where the agent does not care about what the output is. Moreover, the higher is his marginal return for output levels at which he benefits from higher output, the greater will be his incentives to choose higher risk. Hence, a contract that reduces the range of output over which the tenant defaults by reducing the rent and also reduces his marginal return from high outputs is best from the point of view of discourage risk-taking. Hence, we show:

Proposition 3. *If e is contractible, then the optimal contract is a fixed wage contract which achieves the first-best.*

However, if both e and r are non-contractible, Propositions 2 and 3 indicate a trade-off between the landlord's objective of maximizing effort and minimizing risk in terms of setting the optimal value of s. The next proposition shows:

Proposition 4. *When both effort and risk are subject to moral hazard, a fixed wage contract cannot be optimal. Hence, the optimal contract is either a share-cropping or a fixed rent contract.*

The intuition is that while setting $s=0$ and $R<0$, the landlord completely gets rid of the default problem (namely, the tenant does not have to pay anything to the landlord in any state of the world, and so the limited liability constraint does not bind), but the incentive problem is severe because now the tenant has no incentive to supply either e or r. If s is increased by a very small amount (and R adjusted to keep the tenant's expected payoff constant), then the resulting increase in effort always dominates any increase in risk for the landlord, because the range of output over which the limited liability constraint binds is very small.

Next, we characterize conditions under which a fixed rent contract will lead the tenant to choose too much risk, so that the landlord will be better off with a share contract although that will reduce effort. As s tends to 1, $R>0$ and $\hat{x}=R>0$. This is fixed rental contract, which allows for default by the tenant when output is less than the amount of the rent. This contract, which is the same as a debt contract with bankruptcy in finance, has been shown to be the optimal contract when there is moral hazard in the provision of effort. The landlord's marginal return from increasing s at $s=1$ is:

$$\lim_{s\to 1}\frac{\partial U_{\mathrm{L}}}{\partial s}=a_e^{\mathrm{L}}(1,R)\lim_{s\to 1}\frac{\partial e}{\partial s}-a_r(1,R)\lim_{s\to 1}\frac{\partial r}{\partial s}.$$

If $\lim_{s\to 1}(\partial EU^P)/(\partial s)<0$, the landlord will be better off by choosing $s<1$. This condition gives us a formula to check whether the optimal contract is going to be a sharecropping contract or a fixed rent contract in the presence of joint moral hazard in effort and risk. After substituting the expressions for $\partial e/\partial s$ and $\partial r/\partial s$ from Eqs. (7) and (8) in the necessary and sufficient condition for the optimal contract to be a sharecropping contract and rearranging terms, we get:

$$\frac{a_r}{a_e^{\mathrm{L}}}>\frac{\Delta_{rr}A_e-\Delta_{er}A_r}{\Delta_{ee}A_r-\Delta_{er}A_e}.$$

This expression shows that other things being the same, this condition is more likely to be satisfied if Δ_{ee} is higher than Δ_{rr}. The higher is Δ_{ee} relative to Δ_{rr}, the faster the marginal cost of supplying e rises relative to that of r, and hence, the more costly it is to elicit effort from the tenant. For example, if we take separable and quadratic cost functions, $C(e)=1/2ce^2$ and $C(r)=1/2c\gamma r^2$ then as γ decreases, Δ_{ee} goes up relative to Δ_{rr}. Similarly, the condition is more likely to be satisfied if A_e is small relative to A_r or a_e^{L} is small relative to a_r. This will be the case the more sensitive is the distribution of output to changes in risk relative to effort. Let us introduce a parameter $\alpha>0$ in the distribution function of x that captures the sensitivity of the distribution of x to effort. In particular, let

$$F(x)=F(x|\alpha e,r).$$

Hence,

$$F_e(x) = \alpha F_e(x|\alpha e, r)$$

which implies that if $\alpha_1 > \alpha_2$, then $|F_e(x|\alpha_1 e, r)| \geq |F_e(x|\alpha_2 e, r)|$ for all x. As α goes to zero, the productivity of effort goes to zero; there is no FOSD shift in the distribution of x as effort increases. Now

$$a_e^{\mathrm{L}} = -\alpha \int_{\underline{x}}^{\bar{x}} F_e(x|\alpha e, r)\,\mathrm{d}x + s\alpha \int_{\frac{R}{s}}^{\bar{x}} F_e(x|\alpha e, r)\,\mathrm{d}x.$$

