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Epistemic Holes and Determinism in Classical General Relativity

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ABSTRACT

Determinism fails easily if spacetimes with points removed from the spacetime manifold are taken to be physically reasonable representations of a way the world could be according to classical general relativity. I discuss a recently proposed condition for determining which spacetimes have holes—epistemic hole freeness—and show that (a) epistemic hole freeness gives the correct verdict in some (but not all) non-globally hyperbolic spacetimes with a closed subset removed, (b) certain spacetimes with genuinely indeterministic features count as having an epistemic hole, which implies that the requirement of epistemic hole freeness enforces a form of determinism, and (c) there is a large class of spacetimes that intuitively are radically indeterministic and unphysical due to containing a hole (of a kind), but are free from epistemic holes. I show that a few natural ways of remedying (c) are not satisfactory.

1 Holes and Determinism

Consider a relativistic spacetime $\langle M, g_{ab} \rangle$.[1] By cutting and pasting some of its subsets one can easily produce other philosophically interesting spacetimes.

[1] In what follows, I will assume that the reader is familiar with the basic notions, such as a spacetime manifold M and a spacetime metric g_{ab}, definitions of spacelike, timelike and null curves, geodesics, inextendibility, completeness and incompleteness of a curve γ, achronal slices without edge, Cauchy surfaces, global hyperbolicity, the chronological future I^+, the

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For instance, from a given $\langle M, g_{ab}\rangle$ one can produce another spacetime $\langle M', g'_{ab}\rangle$ which agrees with $\langle M, g_{ab}\rangle$ in some regions (say, such as the past of some achronal subset), but disagrees overall—what Earman ([1995]) dubbed 'a dirty open secret' concerning the failure of determinism. I will assume the definition of determinism due to Butterfield ([1989])—that is, DM2 of that paper—according to which:

> [. . .] a theory with models $\langle M, O_i\rangle$ is **S**-deterministic, where **S** is a kind of region that occurs in manifolds of the kind occurring in the models, iff: given any two models $\langle M, O_i\rangle$ and $\langle M', O'_i\rangle$ containing regions S, S' of kind **S** respectively, and any diffeomorphism α from S onto S': if $\alpha^*(O_i) = O'_i$ on $\alpha(S) = S'$, then: there is an isomorphism β from M onto M' that sends S to S', that is, $\beta^*(O_i) = O'_i$ throughout M and $\beta(S) = S'$.

So to find a witness of indeterminism one needs merely to find two models that are isomorphic in region S but are not isomorphic overall—just as standard Laplacian intuitions demand.[2] In this sense one can easily and massively produce indeterministic counterparts of any relativistic spacetime whatsoever. This is a philosophically unsatisfactory situation. To paraphrase Earman and Norton ([1987]), determinism may fail, but (one hopes) if it fails, it should fail because of reasons of physics, not because of a clever mathematical trick.

Note that what is relevant to the issue of determinism is not what one does to the spacetime, but what the resulting spacetime is like. As stressed by Earman ([1995]), any relativistic spacetime can be obtained using cutting and pasting. So one should not object to the procedure of cutting and pasting as such. But perhaps in some cases the resulting spacetime is unphysical. The desire to capture the sense of unphysicality present in such cases motivates the search for hole freeness conditions, which would rule out spacetimes in which a sort of artificial 'hole' has been made.[3] Various such conditions have been

chronological past I^- and the causal future J^+ of a point p and of a curve γ, and what an isometry is. I am following standard definitions and notational conventions, such as in (Malament [2012]; Manchak [2016a]). The only exception is that my brackets are angled, not rounded (in accordance with what I take to be a standard notation for an ordered pair).

2 Note that the choice of kind of region **S** is somewhat tricky: although in both cases one may pick an a slice (achronal closed subset without edge), and 'dirty open secrets' are best understood in these terms—it's just that in one model the slice is additionally a Cauchy surface, whereas its isomorphic counterpart fails to have the Cauchy property—in particular contexts other choices may be more natural. For example, in Section 4 one could take **S** to be the maximal globally hyperbolic development, whereas in Section 5 it could be a slice that is a Cauchy surface. Considerations of Earman ([2007]) strongly suggest that the choice of region **S** is not easy, and many seemingly natural choices trivialize the issue; for further discussion, see (Doboszewski [2019]). For present purposes, commitment to particular kind of regions is tentative.

3 This kind of hole consists of literal holes, that is, one or more points removed from a given spacetime manifold, making this form of indeterminism very different from the kind of holes

proposed in the past; unfortunately, none seems to work.[4] In what follows, I will analyse a recently suggested promising condition, epistemic hole freeness. Upon closer inspection epistemic hole freeness turns out to have highly unsatisfactory features: in particular, it allows for a robust (and intuitively highly artificial) form of indeterminism. It seems, then, that either a very large set of relativistic spacetimes has to be by *fiat* declared to be physically unreasonable, or classical general relativity is radically indeterministic.

2 Spacetimes, Holes, and Epistemic Holes

To introduce the condition of interest, I will briefly discuss some of the attempts at distinguishing spacetimes with holes from hole free ones.

Observe that spacetimes with points removed tend to have incomplete geodesics. Perhaps geodesic completeness should be a hole freeness condition? This would, however, be way too strong: singularity theorems establish that in a wide range of physically reasonable situations (including gravitational collapse) geodesic completeness is violated.

