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Many worlds: decoherent or incoherent?

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Abstract We claim that, as it stands, the Deutsch–Wallace–Everett approach to quantum theory is conceptually incoherent. This charge is based upon the approach's reliance upon decoherence arguments that conflict with its own fundamental precepts regarding probabilistic reasoning in two respects. This conceptual conflict obtains even if the decoherence arguments deployed are aimed merely towards the establishment of certain 'emergent' or 'robust' structures within the wave function: To be relevant to physical science notions such as robustness must be empirically grounded, and, on our analysis, this grounding can only plausibly be done in precisely the probabilistic terms that lead to conceptual conflict. Thus, the incoherence problems presented necessitate either the provision of a new, non-probabilistic empirical grounding for the notions of robustness and emergence in the context of decoherence, or the abandonment of the Deutsch–Wallace–Everett programme for quantum theory.

Keywords Quantum mechanics · Decoherence · Emergence · Probability · Everett interpretation · Many worlds

1 Introduction

In recent years, Everett-type approaches have assumed an eminent, if not quite pre-eminent, position among the various interpretations of quantum mechanics. This rise in the perceived persuasiveness of Everettian arguments can be related to two significant

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new developments. First, it was understood that decoherence could provide a foundation for an interpretation of the wave function in terms of a superposition of branches which, after forking off, remained effectively independent from each other. Second, it was suggested that a satisfactory Everettian interpretation of quantum mechanics could be based on a purely *subjective*[1] perspective on the probabilistic role of the wave function. According to this understanding, a connection between the wave function and probabilities can be extracted from the requirement that agents who find themselves in one branch of the Everettian universe act rationally.

This new, subjectivist understanding of Everettian quantum mechanics was given a formal basis through the work of Deutsch (1999) and Wallace (2002, 2007, 2009, 2012) (see also Saunders 2004); with their proofs demonstrating that, provided Everettian quantum mechanics is true, the Born rule can be established, under certain conditions, as being constitutive of any rational betting behaviour on future outcomes of quantum experiments.

Various critical responses to this Deutsch–Wallace–Everett (DWE) approach to quantum theory have been put forward. Many of these focus upon the the structure of the decision theoretic proof itself, and rely either on disputing the notion of rationality (Price 2010; Dizadji-Bahmani 2013), or on challenging the derivation of the Born rule as a unique/viable subjective probabilistic measure (Lewis 2006; Hemmo and Pitowsky 2007). Such internal criticisms rest upon implicitly accepting the viability of the DWE framework but questioning particular aspects of its implementation. There are also several lines of analysis that provide means for an *external* critique of the approach. One perceived problem relates to the role played by probabilistic empirical phenomena within the the DWE framework. *Prima facie*, within the DWE framework as it stands there seems no natural connection between the part played the Born weighting, as assigned to branches, and empirical statistical data that can be observed within a single world situation, and this might be seen to cast doubt upon the empirical viability of the approach (Albert 2010; Adlam 2014; Dawid and Thébault 2014). A second external line of criticism, closely related to that developed here, seeks to undermine the DWE claim of a *derivation* of the Born rule on the basis of a decoherence related circularity objection (Zurek 2005, 2010; Baker 2007; Kent 2010).

Thus, various authors have already found reasons to claim that the DWE approach has certain, major conceptual flaws. In the present paper, we want to make a stronger claim. We will argue that, as it stands, the DWE approach to quantum theory is *conceptually incoherent*. This charge is based upon the approach's reliance upon decoherence arguments that conflict with its own fundamental precepts regarding probabilistic reasoning in two respects. This conceptual conflict obtains even if the decoherence arguments deployed are aimed merely towards the establishment of certain 'emergent' or 'robust' structures within the wave function: To be relevant to physical science notions such as robustness must be empirically grounded, and, on our analysis this grounding

[1] Here the use of 'epistemic' rather than 'subjective' would perhaps be more consistent with the terminology in use within the philosophy of science. However, we will retain the original terminology to avoid confusion. The more recent argument from Wallace (see in particular p. 249 of Wallace 2012) that within his program the probabilities *are* actually objective, over and above the decision theoretic basis given to the Born rule, will be considered in Sect. 3.2

can only plausibly be done in precisely the probabilistic terms that lead to conceptual conflict. Thus, the incoherence problems presented necessitate either the provision of a new, non-probabilistic empirical grounding for the notion of robustness and emergence in the context of decoherence, or the abandonment of the DWE programme for quantum theory.

2 The Deutsch–Wallace–Everett approach to quantum mechanics

2.1 Probability and decoherence

Assume that an agent carries out a quantum mechanical experiment with a number of different possible outcomes. Broadly speaking, Everett-type interpretations of quantum mechanics relate each one of the possible *distinct* outcomes to a particular branch within a 'many worlds' universe. We understand this universe as including a representation of every physically possible outcome of this and every other experiment (where, under some understandings, 'experiments' are taken to include a variety of spontaneous physical processes); and we understand these representations to exist within an ordered branching tree structure, where a set of worlds following on from every distinct outcome of every experiment can be identified with its own independent 'sub-tree'. If we assume that our experiment is the initial branching event (which is equivalent to focusing in on one particular sub-tree), then immediately subsequently we can think of a many worlds universe as constituted purely by an array of worlds, with one world corresponding to each possible outcome of our experiment.

Now, since an instantiation of our agent exists in each one of these worlds, the agent's chances of finding themselves in one specific world after the experiment might be expected to be directly determined by the proportion of worlds within which the relevant experimental outcome obtained. In the tree picture, this is of course just equivalent to counting the number of branches with the relevant outcome that fork off from the initial state of the experiment, and then dividing this by the total number of branches. Intuitively then, one may expect that within an Everettian picture the probabilities relevant to the outcomes of quantum mechanical experiments can be obtained simply via such a *branch counting* methodology. However, it transpires that if we consider an agent who adopts a branching counting strategy, they will not, in general, reproduce the probabilities provided by quantum mechanics (see e.g. Rae 2009). Thus, one plausible requirement for a full understanding of probability in an Everettian setting would be for arguments that provide a good reason to discount the viability of the (seemingly intuitive) branch counting strategy.