As $\alpha \to 0$, both terms in a_e^{L} go to zero. Therefore, $a_e^{\mathrm{L}} \to 0$. An increase in r induces a mean-preserving spread in the distribution of output even when the mean is 0. Hence, $a_r = s\int_{\frac{R}{s}}^{\bar{x}} F_r(x|\alpha e, r)\mathrm{d}x$ remains finite and positive as $\alpha \to 0$. As $\alpha \to 0$, $\Delta_{rr} > 0$, $\Delta_{ee} \to C_{ee} > 0$, $\Delta_{er} \to C_{er}$. So $\Delta > 0$. Finally,

$$A_e = -\alpha \int_{\frac{R}{s}}^{\bar{x}} F_e(x|\alpha e, r)\,\mathrm{d}x + \alpha F_e\left(\frac{R}{s}\right)\left[E\left(x|x \geq \frac{R}{s}\right) - \frac{R}{s}\right].$$

So $A_e \to 0$ as $\alpha \to 0$. Hence, $A_r = -\int_{\frac{R}{s}}^{\bar{x}} F_r(x|\alpha e, r)\mathrm{d}x + F_r\left(\frac{R}{s}\right) \times \left[E(x|x \geq \left(\frac{R}{s}\right) - \frac{R}{s}\right]$ does not go to 0 as α goes to zero. Then it follows directly that the condition for having a share contract as the optimal contract will be satisfied. The above discussion can be summarized as follows.

Proposition 5. *When both effort and risk are subject to moral hazard, the optimal contract is a sharecropping contract or a fixed rent contract depending on whether* $a_e^L(1, R)(\partial e/\partial s)|_{s=1} - a_r(1, R)(\partial r/\partial s)|_{s=1}$ *is* ≤ 0 *or* ≥ 0. *If the sensitivity of the distribution of output to changes in effort relative to risk is small enough, or, the rate at which the marginal cost of effort rises relative to the marginal cost of risk is high enough, a sharecropping contract is optimal.*

Intuitively, the landlord is balancing two types of moral hazard problems in choosing the optimal crop share. A higher crop share induces a higher effort level, but it also induces the tenant to adopt more risky techniques. The former effect is good from the landlord's point of view but the latter effect is bad. If the rate at which the marginal cost of effort rises as effort goes up is small, or, the distribution of output is not very sensitive to changes in effort, then it is more valuable for the landlord to try to discourage the tenant from choosing risk rather than trying to elicit higher effort from him through the instrument of the crop-share. As a result, the optimal contract is more likely to be a sharecropping contract than a fixed rent contract.

4. An illustrative example: normal distribution

In the previous section, we derived a condition in terms of general functional forms for the distribution function of output and the cost function of the tenant that determines whether sharecropping or fixed rental contracts will emerge under joint moral hazard in effort and risk. In this section, we use a particular parametrized probability distribution of output to show how this general condition can be applied to a specific environment and gain more insight in terms of effects of variation of parameters of the distribution and the cost function on contract choice.

Let output $x = \alpha e + \varepsilon$ where e is the effort level and ε is a zero-mean random variable and $\alpha > 0$ is the marginal product of effort. Suppose that ε is normally distributed with variance r^2 which is our measure of risk. Accordingly, the probability density of output is $f(x) = \frac{1}{r\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left[-\frac{1}{2}\left(\frac{x-\alpha e}{r}\right)^2\right] \mathrm{d}x$ and the distribution function is $F(x) = \frac{1}{r\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}\left(\frac{t-\alpha e}{r}\right)^2\right] \mathrm{d}t$. Let

$$z \equiv \frac{(sx-R)-(s\alpha e-R)}{sr} = \frac{x-\alpha e}{r}$$

which has the standard normal distribution. Also, let the cutoff point at which the limited liability constraint binds be denoted by

$$\underline{z} \equiv \frac{s\hat{x}-R-s\alpha e+R}{sr} = \frac{s\frac{R}{s}-R-s\alpha e+R}{sr} = \frac{R-s\alpha e}{sr}.$$

We show that an increase in e satisfies Eq. (A1) for the normal distribution.

$$F_e(z) = \frac{\alpha}{r}\int_{-\infty}^{z} tf(t)\mathrm{d}t < 0$$

where t is the standard normal variable since $\int_{-\infty}^{\infty} tf(t)\mathrm{d}t = 0$.

