PROBLEM SET 6. JOLLARD MICHELE PIFFER

Firm's maximization problem:

$$\max_{\{I_s\}_{s=t}^{\infty},\ K_{s+1}} \sum_{s=t}^{\infty} (1+r)^{-(s-t)} \left[A_s \cdot F(K_s) - I_s - \chi \cdot \frac{I_s^2}{2} \right]$$

s.t. $K_{s+1} = K_s + I_s$, $\forall s$, $s=t, \dots, \infty$

K_t given

$$\mathcal{L} = \sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} \left\{ A_s \cdot F(K_s) - I_s - \chi \frac{I_s^2}{2} + \right.$$

$$\left. - q_s (K_{s+1} - K_s - I_s) \right\}$$

q_s is current value multiplier

a) FOC I_s: $\cancel{(1+r)^{-(s-t)}} \cdot \left[-1 - \chi I_s + q_s \right] = 0$

$\forall s$, $s=t, \dots, \infty$

FOC K_s: $(1+r)^{-(s-t)} \left[A_s F'(K_s) + q_s \right] +$

$-(1+r)^{-(s-1-t)} q_{s-1} = 0 \qquad \forall s, s=t+1, \dots, \infty$

or $A_s F'(K_s) + q_s - (1+r) q_{s-1} = 0$

b) FOCs are   MB of extra capital

$q_s = 1 + \chi I_s$ — MC of extra capital

$A_s F'(K_s) + q_s - (1+r) q_{s-1} = 0$

From the first one: $I_s = \dfrac{q_s - 1}{\chi}$

$$\boxed{K_{s+1} - K_s = \frac{q_s - 1}{\chi}}$$

From the second one: iterate

$A_{s+1} F'(K_{s+1}) + q_{s+1} - q_s - r q_s = 0$

$q_{s+1} - q_s = r q_s - A_{s+1} F'(K_{s+1})$

$$\boxed{q_{s+1} - q_s = r q_s - A_{s+1} F'\left(K_s + \frac{q_s - 1}{\chi}\right)}$$

c) Assume A constant. System of equations is

$$\begin{cases} K_{t+1} - K_t = \dfrac{q_t - 1}{\chi} \\ q_{t+1} - q_t = r q_t - A \cdot F'\left(K_t + \dfrac{q_t - 1}{\chi}\right) \end{cases}$$

in K, q only

$k_{t+1} = k_t$ for $q_t = 1$

$q_t > 1 \quad k_{t+1} > k_t$

$q_t < 1 \quad k_{t+1} < k_t$

$q_{t+1} = q_t$ for $q_t = \frac{A}{\Delta} F'\left(k_t + \frac{q_t - 1}{\pi}\right)$

$q_t \uparrow \to$ LHS $\uparrow$ and $k_t + \frac{q_t - 1}{\pi} \uparrow$, so $F'(\,) \downarrow$, so