According to the Deutsch–Wallace argument (on this see in particular (Wallace 2007, p. 9; Wallace 2012, p. 4.3) it is decoherence effects—which, as discussed below, are in any case needed to ground the discreteness of the branches—that provide an argument towards the impossibility of extracting probability statements on the outcomes of quantum experiments from branch counting. Since there is no unique way to specify at which stage two branches have fully decoupled and therefore must be counted separately, it is impossible to specify one definitive branching structure for a quantum process. This in turn implies that no definitive probabilistic conclusions can be drawn from branch counting since the number of branches is inherently indeterminate. Once the connection between measurement probabilities and the branching structure is broken, however, the problem of an incompatibility between the two concepts disappears.

Given that the 'naïve' branch counting stratgey has been discounted (either by such a decoherence based argument or by more abstract lines of reasoning such as rejecting the principle of indifference upon which it is based) the Everettian requires an alternative methodology leading to the correct Born rule measure. Since no known strategy is left for the Everettian to extract the probabilistic characteristics of quantum measurements from the *objective* structure of the wave function alone. Where by *objective* structure we mean the structure of the wave function as it appears in a physically realized solution of the equations of quantum mechanics.[2] It then remains to be explained how measurement probabilities are related to Everettian QM at all. The second step in the Deutsch–Wallace argument is to implement probabilities at a *subjective* level by introducing a decision theoretic analysis from the perspective of an agent in one branch of the wave function. Immediately before a branching event it is rational, taking the perspective of an embedded agent, for me as an agent to consider the interests of my future selves in the various branches. Even though there is no fact of the matter about which future self *I* will become, I should believe that I will become exactly one of them and, given it is uncertain which one I will become, my current deliberations (in particular betting behaviour) should then be dictated by the combination of my preference between the various possible eventualities and the extent to which I think they are likely to be relevant to *me*.

The key to the Deutsch–Wallace argument is then to demand that the ordering an agent in such a position of *subjective uncertainty*[3] can apply to their preferences between the various outcomes, is constrained by a set of basic rationality criteria (with the specifics differing slightly between the various proofs). These rationality criteria are taken to be necessary for consistent reasoning and include both seemingly self-evident conditions, such as coherence, and several more disputable principles, such as diachronic consistency. Given this basis, the principal achievement of Deutsch–Wallace proofs is then the demonstration that, up to linear transformations, we can fix the Born rule as the only consistent subjective probability measure available to our deliberating agent.

As noted above, questions have been raised with regard to both the cogency of the rationality criteria chosen, and the uniqueness of the relevant proofs. We will not here add to the already substantial literature in this line. Rather, we will, for the sake of argument, assume that once the first step of breaking the connection between objective branching structure and probabilities via decoherence arguments has been achieved, a derivation of the Born rule as a subjective probability measure along

[2] Once more we note the existence of the more recent argument from Wallace (2012, p. 249) relating to 'objective' quantum probability and refer to the reader forward to §3.2 for detailed consideration of this point.

[3] See Saunders (1998) for detailed philosophical consideration of this idea and Wallace (2012, p. 10.4) for consideration of its changing role within Wallace's arguments. An alternative 'objective determinism' understanding of personal identity in this context has also been considered, see Wallace (2007) and references therein for details. This distinction is not important for the purposes of the arguments given here.

decision theoretic lines can be established. Thus, from our perspective, the crucial issues will turn on whether or not the decision theoretic understanding of quantum probability that is endorsed by the DWE approach is consistent with the approach's appeal to decoherence effects.

2.2 Ontology and decoherence

The central Everettian claim, dating back to the work of DeWitt and Everett himself, is that that theirs is the interpretation which is naturally implied by the bare mathematical formalism of quantum theory—in particular, the unitary evolution of the quantum state is taken to be a complete description of quantum dynamics. As we have seen, the Everettian ontology is constituted by some form of branching tree structure with each branch corresponding to a discrete outcome of events. Quantum mechanically the instantaneous state of such a branching structure closely resembles a proper mixture of pure states: the branches can be understood as decoupled from each other. It is from the perspective of an agent situated within this ontology (or something that closely approximates it) that the Deutsch–Wallace argument, based upon *classical* decision theory, is applied.

Within the quantum formalism there can, of course, exist states which cannot be understood in terms of proper mixtures of pure states. When considering quantum systems we can, and generically do, encounter quantum states which correspond to non-classical interference between distinct outcomes. If such coherence effects occurred between Everettian branches of the wave function, that would amount to 'spooky' deviations from the predictions of quantum mechanics which have not been observed. Large coherence effects would simply destroy the notion of a discrete branching tree structure and, therefore, would be inherently in conflict with any application of classical decision theory on a branch of the Everettian universe. In other words, the absence of substantial coherence effects between branches is a precondition for any notion of a classical rational agent existing within a many worlds framework. Thus, we can insist that proponents of Everettian quantum theory in general, and those who apply classical decision theory within it in particular, must provide a mechanism for removing or discounting coherence effects between branches. In order to remain consistent with their own precepts they must do this in the context of a formalism where the fundamental dynamics is unitary.

For that reason, the Everettian approach crucially relies on the understanding that decoherence does arise in large quantum systems: a quasi-classical branching structure can be shown to *emerge* from a fundamentally quantum mechanical underlying reality based on the unitary evolution of the system. Thus, according to Wallace (2010), the Everettian can claim *both* that the quantum state is all there is *and* that there exist 'worlds' which are mutually isolated structures instantiated within the dynamical state. We can thus understand decoherence as a 'dynamical process by which two components of a complex entity (the quantum state) come to evolve independently of one another, and it occurs due to rather high-level, emergent consequences of the particular dynamics and initial state of our Universe' (Wallace 2010, p. 10).