A spacetime with a hole can be made larger by an inclusion of a missing point. This leads to the demand that spacetime should be inextendible. Recall that the spacetime $\langle M', g'_{ab}\rangle$ is an extension of $\langle M, g_{ab}\rangle$ if there exists a function $\Lambda : M \to M'$ that is an embedding of M in $\Lambda(M')$ (and a diffeomorphism onto its image), such that $\Lambda^*(g'_{ab})_{|\Lambda(M)} = g_{ab}$ and $\Lambda(M) \neq M'$, and that a spacetime is inextendible if and only if it has no extension. This condition rules out some spacetimes with holes, but not all: there are simple mathematical procedures (such as slapping a conformal factor on a spacetime,[5] or moving to the universal cover of a given spacetime) that ensure that in a large set of cases inextendibility can be satisfied.[6] Moreover, Manchak ([2016b]) argued that there are independent reasons to be suspicious about inextendibility.

One could turn for help to conditions inspired by the condition given by Geroch ([1977]), according to which, intuitively, spacetime $\langle M, g_{ab}\rangle$ has a hole

considered in the so-called hole argument and discussions concerning diffeomorphism invariance. Conceptual issues raised by these kinds of holes should be carefully kept apart.

4 Manchak ([2016a]) provides a reader-friendly introduction to the current state of debate concerning hole freeness conditions. Naturally, even if some condition worked, justifying that nature obeys such a condition is another matter entirely (as stressed by Earman ([1995]) in his discussion of what seems to be an invocation of some metaphysical principle of sufficient reason as a justification for positing a particular hole freeness condition). But since none such conditions seem to give the expected verdict, one needs not to worry about providing a justification for them.

5 A simple example of this is the construction of a Malament–Hogarth spacetime by conformal transformation of Minkowski spacetime with a point removed, as in (Welch [2008], pp. 661–2, and Figure 1).

6 Beem ([1976]) has shown that any spacetime in which an inextendible causal geodesic is not fully contained in a compact subset is conformally equivalent to a geodesically complete (and hence inextendible) one.

if for some achronal subset S, the domain of dependence $D(S)$ can be enlarged by finding an isometric embedding ϕ from $\langle M, g_{ab}\rangle$ to some other spacetime $\langle M', g'_{ab}\rangle$. But surprisingly, Krasnikov ([2009]) has shown that according to this condition, even Minkowski spacetime has a hole. Minguzzi ([2012]) and Manchak ([2013]) tried to improve on the 'domains of dependence should be as large as possible' hole freeness conditions. But these conditions seem to be highly contrived and much less natural than the simple and intuitive condition of Geroch ([1977]).

So, it seems that all available hole freeness conditions have undesirable features. Not good, especially if one is worried by questions concerning determinism. Since one is trying to learn whether the theory is deterministic or indeterministic (for some class of physical phenomena of interest), one would like to have a way of excluding 'fake' indeterminism (where for any model whatsoever plenty of alternative counterparts can be produced), without assuming by *fiat* that the theory is deterministic. So a hole freeness condition should be strong enough to rule out some obviously uninteresting instances of indeterminism, while leaving the possibility of some (more interesting) form of indeterminism open. But how to do that?

Recently, Manchak ([2016a]) has suggested the following schema:

> A spacetime with an epistemic hole property (EH): A spacetime $\langle M, g_{ab}\rangle$ has an epistemic hole if there are two future-inextendible timelike curves γ and γ' with the same past endpoint and which satisfy the following condition $(*)$, such that $I^-[\gamma]$ is a proper subset of $I^-[\gamma']$.

The condition $(*)$ can be filled in at least two different ways: $(*)_g$ states that both curves are geodesics, $(*)_f$ states that both curves have finite total acceleration. Epistemic hole freeness (EHF), then, is a property obtained by ensuring that EH does not hold. Depending on which version of the condition $(*)$ is used, spacetime that satisfies the negation of the above condition is free from epistemic holes, in short: satisfies EHF(g) (in case of $(*)_g$), or EHF(f) (in case of $(*)_f$). A spacetime satisfying EH has an epistemic hole in the sense that there are two observers, starting from the same point, and one of them is in a significantly worse epistemic position than the other. For the observer travelling along γ' has access to all the observable content that the observer travelling along γ could see, plus some more.

EHF has been advertised as a condition that has quite a few advantages over other forms of conditions that rule out holes in spacetime, such as inextendibility or hole freeness conditions based on an idea that domains of dependence are 'as large as possible'. First, all these notions have a modal character: they require one to consider mapping between the given spacetime and various other spacetimes (which seems to already presuppose a distinction between physically reasonable and physically unreasonable spacetimes).

In contrast, EHF pertains to a single spacetime, and so it does not require comparison of a given spacetime with some other spacetime. This makes epistemic hole freeness an interesting candidate for a condition that distinguishes between physically reasonable and physically unreasonable spacetimes (assuming that the enterprise of providing such a distinction is worth pursuing, and that a hole freeness condition is useful for this purpose).[7] It also gives intuitively correct verdicts in a large class of examples. And it can be shown, among other results, that any spacetime that is EHF(f) is not (future) nakedly singular.[8] This is good news. So, perhaps epistemic hole freeness could serve as a way of distinguishing 'fake' indeterminism from 'physically reasonable' indeterminism? With this in mind, let me move to observations concerning the strength of epistemic hole freeness in the next three sections.