Next, we show that an increase in r satisfies Eq. (A2).

$$F_r(z) = \frac{1}{r}\int_{-\infty}^{z}[t^2-1]f(t)\mathrm{d}t.$$

As $z \to \infty$, $\int_{-\infty}^{z} t^2 f(t)\mathrm{d}t$ approaches 1 which is the (unconditional) variance of z. Also, $f(t)$ being the normal density function $\int_{-\infty}^{\infty} f(t)\mathrm{d}t = 1$. Hence $F_r(z) \to 0$ as $z \to \infty$. Notice that the function $z^2 - 1$ is symmetric around 0 and so is the normal density function. Also, for $z \leq -1$ or $z \geq 1$, $z^2 - 1 > 0$ while for $-1 < z < 1$, $z^2 - 1 < 0$. These two properties imply

$$\frac{1}{r}\int_{-\infty}^{-1}[t^2-1]f(t)\mathrm{d}t = \frac{1}{r}\int_{1}^{\infty}[t^2-1]f(t)\mathrm{d}t > 0$$

and

$$\frac{1}{r}\int_{-1}^{0}[t^2-1]f(t)\mathrm{d}t = \frac{1}{r}\int_{0}^{1}[t^2-1]f(t)\mathrm{d}t < 0.$$

Since $F_r(\infty) = 0$, this also implies $F_r(0) = 0$.

Given the above characterization, if we plot $F_r(z)$ against z, we see that as $z \to -\infty$, $F_r(z) \to 0$, as z increases $F_r(z)$ increases and reaches a maximum at $z = -1$. After that, it falls, and becomes zero at $z = 0$. It keeps on decreasing, reaching a minimum at $z = 1$. After that, it increases and tends to 0 as $z \to \infty$. Using the symmetry argument, for any positive z, $F_r(z) = \frac{1}{r}(\int_0^z[t^2-1]f(t)\mathrm{d}t)$. For any negative z, $F_r(z) = \frac{1}{r}(\int_{-\infty}^{z}[t^2-1]f(t)\mathrm{d}t) = -\frac{1}{r}(\int_0^z[t^2-1]f(t)\mathrm{d}t)$ since $F_r(0) = 0$. So $F_r(z) = -F_r(-z)$ for any positive z. From this, it directly follows that $\int_{-\infty}^{z} F_r(z)\partial z \geq 0$ for $-\infty \leq z \leq \infty$ and $\int_{-\infty}^{\infty} F_r(z)\partial z = 0$.

Let us assume for simplicity that the costs of e and r are separable and take the quadratic form: $C(e, r) = (1/2)ce^2 + (1/2)c\gamma r^2$ where $c > 0$ and $\gamma > 0$. The parameter $\gamma \in [0, 1]$ captures the relative cost of r relative to e. We are going to show that if γ is sufficiently close to 0 (i.e., the marginal cost of increasing r rises sufficiently less fast compared to the marginal cost of increasing e to the tenant) or if α is sufficiently close to 0 (so that the marginal product of effort is low), then the optimal contract is a sharecropping contract. Otherwise, it is a fixed rent contract.

The tenant chooses $e \in [0, \bar{e}]$ where $\bar{e} > 0$ and $r \in [\underline{r}, \bar{r}]$ where $\bar{r} > \underline{r} > 0$. The tenant's payoff is

$$U_{\mathrm{T}} = (1 - F(\underline{z})(s\alpha e - R) + srf(\underline{z}) - \frac{1}{2}ce^2 - \frac{1}{2}c\gamma r^2.$$

The incentive-compatibility constraints with respect to e and r are:

$$\frac{\partial U_{\mathrm{T}}}{\partial e} = \left(1 - F(\underline{z})\right)s\alpha - ce = 0$$

$$\frac{\partial U_{\mathrm{T}}}{\partial \sigma} = sf(\underline{z}) - c\gamma r = 0.$$

For the standard normal distribution $f'(z) + f(z)z = 0$, and so the term $\{s\sigma f'(\underline{z}) - f(\underline{z})(s\alpha e - R)\}(\mathrm{d}\underline{z})/(\mathrm{d}i)$ $(i = e, r)$ drops out of the first-order conditions. Notice that under the first best, $e = \alpha/c$ and $r = \underline{r}$. Since $0 < s \leq 1$ and $1 - F(\underline{z}) < 1$, e is less than the first-best level.