Crucially, this means that rather than eliminating coherence between branches, we are supposed to think of decoherence processes as merely rendering the contribution of coherence between branches to the quantum state of complex systems *negligible for all practical purposes*. Thus, the discrete branching ontology fundamental to the many worlds picture in general, and the Deutsch–Wallace subjective probability approach in particular, is *not* claimed to be established by a direct decoherence algorithm starting with the quantum state. Rather, it is claimed, the ubiquity of decoherence phenomena for reasonably complex systems on reasonably long time scales means that one may justifiably treat the quantum mechanical description of high-level systems as *effectively* describing a many worlds ontology.

3 Decoherence, circularity and incoherence

It is crucial to the consistency of the DWE framework that by by invoking 'decoherence theory' the Everettian is not introducing formal or conceptual machinery to which they are not entitled. In particular, no notion of probability or ontology that is in direct conflict with the relevant Everettian notions can consistently be relied upon to argue towards those notions, otherwise the scheme would be conceptually inconsistent. Furthermore, given that the proponents of the DWE scheme claim that the decision theoretic proofs enable them to derive the Born rule within the post-decoherence many worlds structure, the non-circularity of such a derivation depends on the possibility of invoking decoherence effects without the need to assume the Born rule. In this section we will investigate three objections—one of circularity, two of incoherence—all based upon a close analysis of the details of decoherence.

3.1 Circularity and the Zurek contention

Let us follow the account given by Zurek (2003) and consider the case of a simple set-up consisting of a two state, spin-half system $\mathcal{S}$ and a quantum detector $\mathcal{D}$. If we represent the two states of the detector as $|\uparrow\rangle$ and $|\downarrow\rangle$ then we can simply consider a detector (also with two states) which is such that:

$$|\uparrow\rangle \mid d_{\downarrow}\rangle \rightarrow |\uparrow\rangle \mid d_{\uparrow}\rangle \tag{1}$$

$$|\downarrow\rangle \mid d_{\downarrow}\rangle \rightarrow |\downarrow\rangle \mid d_{\downarrow}\rangle \tag{2}$$

Before the interaction between system and detector we can represent the initial state of the composite system as:

$$\mid \Phi^{I}\rangle = (\alpha \mid\uparrow\rangle + \beta \mid\downarrow\rangle) \mid d_{\downarrow}\rangle \tag{3}$$

and after the interaction the quantum state of the composite system is:

$$\mid \Phi^{C}\rangle = \alpha \mid\uparrow\rangle \mid d_{\uparrow}\rangle + \beta \mid\downarrow\rangle \mid d_{\downarrow}\rangle \tag{4}$$

The process that connects these two states is simply evolution according the Schrödinger equation; such states are, therefore, given by the quantum formalism on its own. That this quantum state can *not* be understood as a proper mixture of distinct outcomes can be seen easily by considering the relevant density matrix:

$$\rho^C = |\alpha|^2 | \uparrow\rangle\langle\uparrow || d_\uparrow\rangle\langle d_\uparrow | + \alpha\beta^* \, |\uparrow\rangle\langle\downarrow || \, d_\uparrow\rangle\langle d_\downarrow | \\ + \alpha^*\beta \, |\downarrow\rangle\langle\uparrow || \, d_\downarrow\rangle\langle d_\uparrow | + |\beta|^2 | \downarrow\rangle\langle\downarrow || d_\downarrow\rangle\langle d_\downarrow | \quad (5)$$

The off diagonal elements of this matrix would correspond to coherence phenomena which render any separation of branches impossible. Compare this to a reduced destiny matrix without the off diagonal elements:

$$\rho^R = |\alpha|^2 | \uparrow\rangle\langle\uparrow || d_\uparrow\rangle\langle d_\uparrow | + |\beta|^2 | \downarrow\rangle\langle\downarrow || d_\downarrow\rangle\langle d_\downarrow | \quad (6)$$

This we *can* interpret as a proper mixture of pure states and so taking it ontologically seriously would seem to imply the discrete branching structure which the Everettian requires. The idea is then that we can understand decoherence as a process by which $\rho^C \Rightarrow \rho^R$ through the (unitary) interaction of the composite system and the environment. One assumes an interaction between system and environment of the form:

$$| \Phi^C\rangle \mid \mathcal{E}_C\rangle = \alpha \, |\uparrow\rangle \mid d_\uparrow\rangle\mathcal{E}_\uparrow\rangle + \beta \, |\downarrow\rangle \mid d_\downarrow\rangle\mathcal{E}_\downarrow\rangle = | \Psi\rangle \quad (7)$$

The claim is that when the states of the environment corresponding to the detector states are orthogonal—i.e. we have that $\langle\mathcal{E}_i|\mathcal{E}_{i'}\rangle = \delta_{ii'}$—the density matrix for the original detector-system combination post-decoherence is given by the expression:

$$\rho^{PD} = Tr_{\mathcal{E}}|\Psi\rangle\langle\Psi|\Sigma_i\langle\mathcal{E}_i|\Psi\rangle\langle\Psi|\mathcal{E}_{i'}\rangle \quad (8)$$

$$\cong |\alpha|^2 | \uparrow\rangle\langle\uparrow || d_\uparrow\rangle\langle d_\uparrow | + |\beta|^2 | \downarrow\rangle\langle\downarrow || d_\downarrow\rangle\langle d_\downarrow | = \rho^R \quad (9)$$

Environment induced decoherence does not fully *eliminate* the off diagonal elements, but it *re-scales* them to vanishingly small amplitudes *as given by the associated Born weights* (see the explicit formulas of Zurek (2003) as well as references therein). In order for environment induced decoherence to give us licence to ignore quantum effects between branches—and therefore justify an interpretation of the quantum state in terms of a discrete branching structure—we need one more step: we have to find a reason for connecting the smallness of the off-diagonal terms to the privileged status of discrete branching structure within the post-decoherence quantum formalism.