3 Some Non-globally Hyperbolic Spacetimes Have Epistemic Holes

An additional advantage of epistemic hole freeness is that it gives the intuitively correct verdict for some non-globally hyperbolic spacetimes with a closed subset removed.

The following example illustrates this point. Consider anti-de Sitter spacetime $\langle M, g_{ab}\rangle$ and anti-de Sitter spacetime with a point s removed $\langle M-\{s\}, g'_{ab}\rangle$, where g'_{ab} is the restriction of g_{ab} to $M-\{s\}$.[9] It is easy to show that with a point s removed from the bulk of $\langle M, g_{ab}\rangle$, one can find two timelike geodesics γ and γ' with the same past endpoint p such that $I^-[\gamma]$ is a proper subset of $I^-[\gamma']$.

Consider first the spacetime $\langle M, g_{ab}\rangle$. An important feature of that spacetime is that all future-directed timelike geodesics from any point p are contained in a sequence of diamond-shaped regions: all such geodesics refocus on a point q, then expand again from q to refocus on some q', and so on. (See Hawking and Ellis [1973], p. 132, Figures 20(i) and 20(ii), and p. 133; a subset of a conformal patch of anti-de Sitter spacetime illustrating this behaviour is schematically represented in Figure 1, which also summarizes the argument of the next paragraph.)

7 The way I am using the expression 'physically reasonable spacetime' is intended as subsuming related notions, such as physically salient, cogent, significant, and so on. Sometimes a distinction is made between a metaphysical notion of being physically reasonable (under which, presumably, conditions for being physically reasonable obtain lawlike status, since they determine what is physically possible and what is not) and an epistemic notion of physical significance (for a suggestion along these lines, see Fletcher [2016]). Note also that arguably such conditions are to an extent context-dependent (as argued by Earman ([1995]) in the context of cosmic censorship). But it seems that in most contexts, the relevant set of conditions for being a physically reasonable spacetime will include some form of hole freeness.

8 To appreciate that result, recall that a spacetime is future nakedly singular if and only if there exists an inextendible, future-directed timelike or null curve that is fully contained in the chronological past I^- of some point p.

9 In dimension $d=2$, $M=\mathbb{R}^2$ and $g_{ab}=\cosh^2 x \nabla_a t \nabla_b t - \nabla_a x \nabla_b x$.

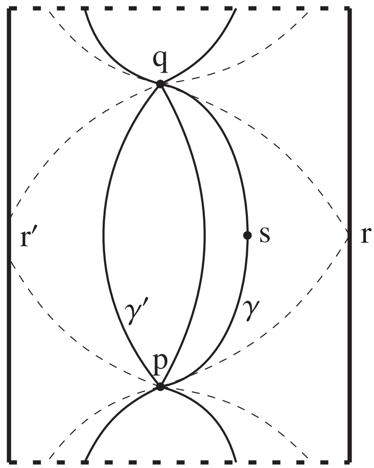

Figure 1. Schematic representation of a single diamond-shaped subregion of a conformal diagram of anti-de Sitter spacetime, illustrating why removal of a point s results in epistemic holes. Dashed lines are null curves, r is the null infinity, r' is a coordinate singularity; solid black curves represent some timelike geodesics that focus at p, expand from p to refocus at q, and so on.

Now, take some point s and consider the punctured spacetime, $\langle M - \{s\}, g'_{ab} \rangle$. Since any point in this spacetime is contained in some diamond-shaped region, without loss of generality it may be assumed that s lies on a timelike geodesic that (in the spacetime $\langle M, g_{ab} \rangle$) goes through some point p in the past of s and then q, which is the first point at which future-directed timelike geodesics from p refocus. For γ, take a timelike geodesic linking (in $\langle M, g_{ab} \rangle$) p and q which passes through s, and consider its counterpart (by map ϕ, which is an identity map on M restricted to $M - \{s\}$) in $\langle M - \{s\}, g'_{ab} \rangle$. Since s has been removed, one connected component of $\phi(\gamma)$ with p as past endpoint is a future-directed, inextendible, incomplete geodesic. For γ', take any future-directed complete timelike geodesic with past endpoint at p (it has to pass through q, since all future-directed timelike geodesics from p refocus at that point). γ is incomplete, and $I^-[\gamma]$ is fully contained in $I^-(q)$, so it is contained in $I^-[\gamma']$. So $\langle M - \{s\}, g'_{ab} \rangle$ has an epistemic hole in the EHF(g) sense.

So even though anti-de Sitter spacetime satisfies EHF(g) and EHF(f) (as shown in Manchak [2016a]), anti-de Sitter spacetime with a point removed does not satisfy EHF(g).[10] Both of these spacetimes are not globally hyperbolic, but one has a hole, while the other does not.

