The Multiverse: a Very Short Introduction

Chapter 4: All the worlds encoded in the quantum state of the cosmos

This Chapter expounds the multiverse proposed by the Everettian interpretation of quantum theory. The first half is largely independent of previous Chapters' philosophical discussions. But philosophical themes will emerge as the Chapter unfolds.

The Chapter proceeds in four stages. First, I introduce quantum theory (Chapters 4.1 to 4.3). I build on the last Chapter's discussion of state-space (Chapter 3.3), so as to emphasize how strange the conception of quantum state is. This leads, in the second stage, to the measurement problem, symbolized by Schroedinger's cat (Chapters 4.4 and 4.5). This problem has no agreed solution. But I will, in the third stage, (Chapters 4.6 to 4.10) develop just one approach: the Everettian interpretation, with its multiverse. In this approach, a physical process called <u>decoherence</u> will be crucial.

These first three stages will all emphasize what one might call 'synchronic issues': issues about the quantum state at a single time. The topic of time, or diachronic issues, will enter only at the last stage (Chapters 4.11, 4.12), which focus on how the Everettian treats probability. There, I will press one philosophical question that this multiverse raises---what exactly is objective probability?

<u>Chapter 4.1: What is matter? From lumps in the void to fields</u>

So far, our rapid review of physics has consisted of: (i) the rise of mechanics, especially Newton's theory of gravity (Chapter 2.3 to 2.6), and the idea of a <u>state-space</u> (Chapter 3.3).
To understand the Everettian multiverse, we need to understand how in quantum physics, the notion of state is very different from the notion in classical physics.

To prepare for that, the clearest and most vivid route is to review how our conception of matter developed historically, from ancient times to the end of the nineteenth century.

In this development, the main theme is that the idea of matter as a lump of stuff surrounded by void (vacuum) gave way to the idea of a <u>field</u> that pervaded all of space. Here, I hasten to explain that 'field' has nothing to do with corn, or any cereals, or the countryside. A field is, rather, there being a physical quantity associated with each place in space; and a state of the field is therefore an assignment of a value to the quantity at each place. An elementary example is the temperature of the air throughout a room: hotter here, cooler there. Strictly speaking, temperature does not make sense at an extensionless point of space: it is an average property of the air in a small volume, say a cubic millimetre, around the point. But let us idealize, and speak of a temperature at every point of space in the room. Then an assignment of temperature values to all points is a state of the temperature field.

So let us begin with lumps in the void. We discussed this conception of matter in Chapter 2.2 and 2.3. We saw that it was advocated not just by ancient atomists like Democritus and Lucretius, but by many seventeenth-century mechanical philosophers, including Newton himself. In particular, we discussed how non-obvious, indeed unclear, it is: as regards both how it might explain the very varied phenomena we see around us, and how such lumps might interact (whether by contact-action, or by action-at-a-distance).

Then in Chapter 3.3, we noted how complicated a collision between two such lumps really is. This led to the idea of the <u>point-particle</u>: mass concentrated at an extensionless spatial point. So on this conception, ordinary objects are clouds, more or less dense, of such point-particles. This idea was introduced as an idealization by Euler (1707-1783), and then advocated as physically real, i.e. the true nature of matter, by Boscovich (1711-1787).

Again, we should pause over how non-obvious, even problematic, the idea is. For any point-particle, its density, i.e. the ratio of mass to volume, is infinite. So if one advocates point-particles with different masses, one must accept different sizes of infinity, in order to describe their mass-densities. Besides: how do point-particles exert force on each other? And what happens if they ever collide? Boscovich himself---writing in an era when Newton's theory of gravity with its action-at-a-distance was accepted---suggested that at very short distances, a repulsive force, that is ever stronger at shorter distances, comes in to play and overcomes the particles' gravitational attraction: so that collisions never occur. But whatever you say about collisions, the question arises: can you give a good account of the contact and interaction of ordinary objects?

Difficult questions like these suggest a rival conception of matter, as continuous. On this view, there is no void anywhere, not even on the tiniest length-scales: matter fills space completely. As we mentioned in Chapter 2, Descartes endorsed this conception. He explicitly identified matter and extendedness; (while in his metaphysics, mind was essentially unextended). During the eighteenth century, this conception went on being developed. Indeed, the exact mathematical description of how continuous matter moves, and how one part of it exerts forces on, and responds to forces from, various other parts, is a very subtle affair. It requires a lot of advanced calculus, as well as physical insight. Unsurprisingly, there was, from the time of Euler onwards, a century-long struggle to achieve this description.

The result, in brief, is to describe matter as a field, in the above sense. For think of a continuous piece of matter. To begin, let us suppose for simplicity that the matter is utterly rigid. That is: the distance between any two of its material parts, no matter how tiny, remains constant over time. So think of a metal bar, and set aside your knowledge that it has layers of microscopic structure, i.e. crystals, atoms etc. Although it is rigid, properties such as mass-density and temperature may well vary across its expanse; and so these properties call for a field description. Besides, for continuous matter that is not rigid---that can be deformed (like an elastic solid: think of a pencil eraser) and-or compressed (like a liquid or gas)---the positions and velocities of its material parts are not "locked-in-step" together. So the parts' positions and velocities, as well as of their density and temperature, also call for a field description. No wonder that some advanced calculus is required.

In the nineteenth century, electricity and magnetism "went the way" of continuous matter such as fluids. That is: it turned out that, whatever the ultimate micro-structure of matter was (point-particles or continuous), electric and magnetic forces take time to propagate across space, between, say, positively and negatively charged matter. As discussed in Chapter 2.3, this is unlike the gravitational force, as it had been described by Newton. According to him, gravitational force propagates instantaneously.

Besides, these propagating electric and magnetic forces call for a field description. That is: one needs to attribute to each point of physical space, a vector, i.e. a line-segment in physical space (given by three real numbers relative to a coordinate system at the point) which is the (value of the) electric field at that point. This vector represents the electric force that would accelerate a stationary point-like electric charge, if the charge were at that point. And similarly for the magnetic field: though with the difference that it represents the force felt by a moving electric charge. (Here, the phrases 'if it were at', and 'would accelerate' signal a counterfactual conditional. This is another example of science being up to its neck in modality: cf. Chapter 3.3, 3.7.)

Besides, this field description of the electric (or the magnetic) field is not "just" a very convenient way of stating an infinite conjunction of counterfactual conditionals: namely, as a mathematical function from spatial points to vectors located there. For Maxwell (1831-1879), in his stupendously successful theory of the electric and magnetic fields, showed that there is much more to these fields that their describing how a charge would accelerate.

His theory unified electricity and magnetism as two aspects of a single field: the electromagnetic field. It also showed light (and later: radio waves etc.) to be waves in this field. That is: light is an oscillating pattern of electric and magnetic vectors at points of apparently empty space. It is a pattern that propagates, like a wave-form on the surface of the ocean. But it propagates at the speed of light.

Furthermore, this field has energy and momentum: quantities previously attributed only to matter, i.e. to stuff that had mass. That is: the field can convey energy and momentum from one place to another. Thus when you listen to the radio, your aerial is energetically excited by the arriving pattern in the electromagnetic field; and the pattern of excitation is then decoded and amplified into sound.

To sum up: by the end of the nineteenth century, classical physics had a broadly dualist ontology of matter and field. The picture was that matter with mass (and with energy and momentum) is localized in space. It was unknown, and controversial, whether it consisted ultimately of point particles or of continuous, space-filling, matter. But in the space between localized pieces of matter, there was: not just Newtonian gravity, with its action-at-a-distance; but also an all-pervading electromagnetic field that is the medium by which electromagnetic interactions between bits of charged matter occur, and that also itself possesses energy and momentum.

In the twentieth century, this dualism was overcome---with all-pervading fields getting the upper hand. This happened in various ways. But we need only state two.

First: Einstein's relativity theory (from 1905) identified mass and energy; so that one speaks of 'mass-energy'. So the quantity, mass, that had from Newton onwards been attributed only to matter, was now seen as also an attribute of the electromagnetic field.

Second and more important for us: from the mid-1920s, quantum theory replaced classical physics' matter---even a single point-particle, not only extended matter---by a field. But it is a very strange field. And it replaces the classical electromagnetic field by another field that is also strange, in a way exactly parallel to the strange fields for matter.

This strangeness is the source of all the problems about interpreting quantum theory, and it will dominate this Chapter.

Chapter 4.2: The quantum state: probabilities for classical alternatives

The clearest way to grasp this strangeness is to go back to the idea of that a theory attributes to the physical systems it describes, instantaneous states.

We saw in Chapter 3.3, that in classical physics, specifically Newtonian mechanics, the state of a point-particle is given by an ordered set of six real numbers, a 6-tuple: three numbers for its position in space, and three for its momentum. This meant that the state-space of a point-particle, that can be anywhere and have any momentum, is the set of all 6-tuples of real numbers. This is a six-dimensional space: where we use the word 'space' because, although this is not physical space, we can use geometrical ideas in describing it. And we saw that for more complicated systems, the state-space rapidly becomes more complicated and intricately structured. Even if we set aside all the momenta, and consider only the positions of the component parts---which is called the configuration of the system---the space of configurations (called 'configuration-space') rapidly becomes complicated.

Now that we have the idea of a classical field, we can also say talk about an instantaneous state of such a field. Think for example of the electric field throughout 3-dimensional physical space. Its state is of course the assignment of an electric field vector at each spatial point. Such a state is also called a field-configuration. So this requires infinitely many real numbers to specify it: because, for each of infinitely many points in physical space, we must specify three real numbers. We say the field's state-space, i.e. its set of instantaneous states or configurations, is an infinite-dimensional space. (Again, we say 'space' and 'dimensional' because we can again use geometrical ideas: much of the intuition, and precise results, about finite-dimensional spaces carries over to infinite-dimensional spaces.)

Now we can state how quantum theory is strange. It lies in a striking contrast between states in classical physics and states in quantum physics. This contrast applies equally to a point-particle, and to a finite set of them---any such set would have a finite-dimensional classical state-space---and even to a field (which has an infinite-dimensional classical state-space).

In short, the contrast is this. A classical state is an assignment of specific values to appropriate quantities. For our purposes here, we can neglect ideas about momentum, and focus only on position and similar quantities, i.e. on configurations. So a classical state is an assignment of specific values: either to the positions of a material object's component parts, or to field-quantities at all the points of physical space. But a quantum state is an assignment of a "square root of a probability" to every possible configuration of the corresponding classical system.

So a quantum state is a function in the mathematical sense. Its inputs (as we discussed: also called 'arguments') are the classical configurations, and its outputs are "square roots of probabilities". Here, what matters most---and what is most revolutionary about quantum theory---is, not the curious "square root of probability" outputs (which I will discuss shortly), but: the fact that a single quantum state mentions, i.e. takes as its domain of inputs, all possible classical configurations.

This fact will be the origin of both the measurement problem, and of the Everettian proposal about how to solve it.

Even for a point-particle, the proposal is hard to get one's mind around. For classical physics posited a point-particle. Its possible configurations were its possible spatial positions. Agreed, the theory is involved in modality (as discussed in Chapter 3.3). But only one configuration is actual: where the point-particle happens to be. Now quantum physics tells us: there are no such point-particles, each with a single actual position. Each such is replaced by what gets called a 'quantum particle'. But this entity hardly deserves the name 'particle'. For it has no single position. Indeed, it seems thoroughly smeared out in space. For the actual state of this entity, at some time, is an assignment to each point of space---i.e. to each possible configuration of yesterday's classical point-particle---of a number, which (once squared) gives a probability. In short, the state of this so-called quantum particle is a field.

But it is not a field like Newton's gravity, or Maxwell's electric field. For the so-called particle is not, as I put it, 'thoroughly smeared out in space', in the sense of being a cloud of mass or of electric charge. It is a field of probabilities (or rather, of their square roots). This field, this function on classical configurations that assigns to each configuration a square root of a probability, is called a wave-function. It is almost always written as the Greek letter **Psi**.

Besides, where classical physics posited two point-particles, and so configurations that are 6-tuples, and so a six-dimensional configuration space: quantum theory says the state is a wave-function on this six-dimensional space. So the 'smearing' of what is (undeservedly) called the 'quantum two-particle system' is a smearing, not in physical space, but in the abstract space of 6-tuples. And so on, for the quantum replacements of more complicated classical systems. That is: the quantum state, the wave-function, has as its domain of inputs (its arguments) the more complicated classical configuration space.