There are two obvious strategies for doing this, the Deutsch–Wallace type Everettian can either: (a) argue directly that smallness is connected to neglectablity, and therefore that a discrete branching structure is implicit in the post-decoherence quantum formalism; or (b) argue (more indirectly) that the relative smallness of the formal feature related to coherence (i.e. off-diagonal elements) implies the *robustness* of an *emergent structure* corresponding to the post-decoherence quantum formalism.

The easiest strategy to counter is (a). On an intuitive level one might think that the elements within a mathematical model for a physical system which are relatively much much smaller can be interpreted as less physically important, and thus neglectable 'for all practical purposes'. However, such an approach only makes sense once we have an understanding as to what the relevant numbers—in our case the entries in the density matrix—actually mean. The simple statement that after evolving a quantum system relative to a background environment we can consistently attribute small numbers to off-diagonal elements, has no relevance as long as we do not know what those numbers stand for. Obviously, we cannot always neglect small parameter values. Clearly, it would not make sense to neglect the early stages of the universe on the grounds that the values of the time parameter are small, nor to consider the electroweak interaction as relevant only when the value of the distance parameter is high. In order to neglect small values in favour of larger values, we have to establish that the magnitude of the corresponding variable is related to the entry's effect on the measurement to be performed. Since experimental testing and the entries in the density matrix are related in terms of the probabilities for measuring certain outcomes, in order to establish the negligibility of small entries in the density matrix we must introduce the Born rule.

Thus we see that strategy (a) obviously leads straight back to the Born rule. The more plausible strategy is clearly (b), and this is, in fact, precisely the argument deployed by Wallace himself (see Wallace 2010, 2012). We will briefly outline our response to (b) here—but delay a detailed discussion, including consideration of the analogy with effective field theory, to Sect. 4. The claim underlying strategy (b) is that decoherence tends to produce an 'emergent' branching structure in the evolving quantum state describing a suitably complex system and its environment: specific components of the quantum state effectively evolve independently of one another and this indicates the persistence of a 'robust' branching structure within the wave function. The key idea is that rather than connecting the smallness of the interference terms directly to negligibility as in strategy (a), the smallness is taken as a measure of effective independence of components corresponding to on-diagonal density matrix elements. In essence, '[d]ecoherence causes the universe to develop an emergent branching structure. The existence of this branching structure is a robust (albeit emergent) feature of reality' (Wallace 2012, p. 101) The key question, of course, must be: what do the terms 'effective independence', 'emergent' and 'robust' mean here? On our analysis (see Sect. 4 for more detailed argument), these notions can only be of use to physical science if the sense in which they may be *empirically grounded* is made precise.

What seems to us the only plausible methodology for such empirical grounding is one in which the scaling of a relevant parameter, in this case time, can be connected to the significance of terms within the formalism describing the 'emergent' ontological domain. To make this connection we need a physical basis for making an interpretational step regarding the treatment of structures as 'effectively independent', 'emergent' and 'robust'. It is difficult to see how such empirical grounding can be provided without assuming some version of the Born rule as a measure of probability of experimental outcomes. Thus, save for the provision of new, non-probabilistic empirical grounding, the proponent of the DWE scheme must, in deploying strategy b), still assume the Born rule at some level.

Following the influential analysis of Zurek (2005, p. 25) we can state the crucial contention implied by our analysis of both (a) and (b) simply as:

Z Decoherence effects cannot be established without an independent prior derivation (or assumption) of the Born rule.

A significant consequence of **Z** for the Deutsch–Wallace type derivation of the Born rule in terms of subjective probability has been emphasised first by Zurek himself (Zurek 2005, p. 25), and then by Baker (2007) and Kent (2010). In essence all three authors argue that, since decoherence must be assumed in order to ground the DWE classical decision theoretic framework, and since by invoking decoherence one is inevitably assuming the Born rule, the derivation of the Born rule within the DWE scheme is inherently circular. Kent summaries this circularity objection as follows:

> Even if one could show, as Wallace claims, that agents defined within that ontology are rationally justified in using the Born rule as a calculus for decisions, it would seem incorrect to portray this argument as a derivation of the Born rule within Everettian quantum theory. Wallace's argument should rather be understood as attempting to show something weaker: that the Born rule re-emerges as output (albeit, to be fair, in an interesting and non-obvious way) if assumed as input. (Kent 2010, p. 17)

Similarly, Baker specifically builds his detailed presentation of the objection around the need to assume decoherence before we can consider the perspective of an agent making decisions with regard to the distinct outcomes of quantum process (i.e. an agent playing a 'quantum game'):

> I claim that, since the employment of decoherence to identify branches depends upon the unlikeliness of low-weight events, the framework of quantum games in which the theorem is formulated presumes its conclusion. Unless the Born rule (or some similar rule permitting inferences from low weight to low probability) holds, there are no quantum games in Everett. (Baker 2007, p. 21)

We thus see that the Zurek-Baker-Kent circularity objection to the DWE programme follows in straightforward terms from the combination of the Zurek contention, **Z** above, together with the fairly self-evidently necessary requirement for proponents of the DWE scheme to assume decoherence prior to launching into decision theoretic calculus that eventually leads to the relevant probability measure: Without decoherence there is no classical agential perspective, and without this there is no basis for a Deutsch–Wallace type derivation of the Born rule. We can formalise the situation as follows:

Z Decoherence effects cannot be established without an independent prior derivation (or assumption) of the Born rule.