[10] Note that despite the fact that anti-de Sitter spacetime is not globally hyperbolic, achronal subsets S do have non-empty domains of dependence; see (Galloway [1982], Figure 1). So it seems that using a judicious choice of S one should be able to show that some hole freeness

Unfortunately, epistemic hole freeness does not distinguish between any non-globally hyperbolic extension of Misner spacetime and the same spacetime with a point removed from a non-globally hyperbolic region (or with a point removed from the globally hyperbolic region, for that matter): all these spacetimes have epistemic holes in the sense of EHF(g). Why? Recall the argument by Manchak ([2016a]) that Misner spacetime has an epistemic hole: in any of the extensions there is a timelike geodesic that terminates at the Cauchy horizon and does not enter the non-globally hyperbolic region. So the past of that curve will be fully contained in the past of some other geodesic that gets extended through the horizon, which leads to an epistemic hole. Since that argument depends only on the existence of an incomplete geodesic terminating at the Cauchy horizon and another geodesic extending through the horizon, the removal of a point that does not lie on any of these two curves is irrelevant for the property of having an epistemic hole; so, no matter how one removes the point, all of these spacetimes will have an epistemic hole. But for some contexts, such as the question of the existence of time machines, it would be useful to have a hole freeness condition that gives the intuitively correct verdict in these cases (for more on which, see the discussion in Earman *et al.* [2009], Section 3).

4 Spacetimes with Certain Indeterministic Features Have Epistemic Holes

A slightly generalized version of epistemic hole freeness imposes an interesting form of determinism.

Consider the following schema (which is just a time-reversed version of EHF, that is, with past and future interchanged), which rules out asymmetry of possible interventions between observers travelling along curves γ and γ':

> A spacetime with inequality of opportunities (IOO)[11]: Spacetime $\langle M, g_{ab}\rangle$ allows for inequality of opportunities if there are two past-inextendible timelike curves γ and γ' with the same future endpoint and which satisfy condition $(*)$, such that $I^+[\gamma]$ is a proper subset of $I^+[\gamma']$.

Again, conditions $(*)$ will be filled in an analogous way as in case of EHF, and equality of opportunities (EOO) is defined by the negation of the above property of IOO. EOO means that for any two observers who meet at some point it is not the case than the observer travelling along γ' could have intervened on everything the other observer could have, plus some more. 'Domains of

conditions based on the idea that domains of dependence are 'as large as possible' provide an intuitively correct classification of holes in anti-de Sitter spacetime and anti-de Sitter spacetime with a point removed.

[11] I am grateful to Jeremy Butterfield for suggestion concerning the name.

possible interventions' are not asymmetric between observers, so to speak. The motivation for introducing EOO is that epistemic hole freeness is a time-asymmetric condition: one cares only about future-directed timelike curves. But for the discussions of determinism in general it may be helpful to have a time symmetric condition, for there could be situations in which determinism to the future holds whereas determinism to the past is violated (and so on; see the discussion of futuristic and historical determinism in Earman [1986], Chapter 2.6). Note also that other standard hole freeness conditions are time symmetric (or can easily be made so). In what follows, I will say that a spacetime satisfies generalized epistemic hole freeness if and only if it satisfies EHF and EOO.[12]

A form of general relativistic indeterminism is demonstrated by the existence of spacetimes that are maximal globally hyperbolic, but are extendible in non-globally hyperbolic and non-isometric ways. This can be naturally interpreted as indeterminism, in the sense that there are situations when the initial value problem has non-unique solutions. In terms of the definition invoked in Section 1, the region **S** is any slice from the maximal globally hyperbolic region (or even the whole region), and various extensions are the two models that fail to be isomorphic. What I take to be the standard interpretation of this situation is expressed concisely by Ringström ([2010], p. 19) as follows: 'they demonstrate that Einstein's general theory of relativity is not deterministic; given initial data, there is not necessarily a unique corresponding universe'.[13] There are other examples of this behaviour in classical general relativity; see (Chruściel and Isenberg [1993]) for the construction of isometric and non-isometric extensions of Misner, Taub-NUT, and polarized Gowdy spacetimes; and (Doboszewski [2017]) for a recent discussion of their philosophical relevance.

Spacetimes such as maximally extended Misner spacetime or Taub-NUT violate EHF(g). (See the discussion of Misner spacetime in (Manchak [2016a]); since the relevant portion of Misner spacetime is isometric to

[12] I assume that for generalized hole freeness the blank is filled in the same way in the conditions for both epistemic hole freeness and equality of opportunities. But of course it does not have to be so. It would be interesting to see what happens if one fills the blank in different ways (obtaining four different conditions): for example, if one requires that EHF holds for any curves with finite total acceleration, but equality of opportunities holds only for geodesics. Are there any spacetimes that would satisfy EHF(g) + EOO(g), but do not satisfy, say, EHF(g) + EOO(f)? And so on. Nevertheless, I do not see interpretational significance of such 'mixed' conditions, and will ignore them in what follows.

[13] Note that one could subscribe to an alternative interpretation, namely, that such spacetimes are not instances of non-unique solutions to the initial value problem, because the extensions are made through a Cauchy horizon, and hence are not in the domain of dependence of the initial data. Yet another possible reading is that indeterminism is not signalled by the lack of uniqueness, but by lack of causal dependency of the extended region on the globally hyperbolic region, and so even an extension of the Misner spacetime (which in a certain sense is unique) leads to the failure of determinism.