So far, I have summarized the mathematical idea of the quantum state as a function on classical configurations. But the picture gets yet stranger, when we ask what is the physical meaning of this function. Again: stranger, even for a point-particle---or rather for what replaces the classical point-particle and is honorifically labelled 'quantum particle'. For one asks: probability of what? And the answer is a mouthful, that refers to the outcome of a possible measurement, if you were to undertake one, on the system.

For the answer is, for a quantum particle: for each place (i.e point) x in space, the value of **Psi** at the argument x gives the probability, were you to measure the quantity position on the system, that you would get the outcome 'It is at x'. Equivalently, we can think of measuring a quantity with just two values 'Yes' and 'No' (or if you prefer: '1' and '0') that is defined in terms of the place x. In effect, to measure this quantity is to ask the system the question 'Are you at x?'. Thus the value of **Psi** at the argument x gives the probability of getting the answer 'Yes' to this question.

The reason why I call this answer 'strange' is that it means the basic interpretation of the theory's most central mathematical notion, its very concept of state, is in terms of measurement. For think what this implies. Suppose I ask the quantum theorist what their theory of, say, an atom, written in their mathematical language, means in physical terms. I ask: what information about the atom is contained in this mathematical notion **Psi** that they ascribe to the atom? And their official reply is that **Psi** gives probabilities of measurement outcomes: measurements using an apparatus that (for an experiment on an atomic system) is typically more than a million million million times bigger than the system being measured.

One naturally asks: how can this interpretation of **Psi** possibly hold up? For it invokes systems, viz. measurement apparatuses, that are not only utterly different from the system we are concerned with, but also vastly larger---and vastly varied. Can such a grossly extrinsic conception of state, for e.g. an atom, really be true?

To this, the short answer is that until now, more than eighty years after this conception of state was formulated, it is indeed still unrefuted. It is unrefuted for the simple but all-important reason that calculating with it, with due care, delivers the right answers to countless experiments---right answers that underpin countless modern technologies. But on the other hand: not only does every newcomer, every student of quantum theory, find this conception of state very hard to believe---indeed, bewildering. Also, most physicists and philosophers who consider in detail this conception, and the questions it raises, conclude that it is not satisfactory.

More precisely: either they conclude that though unsatisfactory, this conception is the best we can now do, and we must hope that the future will bring insight, maybe even a whole theory replacing quantum theory; or they conclude that we already have some special account of the mathematics of quantum theory, and-or how we apply this mathematics to the empirical world, that vindicates this conception. But there are many such special accounts, which get called 'interpretations of quantum theory'. There are about half a dozen main ones, each with many distinctive varieties. The debate between them still rages, decades after quantum theory was formulated---and one such interpretation is the Everettian interpretation, with its multiverse.

But before discussing that, there is more to say about this strange conception of state.

Chapter 4.3: Amplitudes and quantum fields

We can sum up the exposition so far, in terms of how quantum theory replaces the classical physical description of two or more particles. For two particles, the quantum state is an assignment, to each pair of points of physical space, of a number which (once squared) gives the probability, were you to measure the two position quantities, of getting the answer "Yes" to the two specific questions, "Is one of the particles here?", for the two points. And similarly for how quantum physics replaces classical physics' description of N particles. The state is an assignment to each N-tuple of points of physical space, $<x_1,y_1,z_1,x_2,y_2,z_2,\ldots,x_N,y_N,z_N>$ (i.e. each sequence of 3N real numbers) of a square root of a probability.

There are two further comments to make.

(1): The first is about the wave-function's outputs, i.e. the values of the function: which I called "curious square roots of probabilities". The explanation is that there is a kind of number which this book has so far not mentioned, called a complex number. In effect, a complex number encodes a pair of real numbers in ways that are fruitful. In particular: taking the square of a complex number delivers a third real number; (in almost all cases, different from both the given real numbers). So the values of the wave-function are complex numbers. They are called amplitudes (also: probability amplitudes).

Using complex numbers is fruitful for quantum theory because it underpins the treatment of quantities apart from position. Recall how our interpretation of the wave-function, above, was in terms of probabilities for outcomes of measurements of position. I said nothing about other quantities such as momentum. But it is natural to expect quantum theory's conception of state to say something about them. Indeed it does, by encoding the extra information in its use of complex numbers, rather than real numbers.

Amazingly, the system's wave-function gives, for any quantity (momentum, energy, what-not), the probabilities of the various possible outcomes of measuring that quantity on the system. So again, the conception of state is bewildering. For it invokes a gross and extrinsic apparatus. But I should also note that the mathematics of how the wave-function encodes all the probabilities for all the quantities is unified and very elegant. Calculating probabilities for various different quantities turns out to be a matter of expressing a vector (not in physical space, but in an abstract space) as a sum of vectors, in various different ways.

(2): Finally, let me return to the idea of fields. I introduced this idea for classical fields. Recall the classical description of a fluid, taken as being made, not of atoms jostling each other in a void, but as made---on all length-scales, no matter how minuscule---of extended stuff. Or recall the electric field, whose state is an electric vector at each point of physical space. But then we learnt that the quantum replacement of a classical point-particle is a field of (square roots of) probabilities, defined on configuration space. So one naturally asks: what about the quantum replacement of a classical field, such as the electric field?

Amazingly, the same strange idea of state works again; as follows. We saw that the configuration of a classical field is given by an infinite number of real numbers (not by 3N real numbers for some whole number N). Thus the quantum replacement of such a field has as its state an assignment, to each configuration of the classical field (that we used to imagine was physically real), of a complex number: a probability amplitude. This quantum replacement is called, of course, a quantum field; and the theory of them is called quantum field theory.

Note the dizzying mathematical abstraction. The configuration space of the classical field was itself infinite-dimensional; and now quantum theory posits a state space of functions with arguments in that infinite-dimensional space. This makes quantum field theory much more complicated, mathematically, than the theory of quantum particles, i.e. the theory of wave-functions on finite-dimensional classical configuration spaces.

Besides, quantum field theory gives a supremely successful replacement not just of what classical physics called fields, like the electric field, but also of what classical physics called particles. So it revises what I have said so far about quantum particles---though 'revises' means here 'extends' rather than 'overturns'.

Thus consider the electron. As we have seen: classical physics treats it as a point-particle, located in space at some actual position, moving with some momentum. And similarly for a pair, or any finite number N, of electrons. In short: once you fix the number N, a classical state-space is defined. And so far in this Section, we have learnt that quantum physics replaces this with something probabilistic. Namely: the state provides, for any quantity, a probability distribution over its possible values, where 'values' are understood to be outcomes of a possible measurement. But we can deduce all these distributions from a single representation of the state, the wave-function. And as in the classical case: once you fix the number N, the quantum state-space, consisting of wave-functions assigning amplitudes to each possible classical configuration of N point-particles, is defined.

Quantum field theory goes beyond this. Its state-space is "even bigger" than those we just mentioned. Thus we have mentioned: the space of wave-functions for one particle (i.e. complex-valued functions of position in physical space), the space of wave-functions for two particles (i.e. complex-valued functions of pairs of positions in physical space), … , the space of wave-functions for N particles (i.e. complex-valued functions of N-tuples of points of physical space). But for quantum field theory, the state-space contains all these infinitely many spaces of wave-functions. More precisely: there is a way of adding together two or three or … even infinitely many spaces (defined in terms of all the ways of adding together elements of the spaces). So for quantum field theory, the state-space is the sum of all the spaces just mentioned, for all positive whole numbers N. This vast sum state-space is called Fock space.

Thus quantum field theory envisages the number of particles as a property of the system, that can vary from one state to another. That is: it envisages the <u>number of particles as a quantity</u> of the system. This amounts to treating the system as a field; and to treating the quantum particle, discussed above, as <u>itself a state</u>: a state of this field.

So quantum field theory's conception of an electron is that it is an <u>excitation</u> (an "agitation" with an associated energy) of the field. Similarly, two particles are a pair of such excitations; and so on. Thus the treatment of, say, five electrons that a quantum theorist first learns (using wave-functions on 5-tuples of points of physical space) turns out to be just the five-particle part of a treatment that encompasses any number of electrons. (In the jargon: the elementary quantum state-space for five electrons is a <u>subspace</u> of Fock space.)

Again, the degree of abstraction is dizzying. But so is the empirical success. The quantum field theoretic treatment, both of what classical physics called 'fields', e.g. the electric and magnetic fields, and of what classical physics called 'particles', e.g. electrons, yields vastly many precise predictions that have been confirmed. So much so, that quantum field theory is now regarded as the <u>lingua franca</u> of physics.

And yet . . . questions remain. For the official interpretation of the state of a quantum field is exactly parallel to what I reported, and questioned, in the last Section. Thus for the electric field, the state (i.e. the function on configurations of the classical electric field) encodes probabilities for the various possible values of any quantity, such as energy or momentum of the field, that you might decide to measure on it. So our interpretative worries at the end of the last Section---can such a grossly extrinsic conception of state really be true?---persist.

<u>Chapter 4.4: The measurement problem: Schroedinger's cat</u>

The worries raised in the last Section about how to interpret the quantum state can be sharpened into an argument, whose conclusion is that quantum theory makes a wealth of flagrantly false predictions about the macroscopic world around us. This argument is called <u>the measurement problem</u> (also: 'the reality problem'). It is vividly illustrated---indeed, symbolized---by <u>Schroedinger's cat</u>: which is a thought-experiment presented by Schroedinger in 1935. So in this Section, I will expound the measurement problem, and then the cat.

So far, we have seen that a quantum state prescribes, for any quantity, a probability distribution over the possible outcomes of measuring that quantity (on a system in the given state). For example, a quantity might have four possible values, whose probabilities are 1/4, 1/3,1/3, 1/12: (these add up to 1). Of course: a probability equal to 1 for one value, with probability 0 for any other value, counts as a legitimate probability distribution. It is called a <u>trivial distribution</u> (though 'dogmatic' would be a better name). And for a given quantum state, there are quantities that get such a distribution: one outcome is ascribed probability 1, all the others probability 0. Quantum theory has jargon for this. We say the state is an <u>eigenstate</u> of the quantity, and that the outcome that is "favoured", i.e. gets probability 1, is the <u>eigenvalue</u>.

As to the more general situation, viz. each outcome getting a probability less than 1 (maybe some get 0, but all together, they add up to 1): it turns out that quantum states can be added in a way analogous to adding numbers. (And even more analogous to adding vectors: think of adding directed line segments, nose-to-tail, in the plane, or in three-dimensional physical space.) If you add two eigenstates for a quantity, ascribing probability 1 to two different eigenvalues, the result is a state that ascribes probability 1/2 to each eigenvalue. This state is called a <u>superposition</u> of the two given eigenstates. We write a '+' sign for addition of states.

This example was one of equal weighting. (One could also call it '50-50 weighting'.) But we can also increase one weight and decrease the other, like lengthening one directed line-segment and shrinking the other, before we add them, nose-to-tail. The resulting state (which is again called a 'superposition', and written with a '+' sign) will give the two eigenvalues different probabilities: the first one greater than 1/2 (say 6/10), the other less (say 4/10).

Note two further points. (1): For any given quantity, almost all states are superpositions: its eigenstates are a small, special set of states.

(2) Just as classical theories have equations of motion that describe how the system's state changes over time (cf. Chapters 2.6, 3.3, 3.8, 4.1), so does quantum theory. Its equation is called the Schroedinger equation; (he published it in 1926). It has two crucial features.

First: it is deterministic, in the sense of our previous discussions (Chapter 3.3, 3.8). That is: given the system's quantum state at a time, and the forces exerted on it in past and future, the equation prescribes what the state is at all other times. As we put it there: the equation prescribes a unique curve through the quantum state-space, i.e. the space of wave-functions. Later (in Sections 5 and especially 11), we will confront the obvious question: how can this determinism be reconciled with quantum theory's use of probabilities?