RHS $\downarrow$ $\to$ need $k_t \downarrow$ so that RHS $\uparrow$

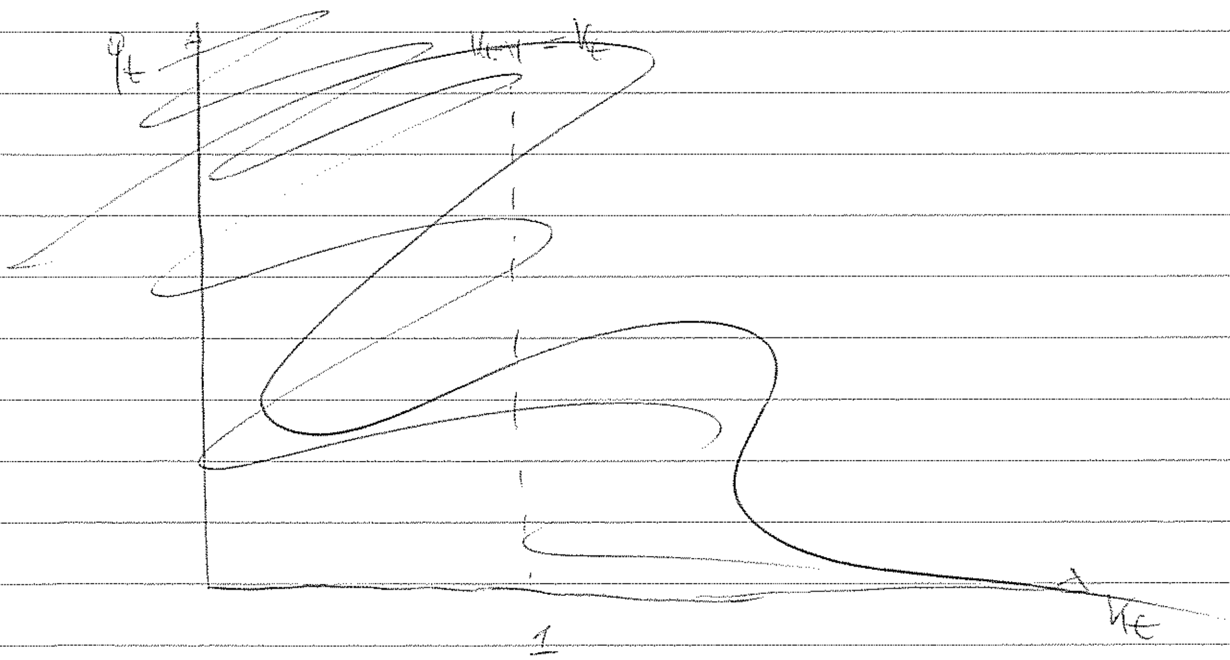
locus (q_t, k_t) is negatively sloped

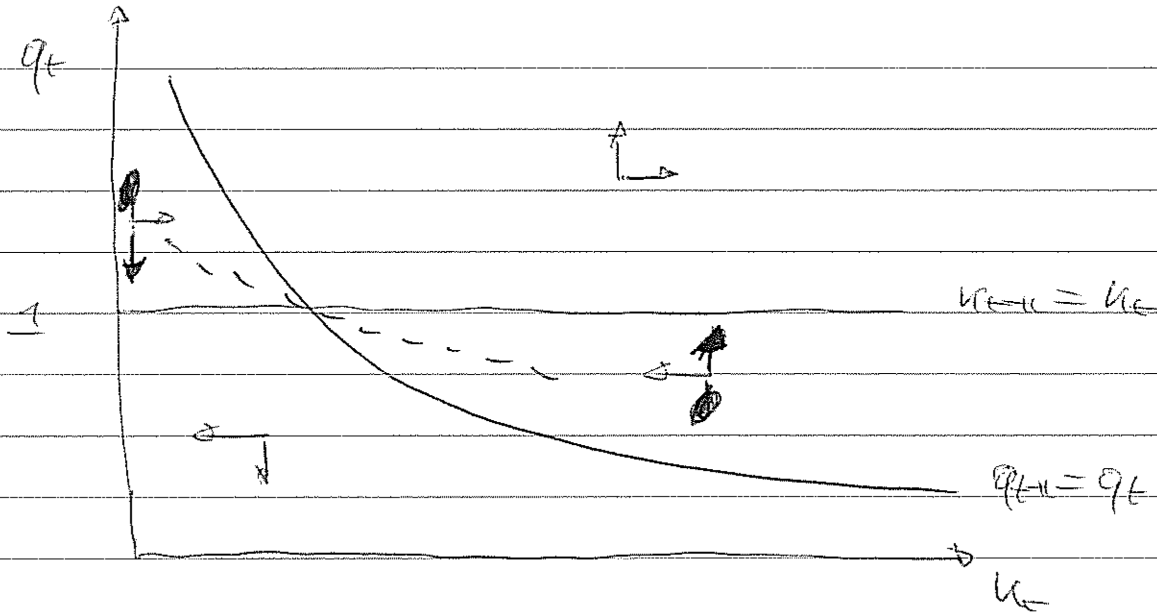
if k_t relatively too big (right of the line)

$\to q_t \uparrow$ (given q, $k_t \to F' \downarrow \to$ RHS $\uparrow \to q_t \uparrow$)

if k_t relatively too small (left of the line)