For the first-order approach to be valid, we must make sure that the second-order conditions of the agent's problem are satisfied. They are (after simplifying, using the incentive constraint for r):

$$\frac{\partial^2 U_T}{\partial e^2} = c(\gamma\alpha^2 - 1) < 0$$

$$\frac{\partial^2 U_T}{\partial r^2} = c\gamma(\underline{z}^2 - 1) < 0$$

and the Hessian

$$\Delta \equiv \gamma c^2(1 - \gamma\alpha^2 - \underline{z}^2) > 0.$$

To ensure that an interior solution exists for some parameter values, we assume that either α or γ is strictly less than 1. Also, when discussing interior solutions, we restrict our attention to subsets of the interval $-1 \leq \underline{z} \leq 1$.[19]

Totally differentiating U_T with respect to R and s, we get:

$$\frac{dR}{ds} = \alpha e + r\frac{f(\underline{z})}{(1 - F(\underline{z}))} = \alpha e + r\lambda(\underline{z})$$

where $\lambda(\underline{z})$ is the hazard rate for the normal distribution.

Totally differentiating these ICCs, and using the above expression for dR/ds, we get:

$$\frac{\partial e}{\partial s} = \frac{c\gamma\alpha(1 - F(\underline{z}))}{\Delta}\left[1 - (\lambda(\underline{z}) - \underline{z})^2\right]$$

$$\frac{\partial r}{\partial s} = \frac{c(1 - F(\underline{z}))}{\Delta}\left[(1 - \gamma\alpha^2)\lambda(\underline{z}) - \underline{z}\{\lambda(\underline{z})(\lambda(\underline{z}) - \underline{z}) - \gamma\alpha^2\}\right]$$

where $\Delta > 0$ is the Hessian of the tenant's optimization problem.

The landlord's objective is to maximize

$$U_L = \alpha e - \{(1 - F(\underline{z})(s\alpha e - R) + srf(\underline{z})\}$$

subject to the incentive constraints and the PC. Differentiating the landlord's objective function totally with respect to s and R and then using the condition $dR/ds = \alpha e + r\lambda$ from the tenant's PC, we get the following first-order condition in terms of s:

$$\frac{\partial \pi_L}{\partial s} = \alpha\{1 - (1 - F(\underline{z})s\}\frac{\partial e}{\partial s} - sf(\underline{z})\frac{\partial \sigma}{\partial s}.$$

[19] This is without loss of generality. Under an optimal contract, to ensure $U_T \geq 0$, $\underline{z}$ must be less than 1, and similarly to ensure $U_L \geq 0$, $\underline{z}$ must be greater than -1. The proof is available upon request.

Substituting the expressions for $\partial e/\partial s$ and $\partial\sigma/\partial s$ from above and evaluating at $s=1$, we get:

$$\left.\frac{\partial\pi_{\mathrm{L}}}{\partial s}\right|_{s=1} = \frac{c(1-F(\underline{z}))}{\Delta}\Big[\gamma\alpha^2 F(\underline{z})\left\{1-\left(\lambda(\underline{z})-\underline{z}\right)^2\right\}$$
$$-f(\underline{z})\{(1-\gamma\alpha^2)\lambda(\underline{z})-\underline{z}\{\lambda(\underline{z})(\lambda(\underline{z})-\underline{z})-\gamma\alpha^2\}\Big].$$