But here in this Section, what matters is the second crucial feature of the Schroedinger equation. Namely: it preserves the addition structure of quantum states. That is: if a system's quantum state, **Psi**, would evolve (i.e. change), in say five seconds, to a state **Psi'**, and another of its states, **Phi**, would evolve in that same five seconds to a state **Phi'**, then any superposition, for example ((6/10)**Psi** + (4/10)**Phi**) would evolve to ((6/10)**Psi'** + (4/10)**Phi'**). The addition structure, and the numbers that are the weights, 6/10 etc., are preserved under time-evolution. This property of the Schroedinger equation is called linearity: the Schroedinger equation is linear. (Here, for simplicity, I have written familiar real numbers, 6/10 and 4/10, for the weights, rather than the complex amplitudes, i.e. 'curious square roots of probabilities', mentioned in (1) of Section 3. The point about linearity is unaffected.)

So far, this Section has just done some stage-setting: the addition of vectors, and the jargon of eigenstates, eigenvalues, superpositions and linearity. But now quantum theory, in its orthodox formulation, makes an interpretative claim. It is a very important claim, since it is restrictive---and it leads directly to the measurement problem.

Namely: quantum theory says that if for a given quantity, the state is a superposition for that quantity (so: not an eigenstate---it ascribes a non-trivial, "non-dogmatic", distribution over possible measurement outcomes), then the system has no value whatsoever of the quantity.

The paradigm case is the quantity position, for a quantum point-particle: as in Section 4.2. The state is a wave-function **Psi** that assigns a complex number to positions in space, whose squares give probabilities of outcomes of position measurements: in picturesque language, probabilities of getting the answer 'Yes' ('1') to asking the system 'Are you at position x?'.

Now imagine that the value of **Psi** is non-zero only in two separated spatial regions, which we call 'L' and 'R'. Here, 'L' and 'R' are mnemonics for 'Left' and 'Right'. For nothing here will depend on space being three-dimensional; so we may as well imagine it as one-dimensional---"life on a railway line". So if we draw a graph of the values of **Psi** (more precisely: their squares), it looks like two humps with a flat line between them. (Think of the road-sign for 'Bumps in the road ahead.')

So quantum theory says that **Psi** is a superposition of two states. One is an eigenstate for being in L; (and so: would with probability 1 be found in L, if measured). The other is an eigenstate for being in R; (and so, correspondingly: would with probability 1 be found in R, if measured). This superposition can be written with a '+' sign for addition of states. We could write: **Psi** = in L + in R.

It is also common to write states between a vertical line and an angle bracket; (a notation invented by Dirac (1902-1984), one of the great quantum physicists). So for the equal or '50-50' weighting of L and R, we write: |**Psi** > = |in L > + |in R > .

But, says quantum theory: a system in state **Psi** has literally no position at all. So the system exists but it has no location. It only has dispositions to be found in L, or in R, if we were to measure position.

Similarly for other quantities, such as momentum. A particle might be in a superposition of momentum eigenstates, for two different values (measurement-outcomes) of momentum, say

'1' and '2'. So for equal weighting of '1 and '2', we can write the state as: |momentum 1> + |momentum 2>. Again, quantum theory says that a system in this state has literally no momentum at all.

To sum up: since for any given quantity, almost all states are superpositions, quantum theory's denial that systems in superpositions have a value (for that quantity) makes the lack of values *endemic*. Obviously, this situation prompts the question: how can this lack of values be reconciled with the apparent fact that objects do have values for position, momentum and other quantities such as energy?

Besides: recall how classical physics gives supremely successful descriptions and explanations of the physical behaviour of macroscopic objects, by ascribing them definite values of position, momentum etc., subject to equations of motion (cf. Chapters 2.6, 3.3, 3.8, 4.1). Once we recall this, the question becomes more pointed: how can quantum theory's denial of values be reconciled with the supreme success of classical physics?

This question is now easily sharpened in to an argument: an argument that the lack of values contradicts countless facts that macroscopic objects have definite values. We only need to transmit the lack of values from the microscopic realm of electrons, atoms etc., for which quantum theory is indeed successful, to the macroscopic realm of tables, chairs etc., where classical physics with its definite values is successful. This is done by describing, within quantum theory, a measurement of a quantity on a microscopic system (say, an electron) that is in a superposition (for that quantity). In such a description, we can see how the lack of values is transmitted to the macroscopic realm---surely contradicting countless facts of definiteness.

So let us assume we have a measurement apparatus for measuring an electron's momentum that is *reliable* on each of the two eigenstates, |momentum 1> and |momentum 2>, in the following sense. Starting the apparatus in an appropriate 'ready' state, the state of the pair of systems changes over time, so that at the end of the measurement interaction, the pointer on the apparatus reads the corresponding eigenvalue of momentum. Thus we think of the pointer as being, at the end of the measurement, in front of the digit, '1' or '2', painted on a dial.

Let us use an arrow, ---->, to symbolize the change of state over the period of the measurement. And let us write the state of the pair of systems by simply juxtaposing their states on the paper. This state of the pair is, in the jargon of logic or philosophy, the conjunction of the two components' individual states. In physics jargon, it is called a *product state* (or 'the product of the individual states').

Then our assumption of reliability, that measuring either of the electron eigenstates yields a veridical reading at the end, can be written as:

|momentum 1> |ready> ----> |momentum 1> |reads '1'>

and

|momentum 2> |ready> ----> |momentum 2> |reads '2'> .

So far, so good. But now suppose the electron's initial state is a superposition: for example, of our two eigenstates, |momentum 1> and |momentum 2>. Then the composite system of electron and apparatus has the initial state: (|momentum 1> + |momentum 2>)|ready>. This can be written as: |momentum 1>|ready> + |momentum 2> |ready>. Then because the Schroedinger equation is linear, we must accept:

(|momentum 1> + |momentum 2>)|ready> ---->
|momentum 1> |reads '1'> + |momentum 2> |reads '2'> .

This is the punchline. For consider the state of the composite system after the measurement: i.e. the second line, or right-hand-side, of this formula. Consider what it says about the apparatus, in particular the position of its pointer. The main point to notice is that it is *not* an eigenstate of pointer-position, i.e. of the quantity position, for the pointer. (There is also another point to notice about this state. It is not a product state, i.e. a conjunction of states for the components. Such a state is called *entangled*; and the theory having such states is called *entanglement*. We will return to this in Section 4.7.)

So quantum theory denies that the pointer has any position. Like in the example above, $|\mathbf{Psi}> = |\text{in L}> + |\text{in R}>$: the pointer only has dispositions to be found in front of the numeral '1' on the dial, or in front of the numeral '2' on the dial---if we were to measure it.

But the pointer <u>not</u> having any position surely contradicts the fact that macroscopic apparatuses give definite readings. Besides, the argument is so simple---depending only on microscopic superpositions and the Schroedinger equation being linear---that it suggests more generally that orthodox quantum theory's denial of values in the microscopic realm will contradict countless facts of definiteness about the macroscopic realm.

So this is the measurement problem. As I said at the start of this Chapter, it has no agreed solution. So our first job, in the next Section, will be to consider some possible solutions: including the Everettian proposal, on which we will then focus.

But before that, it is worth summing up the measurement problem, with Schroedinger's own description of his eponymous cat. In his paper of 1935---which is still worth reading for many reasons, some of which we will touch on later---he writes (at the end of Section 5; 1935 (1980)):

One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, so small, that perhaps in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The first atomic decay would have poisoned it. The wave-function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed in to macroscopic indeterminacy.

<u>Chapter 4.5: Solving the problem: the usual suspects</u>

There are three main strategies for addressing the measurement problem. It will be clear that each is a broad church that includes many versions; and that jointly, they are exhaustive. So one has to endorse one of the three. But I will not try to formulate the strategies precisely, and will only give two or three examples of each strategy. Nor is this trio of strategies, and their versions. original to me. Many surveys of the measurement problem give a similar trio. Hence this Section's title: the usual suspects.

I especially like the survey by Bell, which I will follow (1986). It is a brilliant, and equation-free, introduction to quantum theory and its interpretation. Bell describes, for each of his three strategies, a pair of versions; and for each pair, he gives what he wittily calls a <u>romantic</u> version, and an <u>unromantic</u> version. Unsurprisingly, the Everett interpretation will be the romantic version within its pair.

Bell also makes it clear that for each pair, he prefers its unromantic version. That is perhaps disappointing. But as I discussed in Chapter 1.4, we each have an intellectual temperament which is hard to change, and about which we are obliged only to be self-aware. (John Bell (1928-1990) was a profound quantum physicist, as well as a gifted writer. He also originated what is now called 'Bell's theorem': it is about how correlations between quantum systems defy a very natural form of probabilistic explanation.)

The first strategy is to reject, somehow or other, the formulation of the problem. That is, one rejects the premises of the argument leading to the contradiction. The main idea must be to deny that the quantum state is "physically real", in any sense of that phrase that makes the final post-measurement state (at the end of the last Section) contradict the macroscopic pointer having a definite position.

Bell calls his two versions of this strategy: 'pragmatism' (the unromantic version) and 'complementarity' (the romantic version). Here, 'pragmatism' means---not the philosophical tradition launched more than a hundred years ago by American philosophers such as Peirce, James and Dewey---but 'being practical'. That is: using the theory to calculate probabilities of outcomes, without taking its words and concepts to describe any reality other than those outcomes. (Philosophers often call this 'instrumentalism': the theory is an instrument, a tool for predicting observable facts, but not a description of the world, especially not the world beyond observations.) Obviously, pragmatism, in this instrumentalist sense, shades in to simply not wishing to ponder whether the theory goes beyond predicting observable facts, rather than the firm view that it does not go beyond such predictions.

On the other hand, 'complementarity' is Bohr's word for his views that attempted to go beyond pragmatism and, by explicitly philosophical argumentation, to solve the measurement problem. His core idea was that since measurement outcomes must be stated using the concepts of classical physics, there is no contradiction with, in Schroedinger's phrase, 'macroscopic determinacy'. And this is so, even though: (1) the Schroedinger equation, with its linearity, is always correct; and (2) quantum systems have no values for quantities except the eigenvalue for those quantities for which they are in an eigenstate.

The other two strategies deny, respectively, these claims, (1) and (2). These were, of course, the premises of our formulation of the measurement problem. So one might say that the first strategy aims to dissolve, rather than solve, the problem; while these two strategies accept that the problem is genuine, and then propose to solve it. Bell himself clearly prefers these two strategies (in their unromantic versions) over the first. As he puts it in another paper: 'either the wave-function, as given by the Schroedinger equation, is not everything or it is not right' (1987, p. 201). Here, 'the wave-function is not everything' means that a system has values for quantities additional to the eigenvalues ascribed by orthodoxy, i.e. (2) is wrong. And 'the wave-function is not right' means that the system is being attributed the wrong state, because (1) is wrong. In our paradigm case of the pointer: we should attribute to it a state of definite position, not a superposition of position eigenstates, and to do so, we should revise the Schroedinger equation.

So suppose we deny (1), that the Schroedinger equation is always correct. Again, this strategy comes in various versions.

The simplest version says that at the end of a measurement process (like that at the end of the last Section), the troubling superposed final state is replaced by an eigenstate of pointer position, so that indeed, the pointer has a definite position. (Usually, advocates of this version also say that the measured system goes in to an eigenstate of the measured quantity: in our example, the electron goes in to a momentum eigenstate. But we can concentrate on the measurement apparatus and its pointer.) Besides, <u>which</u> eigenstate replaces the troubling superposed state is said to be a matter of sheer <u>chance</u>, with each eigenstate occurring with a probability equal to its weight (more precisely: the square of its complex amplitude weight) within the superposed state.

Again, there is jargon: this replacement of the superposition by the eigenstate is called '<u>the projection postulate</u>', or '<u>the collapse of the wave-function</u>'. (But this second phrase is also used as a vivid label for the measurement problem, not just for this approach to solving it.) And the prescribed probabilities (the squares of the complex amplitude weights) are called '<u>Born-rule probabilities</u>'. (This is in honour of Max Born (1882-1970), one of the half-dozen co-discoverers of quantum theory who realized this role of probabilities in a 1926 paper.)