upper half of Taub-NUT spacetime, the argument carries over.) Any of the non-isometric extensions of polarized Gowdy spacetime, in which the time orientation is chosen such that the extension is made to the past, satisfies EHF, but not EOO, and similarly in the case of extendible Bianchi IX solutions discussed by Ringström ([2009]). Indeed, all known examples of extensions of extendible maximal globally hyperbolic spacetimes violate generalized epistemic hole freeness.[14] Why is that so? Recall that all known such cases have a globally hyperbolic region with at least two classes of incomplete timelike and null geodesics, each of which gets extended through the Cauchy horizon and completed in one of the non-globally hyperbolic extensions.[15] (For an explicit discussion in case of Misner spacetime, see Levanony and Ori [2011].) Note that the requirement that a spacetime manifold is Hausdorff forbids one to perform the multiple extensions simultaneously (for discussion of these topics, see Hawking and Ellis [1973], Section 5.8), which results in non-uniqueness. To find a witness for an epistemic hole in an extension of such a spacetime it is sufficient to take one of the geodesics that remains incomplete in the extended (whether maximally or not) spacetime (that is, a geodesic that does not get extended across the Cauchy horizon) as γ, and one of the geodesics that gets continued in the extension as γ'. The idea of the EOO can be summarized as follows: Extensions of spacetimes that demonstrate that the theory is future indeterministic in this way (that is, spacetimes with a Cauchy horizon in the future of the globally hyperbolic region) always have epistemic holes. Similarly, extensions of spacetimes that demonstrate that the theory is past indeterministic can be EHF, but will not satisfy EOO.

Which of the three conditions, EHF or EOO or generalized epistemic hole freeness, is violated depends on the whether the spacetime has future, past, or both future and past Cauchy horizons. (Generalized) epistemic hole freeness should be contrasted with previous variants of hole freeness, according to which extendible maximal globally hyperbolic spacetimes do not have holes, since points in the extended regions cannot be in the domains of dependence of any achronal subset S. (Since non-globally hyperbolic regions of these spacetimes violate chronology, there is no achronal subset S in the additional region, and achronal subsets of the maximal globally hyperbolic region already have, because of that maximality, domains of dependence that are ‘as large as possible’.)

[14] I stress this aspect since there is no theorem to the effect that any extendible maximal globally hyperbolic spacetime needs to exhibit qualitatively similar behaviour.

[15] Recall that the Cauchy horizon is defined as the boundary of the domain of dependence of Cauchy surface S. If a spacetime contains the maximal globally hyperbolic development of the initial data and the Cauchy horizon is empty, then its uniqueness up to isometry (and hence a form of determinism) is guaranteed by the theorem of Choquet-Bruhat and Geroch ([1969].

Demanding that generalized epistemic hole freeness is satisfied removes a number of indeterministic examples from the set of physically reasonable spacetimes. There are two ways to react to this situation:

(1) Argue that all is perfectly fine, for extendible maximal globally hyperbolic spacetimes display strange forms of curve incompleteness, which justifies calling these spacetimes pathological and unphysical. After all, any extension made to the future is future nakedly singular, and maximal extensions violate chronology—and thus it is the strength and an advantage of (generalized) epistemic hole freeness that such examples are no longer allowed.[16] Moreover, assuming that a physically reasonable spacetime has to be stable—a classic version of this view is expressed in (Hawking [1971]), but see (Fletcher [2016]) for a discussion of difficulties associated with such proposals—then perhaps not much is lost, since the property of being an extendible maximal globally hyperbolic spacetime seems to be unstable anyway (for arguments to that effect, see Misner and Taub [1969]; Thorne [1993]; Ringström [2010]).

(2) Say this verdict is unacceptable, for extensions of maximal globally hyperbolic spacetimes are physically reasonable examples of indeterminism: that a Cauchy horizon forms in the spacetime is a physically important dynamical feature of certain initial data sets, and it seems wrong to dismiss them as unphysical on the grounds of having a hole. Moreover, it may already be suspicious that EHF spacetimes are not nakedly singular. After all, there are physical situations in which naked singularities arise dynamically (Joshi and Dwivedi [1993]; Christodoulou [1994]; Dwivedi and Joshi [1994]), which seem very different from spacetimes with holes of the kind that motivates looking for hole freeness conditions. It seems that differences between dynamically arising naked singularities and artificial naked singularities cannot be captured using EHF. And in case of a naked singularity an asymmetry between observers (forbidden by EHF) seems natural and acceptable: after all, one observer hits the singularity, the other one does not. The asymmetry is physical, in that it is introduced by the gravitational collapse resulting in the naked singularity. Ruling out naked singularities, then, is at best a mixed blessing. It may be a virtue if nature indeed 'abhors naked singularities'. But

[16] Note that there are still multiple forms of indeterminism present in spacetimes that satisfy EHF. For instance, anti-de Sitter spacetimes have no epistemic holes, but seem to display a relativistic version of space invaders-like indeterminism (see Earman and Norton [1993]). Moreover, if determinism is conceptualized in such a way that spacetimes violating chronology or singular spacetimes count as a form of indeterminism, ample amount of indeterminisic cases arises.

until this has been convincingly demonstrated, demanding epistemic hole freeness rules in favour of determinism—unfairly.

In the light of the next section, I think it is right to react in the second way. For it will turn out that even though epistemic hole freeness does not allow for indeterminism that can be associated with non-unique extensions of maximal globally hyperbolic spacetimes, it does allow a Doomsday kind of indeterminism, that is, a situation in which whether spacetime abruptly ends at any given moment of time is not determined by the initial conditions and dynamical laws. In my opinion, this seems much more artificial and unreasonable than the indeterminism induced by extensions of maximal globally hyperbolic spacetimes.

5 Some Truncated Spacetimes Are Epistemically Hole Free

In the light of the various nice features of EHF, one may be tempted to claim that some version of EHF is a necessary condition for a spacetime to be physically reasonable. Could EHF also be a sufficient condition?