Suppose α is very small. Concentrating on the region for which an interior solution exists to the tenant's problem, i.e., $-1\leq\underline{z}\leq 1$, each term in the above expression is bounded. Hence, the above expression will be negative if and only if $\lambda(\underline{z})-\underline{z}\{\lambda(\underline{z})(\lambda(\underline{z})-\underline{z})\}>0$. Now, since $\lambda(\underline{z})\geq 0$ and $\lambda(\underline{z})(\lambda(\underline{z})-\underline{z})\in(0,1)$ for the normal distribution, for $\underline{z}\leq 0$ the above expression is positive. For $\underline{z}>0$, we have $\lambda(\underline{z})-\underline{z}\{\lambda(\underline{z})(\lambda(\underline{z})-\underline{z})\}>\lambda(\underline{z})-\underline{z}$. However, $\lambda(z)-z\geq 0$ for all z and so the proof is complete.[20] Now, suppose γ is small. Since γ also enters Δ, the only modification to the above argument is as follows: as $\gamma\to 0$ now the first term in the expression stays finite (it is positive for $\underline{z}\geq -0.4$ and negative otherwise) but the second term is negative and goes to infinity. Hence, the entire expression is negative. Using the tables for the standard normal distribution, we can evaluate this expression for various values of $\underline{z}$. In particular, for $\gamma\alpha^2\leq 0.23$, a share contract is always optimal. On the other hand, for $\gamma\alpha^2\geq 0.79$ a fixed rent contract is always optimal even though the tenant chooses a level of risk which is greater than the socially optimal level. For intermediate values of $\gamma\alpha^2$, both share and fixed rental contracts are possible candidates for optimal contracts.

5. Conclusion

We conclude with a discussion of some implications of our model in terms of observable characteristics of farms cultivated under alternative contractual arrangements. A direct implication of the joint moral hazard model is that controlling for factors such as the characteristics of the tenant and landlord and land quality, both the mean and the variance of output will be higher in farms that are cultivated under fixed rent contracts as opposed to sharecropping contracts.[21]

[20] For properties of the hazard rate of the normal distribution, see Johnson and Kotz (1970).

[21] It should be clarified here that we are talking about output per unit of land and not net surplus (that takes into account the cost of non-contractible inputs by the tenant) here. Since we derive contracts that maximize one party's payoff subject to informational and transactional constraints for a given level of payoff of the other party, they are by construction constrained Pareto-efficient. Hence, if we observe a plot of land being cultivated under sharecropping, it has to be the case that it is more efficient compared to wage or fixed rent contracts, since the landlord and the tenant were free to choose any contract.

While the insurance vs. incentives model has the same implication in terms of mean output, since our model allows for endogenous choice of risk, it also generates predictions about the relationship between contractual structure and output variability.

Another interesting implication of our analysis is regarding what kind monitoring technologies are likely to be adopted by landlords. Suppose a landlord has access to monitoring technologies that enable her to observe various actions chosen by the tenant, namely, effort and risk. We assume that by incurring a fixed cost of K, the landlord can receive a perfectly accurate (and verifiable) signal of effort. Similarly, by spending K', the landlord can perfectly monitor risk. Two straightforward implications of Propositions 1–4 follow. First, the landlord will, in equilibrium, never invest in both of these technologies. Because by investing in monitoring effort perfectly, the landlord can elicit the first-best level of risk taking by offering a wage contract. Second, so long as $K \leq K'$, if the landlord decides to invest in a monitoring technology at all, it will be to monitor effort. The reason is, even when risk is perfectly contractible, the landlord cannot achieve the first-best level of effort using contractual means. However, if effort is perfectly contractible, the landlord can indeed achieve the first-best level of risk using a fixed wage contract.

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Appendix A

A.1. Derivation of the incentive constraints for effort and risk

Differentiating the tenant's payoff function with respect to e, we get:

$$\frac{\partial U_{\mathrm{T}}}{\partial e} = \int_{\hat{x}}^{\bar{x}} (sx - R) f_e(x|e,r) \mathrm{d}x - C_e(e,r).$$

Integrating the first term of the right-hand side by parts, we get

$$\int_{\hat{x}}^{\bar{x}} (sx - R) f_e(x|e,r) \mathrm{d}x = \left[(sx - R) F_e(x|e,r) \right]_{\hat{x}}^{\bar{x}} - s \int_{\hat{x}}^{\bar{x}} F_e(x|e,r) \mathrm{d}x$$

$$\int_{\hat{x}}^{\bar{x}} (sx - R) f_e(x|e,r) \mathrm{d}x = -s \int_{\hat{x}}^{\bar{x}} F_e(x|e,r) \mathrm{d}x$$

as $s\hat{x} - R = 0$ from the definition of $\hat{x}$ and $F_e(\bar{x}|e, r) = 0$ as $F(\bar{x}|e, r) = 1$. Repeating the same steps for r, we are able to derive the tenant's first-order conditions with respect to e and r. ■