This simplest version of denying (1) occurs in many textbooks of quantum theory. But evidently, it is vague and contentious. For when exactly is the state calculated from the Schroedinger equation to be 'replaced'? Or in other words: what is the exact definition of 'the end of the measurement process'? And since a measurement is, after all, a physical process, how is this suspension of the theory's equation of motion to be justified? Clearly, this version is close to what Bell called 'pragmatism': and we are back at our initial bewilderment that the notion of a system's state should invoke such an extrinsic idea as measurement.

Two other versions of this strategy, developed in response to these difficulties, are worth mentioning. The first is very speculative; the second is down-to-earth. So Bell calls them, respectively, romantic and unromantic; (and as I said, he prefers the latter).

The first says, in a slogan, that consciousness collapses the wave-function. The idea is that the Schroedinger equation only falters at the interface between mind and matter. Although the inanimate physical world may get in to states superposing macroscopically distinguishable alternatives (e.g. of positions of a pointer), once a conscious being "looks", the state changes to a macroscopically definite state.

Obviously, people will differ about how plausible they find this proposal. Someone who is a physicalist (cf. Chapter 3.8) will almost certainly reject it. Yet it might at first sight appeal to the practically-minded physicist, on the grounds that it makes the measurement problem "someone else's problem". They might say: 'surely, physics has no responsibility to describe the relation between mind and matter'. But I would say that this response is just a verdict about disciplinary boundaries, or the division of cognitive labour: about who needs to worry. Whichever discipline one takes the problem to fall within, the proposal is obviously hard to make precise, hard to gather evidence for, and indeed: hard to believe. In particular, why should these collapses of the wave-function due to consciousness respect the Born-rule probabilities?

The other version of this strategy is much more down-to-earth: as Bell says, unromantic. It seeks, as quantum theory's fundamental equation of motion, a "cousin" of the Schroedinger equation. This equation is to be chosen so as: (i) to agree with the Schroedinger equation for microscopic systems like atoms, so that it also gets the vast amount of confirmation that the Schroedinger equation has gathered over the last ninety years; and yet (ii) to disagree with the Schroedinger equation for macroscopic systems like pointers, and cats. So according to this version, the wave-function of a quantum system does indeed collapse i.e. transit to an appropriate eigenstate, in suitable---in particular suitably large---systems. And this occurs in the inanimate world, wholly irrespective of consciousness: the collapse of the wave-function is an indeterministic physical process. So our task is to find the equation that describes these collapses precisely, in a way that meshes with the established successes of the Schroedinger equation. This will include recovering the Born-rule probabilities for experimental outcomes. In short: our task is "theoretical physics as usual: find the right equations".

In the last forty years, a great deal has been learnt about such cousin equations, both mathematically and physically. But no single proposed equation has yet won the allegiance of physicists, on either theoretical grounds or by being confirmed by experiment. So the question whether the Schroedinger equation will indeed be overturned remains open. For this, we must wait upon the future of physics.

Finally, suppose we deny (2). That is: we say that quantum systems <u>do</u> have values for quantities for which their state is not an eigenstate. The motivation for saying this is of course to keep the post-measurement state given at the end of the last Section, but nevertheless to ascribe a definite position to the pointer.

One version of this strategy is, again, "theoretical physics as usual". It was invented by de Broglie (1892-1987). In the mid-1920s, he was another of the half-dozen creators of quantum theory; and his formulation of the theory explicitly attributes values to quantities additional to eigenvalues (of quantities for which the state is an eigenstate). To be precise, let us consider a system that orthodox quantum theory (the textbook) calls '<u>N</u> quantum particles'. De Broglie proposed that in addition to the wave-function **Psi** on configuration space, that always obeys the Schroedinger equation, there are also---our "old friends": <u>N</u> point-particles.

In themselves, these point-particles are as described by classical physics: at any instant, each has a definite position in space, and over time each moves in a continuous trajectory. The difference from classical physics lies in how they move. Namely: at each instant, their velocity is determined by a combination of: (i) the wave-function, and (ii) the positions of the other point-particles in the system (which contribute in an action-at-a-distance manner, somewhat similar to Newtonian gravity). The idea in (i), that the wave-function, though it lacks mass and energy, "guides" all the particles in the system, has given rise to the theory's name: <u>pilot-wave theory</u>. (Think of how a pilot guides a ship, but not by the effects of his mass or energy.)

We do not need the details of this theory. What matters for us is that the pilot-wave description of how the point-particles move gives the key idea for solving the measurement problem. (This merit became clearer in the work of David Bohm (1917-1992), who in 1952 re-discovered the pilot-wave theory.) In the last Section's measurement scenario, the pilot-wave theory takes the pointer to really be a cloud of point-particles. Then it gives a detailed description of how, in each individual measurement, this cloud of particles is guided by the wave function (which always obeys the Schroedinger equation), so as to be either in front of the digit '1' painted on the dial, or in front of the digit '2'. In short: the pilot-wave theory, by ascribing values of position additional to those ascribed by orthodox quantum theory, secures a definite outcome for each individual measurement---solving the measurement problem.

The second version of this strategy, i.e. the second way to claim that quantum systems <u>do</u> have values for quantities for which their state is not an eigenstate, while the Schroedinger equation is always correct, is the Everettian proposal---with its multiverse. It will be our topic for the rest of this Chapter.

<u>Chapter 4.6: Everett's proposal: a bluff?</u>

Let me start by stating baldly the Everettian proposal. This will show how it counts as a version of this last strategy. (From now on, I will talk about 'the Everettian', rather than Everett himself, since the proposal has developed a good deal since Everett's paper in 1957; besides, there is controversy about whether the version of the proposal that is nowadays dominant---on which I will concentrate---matches Everett's own ideas.)

The key ideas are as follows. The cosmos as a whole has a quantum state, which always evolves according to the Schroedinger equation: indeed, a <u>very</u> grand version of the equation that describes quantitatively how all the cosmos' component parts interact, exerting forces on one another. Needless to say, no one has come close to writing down this version of the Schroedinger equation. But the Everettian proposes that it is, as usual, deterministic; so that the wave-function of the cosmos never collapses. (In Section 11, we will confront the obvious question: how can this determinism be reconciled with quantum theory's use of probabilities?)

This state is usually written as <u>**Psi**</u>: where use of the capital letter is, so to speak, honorific, since no one has the faintest idea how to write it down in detail.

<u>**Psi**</u> is usually called 'the universal state', or 'the universal wave-function': hence the title of this Chapter. But in this book, I have adopted, since the Introduction: 'multiverse' as the name of all of reality in the most inclusive sense; and 'world' or 'universe' as the name for its "more familiar" parts, where as we discussed, each part is understood to mean 'throughout all of time and space'. So using my jargon: the cosmos' quantum state <u>**Psi**</u> is the multiverse's quantum state. But since almost no one says 'the multiversal state' or 'the multiversal wave-function', I will in this Chapter talk of 'the cosmos' quantum state'.

Our knowledge of quantum theory, and our empirical success in applying it to small systems, suggests that superpositions will promulgate, in the way we saw in the measurement problem at the end of the Section 4. So we have good reason to think that the cosmos' quantum state is a vast superposition, i.e. a vast sum, of product states. Each of these is a long "conjunctive state" for countless component systems---not just an electron and a measurement apparatus or its pointer, but countless electrons, quarks, molecules, specks of dust etc.

Again, there is jargon: when items, like numbers or vectors or these product states, get added together, they are called <u>terms</u>, or <u>summands</u> (for 'item that gets summed'). So these product states are terms of the vast superposition.

The Everettian proposes a literal interpretation of this vast superposition. Just as this state contains vastly many product states, so also the cosmos (i.e. the multiverse) contains a plethora of Everettian 'worlds'. (These are also called 'branches'.) Each is represented by such a product state. Some (perhaps many) of these worlds are something like the macroscopic realm familiar to us: with all macroscopic objects (pointers, tables etc.) in definite positions.

But the worlds differ among themselves about these positions; and it is relative to each such world that there are extra values, i.e. values additional to orthodox quantum theory's attributing only eigenvalues. Think, for example, of Section 4.4's toy-model of a momentum measurement. The two possible outcomes were distinguished by two different positions of the pointer: in front of '1', or in front of '2'. Everett proposes that the two outcomes, the two positions, are in different worlds.

This bald statement of the Everettian proposal prompts the obvious question: 'If this is so, how come I have no evidence of the other worlds? In particular, how come I experience a single definite macrorealm?'.

As I see matters, there are two main Everettian answers to this question: a traditional one, which dominated discussion till about 1990; and a modern one which has dominated discussion since the mid 1990s. I think the first answer is unsatisfactory. It seems like a bluff, or a mere debating tactic; I will discuss it in this Section. But the second answer <u>is</u> satisfactory; although not wholly convincing. It is an appeal to an important phenomenon, <u>decoherence</u>, that I have so far not mentioned. I will take it up in the next Section.

So first, here is what I called the traditional answer. It says: according to the Everettian proposal, appearances are indeed definite---just as much as they are on a proposal that the wave-function collapses, as discussed in the last Section. For the objects involved in the problematic superposition <u>split</u> into many <u>copies</u>, corresponding to the various worlds. So in our toy model of measurement with two outcomes, the apparatus' pointer splits into copies, some with the '1' outcome, and some with the '2' outcome. (Authors differ about exactly how many copies. Some say that for two outcomes, there are just two copies; some say that for each outcome, there are many copies, perhaps infinitely many. I will return to this in later Sections.) This splitting, it is claimed, secures definite appearances. For appearances are only "seen" from within a world. And within a world, the wave-function is, by definition, the corresponding term—which <u>is</u> an eigenstate of the quantity, such as pointer position, that in order to solve the measurement problem, we want to have a definite value.

The trouble with this answer is not that the idea of splitting is plain wrong. It is that, stated so briefly, the answer is too programmatic: it raises more questions than it answers. If the splitting is a <u>bona fide</u> physical process, we need to hear details: for example, about how it can be consistent with laws like the conservation of mass or of energy. If it is somehow a conceptual splitting, without a physical description, then there is philosophical work to do, to explain what it involves. In particular, if the splitting is 'conceptual' in the sense of being a distinction made by a conscious mind, then presumably, there is no splitting in those regions of the cosmos without conscious minds. In that case, the proposal is similar to that considered in the last Section, that consciousness collapses the wave-function; and it is thereby similarly hard to make precise and to gather evidence for.

In my opinion, until about 1990 most of the Everettian literature did not adequately answer such questions. Hence my accusation that it seems like a bluff. Of course, I am not alone in my misgivings: many authors pressed such questions. In particular, Bell was very doubtful. In his 1986 paper, he says that the Everettian proposal 'is surely the most bizarre' and 'extravagant, and above all extravagantly vague, hypothesis. I could almost dismiss it as silly' (1986, pp. 192, 194). But as I announced above: since about 1990 (when Bell died), the Everettian literature has appealed to decoherence, which does address the questions.

That will be the topic of the next Section. But first, it is worth discussing---and criticising---an analogy that is sometimes suggested in defence of the suspiciously programmatic 'splitting' answer above.

The alleged analogy is that the 'splitting' answer is like what is surely the right reply to the objection (perhaps apocryphal) against the proposal by Galileo and other advocates of a heliocentric astronomy, i.e. the proposal that the Earth goes around the Sun. The idea of the objection (i.e. in defence of the traditional, Aristotelian geocentric astronomy, that the Sun goes around the Earth) is to appeal to appearances. Namely: it looks (especially at sunrise and sunset) as if the Sun goes around the Earth. So the objection is that the heliocentrists' radical proposal seems to conflict with the appearances.

The reply that Galileo and the heliocentrists are said to have made is that in fact, assuming that the Earth goes around the Sun leads to the <u>very same appearances</u>. That is: the <u>appearance</u> at sunrise is exactly the same whether you describe this as the sun rising above the horizon, or as the horizon sinking below the sun. (And similarly, of course, at sunset: the appearance is exactly the same whether you say the sun is sinking, or the horizon is rising.)

Similarly, it is alleged, for the Everettian's "splitting" answer. If we assume such a splitting, the appearances will be just as the objector says they are: that is, perfectly definite.

But I submit that this analogy has a merely rhetorical force. Agreed: the broad logic o f the two disputes is the same. In both cases, the radical proposal (Galileo's, or Everett's) replies to the objection that it conflicts with appearances, by saying: 'No, I do not: I accord perfectly well with appearances'. But there is a big difference between the two cases.

