Quiz 1, question 2

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$P(A) = P(B) = P(C) = P(D) = 0.4$

$P(D \cap C) = P(A \cap C) = P(D \cap B) = 0$

$P(A \cap B) = 0.1$

a)

$P(A \cup D) = P(A) + P(D) - P(A \cap D) = 0.4 + 0.4 - P(A \cap D)$

if independent

$= 0.4 + 0.4 - 0.16 = 0.64$

Event C is disjoint of A and D, hence

$P(A \cup D \cup C) = P(A \cup D) + P(C) = 0.64 + 0.4 = 1.04 > 1$

impossible, hence A and D not independent

b)

Suppose $P(B \cap C) = 0.2$. Then

$P(B \cup C) = 0.4 + 0.4 - 0.2 = 0.6$

D is disjoint, so

$P(B \cup C \cup D) = P(B \cup C) + P(D) = 0.6 + 0.4 = 1$

ok, true

c)

- Step A: $P(C \cup D) = P(C) + P(D) - P(C \cap D) = 0.8$, hence $P(C \cup D)^c = 0.2$

  this means that the intersection of event $(C \cup D)^c$ with any event cannot be bigger

- Step B: We are given $P\big((B \cap C) \cup (A \cap D)\big) = 0.45$. Note, $(B \cap C) \cup (A \cap D) = (A \cup B) \cap (C \cup D)$, so $P\big(A \cup B) \cap (C \cup D)\big) = 0.45$

- Step C: $P(A \cup B) = 0.7$. Since for any event N and M, $P(N) = P(N \cap M) + P(N \cap M^c)$, so $P(A \cup B) = P\big((A \cup B) \cap (C \cup D)\big) + P\big((A \cup B) \cap (C \cup D)^c\big) = 0.7$. Combine this with step B and get $P\big((A \cup B) \cap (C \cup D)^c\big) = 0.25$. This is incompatible with step A

d)

$P(A^c) = 1 - 0.4 = 0.6$. Since event $A^c \cup D$ is bigger than A^c, $P(D \cup A^c)$ cannot be smaller than 0.6

e)

If $P(D \cup A^c) = 0.65$, $P(D \cap A^c) = 0.35$. It follows that $P(D \cap A) = P(D) - P(D \cap A^c) = 0.05$. Hence $P(D \cup A) = 0.75$. Event C is disjoint, so $P(A \cup D \cup C) = P(D \cup A) + P(C) = 1.15$. Cannot be