P1 Decoherence effects are a necessary precondition for one to assume the perspective of an agent making decisions (i.e. playing quantum games) within a world of distinct outcomes, corresponding to the discrete branches of the many worlds ontology.

P2 Adopting this agential perspective is necessary prior to any Deutch–Wallace type derivation of the Born rule as a subjective probability measure.

Therefore:

C Deutsch–Wallace type derivations of the Born rule as a subjective probability measure are inherently circular since they involve the prior assumption of the Born rule in order to establish the Born rule

Thus there exists a seemingly robust challenge to any Everttian claim of being able to *derive* the Born rule based upon arguments of the Deutch–Wallace type. For all that, even if the Everettians are forced to accept that they cannot, as claimed, *derive* the Born rule[4] the Deutsch–Wallace argument might seem to at least suggest consistency within the framework, and to possibly also give an insight into the much discussed connection between objective and subjective notions of probability (Saunders 2005). However, as claimed above, the conceptual problems posed by decoherence for the neo-Everettian approach go beyond an objection merely on the grounds of circularity.

3.2 Not just circular but incoherent

In the analysis above argued that decoherence processes do not provide the foundations necessary to establish the DWE scheme without the prior assumption of a notion of probability as given by the Born rule. This is because, although such processes may act to re-scale the weightings of the off-diagonal elements of the density matrix to be very small, the interpretation of the smallness of those values as indicating either: (a) the neglectability of the corresponding component of the wave function; or (b) the robustness of branching structures within the wave function, will in the end rely on the prior assumption of the Born rule. (We again refer the reader forward to Sect. 4 for further discussion of option b). Thus we see the basis for the circularity charge regarding the Born derivation, as made in the literature.

Closer analysis reveals, however, that the problems for the DWE scheme are actually worse than circularity: the derivation of the Born rule within the DWE framework is in fact incoherent with the derivation of decoherence. The crux lies in the specific character of the decision theoretic notion of probability used in the DWE approach.

The decision theoretic approach defines quantum probability via the specification of rational betting behaviour for an agent on an approximately separated branch of the wave function. Since, according to Wallace, the only consistent betting behaviour corresponds to the Born rule, Born weights can be identified with probabilities. However, such a decision theoretic notion of quantum probability constitutes a meaningful concept only in contexts where it is possible to take the perspective of an agent on a branch of the wave function that shows near-classical behaviour. Whilst we can reasonably consider generalising the argument to branches which potentially—rather then actually—contain agents, we *cannot* consistently apply the argument to ‘branches’ which are not approximately separated. In such cases strong effects based on coherence with other ‘branches’ would destroy the discrete structure and render the concept of a bet by an agent on a distinct outcome inappropriate. This means

[4] They would, in this respect, then be at the disadvantage of their Bohmian rivals who’s own derivation would not seem to be susceptible to the same objections on the grounds of circularity (Valentini and Westman 2005).

that a scope of a version of the Born rule derived based on the decision theoretic argument is limited to comparing outcomes on (approximately) separated branches of the wave function. It offers no basis for a probabilistic assessment of components of the wave function which do not show semiclassical behaviour. The definition of the Born rule in terms of rational betting cannot be extended in a consistent way to cover those contexts since the betting behaviour of a non-semi-classical agent is not well defined.

The decision theoretic approach thus must distinguish between two kinds statements. (i) There are statements about the observations and predictions of agents living (or potentially living) in branches of the wave function. In that regime the decision theoretic concept of probability can be applied. (ii) There are statements which compare components of the overall wave function including those which do not correspond to approximately separated branches. Such comparisons cannot be based on the decision theoretic concept of probability. The separation between (i) and (ii) introduced by DWE distinguishes the approach from other interpretations of quantum mechanics such as spontaneous collapse or Bohmian QM. In the latter cases, the viability of the Born rule is understood as a characteristic feature of the dynamics of the entire physical system without limitations to a selective part of the wave function.

As discussed in the previous subsection, the derivation of decoherence is based on an application of the Born rule. More specifically, it is based on applying the Born rule in order to get a probabilistic evaluation of all components of the overall wave function. The crucial step is to establish that off-diagonal elements in the density matrix get so small that they can be neglected. This application of the Born rule, however, is clearly of kind (ii), that is of the kind that cannot be based on the Born rule extracted from the decision theoretic argument.

We have thus established that DWE does not face a simple circularity problem. Circularity would mean that DWE offers a deduction of the Born rule that is based on assuming the very same rule already in the context of decoherence. But DWE merely deduces a limited version of the Born rule, call it *Born rule*$_{weak}$ that is insufficient for deducing decoherence. The latter deduction requires the more powerful full version of the Born rule as it is assumed in other interpretations of QM.

Why does this amount to an outright inconsistency? Couldn't one still rely on a broader notion of the Born rule that does not hinge on the decision theoretic argument and is strong enough for providing the basis for decoherence? The most direct way of identifying an inconsistency in DWE's treatment of the Born rule is based on taking Wallace's exposition of DWE at face value. In Wallace (2012) Wallace explicitly rules out an implementation of the Born rule that does not rely on the decision theoretic argument. The decision theoretic argument is taken to be the only basis for introducing probabilistic statements in an Everettian framework. This understanding is most clearly exemplified by Wallace's idea to call the quantum probabilities extracted from the decision theoretic argument 'objective' since there is no other basis for identifying probabilities in quantum processes (Sections 4.9–4.12 of Wallace 2012). The argument is based on the *Principal Principle* due to Lewis (1980) that can be understood as a means of restricting the subjective probabilities, when these are understood as rational credence:

Principal Principle: For any real number x, a rational agent's personal probability of an event E conditional on the objective probability of E being x, and on any other background information, is also x.