For Galileo and the other advocates of a heliocentric astronomy can readily spell out how they accord with appearances. It is a matter of optics, i.e. the paths of light rays. In particular, it is straightforward to argue that: (i) the appearance at sunrise is a matter of the angle between a ray of sunlight and one's line of sight to the horizon increasing (and similarly, at sunset: decreasing); and (ii) this increase (respectively: decrease) depends only on a <u>relative</u> motion of sun and horizon. But the Everettian's idea of splitting gives no such straightforward argument for recovering definite appearances: it leads only to the questions I pressed above.

<u>Chapter 4.7: Doing better with decoherence</u>

However, as I said: by appealing to a process called <u>decoherence</u>, the Everettians can make much better sense of their proposed splitting; and since about 1990, they have done so. So in this Section and the next two, I will spell out what decoherence is, and how it clarifies what splitting involves. It will also be clear that decoherence is important for all approaches to quantum theory, not just for Everett's: any of Section 5's interpretations need to accommodate it.

There will be three stages, one in each Section. This Section gives the basic idea of decoherence. In the next, decoherence helps make more precise the definition of an Everettian world (or 'branch'). After that, decoherence will suggest that macroscopic objects such as cats or pointers are---not aggregates of stuff, but---enduring and stable patterns. (Then in the Chapter's final Section, I will turn to the topic of probability.)

'Decoherence' means, in this context, the <u>diffusion of coherence</u>. Here, 'diffusion' means spreading: namely, spreading from the system of interest to its environment with which it is interacting.

'Coherence' is physics' jargon for some characteristic differences between (i) the probability distributions prescribed by quantum states, especially superpositions, and (ii) those prescribed by classical states. We saw already in Section 2 that quantum superpositions prescribe probability distributions over a measurement's positive outcomes; (while eigenstates are "opinionated"---they give probability 1 to just one outcome). Classical states---meaning states prescribed by classical physics---also prescribe distributions, once we include <u>probabilistic mixtures</u> of states. Thus imagine being given, say, three states, and taking a quarter-quarter-half mixture of them. This means, for example, taking a thousand systems: of which 250 are in the first state, 250 are in the second state, and 500 in the third. Then the predicted statistics for measuring any quantity on a randomly selected member of the set of a thousand systems would be the average, with weights ¼, ¼, ½, of what the three given states prescribe.

The important point here is that superpositions cannot be understood as such classical mixtures. For although for a single quantity, a superposition and a classical mixture might prescribe exactly the same distribution: there will be other quantities (which are said to <u>not commute</u> with the first one), about which their distributions differ. The numerical differences, outcome by outcome, between these distributions (for this other quantity) are called <u>interference terms</u>. In short: interference terms are the signature of a state being a superposition. (The word 'interference' comes from the physics of waves. When the peaks of two water waves meet and make yet higher peak, we say that the waves interfere constructively; when a peak of one wave meets a trough of the other, and they cancel out to give a level surface, we say that the waves interfere destructively.)

So imagine a quantum system interacting with its environment. Given our interest in the measurement problem, a paradigm case is the pointer of an apparatus (or a cat) interacting with the air around it, by air molecules bouncing off it. So quantum theory tells us, of course, that the quantum state of the composite system, pointer plus air, prescribes probabilities for the various possible outcomes of measuring any quantity on either component, the pointer or the air. And in general, this composite quantum state will be a superposition of product states, so that the probabilities it prescribes will include interference terms---even for a quantity on a component system. That is: one expects, <u>a priori</u>, that the component system's state will be superposed, i.e. its probabilities will have interference terms.

Now we can state the punch-line about decoherence. Namely: according to many realistic models of how a macroscopic objects like a pointer interacts with its environment like air molecules, the interaction establishes very rapidly a composite quantum state that <u>falsifies</u> this <u>a priori</u> expectation, as regards the macroscopic object. That is: very soon after the interaction starts, the composite quantum state prescribes probabilities for the macroscopic object whose interference terms are negligible. In other words: the composite quantum state determines a state for the pointer that is almost a mixture: it differs from a mixture only by tiny interference terms.

More is true. The states within this mixture, that the pointer is "almost in", are the states that we intuitively want, in order to solve the measurement problem. For they are, roughly speaking, states of definite position: for example, position of the centre-of-mass of the pointer. Thus in our toy-model of measurement: once we include the air molecules in our analysis, the post-measurement state of the pointer, as determined by the state of the whole composite system, is (apart from tiny interference terms) a mixture of the centre-of-mass being in front of the numeral '1', and it being in front of the numeral '2'.

And yet more is true. One does not just get such promising-looking mixtures in situations of explicit measurement, involving everyday-sized objects like pointers. Nowadays, there are detailed models of much smaller objects, immersed in their environment, that give mixtures of states that are definite in the quantities we want definite. For example: a tiny dust-particle, a tenth of a millimetre in diameter, in outer space will be in a mixture of states of definite position, thanks just to its interacting merely with the dim light of the stars. (I said 'roughly speaking, states of definite position', because in many models, the states in the mixture obtained are so-called <u>coherent states</u>. These are states whose probability distributions are sharply peaked for both position and momentum, so that a system in such a state seems almost definite in both position and momentum. I say 'sharply peaked' because the distributions have enough spread so as to obey quantum theory's Uncertainty Principle, which vetoes having absolutely precise values for both position and momentum.)

To sum up so far: decoherence is the fast and ubiquitous process whereby, for appropriate physical quantities on a system immersed in its environment, the interference terms that are characteristic of a quantum state (a 'superposition') as against classical states (a 'mixture'), become tiny. In effect, the coherence has diffused from the system to its environment.

We will see in the next two Sections how with decoherence the Everettian can give a much better account of the splitting of worlds. I end this Section by stressing two important features of decoherence, that apply not just to Everettians, but to all approaches to the measurement problem. The first is positive; the second is a limitation.

The positive feature is flexibility. For to solve the measurement problem, we need the classical physical description of the world to be vindicated by quantum theory—but only approximately. We need only some subset of quantities, not all quantities, to have definite values. And maybe that subset should only be specified contextually, even vaguely. And maybe the values should only be definite within some margins of error, even vague ones. Decoherence secures this sort of flexibility. For which quantity on the system is "preferred" (i.e. rapidly becomes definite, in that the state is a mixture of its eigenstates) is determined by the physical process of interaction---whose definition in the model can be legitimately varied in several ways. Here are three examples of such ways. One can vary the definitions of: the system-environment boundary; the time at which the interaction is taken to end; and what counts as the state being 'sharply peaked' for a quantity.

The limitation is that decoherence does <u>not</u> just by itself solve the measurement problem. More precisely: it does <u>not</u> imply that in any individual case, the system actually is in one of the states in the mixture. It implies only that the quantum probabilities for any quantity are <u>as if</u> the system were in one such state.

Furthermore, quantum theory implies that the system is in fact <u>not</u> in one of those states. This last is a subtle point, which many textbook discussions miss. Some authors signal it by calling the mixture that the system is in an <u>improper mixture</u>; while calling mixtures for which the system in any individual case is indeed in one of the states getting mixed: a <u>proper mixture</u>, or an <u>ignorance-interpretable mixture</u>. But whatever one's jargon, the point is that the system being in one of the states getting mixed would contradict the original hypothesis that the total system-plus-environment is in a superposition, not a mixture. And though the point is subtle, it is uncontroversial. In fact, it is already clear in Schroedinger's 1935 paper, in which he introduced the cat. For in Sections 10 to 13 of that paper, he discusses entanglement (indeed, he introduces this word), and expounds the point.

This limitation of decoherence can be made vivid in terms of Schroedinger's cat. Namely: at the end of the measurement and after the decoherence process, the quantum state still describes two cats, one alive and one dead. It is just that the two cats are correlated with very different microscopic states of the surrounding air molecules. For example: an air molecule will bounce off a wagging upright tail, and a stationary downward one, in different directions.

ADD HERE IF POSSIBLE, honouring **Roger Penrose** for this vivid and witty way of making the point, using overhead transparencies: six Pictures, as follows:---
1: cat alive, smiling and standing, with vertical legs and vertical tail;
2: cat dead, frowning and lying down, with horizontal legs and horizontal tail;
3: superposition of 1. with 2. i.e. cat alive combined with cat dead: as in the usual pictures of the measurement problem
… then someone advocates including the environment in one's analysis, leading to…
4: cat alive, smiling and standing etc: and with many air molecules bouncing off it with such-and-such trajectories;
5: cat dead, frowning and lying down etc: and with many air molecules bouncing off it with <u>different</u> 'so-and-so' trajectories;
… but including the environment in one's analysis does not solve the basic problem that there seem to be two cats, one alive and one dead: as shown by…

6: superposition of 4. with 5. i.e. cat alive, smiling, and air molecules with such-and-such trajectories, combined with cat dead, frowning, and air molecules with so-and-so trajectories.

Chapter 4.8: A sketch definition of 'world'

In this Section and the next, we will see how decoherence clarifies the Everettian's idea of worlds and thus of how they "split". In this Section,

At the beginning of Section 6, I introduced the Everettian proposal as saying that the cosmos as a whole has a quantum state **Psi** which is a vast superposition, i.e. a vast sum, of product states describing countless component systems---not just an electron and a measurement apparatus or its pointer, but countless electrons, quarks, molecules, specks of dust etc. So the Everettian now needs to be more precise about how to extract from this state a set of worlds: each of them (or maybe just: some of them) like our familiar macroscopic realm, with all tables etc. in definite positions.

In this endeavour, decoherence is a promising resource, not least because as we discussed near the end of the last Section: it is flexible. The idea will be that to get something like our familiar macroscopic realm, the Everettian will adopt a definition of 'macroscopic object', or for short 'macrosystem', and will then take the cosmos as the vast composite system of all the macrosystems, and the rest of cosmos. Accordingly, the quantum state-space of the cosmos is broken down, in the mathematics of the theory, into the state-spaces of its components: all those of the macrosystems, and the state-space of the rest of cosmos. (In the mathematics, the state-spaces combine rather like numbers being multiplied, rather than added. So the component state-spaces are called factor-spaces, and the state-space of the cosmos is a product space.) I shall assume there is some finite number N of macrosystems, and so exactly N+1 factor-spaces in all.

So far, so mathematical. But now decoherence prompts two distinctively physical suggestions.

First: the Everettian can legitimately define 'macrosystem' along the lines: 'any system whose interaction with its environment rapidly makes its state almost a mixture in quantities such as position (and momentum), i.e. the quantities whose values, as attributed by classical physics, gave such a successful description'. For we learnt in the last Section that nowadays physicists' models of decoherence are so many and varied that a definition along these lines will encompass, not just everyday objects like a table or a pointer of an apparatus, but also a tiny dust-particle floating in outer space---and countless objects in between, such as a lock of hair or a dew-drop. So in effect, every object that anyone has described, or could successfully describe, with classical physics will be among the Everettian's vast set of N macrosystems.

Second: I have so far considered only a single macrosystem interacting with its environment: as the saying goes, being decohered by (i.e. its state becoming a mixture, due to) its environment. But such decoherence interactions are happening continuously, to each of the N macrosystems. For there is a vast "common environment" of them all: of the table, the pointer, the lock of hair, the dew-drop, the tiny dust-particle in outer space. Where the air molecules---as a sink into which the interference terms can diffuse---give out, the dim light of the stars can take over.

Now, with a bit more stage-setting, I can state an Everettian definition of 'world'. There are two preliminary steps, and then the definition itself. (After stating it, I will comment on how it remains only a sketch definition.)

(1): We adopt a definition of 'macrosystem' along the lines above. Suppose that throughout the cosmos, there are N macrosystems, as thus defined. Then we factorize (break into its factors) the cosmos' state-space into N+1 factor spaces: one factor for each of the N macrosystems (dust-particles etc.) that gets decohered by its environment, and one factor for their "common environment"---the vastly complicated and dispersed "rest of the universe".

(2): Then we express the cosmos' quantum state, **Psi**, as a superposition whose components (i.e. summands) are (N+1)-fold product states, i.e. products of N+1 states, one in each factor-space. We choose these product states by: (i) for each of the first N factors, taking an eigenstate, on the corresponding macrosystem, for the quantity that gets "preferred" (i.e. selected) by the decoherence process; and (ii) for the last factor, i.e. the (N+1)-th factor associated to the rest of universe, taking what is called the relative-state of the rest of universe.