This is also related to the issue of determinism, in the following sense. A natural interpretation of the 'dirty open secret' of Earman ([1995]) is the following: unless a hole freeness condition (considered there in the sense of '$D(S)$ is as large as possible' approaches) is assumed, determinism fails very easily. But, as I argued in the first section of this article, there are good reasons to dismiss these approaches to hole freeness as unsatisfactory. This leads to a natural question: what is the fate of determinism (and the 'dirty open secret') under the assumption of EHF?

It turns out that there are some intuitively physically unreasonable spacetimes, obtained by excision of certain subsets of M, which do satisfy both EHF(g) and EHF(f). Let $\langle M, g_{ab}\rangle$ be two-dimensional Minkowski spacetime,[17] and consider three different truncations:

(1) $\langle M'^1, g'^1_{ab}\rangle$, where M'^1 is M with a null slice without edge S and its future $I^+(S)$ removed; the metric is $g'^1_{ab} = g_{ab|M'^1}$ (null-truncated Minkowski spacetime; see Figure 2),

(2) $\langle M'^2, g'^2_{ab}\rangle$, where M'^2 is M with a zigzagging achronal (but not acausal) slice without edge S and its future $I^+(S)$ removed. That is, the achronal slice S has some spacelike and some null components, and the metric is, again, $g'^2_{ab} = g_{ab|M'^2}$ (zig-zag-truncated Minkowski spacetime; see Figure 3),

[17] This choice is made just for the sake of simplicity; the examples will work just as fine for higher dimensions and other choices of the metric.

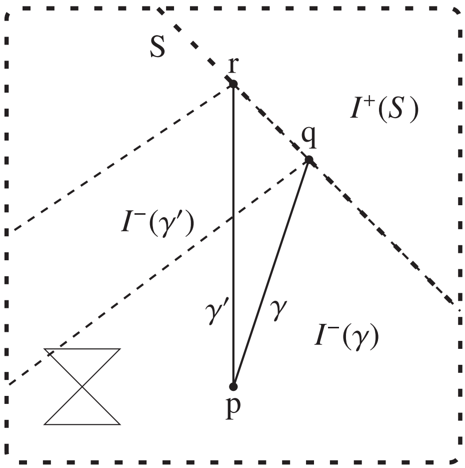

Figure 2. If S has a null component, the truncated spacetime violates EHF(g): the case of a null-truncated Minkowski spacetime $\langle M'^1, g'^1_{ab}\rangle$.

(3) $\langle M'^3, g'^3_{ab}\rangle$, where M'^3 is M from which a spacelike slice without edge S and its future $I^+(S)$ have been removed; the metric is $g'^3_{ab} = g_{ab|M'^3}$ (future-truncated Minkowski spacetime; see Figures 4 and 5).

Intuitively, these spacetimes are clearly physically unreasonable: there is no physical reason for spacetime to come to an end abruptly in this way, which I will call Doomsday indeterminism.[18,19]

[18] A digression: Barnes and Cameron ([2009]), when arguing against the branching time approach to the open future, subscribe to the view that situations in which time ceases to exist are genuine metaphysical possibilities that cannot be represented by a branching model. Such ceasing of time seems to be represented by a removal of the upper part of the branching model (that is, the future of some point x). It seems that what they are after is a non-relativistic analogue of the truncations I am discussing here. However, even if such situations should be counted as metaphysical possibilities, it is not clear that such a ceasing of time should be counted as a physical possibility in classical general relativity (even though other forms of ceasing of time, such as a big crunch, are clearly physically possible). The difference between the truncated version of the ceasing of time and other forms (such as a big crunch) is that in the latter case there is a physical description of spacetime structure leading to the ceasing (a physical mechanism, if you like). The lack of such description in the case of truncated spacetimes is the reason why, intuitively, they should not represent genuine physical possibilities. In short: I accept that Doomsday indeterminism is a possibility that should be seriously entertained, but if (and only if) it is accompanied by a physical mechanism.

[19] Note also that even though these spacetimes present us with a somewhat radical form of indeterminism, it is not obviously of the 'multiple continuations' variety required by some formal statements of indeterminism: the disagreement between these models lies in the fact that some of them continue a given region with an empty set, whereas some others continue with another spacetime region (and whether this counts as disagreement of worlds or models depends on additional considerations). This could be seen merely as a glitch in a definition, and the one adopted here (due to Butterfield [1989]) does not suffer from that glitch: in case of the original

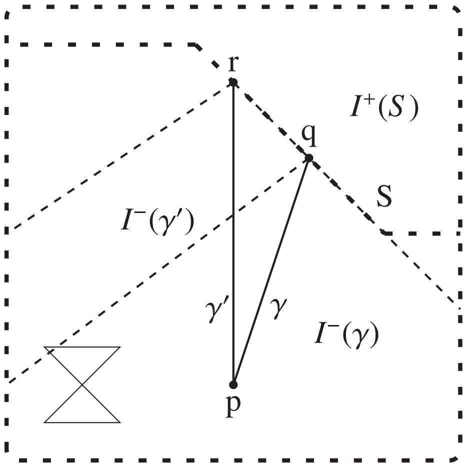

Figure 3. If S has a null component, the truncated spacetime violates EHF(g): the case of a zig-zag-truncated Minkowski spacetime $\langle M^{'2}, g^{'2}_{ab}\rangle$.