Wallace argues that the Principal Principle can serve as a definition of *objective* probability in terms of *subjective* probability if no independent basis for defining *objective* probability is available. He concludes that, since there is indeed no way to implement *objective* quantum probabilities directly within the Everettian scheme, we are allowed to call objective quantum probability any variable i) that is attributable to Everettian structure and ii) the values of which are consistent with the deployed subjective quantum probabilities according to the Principal Principle. Specifically, this would justify calling Born weights objective probabilities. The idea is that the 'branch weights are objective', due to their role in the theory's structure, and 'what makes it true that branch weight = probability is the way in which branch weights figure in the actions of (ideally) rational agents' (Wallace 2012, p. 249).

That means, however, that DWE *must not* rely on any version of the Born rule that is more widely applicable than the *Born rule*$_{weak}$ that was deduced based on the decision theoretic argument.

Let us spell out carefully the implications of Wallace's position. Every component of the wave function can be characterized by its Born weight. Wallace suggests that we are allowed to call these Born weights objective quantum probabilities. Still, an objective quantum probability in this sense has genuine probabilistic implications *only* in the decision theoretic sense, that is, *only* from the perspective of an agent on a semi-classical branch with respect to events on the same branch. *In all contexts other than that, the Born weight, vulgo objective probability, must not be used for extracting probability statements on quantum processes.*

Thus, we have a clear case of incoherence. Decoherence requires a strong form of the Born rule. The DWE approach, however, only provides the insufficient $Born\ rule_{weak}$ and does not allow assuming the Born rule at a more general level. Formalising this incoherence in analogy with our previous formulation of of the Zurek contention, we get:

Z* Decoherence effects cannot be established without an independent prior derivation (or assumption) of the Born rule *with respect to a full probabilistic assessment of the wave function*.

P1 Decoherence effects are a necessary precondition for one to assume the perspective of an agent making decisions (i.e. playing quantum games) within a world of distinct outcomes, corresponding to the discrete branches of the many worlds ontology.

P2 Adopting this agential perspective is necessary prior to any Deutsch–Wallace type derivation of the Born rule.

P3 A decision theoretic argument can only be the basis for a $Born\ rule_{weak}$ the viability of which does not provide a basis for a full probabilistic assessment of the wave function.

P5 The DWE scheme explicitly assumes that there is no other basis than the decision theoretic argument for implementing the Born rule in the Everettian framework.

Therefore:

I1 The proponent of the DWE scheme must simultaneously assume: (i) that the Born rule provides a probability measure valid with respect to the entire wave function in order to appeal to the decoherence effects necessary to establish the Everettian ontology; and (ii) that there is no probability measure available whose viability extends beyond semi-classical branches which contain (or potentially contain) agents.

The interpretation of the wave function, necessary for decoherence, as associating a probability to all of its components, thus is inconsistent with the core tenet of the subjectivist approach: the provision of a decision theoretical foundation for a notion of probability whose viability is constrained to semi-classical branches which contain (or potentially contain) agents branch.

It still seems, however, that there might be an easy way our for the proponent of DWE: could they not just concede the necessity of some more general Born rule which provides the basis for decoherence and is independent of the decision theoretic argument? In that case, we would have to call that general Born rule 'objective', drop the principal principle argument and call the Born rule derived based on decision theory the 'subjective' Born rule. The problem is that such a step would render the entire subjective approach superfluous. Once we assume that a more general Born rule must be assumed beside the weak Born rule derived based on decision theory, that general Born rule must have probabilistic implications for the observations of an agent on a branch as well. The resulting probabilities of measurement outcomes would determine the agent's betting behaviour without any recurrence to Wallace's decision theoretic argument.

3.3 Incoherence and ontological prejudice

A further, incoherent, consequence of the chain of reasoning just given is that by appealing to a probabilistic notion of Born weights in order to justify neglecting off diagonal elements, one would seem to have accepted such an interpretation of the Born weights of distinct states also. Thus we have that:

I2 In (effectively) eliminating off diagonal elements due to their low Born weight the Everettian must either also (effectively) eliminate similarly low weighted distinct states and thus subvert their own position or simply apply a principle of *ontological prejudice*, such that coherence effects are eliminated simply on the grounds of being coherence effects, irrespective of their Born weighting.

Like other forms of prejudice, the essential problem is one of arbitrary discriminatory judgment: there is no basis for privileging one state over the other apart form belonging to a class that has been identified as privileged in principle. This second incoherence problem thus relates to the conceptual basis for justifying the many worlds ontology in the context of decoherence: On the one hand, in order to allow throwing out the off-diagonal elements and therefore provide the framework for defining an agent in a branch of the wave function, the Everettian must assume that the Born weights represent probabilities that can, if very small, justify neglecting the corresponding states. On the other hand, in order to justify a many worlds branching ontology suitable for the relevant decision theoretic proof, they must insist that all distinct states—even those with vanishingly small Born weights—must be considered when analysing the objective branching structure of the wave function. In order for the wheels of the *classical* decision theoretic proof to even start turning it is essential for that off-diagonal elements to have been neglected; but in order for the proof to reach the desired result it is essential that all decisions corresponding to discrete outcomes are included in the preference ordering—even those of extremely small weighting. Specifically, this is because neglecting any of the possible discrete outcomes would violate Wallace's 'Branching availability' axiom, see (Wallace 2012, p. 166).

Thus, prima facie, the situation seems completely inconsistent unless the Everettian can provide an alternative description of decoherence in which neglecting off-diagonal elements is not based on the probabilities given by the Born rule. Without such an alternative understanding of decoherence they would seem to either have to reject their own ontology (and thus give up their own interpretation!) or fall back to the endorsement of outright ontological prejudice: irrespective of Born weights the off-diagonal elements are neglected in favour of the distinct states merely on the virtue of the fact that they are coherence effects. Thus, we have our point **I2**.