(We do not need to pause on the notion of relative-state, although it was a centre-piece of Everett's original paper (in 1957). Suffice it to say that, stated for a two-component system (rather than a N+1-component system), the relative-state is roughly: what the component system "looks like", assuming the other component system is in a given state. More precisely: it is the state prescribed after a projection-postulate measurement resulting in the given state on the other component.)

Finally, a world is defined as: the physical reality described by a summand, in this way of writing **Psi.**

In other words, again using some of the jargon we have introduced: (i) for all N macrosystems, we take a component of its "post-decoherence" improper mixture (i.e. an eigenstate of the quantity selected by decoherence; and then (ii) for the rest of universe, we take the relative state. We then take the product of (i) and (ii): which is an (N+1)-fold product state. Then the world is to have as values for quantities just what that product state orthodoxly ascribes, i.e. the corresponding eigenvalues.

It is clear that in a world, as I have just defined it, each macrosystem has, by construction, a definite value for the quantity on that macrosystem that was selected by the decoherence process. Thus the promise of decoherence for the Everettian lies in the fact that in many models of many sorts of macrosystem, the definite-valued quantity is calculated to be position or momentum, or something "close" to these---in short, the sort of quantity that, to solve the measurement problem, we want to have definite values.

So this definition makes 'world' precise in a way that meshes with the basic ideas of Everett's proposal. It helps the Everettian to answer Section 6's accusation of bluffing. But I stress that this definition of 'world' is very much a sketch definition. There are at least two broad ways in which one would like to see it improved. In listing these ways, I am not being controversial. For the physics of decoherence remains an active area of research, both theoretically and experimentally. Besides, it will be clear that not only the Everettian, but anyone interested in the interpretation of quantum theory, will want to see progress about these two ways.

First: this definition assumed a notion of macrosystem, which I initially said would be defined in terms of: 'getting a mixture in quantities … whose values, as attributed by classical physics, gave such a successful description'. Such an assumption seems suspiciously close to postulating what one wants, rather than arguing for it. I think this suspicion lessens once one sees the detail in the definition of 'world': the factorization of the cosmos' state-space, and the decoherence of each component system. But I agree that much more detail is needed. Not just more than I have given here; but more than the research literature has so far achieved. As I said: this is an active area of research.

In broad terms: one would like to see models of decoherence that are more rigorous and of wider scope (i.e. cover more systems) and that make definite the "right" quantities. In this endeavour, one can of course appeal to the flexibility of decoherence which I praised at the end of the last Section. Thus I said that one can vary the definitions of: the system-environment boundary; the time at which the interaction is taken to end; and what counts as the state being 'sharply peaked' for a quantity. So in seeking a better definition of 'macrosystem', in terms of which to make the factorization, one can hope to exploit this variety or "wiggle-room".

So to sum up this first issue: the Everettian hopes that the physics of decoherence will enable us to avoid taking 'macrosystem' as a primitive concept; but there is much work yet to do.

Second: there is an issue about the fact that our sketch definition of world appeals to a factorization of the cosmos' state-space, prior to and independent of what the state of the cosmos **Psi** actually is. This issue is independent of the first. For even if that was completely dealt with by a satisfactory rigorous definition of 'macrosystem' etc., it will surely still be true that what macrosystems there are, and how many there are, will be very much a matter of happenstance, a matter of contingency. Indeed, there are two points here.

The first is a matter of everyday belief, and regardless of quantum theory, Everettian or otherwise. It goes back to Chapter 3.3, about our being up to our necks in modality. Namely: there surely could have been different macroscopic objects than there in fact are; not just different tables, or locks of hair, or stars, but also more mundanely, different dust-particles. Secondly, even if one agrees with the Everettian in boldly postulating a quantum state of the cosmos **Psi**: still it is presumably a contingent fact what that state is. After all, **Psi** is an element of a vast state-space, with countless other elements; and nothing in the Everettian proposal, as so far stated, forbids the cosmos being instead in one of those other states.

Putting these points together, the obvious suggestion is that we should allow the factorization of the cosmos' state-space to depend on the state. By doing that, the second point---the happenstance about what the cosmos' state in fact is---could accommodate the first point, that we need to accept happenstance about what, and how many, macrosystems there are. At least, so it seems: we will return to this in Chapter 6. But nothing we say there will undermine the present conclusion: that we should let the Everettian's factorization of the cosmos' state-space to depend on the state---which our sketch definition does not do.

Chapter 4.9: On what there is: objects as patterns

'On what there is' is the title of a much-cited article about deciding what truly exists on the basis of logic. In this Section and the next, I will pursue this theme, as regards the Everettian multiverse. This Section will be positive, i.e. pro the Everettians. The next Section will raise doubts; (and it will lead in to the final Section).

This Section's main idea is in the title: 'objects as patterns'. Nowadays, Everettians can, and do, build on the previous Sections' account of decoherence, so as to justify their metaphor of the world splitting into many alternatives corresponding to, for example, the various outcomes of a measurement process. In this justification, their main new idea is that a macroscopic object is not a lump of stuff, or an aggregate of tiny lumps, or even a cloud of point-particles. (Recall Section 1 above, and Chapter 2's discussion of the seventeenth-century mechanical philosophers.) It is really a pattern in the quantum state: in the quantum state of the cosmos.

As we will see, this idea promises to overcome the limitation I explained at the end of Section 7 above: that decoherence does not by itself solve the measurement problem, since it does not imply that in any individual case, the system actually is in one of the states in the mixture obtained after the decoherence process. I made this limitation vivid in terms of Schroedinger's cat. Thus I complained that this mixture still describes two cats, one alive and one dead: it is just that the two cats are correlated with very different microscopic states of the surrounding air molecules. Now, the idea that macroscopic objects are patterns will vindicate the proposal that at the end of a Schroedinger-cat experiment, there are indeed two cats. For there are two patterns in the quantum state of the whole system (say: the cat, the apparatus, and air). Therefore, there are two cats. In other words, the Everettian claims: the final quantum state being a mixture describing two cats, one alive and one dead, is a matter of the state encoding two patterns---and that description is entirely right.

This claim is certainly dizzying. But it is, I think, completely coherent. (My misgivings in this Section's second stage, below, will not refute it, but at most make it less plausible.) It becomes clearer if we assume that the quantity selected by decoherence (i.e. having negligible interference terms in the final mixture) is a familiar one that, to solve the measurement problem, we intuitively want have a definite value. Let it be our old friend: position. That assumption is reasonable. For as hinted in Section 7: the final mixture typically contains coherent states, in which both position and momentum are very close to definite in value.

Of course, there is no single position quantity for the cat. For the cat has very many parts, both macroscopic like legs and tail, and tiny like individual cells and molecules. (Similarly of course for the pointer of a measurement apparatus: when we discussed its being in front of the numeral '1' on the dial, or in front of '2', we were exploiting the assumed rigidity of the pointer so as to get by with a description using a single position quantity, viz. the position of the centre of mass.) So we need to recall from Section 2 above (and Chapter 3.3's discussion of state-spaces in classical physics) that even the <u>classical</u> configurations of a composite system, with say <u>N</u> point-particles as its components, form a vast state-space. With each point-particle placed somewhere in three-dimensional physical space, the configuration space is the set of (3N)-tuples of real numbers. For one needs three real numbers for the spatial position of each point-particle: so in all, one needs 3<u>N</u> real numbers. The state of the corresponding quantum system is then a wave-function (a complex-valued function) on this classical configuration space. (As we saw in Section 2, this system is called '<u>N</u> quantum particles'. But as I lamented there: the word 'particle' is misleading. For it strongly suggests the system is localized: whereas in fact it only has tendencies (measured by probabilities) to be found somewhere, if measured by a position apparatus.)

So we need to ask: how many parts should we take a cat to have? In other words: what is a good guess for the number <u>N</u>, such that a successful quantum description of a cat can use a wave function on the set (3N)-tuples of real numbers? (Again, the factor 3 just encodes the fact that physical space is three-dimensional.) Let us for simplicity think of an atom as a single particle; (in the Pickwickian quantum sense that we take it to define a location in physical space, in our effort to estimate the number <u>N</u>). But chemistry teaches us that atoms are minuscule. For example, in twelve grams of carbon, the number of carbon atoms is 6 followed by twenty-three noughts. This is written: 6×10^{23}. (This is called 'Avogadro's number'.) So for a cat weighing say a thousand grams (1 kilogram: roughly 12 x 100), the number of atoms---most of which will weigh less than carbon---will be enormous. It will be about $6 \times 10^{23} \times 100$. Which is roughly: 1000×10^{23}. So we can take as a guess for <u>N</u>: 10^{26}. So even with our simplifying assumption to think of an atom as a single particle, the classical configuration space is stupendously large. It consists (3×10^{26})-tuples of real numbers. So it has dimension 3×10^{26}: a vast number—in which the '3' is hardly worth keeping track of. So the quantum state is a wave-function whose arguments (inputs) are elements of this space. Each argument is a (3×10^{26})-tuple, i.e. an exact <u>classical</u> configuration for all the (approximately) 10^{26} atoms.

I can now say how the Everettian argues that in the myriad complexity of such a wave function, there is a pattern that deserves to be called 'a living cat', and another pattern that deserves to be called 'a dead cat'. (I say 'argues', since it will be clear that there remains a lot of intellectual work to do.)

The idea is to focus on the fact that in a <u>classical</u> description, there is a set of configurations that would all count as a living cat. Indeed, there are vastly many configurations of 10^{26} classical point-particles that would count as constituting a living cat. We can put it in well-nigh cartoon form: think of being alive as having vertical legs, and a vertical tail, and a smile on the mouth. Similarly, we can think of being dead as having horizontal legs, and a horizontal tail, and a frown on the mouth. So there is a similarly vast set of configurations that would all count as a dead cat. Then the Everettian's point is these two sets, though both vast, do not overlap at all. No configuration of a classical cat with the point-particles composing its legs and tail aligned vertically is also a configuration in which the legs and tail are aligned horizontally.

And so---now returning to quantum theory---the quantum state at the end of Schroedinger's experiment is, as regards the cat, a wave function with two <u>peaks</u>. That is: there are two regions of the configuration space, i.e. the set of arguments of the wave-function, where the function's value, i.e. the output or amplitude, is non-negligible. (For countless other configurations, the amplitude is indeed negligible. Bear in mind that, a priori, these 10^{26} point-particles could be configured to be any classical object of the same total mass: e.g. a puddle, or a saucepan with risotto, or a small dachsund, or any of myriad nameless and often monstrous combinations that only a horror movie might devise, such as a half-cat-half-dachshund.)

Summing the amplitudes for all the configurations in each of these regions, we get a (square root of) a probability that is substantial. In the traditional version of the experiment, where there is about a 50% probability of an atomic decay causing the release of the poison, the probability for each of the two regions is about 50%: 50% for being alive, and 50% for being dead.

But whether the probabilities are 50-50, or nearly so, doesn't matter here. What matters is that there are two such patterns in the quantum state: two regions of the configuration space where the amplitude is vastly greater (as a ratio) than it is for points outside both regions. So if we accept that a cat is such a pattern, then there really are two cats.

Note that the essential idea here is independent of quantum theory's details; (as Everettians note). For the idea is closely analogous to one which we all unhesitatingly endorse for several other physical theories. Namely, theories in which states can be added together to give a sum-state, in which the component states do not influence each other, or only do so negligibly. (This is called being are dynamically isolated from each other.) Examples include the theory of water-waves, or electromagnetism.

For example: the water in Portsmouth harbour can get into a state which we describe as, e.g. a wave passing through the harbour's centre heading due West; or into a state which we describe as a wave passing through the centre heading due North; or into a state which is the sum of these. But do we face a 'Portsmouth water paradox'? Do we agonize about how the Portsmouth harbour water-system can in one place (viz. the harbour's centre) be simultaneously both Westward and Northward? Of course not! Rather, we say that waves are patterns in the water-system. (Agreed, such patterns can be called 'objects'; in the jargon of philosophy, they are often called higher-level objects.) And so we say that there are two waves, with the contrary properties, one Northward and one Westward. Similarly for the electromagnetic field in a certain region, and e.g. pulses of laser light travelling in different directions across it, as happens in a light-show at a rock concert. There is no 'laser light-show paradox'.

