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1] $t=1,2$ $\{y_1, y_2\}$ income endowment

Life-time utility: $U(c_1, c_2) = \ln c_1 + \beta \ln c_2$

a) $t=1$: BC $c_1 + a_1 = y_1$

$t=2$: BC $c_2 = y_2 + (1+r) a_1$

a_1 = asset holding at end of period $t=1$

$a_1 > 0$ saves/lends
$a_1 < 0$ borrows from future income

Of course it's the same a_1. Substitute out a_1

From BC at $t=2$ $a_1 = \frac{c_2}{1+r} - \frac{y_2}{1+r}$

$$c_1 + \frac{c_2}{1+r} - \frac{y_2}{1+r} = y_1$$

$$c_1 + \frac{c_2}{1+r} = \underbrace{y_1 + \frac{y_2}{1+r}}_{\text{life time income}} \quad \text{IBC}$$

b) $\max_{c_1, c_2} U(c_1, c_2)$

s.t. IBC

$$\max_{c_1, c_2} \ln c_1 + \beta \ln c_2$$

$$\text{s.t.} \quad c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$\mathcal{L} = \ln c_1 + \beta \ln c_2 + \lambda \left\{ y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} \right\}$$

FOC c_1 : $\frac{1}{c_1} = \lambda$

FOC c_2 : $\frac{\beta}{c_2} = \frac{\lambda}{1+r}$ $\qquad \lambda = \frac{\beta(1+r)}{c_2}$

$$\Rightarrow \frac{1}{c_1} = \overset{\text{discount FACTOR}}{\beta} \cdot (1+r) \cdot \frac{1}{c_2}$$

or

$$\frac{1}{c_1} = \frac{1+r}{1+\underset{\text{discount RATE}}{\rho}} \frac{1}{c_2}$$

Interpretation: given life-time income, set c_1 & c_2 s.t. their ratio is

$$\frac{c_2}{c_1} = \frac{1+r}{1+\rho}$$

and you spend entirely you life-time income

Note, $\frac{c_2}{c_1} = \frac{1+r}{1+\rho}$

independent on income.

$c_2 > c_1$ if $r > \rho$: the market values the future more than us, so we postpone consumption to take advantage of r despite the fact that we value 1 € at $t=2$ only β € at time $t=1$

$c_2 < c_1$ if $r < \rho$: the other way around

c) To pin down $\{c_1^*, c_2^*\}$,

$$\begin{cases} \frac{c_2}{c_1} = \frac{1+r}{1+\rho} \\ c_1 + \frac{c_2}{1+r} = \underbrace{y_1 + \frac{y_2}{1+r}}_{W} \end{cases}$$

$$c_1 + (1+\rho)^{-1} c_1 = W$$

$$\begin{cases} c_1^* = \frac{1+\rho}{2+\rho} W < W \text{ . ok} \\ c_2^* = \frac{1}{2+\rho} (1+r) W \end{cases}$$

check IBC:

$$\frac{1+\rho}{2+\rho} W + \frac{1}{\cancel{1+r}} \frac{\cancel{1+r}}{2+\rho} W =$$

$$= \frac{1+\rho+1}{2+\rho} W = W \quad \text{ok}$$

check EE

$$\frac{\frac{1+r}{2+\rho} \cancel{W}}{\frac{1+\rho}{2+\rho} \cancel{W}} = \frac{1+r}{1+\rho} \quad \text{ok}$$

Hence, given $\{y_1, y_2\}$

$$a_1^* = y_1 - c_1^*$$

$$= y_1 - \frac{1+\rho}{2+\rho} \underbrace{\left(y_1 + \frac{y_2}{1+r} \right)}_{W}$$

$$= \frac{2+\cancel{\rho}-1-\cancel{\rho}}{2+\rho} y_1 + \frac{y_2}{1+r}$$

$$= \frac{y_1}{2+\rho} + \frac{y_2}{1+r}$$

Note: interpret savings behavior from Euler equation, not from levels, since as $r \uparrow$ you postpone consumption, so you would imagine to increase savings. But at the same time W decreases since discount y_2 by more. So, c_1^* is decreasing in r! This is because as $r \uparrow$

1) Given W you consume it more in $t=2$ relative to time 1 (Euler equation)

2) You have a smaller W, which is going to decrease c_1 and c_2

2) $U(c_1^j, c_2^j) = \ln c_1^j + \beta \ln c_2^j \qquad j = d, f$

endowments: $y_1^d, \quad y_2^f$

Of course without trade we would have

$$c_1^d = y_1^d \qquad c_1^f = 0$$
$$c_2^d = 0 \qquad c_2^f = y_2^f$$

(Note, assume no capital markets within countries, only across countries)

a) Same utilities as in question 1, only here

$$W^d = y_1^d \qquad W^f = \frac{y_2^f}{1+r}$$

Euler equations:

$$\frac{c_2^d}{c_1^d} = \frac{1+r}{1+\rho}$$

$$\frac{c_2^f}{c_1^f} = \frac{1+r}{1+\rho}$$

Since face the same interest rate and discount future values at same ρ, optimality implies

$$\frac{c_2^j}{c_1^j} = \frac{1+r}{1+\rho} \quad , \forall j$$

$$c_1^j = \frac{1+\rho}{2+\rho} W^j = \begin{cases} \frac{1+\rho}{2+\rho} \cdot y_1^d & \text{if } j=d \\ \frac{1+\rho}{2+\rho} \frac{y_2^f}{1+r} & \text{if } j=f \end{cases}$$

$$c_2^j = \frac{1+r}{2+\rho} W^j = \begin{cases} \frac{1+r}{2+\rho} y_1^d & \text{if } j=d \\ \frac{1}{2+\rho} \cdot y_2^f & \text{if } j=f \end{cases}$$

Since here domestic agent lends to foreign one, an higher r will benefit domestic guy and penalize foreign guy

Formally

$$U^{d*} = \ln\left(\frac{1+\rho}{2+\rho}\, y_1^d\right) + \frac{1}{1+\rho} \cdot \ln\left(\frac{1+r}{2+\rho}\, y_1^d\right)$$

increasing in r

$$U^{f*} = \ln\left(\frac{1+\rho}{2+\rho}\, \frac{y_2^f}{1+r}\right) + \frac{1}{1+\rho} \ln\left(\frac{1}{2+\rho}\, y_2^f\right)$$

decreasing in r

c) Market clearing:

$$t=1 \quad \frac{1+\rho}{2+\rho}\, y_1^d + \frac{1+\rho}{2+\rho}\, \frac{y_2^f}{1+r} = y_1^d$$

$$y_1^d + \frac{y_2^f}{1+r} = \frac{2+\rho}{1+\rho}\, y_1^d$$

$$\frac{1}{2+\rho}\, y_1^d = \frac{y_2^f}{1+r}$$

$$1+r = (1+\rho) \cdot \frac{y_2^f}{y_1^d}$$

foreign is borrower. the richer he is and the higher his levels of consumption will be, so will have to borrow more in t=1

$\rho \uparrow \rightarrow$ both want to consume more in $t=1 \rightarrow$
$\rightarrow$ credit supply $\downarrow$ & credit demand $\uparrow$