Proof of Proposition 1. The limited liability constraint is binding if there are states of the world when the tenant cannot pay the due rent, $(1-s)x + R$. This will happen if $\exists x \in [\underline{x}, \bar{x}]$ such that $sx - R < 0$ or $x < \hat{x}$. So long as $\hat{x} > \underline{x}$ such states will occur with positive probability. By Eq. (A2), $\int_{\underline{x}}^{\bar{x}} F_r(x|e,r)\mathrm{d}x = 0$. Also, so long as $\hat{x} > \underline{x}$, $-s\int_{\hat{x}}^{\bar{x}} F_r(x|e,r)\mathrm{d}x > 0$. As a result $r > \bar{r}^* = \underline{r}$ for $0 < s \leq 1$. If $s = 0$ then $r = \bar{r}^* = \underline{r}$. On the other hand, by Eq. (A1), we get $-s\int_{\hat{x}}^{\bar{x}} F_e(x|e,r)\mathrm{d}x < -s\int_{\underline{x}}^{\bar{x}} F_e(x|e,r)\mathrm{d}x$ for $0 \leq s \leq 1$ and for any $r \in [\underline{r}, \bar{r}]$ so long as $\hat{x} > \underline{x}$. Since $\int_{\underline{x}}^{\bar{x}} F_r \mathrm{d}x = 0$ by Eq. (A2), differentiating both sides with respect to e and noting that $\underline{x}$ and $\bar{x}$ are independent of e, we get $\int_{\underline{x}}^{\bar{x}} F_{er}\mathrm{d}x = 0$. This implies that $-\int_{\underline{x}}^{\bar{x}} F_e(x|e,r)\mathrm{d}x = -\int_{\underline{x}}^{\bar{x}} F_e(x|e,\underline{r})\mathrm{d}x$ for any $r \in [\underline{r}, \bar{r}]$. As a result, $-s\int_{\hat{x}}^{\bar{x}} F_e(x|e,r)\mathrm{d}x < -\int_{\underline{x}}^{\bar{x}} F_e(x|e,\underline{r})\mathrm{d}x$. Hence, $e < e^*$. ■

A.2. Derivation of the landlord's first-order condition

First, we totally differentiate the tenant's PC with respect to s and R. Any changes in s and R on the tenant's payoff via e or r are ignored given the tenant's first-order conditions.

$$\int_{\hat{x}}^{\bar{x}} xf(x|e,r)\mathrm{d}x - \frac{\partial R}{\partial s}\int_{\hat{x}}^{\bar{x}} f(x|e,r)\mathrm{d}x - s\frac{R}{s}f\left(\frac{R}{s}|e,r\right)\frac{s\frac{\partial R}{\partial s} - R}{s^2} + Rf\left(\frac{R}{s}|e,r\right)\frac{s\frac{\partial R}{\partial s} - R}{s^2} = 0$$

The last two terms cancel out. Hence,

$$\frac{\partial R}{\partial s} = \frac{\int_{\hat{x}}^{\bar{x}} xf(x|e,r)\mathrm{d}x}{1 - F(\hat{x}|e,r)} \equiv E[x|x \geq \hat{x}]$$

where $E[x|x \geq (R/s)]$ is the mean of the conditional distribution of x for $x \geq \hat{x}$. The first-order condition of the landlord with respect to s is:

$$\frac{\partial U_{\mathrm{L}}}{\partial s} = \left(\int_{\underline{x}}^{\bar{x}} xf_e(x|e,r)\mathrm{d}x\right)\frac{\partial e}{\partial s} + \left(\int_{\underline{x}}^{\bar{x}} xf_r(x|e,r)\mathrm{d}x\right)\frac{\partial r}{\partial s} + s\left(\int_{\hat{x}}^{\bar{x}} F_e(x|e,r)\mathrm{d}x\right)\frac{\partial e}{\partial s} + s\left(\int_{\hat{x}}^{\bar{x}} F_r(x|e,r)\mathrm{d}x\right)\frac{\partial r}{\partial s} - \int_{\hat{x}}^{\bar{x}}\left(x - E\left[x|x \geq \frac{R}{s}\right]\right)f(x|e,r)\mathrm{d}x.$$