Analogously, says the Everettian, we should endorse this idea when it is applied to the end of Schroedinger's experiment. In short: the quantum state is a sum of two waves, and so we should accept that there are two cats. (And besides, the quantum state-space contains myriad other states, the vast majority of which do not represent macroscopic objects (patterns!) which we could recognize---as cats or puddles or saucepans or dachsunds or combinations of these.)

So to sum up: the Everettian claims to overcome the limitation at the end of Section 7 above. For with macroscopic objects as patterns in the quantum state, not lumps of stuff, we see that solving the measurement problem does not require that in any individual case, the system is actually in just one of the states in the mixture obtained after the decoherence process. The system is in none of them: but each pattern is one of these states---and that is what a macroscopic object such as a cat really is.

I said above that this is how the Everettian 'argues' for this claim, since there would remain a lot of intellectual work to do. I meant of course that the Everettian owes us details about which classical configurations are to count as 'being alive': or, setting aside cats and Schroedinger's experiment and biology, which classical configurations are to count as the system which we are concerned with, having any of a host of properties that we ascribe to macroscopic objects. 'Being alive' is of course a property ascribed in both everyday life and scientific work. But we can concede that it would be enough for the Everettian to give a "translation-manual" from the regions of their vast configuration space to just scientific properties, even just properties used in physics. For example, which regions (and so which quantum states, with a peak, a non-negligible amplitude, over those regions) count as: having a density of 20 grams per cubic centimetre, or being made of lead, or being a fluid, or being a good electrical conductor? Only if the Everettian can give us such a translation-manual will it be plausible that the familiar macroscopic realm---and more precisely, the vast empirical success of classical physics---can be understood as emergent from the quantum state of the cosmos.

Obviously, this is a vast challenge. But I do not say it is impossible; and I stress that in fulfilling it, decoherence will play a crucial role. We can illustrate this by going back to the cat, i.e. to biology. Recall from, for example, biology lessons that biochemistry successfully describes the metabolism of cats (and of course all other organisms) in a completely classical way. Its models of chemical reactions in the cell assume that the proteins, DNA-sequences etc. are localized: these molecules are modelled as minuscule cousins of the ball-and-stick models on the biochemist's table-top; (balls for the atoms, sticks for the bonds between them).

That this classical description of what is after all a quantum reality can succeed so well reflects the efficiency and ubiquity of decoherence. For the protein and DNA molecules are, on the atomic scale, very large: they often contain well over 10,000 atoms (and so are often called 'macromolecules'). And they are constantly bombarded by tiny molecules such as water molecules that decohere them---that is, localize them. The upshot is that at the length-scale of macromolecules and at longer lengths, a classical description of protein molecules, DNA-sequences etc. as having well-nigh definite positions can succeed: a success well illustrated by the models filling biochemistry textbooks. (With these remarks about biology managing well while treating quantities like the position of a macromolecule with classically, without regard to quantum theory, I do not mean to deny that some important biological processes "cue in" to quantum aspects. Examples of this, including crucial processes like photosynthesis, are nowadays a focus of research in the new field called 'quantum biology'.)

Chapter 4.10: A reversal of ideas

So much by way of expounding the Everettians' claim that indeed there are two cats, just because there are two patterns: or more generally, that indeed the world splits---there is a multiplicity of objects---at the end of a decoherence process, since the state is then a mixture with two or more components corresponding to macroscopic realms with different values for the quantities selected by decoherence. As I said: I think this claim is, though dizzying, coherent.

Should we therefore conclude that the Everettian is home free? That is: does their solution to the measurement problem (specifically: their appeal to decoherence to justify talk of splitting) have no internal difficulties? I say 'internal' because we may prefer a rival solution---perhaps one of those reviewed in Section 4 above---for other reasons; (including perhaps reasons that boil down to our intellectual temperament, as discussed in Chapter 1.4).

I say: No. I submit that there are two main difficulties remaining. Both are distinctively philosophical, or interpretative, rather than physical. One difficulty is about the topic of probability: I will address it in the next Section. Here, I address a difficulty about the quantity selected by decoherence. I do not claim that it is a knock-down objection: it is a conceptual tension or embarrassment facing the Everettian, rather than an outright problem. But it is worth articulating for two reasons. So far as I know, it is not addressed in the Everettian literature. And there is an interesting analogy between it and a criticism of Bohr's complementarity interpretation (cf. Section 5 above) that Schroedinger made in the great 1935 paper that formulated the cat paradox (and that also, as I mentioned, analysed entanglement).

To explain this difficulty, I need first to stress the striking conceptual unity of classical physics' successes from the time of Newton till about 1900 (reviewed in Chapters 2.3, 2.6, 3.3 and Section 1 above). In classical physics, each of a very small set of quantities fulfils two roles that are, a priori, disparate. Namely: (i) being postulated as basic for the description of matter's tiniest components (whether point-particles or small extended pieces of matter); and (ii) being used to describe composite systems with vastly many such components. The paradigm examples are position and momentum within mechanics. In classical physics, a single quantity's fulfilling these disparate roles (i) and (ii) was unified by various procedures: especially summing or averaging of the values for the tiny components in (i), to get the values for composite systems in (ii). The simplest case is elementary and familiar: the centre-of-mass of a composite object is a weighted average of the positions of its components, with weights equal the components' masses.

Agreed: with the development of other branches of physics, especially the rise of field theory (Section 1 above), quantities other than position and momentum, such as electric charge and electric field, had to also be accepted. Nevertheless, classical physics manages with a very small set of quantities. (Depending on how finely you distinguish quantities, there are between about a dozen and about fifty of them.) And most of the quantities fulfilling role (i) also fulfil role (ii): again with various procedures, especially summing and averaging, unifying the roles.

Against this background, we can now see the conceptual tension or embarrassment facing the Everettian. The Everettian makes two claims that are in tension with each other. Namely, they say:

(a): Although classical physics took quantities such as position and momentum to be exact, and to always have exact values: such quantities are in fact only definable approximately, through the process of decoherence. (Jargon: philosophers might call them 'emergent'; physicists also say 'effective'). But also:---

(b): These classical notions are needed to define the quantum state-space. For as we saw: the quantum state is a wave-function defined on <u>classical</u> configuration space. Agreed, I have hitherto simplified. For quantum states can be represented as complex-valued functions (also called 'wave-functions') with <u>other</u> sets of arguments than components' positions. The main such alternative representation uses momentum. That is: the set of arguments for a <u>N</u>-particle system are the 3<u>N</u>-tuples of all the possible values of the components' classical momentum in each spatial direction. But here again, it is the <u>classical</u> notion, viz. of momentum, that needs to be invoked; and so the same conceptual tension arises.

So the difficulty, or tension, is that notions which according to the Everettian is really approximate (emergent, effective) must be appealed to, in order to interpret the theory at the smallest and most basic level.

A final comment. In Section 5's quick review of the main strategies for solving the measurement problem, I mentioned Bohr's complementarity interpretation. I can now explain how this difficulty for the Everettian is analogous to a striking criticism of Bohr that Schroedinger made in his "cat" paper of 1935.

Schroedinger's criticism is based on a historical fact about classical physics being in tension with Bohr's (as one might say: "the Complementarian's") claim that classical concepts are indispensable within quantum physics. Thus with labelling to show the analogy with my (a) and (b) above, the fact and the claim are, respectively:

(a'): Despite the conceptual unity of classical physics' successes (described above), classical physicists did not claim that the classical quantities were indispensable for physics; nor did they claim that indispensability would be shown by future physics. Indeed, many expected these quantities to be superseded by future physics. But Bohr claims that:--

(b'): Classical quantities are indispensable for physics, although of course quantum physics has shown they do not always have exact values. For Bohr, the principal reason for this "lesson" from quantum theory is that position and momentum cannot be measured simultaneously with arbitrary accuracy. (In the mathematics of quantum theory, this is represented by position and momentum <u>not commuting</u> with each other.) And by 'indispensable', Bohr means, roughly speaking: indispensable for reporting experimental results in an objective language. (But here, we do not need the details of Bohr's doctrine, or of why he held it: which are controversial.)

So the difficulty, or tension, is that notions which classical physicists expected to be superseded by future physics, and which according to the Complementarian are indeed really limited (by a lesson from quantum theory), must be appealed to, in order to interpret quantum theory---indeed, in order to report experimental results objectively.

Let me end by quoting Schroedinger's own words against Bohr; (at the end of Section 2 of his 1935 paper). He begins with (a'). He praises classical physicists' intellectual modesty about their theory (which he calls 'model' and 'picture'): in particular, about whether its quantities (which he calls 'determining parts') can be measured on a microscopic object in nature (which he calls 'natural object'). He writes:

'Scarcely a single physicist of the classical era would have dared to believe, in thinking about a model, that its determining parts are measurable on the natural object. Only much remoter consequences of the picture were actually open to experimental test. And all experience pointed toward one conclusion: long before the advancing experimental arts had bridged the broad chasm, the model would have substantially changed through gradual adaptation to new facts.'

Then he goes on to criticize what he calls the 'reigning doctrine' (i.e. Bohr's complementarity) for declaring that only familiar classical quantities (position, momentum) are measurable. He writes:

'Now while the new theory [i.e. quantum theory: JB] calls the classical model incapable of specifying all details of the <u>mutual interrelationship of the determining parts</u> (for which its creators intended it), it nevertheless considers the model suitable for guiding us as to just which measurements can in principle be made on the relevant natural object. ... This would have seemed to those who thought up the picture a scandalous extension of their thought-pattern and an unscrupulous proscription against future development. Would it not be pre-established harmony of a peculiar sort if the classical-epoch researchers, those who, as we hear today, had no idea of what <u>measuring</u> truly is, had unwittingly gone on to give us as legacy a guidance scheme revealing just what is fundamentally measurable for instance about a hydrogen atom!?'

<u>Chapter 4.11: Probabilistic *angst*: what is objective probability?</u>

As this Chapter's Preamble announced: I have so far emphasized synchronic issues, i.e. issues about the quantum state at a single time. I have neglected issues about time and change, except to say that I postpone till this Section the question of how the deterministic Schroedinger equation can be reconciled with quantum theory's use of probabilities. So I now focus on how the Everettian answers this question. This will raise the philosophical question: what exactly is <u>objective probability</u> (also known as: <u>chance</u>)?

The first point to make is that the question really breaks down into two problems that Everettian faces. The Everettian literature calls them 'the <u>qualitative</u> problem of probability', and 'the <u>quantitative</u> problem of probability'. Discussing the first will lead in to the second, which I address in the next Section.

The qualitative problem is that probability surely makes no sense, if all possible outcomes of a putatively probabilistic process in fact occur. But this is what the Everettian claims, at least for quantum measurements and the other processes, such as radioactive decay (remember the poison for Schroedinger's cat), in which the quantum state evolves to include a term, i.e. a summand in the sum, for each outcome. (Here, 'outcome' was made more precise by Section 8's sketch definition of 'world'.)

The Everettian's answer to this question is to invoke <u>subjective uncertainty</u>. Their idea is an analogy with how probability is taken as subjective uncertainty, for deterministic processes of the kind familiar within classical physics, e.g. Newtonian mechanics. So let us begin with these.

For such a process, a unique future sequence of states is determined by the present state. (More precisely: by the present state, together with the process' deterministic law: which in mechanics would be a specification of the future forces exerted on the system.) But a person, for example an experimenter, may not know this future sequence of states in advance, either because she does not know the present state in full detail or because it is too hard to calculate from it the future sequence. Given our present interest in how to understand probabilities, let us set aside the latter cause of uncertainty, since it is a matter of calculational intractability rather than ignorance of which among several alternatives occurs. It is ignorance of this kind which gives scope for the idea of probability. So in the context of classical physics, probability is reconciled with determinism by subjective uncertainty: by the idea of a person not knowing which alternative really occurs, but having various <u>degrees of belief</u>, i.e. subjective probabilities (cf. Chapter 3 Section 3), about the matter. In the context of deterministic physics: these will be subjective probabilities about what exactly is the present state.

