Statistics, question 2, 2010/2011 exam paper: alternative solutions

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Sample space is composed by 2 favorites songs and 10 non favorites songs. Call the former F and the latter F^c. $\Omega = 2$ x $F + 10$ x F^c. For timing, assume $t = 1, 2, ..12$. Call event $F_t = \{\text{F is extracted at t}\}$, and event $F'_t = \{\text{Second F extracted at t}\}$

The event we are looking for is

$$\begin{aligned} P(F'_3) &= P\Big(\big(F'_3 \cap (F_1 \cap F_2^c)\big) \cup \big(F'_3 \cap (F_1^c \cap F_2)\big) \cup \big(F'_3 \cap (\text{other combinations})\big)\Big) \\ &= P\Big(\big(F'_3 \cap (F_1 \cap F_2^c)\big) \cup \big(F'_3 \cap (F_1^c \cap F_2)\big)\Big) + 0 \end{aligned}$$

Call event $A = (F_1 \cap F_2^c) \cup (F_1^c \cap F_2)$. Above result means $F'_3 = F'_3 \cap A$ Then

$P(F'_3 \mid A) = 1/10$

$P(A) = 40/12 \cdot 11 = 10/33$

1. Given that $(F_1 \cap F_2^c), (F_1^c \cap F_2)$ are disjoint

$$P(F'_3) = P\big(F'_3 \cap F_1 \cap F_2^c\big) + P\big(F'_3 \cap F_1^c \cap F_2\big) = \frac{2}{12}\frac{10}{11}\frac{1}{10} + \frac{10}{12}\frac{2}{11}\frac{1}{10} = \frac{1}{33}$$

2. Alternatively, using the formula for conditional probabilities

$$
\begin{aligned}
P(F_3') &= P\big(F_3' \cap F_1 \cap F_2^c)\big) + P\big(F_3' \cap F_1^c \cap F_2)\big) = \\
&= P\big(F_3' \mid F_1 \cap F_2^c)\big) P(F_1 \cap F_2^c) + P\big(F_3' \mid F_1^c \cap F_2)\big) P(F_1^c \cap F_2) = \\
&= \frac{1}{10}\Big(\frac{2}{12}\frac{10}{11}\Big) + \frac{1}{10}\Big(\frac{10}{12}\frac{2}{11}\Big) = \frac{1}{33}
\end{aligned}
$$

3. Alternatively, if one wants to use Bayes rule,

$$
P(F_3' \mid A) = P(F_3')\frac{P(A \mid F_3')}{P(A)} = P(F_3')\frac{P(A \mid F_3' \cap A)}{P(A)} = P(F_3')\frac{1}{P(A)}
$$

Substitute $P(F_3' \mid A) = 1/10$ and $P(A) = 10/33$ and get $P(F_3') = 1/33$

It makes sense that $P(F_3') < P(F_3' \mid A)$, since I am not conditioning on having already extracted A. F_3' will never occurs if A does not occur.