JULLIARD, PS 4

MACROECONOMICS FOR MSC
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$$E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s})$$

$r$ is deterministic and constant; $\{Y_{t+s}\}_{s=0}^{\infty}$ is exogenous and stochastic. The government has to finance an exogenous stochastic $\{G_{t+s}\}_{s=0}^{\infty}$ using a $\{T_{t+s}\}$. He solves for Taxation maximizing HH's utility

HH constraint

$$c_t + T_t + B^p_{t+1} = Y_t - \alpha \frac{T_t^2}{2} + B^p_t$$

with $B^p_{t+1}$ = assets ~~for~~ left at time $t$

$$c_{t+1} + T_{t+1} + B^p_{t+2} = Y_{t+1} - \alpha \frac{T_{t+1}^2}{2} + B^p_{t+1}(1+r)$$

$$\to B^p_{t+1} = \frac{1}{1+r}\left(c_{t+1} + T_{t+1} + B^p_{t+2} - Y_{t+1} + \alpha \frac{T_{t+1}^2}{2}\right)$$

$$\Rightarrow c_t + \frac{c_{t+1}}{1+r} + T_t + \frac{T_{t+1}}{1+r} + \frac{B_{t+2}}{1+r} = Y_t + \frac{Y_{t+1}}{1+r} - \alpha\frac{T_t^2}{2} - \alpha\frac{T_{t+1}^2}{2(1+r)} + B^p_t$$

$$\text{IBC}^{HH} \quad \sum_{s=0}^{\infty}\left(\frac{c_{t+s}}{(1+r)^s} + \frac{T_{t+s}}{(1+r)^s}\right) + \lim_{q\to\infty} \frac{B^p_{t+q+1}}{(1+r)^q} \leq$$

$$\leq \sum_{s=0}^{\infty}\left(\frac{Y_{t+s}}{(1+r)^s} - \frac{\alpha T_{t+s}^2}{2(1+r)^2}\right) + B^p_t$$

$\{T_{t+s}\}_{s=0}^{\infty}$ must also satisfy the budget constraint of the government:

$$T_t + B_t^G = G_t + B_{t+1}^G$$

iterate, get

$$\sum_{s=0}^{\infty} \frac{T_{t+s}}{(1+r)^s} + B_t^G = \sum_{s=0}^{\infty} \frac{G_{t+s}}{(1+r)^s} +$$

$$+ \lim_{s\to\infty} \frac{B_{t+s+1}^G}{(1+r)^s}$$

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$$\max_{\{C_{t+s},\, T_{t+s}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s})$$

$$\text{s.t. } \sum_{s=0}^{\infty} \frac{1}{(1+r)^s}\left(Y_{t+s} - \frac{\alpha T_{t+s}^2}{2} - C_{t+s} - T_{t+s}\right) + B_t^t \geq 0$$

$$\text{s.t. } \sum_{s=0}^{\infty} \frac{1}{(1+r)^s}\left(T_{t+s} - G_{t+s}\right) + B_t^G \geq 0$$

$$\mathcal{L} = E_t \left\{ \sum_{s=0}^{\infty} \beta^s u(C_{t+s}) + \lambda \left[ \sum_{s=0}^{\infty} \frac{Y_{t+s} - \frac{\alpha T_{t+s}^2}{2} - C_{t+s} - T_{t+s}}{(1+r)^s} + B_t^P \right] \right.$$

$$\left. + \mu \left[ \sum_{s=0}^{\infty} \frac{T_{t+s} - G_{t+s}}{(1+r)^s} + B_t^G \right] \right\}$$

FOC $T_{t+s}$ : $E_{t+s}\left\{-\lambda \frac{\alpha T_{t+s}}{(1+r)^s} + \frac{\mu}{(1+r)^s}\right\} = 0$

$$E_{t+s}\left\{-\lambda\left[\alpha T_{t+s+1}\right] + \mu\right\} = 0$$

$$\mu = \lambda\left[\alpha T_{t+s+1}\right]$$

FOC $T_{t+s+1}$ $\quad E_{t+s}\left\{-\lambda\left[\alpha T_{t+s+1}\right] + \mu\right\} = 0$

$$\lambda \cdot \alpha E_{t+s}\left[T_{t+s+1}\right] = \mu$$

$$\overset{!}{=} \lambda\left[\alpha T_{t+s+1}\right]$$

$$T_{t+s} = E_{t+s}\left[T_{t+s+1}\right]$$

FOC $C_{t+s}$ & $C_{t+s+1}$ $\Rightarrow u'(C_{t+s}) = E_{t+s}\left[u'(C_{t+s+1})\right]$

Government is not indifferent with Taxes, since he will follow $T_t = E_t\left[T_{t+s}\right]$, $\forall s$

$$\begin{cases} \sum_{s=0}^{\infty} \frac{T_{t+s}}{(1+r)^s} + B_t^G = \sum_{s=0}^{\infty} \frac{G_{t+s}}{(1+r)^s} \\ E_t\left[T_{t+s}\right] = T_t \end{cases}$$

$$\Rightarrow T_t = \frac{r}{1+r}\left\{\sum_{s=0}^{\infty} E_t \frac{G_{t+s}}{(1+r)^s} - B_t^G\right\}$$

2) Yes: $T_t$ follows a martingale, while $G_{t+s}$ is exogenous, so government runs into surplus and deficits. Formally, assume

$$G_{t+s} = \bar{G} + \varepsilon_{t+s} \quad , \quad E_t(\varepsilon_{t+s}) = 0$$

Assume $B_t^G = 0$

$$T_t = \frac{r}{1+r} \sum_{s=0} ~~E_t~~ \frac{G_{t+s}}{(1+r)^s}$$

$$= \frac{r}{1+r} \sum_{s=0} \frac{\bar{G}}{(1+r)^s} = \frac{r}{1+r} \frac{1+r}{r} \bar{G} = \bar{G}$$

so, always balanced in expectation

Suppose $\varepsilon_t \neq 0$

$$T_t = \frac{r}{1+r} \sum_{s=0}^{\infty} \frac{\bar{G} + \varepsilon_t}{(1+r)^s} = \frac{1+r}{1+r} \left\{ \frac{1+r}{r} \bar{G} + \varepsilon_t \right\} =$$

$$= \bar{G} + \frac{r}{1+r} \varepsilon_t$$

The government discovers that he has to finance an extra $\varepsilon_t$. To do this it does not tax it entirely on the same period but smoothes this extra $\varepsilon_t$

4) Substitute $IBC^G$ into $IBC^{HH}$, combine with ~~EE~~:

$$C_t = \frac{r}{1+r} \left\{ \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \left[ Y_{t+s} - G_{t+s} - \frac{\alpha T_{t+s}^2}{2} \right] \right.$$

Timing of taxes matters due to deadweight loss