However, $\int_{\underline{x}}^{\bar{x}} x f_r(x|e, r)\mathrm{d}x = [xF_r(x|e, r)]_{\underline{x}\bar{x}} - \int_{\underline{x}}^{\bar{x}} F_r(x|e, r)\mathrm{d}x = 0$ and $\int_{\underline{x}}^{\bar{x}} x f_e(x|e, r)\mathrm{d}x = [xF_e(x|e, r)]_{\underline{x}\bar{x}} - \int_{\underline{x}}^{\bar{x}} F_e(x|e, r)\mathrm{d}x = -\int_{\underline{x}}^{\bar{x}} F_e(x|e, r)\mathrm{d}x$. Finally, $-\int_{\hat{x}}^{\bar{x}}(x - E[x|x \geq (R/s)]f(x|e, r)\mathrm{d}x = -\int_{\hat{x}}^{\bar{x}} x f(x|e, r)\mathrm{d}x + E[x|x \geq (R/s)]\int_{\hat{x}}^{\bar{x}} f(x|e, r)\mathrm{d}x = 0$. ■

Proof of Proposition 2. The effect of an increase in r on the landlord's payoff is $s(\int_{\hat{x}}^{\bar{x}} F_r(x|e,r)\mathrm{d}x) < 0$. Setting r is a zero-sum game between the landlord and the tenant due to the presence of limited liability. It does not affect the mean, and the tenant's gain, $s(\int_{\hat{x}}^{\bar{x}} F_r(x|e,r)\mathrm{d}x)$ is exactly equal to the landlord's loss, $s(\int_{\underline{x}}^{\hat{x}} F_r(x|e,r)\mathrm{d}x)$ by Eq. (A2). In addition, since e and r are (weak) substitutes in the tenant's cost function, a value of r higher than 0 also increases the cost of eliciting e for the landlord. Hence, for any given value of e, the landlord will choose $r = \underline{r}$.

If r is not chosen by the tenant, then $\partial e/\partial s = (A_e)/(\Delta_{ee})$ from Eq. (7). We want to show that the following condition is satisfied:

$$A_e - \int_{\frac{R}{s}}^{\bar{x}} F_e(x)\mathrm{d}x + F_e\left(\frac{R}{s}\right)\left[E\left(x|x \geq \frac{R}{s}\right) - \frac{R}{s}\right] > 0.$$

Re-arranging terms in the tenant's PC, we get:

$$sE\left[x|x \geq \frac{R}{s}\right] - R = \frac{C(e,r) + \bar{u}}{1 - F\left(\frac{R}{s}\right)}.$$

The right-hand side is positive and hence, $E(x|x \geq R) - R > 0$. By Eq. (A1), $F_e(R) < 0$ and so the second term is negative. However, by Eq. (A1), the first term is positive. We show below that for $r = \underline{r}$ and $s = 1$, $A_e > 0$ and hence, the optimal contract is a fixed rental contract.

As $R \to \underline{x}$, $A_e \to -\int_{\underline{x}}^{\bar{x}} F_e(x|e, \underline{r})\mathrm{d}x > 0$ because $E(x|x \geq \underline{x}) - \underline{x} = E(x) - \underline{x}$ is bounded and $F_e(\underline{x}|e, \underline{r}) = 0$ (as $F(\underline{x}|e, \underline{r}) \equiv 0$). Also, as $R \to \bar{x}$, $A_e \to 0$ (notice that this is true even if $\bar{x} = \infty$). Hence, if we can show $(\mathrm{d}/\mathrm{d}R)A_e \leq 0$, then A_e is always non-negative. Now, $(\mathrm{d}A_e/\mathrm{d}R) = F_e(R) + f(R)((f_e(R))/(f(R)) + (F_e(R))/(1 - F(R)))[E(x|x \geq R) - R]$. Since $f(\cdot)$ is a probability density function,