One possible line of response that could be made in defence of the DWE scheme is that the specifics of the decoherence effects are such that our second incoherence argument, relating to ontological prejudice, can be rebuffed. Our ontological prejudice charge is based upon the combination of the effective elimination of off-diagonal elements, with the retention of discrete states of comparably low Born weighting. Now, here the defender of the DWE approach might respond that, in practice, even the most unlikely discrete states will have a Born weighting orders of magnitude higher than the highest weighted off-diagonal elements due to the extremely high level of suppression that decoherence effects enact. Whereas, the weightings of the off-diagonal elements fall off sharply with time, those associated with the least likely on-diagonal elements will only decrease according to the proliferation of branching due to measurement. Thus, there is no need for prejudice to establish as emergent a macro-regime that excludes off-diagonal elements but includes very unlikely discrete states.

Such a chain of reasoning seems to us plausible—and might, in fact, even provide a legitimate basis for grounding *some* variants of a many worlds ontology. However, in the specific context of the notions of branching and measurement defended by Wallace (2007) such a counter argument to our second incoherence charge is simply not available. Within the Wallace approach, branching is not understood as being solely (or even principally) linked to deliberate human measurement events, rather it is understood as 'completely ubiquitous' (Wallace 2007, p. 21) and to be mainly driven by random physical processes (Wallace 2010). It is this notion of measurement that, when combined with decoherence, is intended to ground a discrete branching ontology within which the actual number of branches is fundamentally indeterminate. Given such a framework, the counter-argument to the prejudice charge outlined above fails: it is only by having an estimate of the number of branches on a given timescale (and thus also a concrete model of the branching process) that we can establish a quantification of the Born weighting associated with the lowest weighted discrete branch at that timescale, and without this there is no sense in which a comparison with the (suitably suppressed) coherence effects can be made. And, of course, if one *does* admit a definite, enumerable branching structure, one then needs a new strategy for addressing the conflict between probabilities based upon branch counting, and those based upon the Born rule. Thus, even if one can construct an explicit model that justifies neglecting coherence effects without prejudice, because it is predicated upon establishing a definite number of branches, the counter-argument would also serve to weaken the conceptual foundations upon which the scheme is built by re-admitting a rival probabilistic scheme into the ring.

A simple example will further illustrate each of our two strong claims of incoherence. Let us again consider a simple two state system together with the detector as described above. Given that we have an appropriately repeatable preparation methodology, we can consider an arbitrarily large number of identical trials, n. After the n-trials, and assuming a relatively small n, decoherence involves us associating to components of the quantum state which display extremely small interference effects relatively high Born weights, and to components of the quantum state which display interference effects which are *not* extremely small (i.e. those displaying coherence effects) very low Born weights. By deploying these Born weights for a probabilistic comparison of all components of the wave function, we could reasonably argue that the components of the quantum state which displaying coherence effects should be neglected. However if we do this and then develop a DWE type understanding of quantum probabilities on the basis of the decision theoretic argument, it seems we would have to *forget* about the role of quantum probabilities beyond a semi-classical branch which contains (or potentially contains) agents. This is the inconsistency problem **I1**. Let us then consider the situation for very large n, in that context we would have some components of the quantum state which display extremely small interference effects but which still have vanishingly small Born weights—e.g. all trials result in $|\uparrow\rangle$. Neglecting for the moment the inconsistency problem, if the Everettians rely on Born weights in order to effectively eliminate certain components of the quantum state on the grounds of the low probability, then they must also eliminate, for large n, 'all up' components on the grounds of them also having very low probability. But this the Everettian cannot do if they are to maintain their many world ontology. Moreover, as noted above, to exclude such branches would explicitly contradict the Wallace's 'branching availability' axiom (Wallace 2012, p. 166). Thus, ontological prejudice as described above above becomes a necessity and we have **I2**.

4 Emergent worlds?

The principal neo-Everettian bulwark against charges of conceptual incoherence, such as those presented above, must be expected to be some form of argument from emergence. In his Wallace (2012), Wallace specifically responds to the weaker circularity version of our objection in such terms. In essence, the counter argument is that one may understand the Born weights as 'telling us when some *emergent structure* really is *robustly present*' [italics added]. This coincides with the treatment of Wallace (2010), where a discrete branching structure is understood as a *robust yet emergent feature of reality* (p. 15).

One may thus naturally anticipate a counter argument to our charges of incoherence that seeks to deflate the prior use of the Born measure by connecting it to the robustness of an emergent structure, rather than a universally applicable probability rule. The notion is that one may say that the many worlds branching structure is robustly present in the quantum mechanical wave function as a structural characteristic in the same sense as tigers, phonons and galaxies are robustly present in the relevant underlying microphysics, but in making such a statement one need not apply the Born *rule*—i.e. interpret the Born measure in a probabilistic sense. One claims that decoherence tends to produce an 'emergent' branching structure in the evolving quantum state describing a suitably complex system and its environment precisely because specific components of the quantum state 'effectively evolve independently' of one another and this indicates the persistence of a 'robust' branching structure within the wave function at an 'emergent' level on ontology.