And indeed, all these spacetimes have a hole in the '$D(S)$ is as large as possible' approach and are extendible (but could easily be made inextendible with the help of a suitable conformal transformation); and intuitively they have a gigantic hole looming to the future. What is the verdict of EHF?

Spacetimes $\langle M^{'1}, g^{'1}_{ab}\rangle$ and $\langle M^{'2}, g^{'2}_{ab}\rangle$ have an epistemic hole (both in the EHF(g) and EHF(f) sense). The argument is summarized graphically on Figures 2 and 3 (note that on all Figures S and $I^+(S)$ are removed from spacetime; thin dashed lines are null curves). Consider two curves, γ that goes to some q on the null component of S, and γ' that goes to some $r \in S$, $r \in J^+(p)$. Then $I^-[\gamma]$ is a proper subset of $I^-[\gamma']$, and by applying this line of reasoning to the truncated spacetime, one immediately obtains epistemic holes.[20] So in the null and zig-zag-truncated Minkowski spacetimes one can always find an epistemic hole.

In the future-truncated Minkowski spacetime, however, one cannot do so—$\langle M^{'3}, g^{'3}_{ab}\rangle$ does not have an epistemic hole (either in the EHF(g) or EHF(f) sense). This is displayed on Figures 4 and 5. There are two possible situations: in each case the truncated spacetime has no epistemic holes. Consider two curves, γ and γ', that would intersect S had the truncation not been performed. They would either intersect S at the same $q \in S$, in which case γ and γ' have the same chronological past (as in Figure 4—note that this case occurs only if one

spacetime and a truncated spacetime there is no isomorphism between them—there is only an embedding of the truncated one into the original one—so this example counts as indeterministic.

20 And an anti-de Sitter spacetime truncated by removal of $(J^-(p) \setminus I^-(p)) \cup (I^+(J^-(p) \setminus I^-(p)))$ for some p does have an epistemic hole.

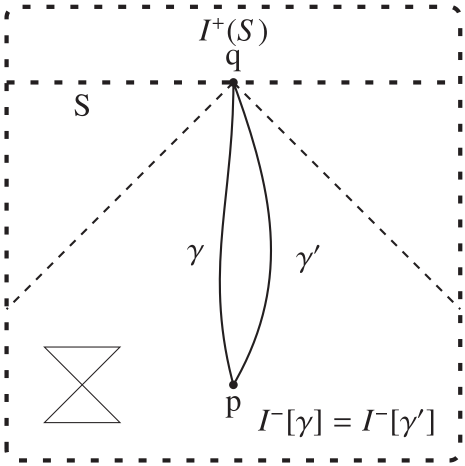

Figure 4. Future-truncated Minkowski spacetime $\langle M'^3, g'^3_{ab}\rangle$, case one: $I^-[\gamma] = I^-[\gamma']$.

considers EHF(f)), or at two distinct points (some q and r). If they would both intersect S at some q, then $I^-[\gamma] = I^-[\gamma']$ in the truncated spacetime. And if they would intersect S at two distinct points, then the symmetric difference $I^-[\gamma] \triangle I^-[\gamma'] \neq \varnothing$,[21] and is not contained in $I^-[\gamma]$ or in $I^-[\gamma]$ (Figure 5). Thus, whether a truncated Minkowski spacetime counts as having an epistemic hole or not is a matter of how the truncation has been performed.

Together with the preceding section this implies that although EHF rules out some instances of indeterminism that are associated with, so to speak, some physical goings-on (dynamically forming naked singularities, extensions through Cauchy horizons) on the grounds of having a hole, it allows for (intuitively) much more unreasonable and radical Doomsday-type indeterminism. This seems to be unfortunate: the hole freeness condition does significant physical work here, and of the sort that hole freeness should not do.

What else do these examples show? There are examples of spacetimes that are inextendible and hole free (according to '$D(S)$ is as large as possible' conditions), but intuitively do have some kind of a hole (see Manchak [2016a]); inextendibility and hole freeness (again, in the sense of '$D(S)$ being as large as possible') are thus insufficient to rule out all artificial examples. I have constructed examples of spacetimes (which are extendible and have a hole according to '$D(S)$ is as large as possible' conditions) that intuitively are physically

[21] Recall that a symmetric difference of two sets consists of elements that are in either of the sets, but not in their intersection.

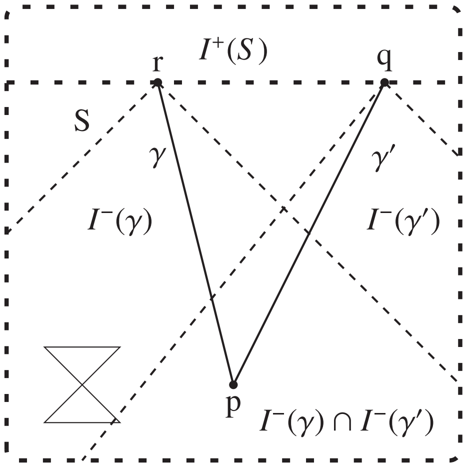

Figure 5. Future-truncated Minkowski spacetime $\langle M'^3, g'^3_{ab} \rangle$, case two: $I^-[\gamma] \triangle I^-[\gamma'] \neq \varnothing$, and is not contained in $I^-[\gamma]$ or in $I^-[\gamma']$.

unreasonable, but do not have an epistemic hole.[22] So, if EHF is to be used as the sole condition for being a physically unreasonable spacetime (in the fairly minimal sense of not being artificially obtained by cutting and pasting), then some truncated spacetimes would be classified as physically reasonable. Hence, epistemic hole freeness on its own is insufficient to rule out all artificially obtained spacetimes.