$$\int_{\underline{x}}^{\bar{x}} f(x)\mathrm{d}x = 1$$

and hence,

$$\int_{\underline{x}}^{\bar{x}} \frac{f_e(x)}{f(x)} f(x)\mathrm{d}x = 0.$$

Given the MLRP, this implies there exists $x_0 \in (\underline{x}, \bar{x})$ such that $(f_e(x))/(f(x)) > 0$ for $x > x_0$, $(f_e(x_0))/(f(x_0)) = 0$, and $(f_e(x))/(f(x)) < 0$ for $x < x_0$. Now, $-F_e(R) = (\partial/\partial e)[1 - F(R)] = (\partial/\partial e)\int_R^{\bar{x}} f(x)\mathrm{d}x$. Hence, $(-F_e(R))/(1 - F(R)) = \int_R^{\bar{x}} (f_e(x))/(f(x))\ (f(x))/(1 - F(R))\mathrm{d}x$. Observe that $(f(x))/(1 - F(R))$ is the conditional density function of x for $x \geq R$ and $\int_R^{\bar{x}} (f(x))/(1 - F(R))\mathrm{d}x = 1$. That is, $(-F_e(R))/(1 - F(R))$ is the conditional expected value of likelihood ratios for $x \geq R$. By the MLRP $(f_e(x))/(f(x)) \geq (f_e(R))/(f(R))$ for all $x \geq R$. Hence, $(f_e(R))/(f(R)) - (-F_e(R))/(1 - F(R)) \leq 0$.
Finally, effort will be less than the first-best because $-\int_R^{\bar{x}} F_e(x|e, \underline{r})\mathrm{d}x < -\int_{\underline{x}}^{\bar{x}} F_e(x|e, \underline{r})\mathrm{d}x$. ■

Proof of Proposition 3. To discourage risk-taking, the landlord should set $s = 0$ which ensures $r = \underline{r}$ from Eq. (5). Since e and r are (weak) substitutes from the tenant's cost function, this does not raise the cost of eliciting e (even if e is contractible). Then, if the landlord simply stipulates the first-best effort level e^* in the contract, that will achieve the first-best. ■

Proof of Proposition 4. As s goes to 0, R will have to be negative to satisfy the PC, Eq. (3). From the PC, $\lim_{s \to 0} R = -[C(e, r) + \bar{u}]$. Hence, as $s \to 0$, $\hat{x} = (R/s) \to -\infty$. Since $\lim_{s \to 0} \int_{(R/s)}^{\bar{x}} F_e(x|e, r)\mathrm{d}x = \int_{\underline{x}}^{\bar{x}} F_e(x|e, r)\mathrm{d}x$ which is negative and finite, $\lim_{s \to 0} s\int_{(R/s)}^{\bar{x}} F_e(x|e, r)\mathrm{d}x = 0$. Hence, from the ICC of the tenant for e, we see that as $s \to 0$, $e \to 0$. As $\lim_{s \to 0} \int_{(R/s)}^{\bar{x}} F_r(x|e, r)\mathrm{d}x = \int_{\underline{x}}^{\bar{x}} F_r(x|e, r)\mathrm{d}x = 0$, an analogous argument establishes that as $s \to 0$, $r \to 0$. Evaluated at $e = \underline{e}$ and $r = \underline{r}$, $R = -\bar{u}$ and hence, the landlord's expected payoff is $\int_{\underline{x}}^{\bar{x}} xf(x|0, 0)\mathrm{d}x - \bar{u}$.
Since by Eq. (A1), $F_e(x) \leq 0$ for all $x \in [\underline{x}, \bar{x}]$, $-\int_{\hat{x}}^{\bar{x}} F_e(x|e, r)\mathrm{d}x$ is decreasing in $\hat{x}$ and achieves the maximum value for $\hat{x} = \underline{x}$. A small increase in s accompanied by a corresponding increase in R (i.e., a reduction in its absolute value) will increase e from the tenant's ICC for e, Eq. (4), because now he gets a (very small) fraction of the marginal product of e, $-\int_{\underline{x}}^{\bar{x}} F_e(x|e, r)\mathrm{d}x$ which is very high. Since the landlord gets a very high fraction of the marginal product of e, his expected payoff goes up. By a similar logic, r goes up above 0, and that is a loss to the landlord. However, given that s is very small, and $-\int_{\underline{x}}^{\bar{x}} F_r(x|e, r)\mathrm{d}x = 0$ by Eq. (A2), this effect is outweighed by the positive effect of an increase in e on the landlord's profits. Formally, looking at Eq. (6), we see that for s positive but very small, $(\partial U_L)/(\partial s) > 0$. ■

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