$\to q_t \downarrow$





Note, in SS $q^{SS} = 1$, so

$$r \cdot 1 = A \cdot F'\left(k_t + \frac{1-1}{\sigma}\right)$$

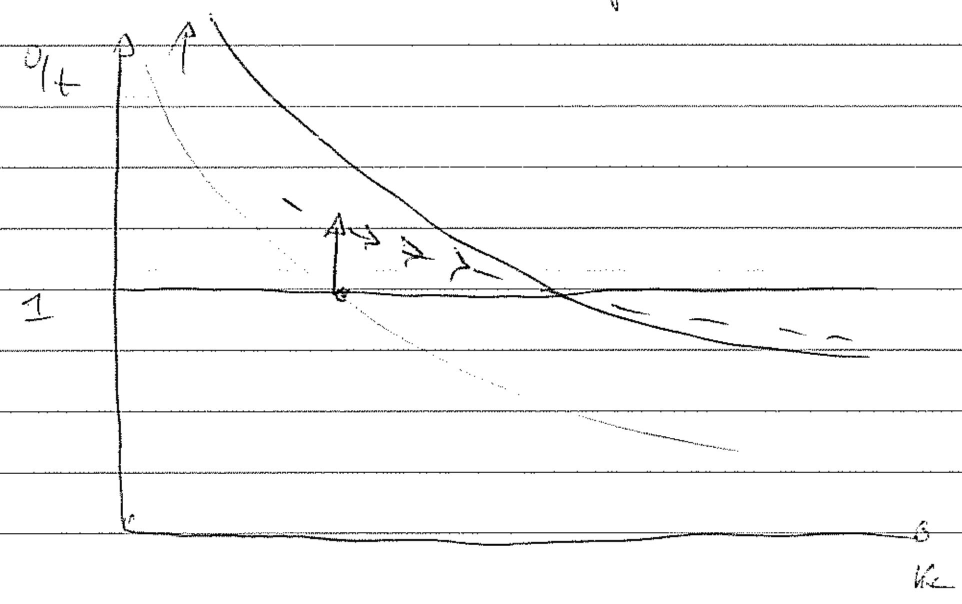
$$r = A \cdot F'(k_{SS})$$

↳ constant, so r constant

Standard optimality condition

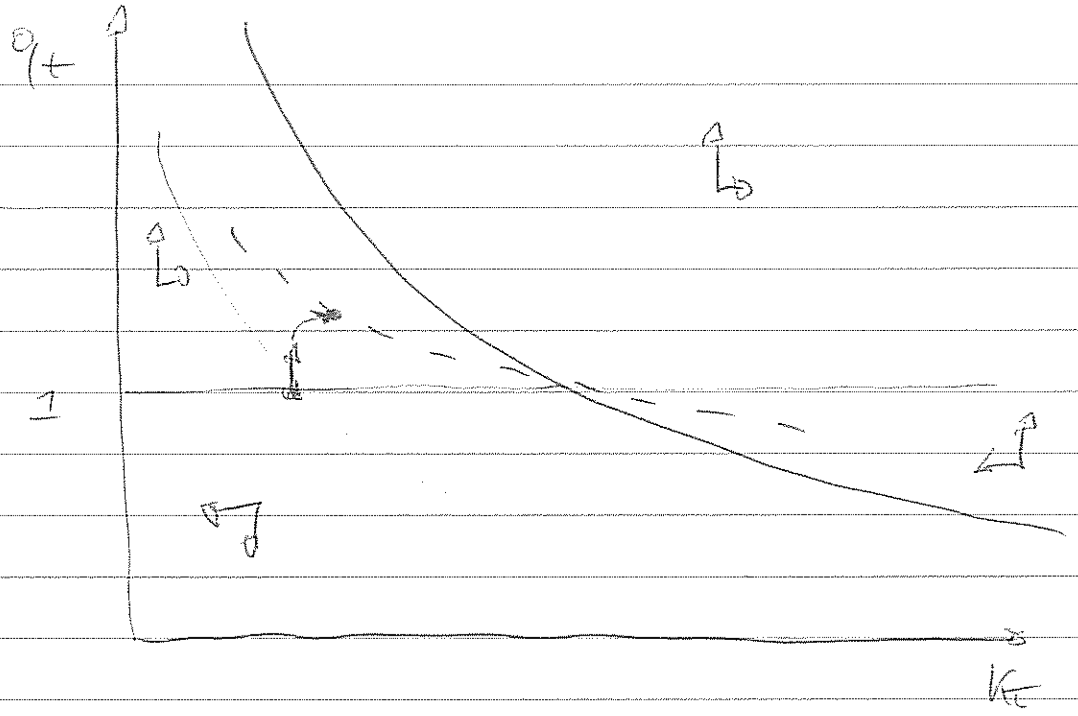
d) $A \uparrow \Rightarrow$ given q, need a k s.t. $F'(\,)\downarrow$, so

$k\uparrow \Rightarrow$ curve shifts to the right



q_t jumps, then follow saddle path

e)



f) No, since adjustment cost is not linear homogeneous (See O.R p.113)