But here, the phrases 'degrees of belief' and 'subjective probabilities' should be understood in a logically weak sense. They do <u>not</u> imply that the probabilities, i.e. the numbers ½, 1/3 etc. assigned by the person, are a matter of idiosyncratic taste or temperament: that is, are undetermined by all the objective evidence. For there is a branch of classical physics, called 'statistical mechanics', that studies composite systems with vastly many components: a large and important branch, though I have not yet had occasion to mention it. (It was developed from the late nineteenth century onwards: among its main figures were Maxwell (1831-1879) who we met in Section 1, Gibbs (1839-1903) and Boltzmann (1844-1906).) Thus statistical mechanics studies systems like a sample of gas taken as composed of classical molecules, tiny "lumps in the void". It surmounts the utter unknowability of the exact microscopic state---the exact positions and momenta of all the classical molecules---by postulating a probability distribution over the possible states, and then calculating average or expected values of quantities like energy etc. The many resulting predictions meet with great empirical success.

Now the point is: a very good case can be made for calling this distribution 'objective', even though it is <u>not</u> determined by the exact microscopic state. This case invokes technical notions which go by names like '<u>mixing</u>' and '<u>ergodicity</u>'; but we need not go into details. For us it is enough that some rather natural assumptions about these notions select the empirically successful probability distribution from the countless horde of mathematically possible distributions: a selection that has nothing to do with idiosyncratic taste or temperament.

Besides, a similar strategy for reconciling probabilities to determinism, and justifying them as objective, occurs in the pilot-wave theory that we mentioned at the end of Section 5. Recall that like the Everettian, the pilot-wave theorist says that the Schroedinger equation is always 'right' (as Bell vividly put it: 1987, p. 201); but unlike the Everettian, the pilot-wave theorist says a quantum system has values for quantities other than its state's eigenvalues. In particular, there are point-particles with exact positions. With this as background, the pilot-wave theorist goes on to say that: the apparent indeterminism of quantum theory arises from the utter unknowability of these positions; and the (again: empirically successful) Born-rule probability distribution over those positions can be derived from rather natural assumptions about mixing and ergodicity applied to the quantum state.

To sum up: both classical statistical mechanics and the pilot-wave theory reconcile the use of probabilities with determinism by the microscopic state, by: (i) invoking the utter unknowability of aspects of that state (in short: particles' positions), and (ii) arguing that the empirically successful probability distribution over these aspects is not a matter of taste or temperament, but of natural physical assumptions.

Now let us see how the Everettian proposes an analogy with all this. So they claim that also in the Everettian framework, probability can be taken as subjective uncertainty: but now, uncertainty about a deterministic process of the unfamiliar Everettian kind. For such a process, a unique future sequence of states for the composite system---in principle, the entire cosmos---is determined by the present quantum state (together with the Schroedinger equation encoding all the forces that are acting). Yet, says the Everettian, there can still be subjective uncertainty. But the situation differs from the classical one in that this uncertainty arises, even if we assume the person, e.g. the experimenter, does know the present state in all its detail, and also how to calculate from it the entire future sequence of states.

Here, I should clarify that Everettians have an account connecting their state of the cosmos **Psi** (which of course no one knows) with the various quantum states we ascribe, with great empirical success, in real-life experiments. So although of course no one can write down **Psi**, the Everettian framework can recover the real-life ascriptions of quantum states to electrons, atoms and even dust-particles. (We can skip the details of this account: suffice it to say that it uses the ideas in Sections 7 and 8.) Thanks to this account, the Everettian can recover the idea, which is realized every day in real-life experiments, of the experimenter ascribing a quantum state to a microscopic system such as an atom that is about to be measured, and thereby deducing from that state the Born-rule probabilities, i.e. those numbers.

But do those numbers deserve to be called 'probabilities'? After all, according to the Everettian, each of the various measurement outcomes truly occurs. To this the Everettian answers: 'Yes: the experimenter is uncertain since, thanks to the impending 'splitting' during the process of measurement, she will experience, not all the outcomes, but just one---and so she can ask 'Which outcome will I see?'. And that is enough for the numbers to be called 'probabilities'.'

I think this answer is tenable. But it is incomplete in that it raises philosophical issues: indeed, at least three. First: the answer leads to the issue of the identity over time of persons and-or consciousnesses. For it clearly depends on taking the question 'Which outcome will I see?' as analogous to the question in the classical or pilot-wave context, 'Which alternative (among the many microscopic states compatible with my knowledge) actually occurs?'. So the analogy involves accepting that the 'I' which sees just one outcome, could---in some good sense of 'could'---see another outcome.

The second issue is related to the first. In recent decades, quite independently of these quantum conundrums, philosophers of mind and metaphysicians have identified the need for a notion of possibility that generalizes Chapter 3's idea of a possible world. It is sometimes called a 'centred world'. It is needed to understand the content of sentences that contain (and mental states that are naturally verbalized using) worlds like 'I', 'you', 'now', 'then', 'here', 'there', 'this' and 'that'---i.e. words whose referent depends on the context of utterance (cf. Chapter 3, Section 5). (Such words are called 'indexicals' or 'token-reflexives'; and the need for the notion of possibility shown by such sentences is called 'the essential indexical'.) We need not go into this issue in detail: both here and in Chapter 3, we have had enough to do. I just note that the uncertainty that the Everettian's answer invokes---viz. uncertainty despite full calculational ability, and full knowledge of the composite system state, and of the forces encoded in the Schroedinger equation---invites comparison with the kind of indexical uncertainty that philosophers nowadays address using centred worlds.

The third issue is: why should this uncertainty be quantified by the Born-rule probabilities derived from the quantum state? Why are they the right, or somehow appropriate, degrees of belief for the experimenter to have about 'which outcome I will see'? So this is what the Everettian literature calls 'the quantitative problem of probability'.

Chapter 4.12: Subjective probability to the rescue?

Indeed, this problem can be made sharper by imagining that a quantum system is subjected to a sequence of measurements. This prompts a tempting line of thought that the Everettian should regard the Born-rule probabilities as wrong: as follows.

According to the Everettian, the quantum state evolves over the course of a sequence of measurements, so as to encode all possible sequences of outcomes. Formally, the final state has a term (i.e. a summand in the sum) representing each sequence of outcomes. For example, consider a toy-model in which there are ten measurements, each with two outcomes (say, H and T, for 'heads' and 'tails'). Then there are $2^{10} = 1024$ sequences of outcomes; and so the Everettian must say there are 1024 terms in the quantum state.

Since according to the Everettian, each such sequence actually occurs, it seems at first that the Everettian probability of a sequence should be given by the naïve <u>counting measure</u>. That is, the Everettian should say: each sequence has probability 1/1024. And so more generally, it seems that the probability of an event corresponding to a set of sequences, such as three of the ten measurements having outcome H, is the sum of the elementary probabilities of its component sequences. But this amounts to assuming that the two outcomes H and T are equiprobable; (and that the measurements form independent trials in the sense of probability theory). And this spells disaster for the Everettian. For the counting measure probabilities bear no relation to the quantum Born-rule probabilities, and so the procedure of counting Everettian worlds by their outcomes seems to conflict with quantum theory's treatment of probability.

So much by way of sharpening the quantitative problem. Nowadays, Everettians have a twofold answer to this. The first part is to point out that decoherence, thanks to its flexibility, refutes the toy-model with its naïve counting measure. (Recall the end of Section 7, and Section 8**.**) That is: on any precise definition of 'world' for the systems concerned, there will be many trillions of worlds, wholly independently of the number of kinds of outcome registered by the measurement apparatus (in my example: just two, H and T). And more important: because one can vary the exact definitions of decoherence's crucial notions (like 'system-environment boundary'), there is <u>no</u> definite number, not even in the trillions, of worlds which we need to—or could!---appeal to in order to give an account of probability. In short: the naïve counting measure is a mirage: it is woefully ill-defined, and the Everettian can just reject it.

The second part is a remarkable recent development, that is wholly unlike anything in the previous discussion (either by me in this Chapter or in the Everettian literature I have so far drawn on). In terms of the previous Section's discussion, it is an analogue of the arguments within classical statistical mechanics and pilot-wave theory that I mentioned. Recall that they justify those theories' probability distributions, not by their empirical success, but by their following from natural assumptions. Analogously, Everettians have recently developed theorems that justify the Born rule, not by its empirical success, but by its uniquely following from certain general assumptions. But there are also two striking differences between the two cases.

(1): The arguments within statistical mechanics and pilot-wave theory make assumptions about how the system changes over time, albeit general ones. In the jargon: the assumptions about mixing and ergodicity are assumptions about dynamics. But the assumptions of the recent Everettian arguments do not refer to how the system changes: they are synchronic, or kinematic. We do not need details: but in short, they turn on the linear structure of the quantum state-space.

(2): The second difference is even more striking. The arguments within statistical mechanics and pilot-wave theory, and their assumptions, make no mention of subjective probability, or of what principles, e.g. of rationality, should govern a person's subjective probabilities. That is as we saw in the previous Section. Although I introduced the reconciliation of probability with classical determinism by invoking subjective probabilities about what is the exact state, it is details of physics, such as assumptions about how a system changes over time, that are the dominant considerations determining which probability is correct (and in particular, empirically successful). But the Everettians' assumptions <u>are</u> about what principles of rationality should govern a person's subjective probabilities.

This is very remarkable since such principles are formulated and compared in a discipline, <u>decision theory</u> (briefly discussed in Chapter 3 Section 3), that belongs to psychology and economics, and has apparently nothing to do with physics. So the Everettians' idea is remarkably inventive: to appeal to such principles of rationality as applied to a person's degrees of belief in outcomes of a quantum measurement, and thereby prove that their degrees of belief are given by the Born rule. So let me end this Chapter with some details about this.

In decision theory, there is a tradition of proving what are called 'representation theorems'. They are so-called because they show that under certain conditions a rational person's behaviour reveals that their degrees of belief can be represented as numerical probabilities. That is: their degrees of belief must conform to the usual rules about probabilities, viz. that the probabilities of all the envisaged alternatives must add up to 1, and that the probability of either Alternative A or Alternative B, where A and B are incompatible (cannot both be true) is the sum of the individual probabilities. Thus there are theorems to the following effect. Imagine a person whose preferences for gambles (encoding certain degrees of belief and certain desires) conform to a certain set of axioms that seem rationally compelling. The axioms say, for example, that a person who prefers A to B and B to C must also prefer A to C, and that a person would not enter a bet (or a collection of bets) that is guaranteed, whatever the outcomes, to yield a loss. Then the person must have degrees of belief that conform to the rules of probability. In other words, their degrees of belief are represented by a probability distribution.

We do not need further details, technical or even philosophical, about such theorems. But let us note that these theorems do not dictate a unique probability distribution over the various alternatives. This is of course as one would expect. Imagine two people are offered bets on horses in a race, and so reveal their degrees of belief in the alternative propositions about which horse wins. Even if the two people are rational in the sense of the listed axioms, we surely do not expect them to accept bets at exactly the same odds. In short: we do not rationality to dictate specific degrees of belief in arbitrary propositions. (We touched on this in Chapter 2 Section 5's discussion of Hume and inductive logic.)

But the recent Everettian theorems secure precisely this uniqueness, about the specific scenario of a person making gambles on the outcomes of quantum measurements. Besides: the probability distribution that is uniquely dictated by the axioms about the person---which, as in the tradition of decision theory, seem to encode their being rational---is indeed quantum theory's Born-rule probability distribution over the various outcomes.

A bit more precisely: the theorems show that a person who: (i) is an Everettian and is about to observe a sequence of quantum measurements, and also (ii) knows the initial state of the quantum system to be measured, and (iii) is forced to gamble on which outcomes she will see (using the Everettian sense of 'splitting', to interpret the phrase 'she will see'), and (iv) whose gambles are subject to certain rationality axioms---must apportion her degrees of belief (as shown by her betting behaviour) in accordance with the Born-rule.

To sum up: this Section began with an objection to the Everettian, saying they could not answer the quantitative problem of probability, since they seemed to endorse the naïve counting measure, and so be doomed to disagreeing with the empirically successful Born-rule probabilities. But thanks to these theorems, the Everettian can say their framework not only accommodates, but even implies, the Born rule probabilities. Remarkable indeed.