6 An Additional Condition for Rescue?

A natural reaction to these examples of truncated spacetimes would be to write down an additional condition $\mathfrak{C}$ and declare that a physically reasonable spacetime satisfies EHF, EOO, and $\mathfrak{C}$. There are some intuitive candidates for $\mathfrak{C}$, but I will argue that none of them is satisfactory.

First, future-truncated Minkowski spacetime is extendible: perhaps epistemic hole freeness should be combined with inextendibility? $\mathfrak{C}$ would then simply be inextendibility. Note that in this case one would lose what is advertised as an important conceptual advantage of epistemic hole freeness: to apply inextendibility one has to consider some spacetime $\langle N, h_{ab} \rangle$ and its

[22] One way of interpreting the situation is the following: Many other hole freeness conditions are deemed unacceptable on the grounds of classifying Minkowski spacetime as having a hole, and thus having Minkowski spacetime as a counterexample. Epistemic hole freeness does not have Minkowski spacetime as a counterexample. But certain proper subsets of Minkowski spacetime do, intuitively, have holes; they do not, however, have epistemic holes. So epistemic hole freeness should be deemed unacceptable on the grounds of classifying a proper subset of Minkowski spacetime as hole free.

extension $\langle N', h'_{ab}\rangle$, making use of the conceptually problematic class of all physically reasonable spacetimes. And since avoidance of implicit commitment to such a distinction is (according to Manchak [2016a]—see also Manchak [2016b] for criticism of the inextendibility condition) among the good features of EHF, inextendibility with its modal character seems to be an unattractive candidate for $\mathfrak{C}$.

Second, note that all future-directed timelike curves in spacetimes $\langle M'^{i}, g'^{i}_{ab}\rangle$ for $i \in \{1, 2, 3\}$ are incomplete. One may then be tempted to take $\mathfrak{C}$ to be existence of at least one complete (in the standard sense that generalized affine parameter of the curve has unbounded range) inextendible, timelike geodesic. To see that this demand is too strong, consider (a) a big crunch scenario (in which the spacetime contracts and ends with a curvature singularity), or (b) the maximal globally hyperbolic region of Misner spacetime (oriented in such a way that the Cauchy horizon is to the future). In such spacetimes all causal geodesics are future incomplete, and thus are ruled out as physically unreasonable on the grounds of violating $\mathfrak{C}$. Even worse, a spacetime with a big crunch singularity and a point removed would be unreasonable, as would also be the original spacetime, both on the grounds of violating $\mathfrak{C}$.

Third, $\mathfrak{C}$ could be 'blowup-or-completeness': the demand that any inextendible past or future-directed timelike geodesics γ is either complete, or some curvature component is unbounded along γ. This certainly gets rid of truncated examples, in which there is no blowup along future incomplete geodesics (indeed, all curvature invariants vanish in the examples I have explicitly discussed); and it may allow for a big crunch-type scenario (since by definition one expects a curvature blowup in the big crunch). This condition by itself has no modal taint, but implies inextendibility, in the following sense: if curvature blows up along any future-directed timelike geodesic in spacetime $\langle M, g_{ab}\rangle$, then $\langle M, g_{ab}\rangle$ is (C^2-)inextendible. The converse is not the case: a spacetime may have no curvature blowup, but be future incomplete and inextendible. (The spacetime obtained by identifying past null lightcone of a point on S on Figure 4 and the annulus spacetime of Geroch and Horowitz ([1979], p. 258) are examples of that.)[23] It would be interesting to determine the exact strength of these sort of conditions as a hole freeness condition. Conditions in plural, for this quickly leads to a family of conditions: why not all causal geodesics, or all curves with finite acceleration? The trouble, however, is in (a) variety of examples of spacetimes that are (say) timelike complete but null incomplete (see discussion of related issues in Curiel [1998], Section 2), in which intuitions concerning holes become rather vague, and (b) apart from the question of

[23] I am grateful to an anonymous referee for reminding me of this fact.

whether this condition does the job, there is a separate and thorny issue of providing a compelling justification for it.

Whatever the outcome, note that even though epistemic hole freeness is satisfied in the globally hyperbolic region of Misner spacetime, 'blowup-or-completeness' is not, and so even the globally hyperbolic region of Misner spacetime would not be physically reasonable under such a $\mathfrak{C}$ (thus restricting further the set of physically reasonable spacetimes allowed by EHF). So for some initial data sets, their maximal globally hyperbolic developments are not physically reasonable according to 'blowup-or-completeness'; and so are Cauchy horizons associated with these data. In my opinion this means going back to square one: a strong form of determinism is being assumed *ab initio*.

This leads to the situation in which (generalized) EHF needs to be supplemented by some additional condition $\mathfrak{C}$ that (a) does not rely on modal clauses, (b) is satisfied in big crunch spacetimes, and (c) is violated in future-truncated Minkowski spacetimes. Unfortunately, I do not see a promising candidate for such a condition. Despite progress being made with the introduction of epistemic hole freeness, there is no satisfactory hole freeness condition available. Analogously to a sceptical analysis of the significance of energy conditions by Barcelo and Visser ([2002]), it seems that we are entering a twilight for hole freeness conditions.

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