Wallace in fact explicitly makes the claim that emergence of robust properties can justify neglecting small contributions without reference to probability. He does this on the basis that small contributions have little impact on the robust properties. Consider the following dialogue in Wallace (2012, pp. 253–254):

> SKEPTIC: Isn't there something a bit circular about your whole position here? First you appeal to decoherence theory to argue that a branching structure is approximately realised—which is to say, is realised to within errors of small mod-squared amplitude. Then you appeal to decision theory, or symmetry, or whatever, to explain why the mod-squared amplitudes in that branching structure are probabilities. But surely the only thing that justifies regarding an error as small if it's of small mod-squared amplitude is the interpretation of mod-squared amplitude as probability.
> AUTHOR: Not really. We can think of the significance of the Hilbert space metric as telling us when some emergent structure really is robustly present, and when it's just a 'trick of the light' that goes away when we slightly perturb the microphysics. (Remember that the Hilbert space norm is a perfectly objective feature of the physics, prior to any considerations of probability.)
> SKEPTIC: But that's just the same thing again. What makes perturbations that are small in Hilbert-space norm 'slight', if it's not the probability interpretation of them?
> AUTHOR: Lots of dynamical features of the theory. Small changes in the energy eigenvalues of the Hamiltonian, in particular, lead to small changes in quantum state after some period of evolution. Sufficiently small displacements of a wavepacket lead to small changes in quantum state too. Ultimately, the Hilbert-space norm is just a natural measure of state perturbations in Hilbert space, and that naturalness follows from considerations of the microphysical dynamics, independent of higher-level issues of probability.

Let us spell out the hypothetical skeptic's worry a little more clearly for him. The skeptic argues that neglecting small corrections to a result (in the given case, the emergence of a branching structure) must be based on an interpretation of those small corrections in terms of their small empirical implications. Knowing that quantum mechanics makes probabilistic predictions of measurement outcomes, the skeptic infers that any such relation between the smallness of corrections and the smallness of empirical implications should be based on on a probabilistic interpretation already, which leads to a circularity.

Wallace answers that neglecting small corrections may be justified without reference to probability just by noticing that those corrections have only slight influence on 'lots of dynamical features of the theory'. The robustness of those features allows us to neglect contributions which have little effect on them.

What we are to make of this claim is far from clear. We might read it in two different ways. First, we may take Wallace to suggest that the fact that the corrections in question have only small effects on the stated 'dynamical features of the theory' justifies neglecting these corrections based on the understanding that the stated dynamical features are themselves empirically relevant and small changes of the dynamical features translate into small empirical changes. On that reading, the question arises how the empirical relevance of the dynamical features can be understood. If (as canonical quantum mechanics would suggest, of course) those features are connected to empirical predictions based on a probabilistic interpretation of the wave function, we are back to square one and nothing has been gained. So this does not seem to be what Wallace wants to say. The alternative would be to assume some entirely new way of relating the dynamical features of a quantum mechanical description to empirical data. This new non-probabilistic concept of empirical implications of quantum mechanics then might justify assuming a branching structure, which, in turn, would provide the basis for the decision theoretic argument. No new interpretation of quantum mechanics that establishes empirical implications of dynamical features before considering a branching structure is suggested in DWE, however. Quite to the contrary, the whole idea of DWE is to base the interpretation of the wave function exclusively on the decision theoretic argument, which only works after the branching structure has been established.

This leaves us with the second possible understanding of Wallace's claim. On that second reading, Wallace wants to argue that the robustness of dynamical features of a theory justifies neglecting contributions that only give very small corrections to those features even if we are in no position to relate small changes of those features to small empirical changes. This claim is quite clearly erroneous, however. It violates the fundamental scientific principle that any approximation must be motivated by first specifying the empirical quantity one is interested in and then demonstrating that the approximation applied does not have a significant impact on the calculation of that quantity. Without understanding the connection between a robust structure and the empirical data one wants to explain, contributions neglected based on robustness arguments may in the end turn out important. If I neglect the Sahara based on the robust structure of minimal quantities of rain, I may have a reasonable basis for estimating the population of Algeria. But my robustness argument would be quite misleading if I wanted to find out the country's size. If all parts of Hilbert space where the state vector is non-zero are realized, neglecting those parts where the state vector is very small will be a good approximation when calculating measurable quantities which are related to the size of the state vector. But it would be just false when calculating quantities that are not related to the state vector in that way. Robustness and a consistently small impact on features of the theory that cannot be interpreted in terms of empirical outcomes provide no sufficient justification for neglecting small contributions.

The requirement that any ‘robust structure’ must be empirically grounded, can be clearly seen in physical theory when ‘robust emergent structure’ is discussed. As a specific example for a physically viable use of robustness and emergence, we want the have a look at the framework of effective field theory (EFT) (Georgi 1993; Hartmann 2001; Bain 2012). This is a useful case to consider since it: (a) has an accompanying well developed scientific and philosophical literature from which a precise notion of emergence can be drawn; and (b) is reasonably analogous to the case of decoherence/many worlds with which we are concerned.

We should emphasise here that our aim is not to give a *general* model for the notion of emergent ontology within science, or even within physics. Since it seems unlikely that there might be one simple and unified picture of emergence consistent throughout the heterogeneous applications to which the concept has been put, for our purpose it seems best to look to find a notion matched to the particular context at hand: clearly ‘different cases require different conceptions of emergence’ (Silberstein 2012, p. 11). Thus, what we aim to do below is extract from a relevantly similar case, a minimal model for emergence that is well suited to the analysis of emergence in the context of many worlds and decoherence.

Following Bain (2012), we can consider an EFT of a physical system as a theory of the dynamics of the system at energies small compared to a given *cutoff* (or characteristic energy threshold). In certain cases, the low energy states with respect to the cutoff are effectively independent of (*decoupled* from) states at high energies, and thus one may study the low-energy regime of the theory without the need for a detailed description of the high-energy regime. Many important physical systems admit a description in terms of EFTs (significant cases are found within particle physics, nuclear physics and condensed matter physics) and it is within the context of the *ontological decoupling* found within EFTs that we can consider a basic model for the concept of emergence that will be very useful to our analysis.

If we consider the pairing of an EFT and the relevant high-energy theory, then a precise notion of emergence is encapsulated within the the following (much simplified) framework: (i) At energies low compared with the relevant cutoff a particular Lagrangian density provides a good description of the system; (ii) At energies comparable or higher than the cutoff a different Lagrangian density provides a good description of the system; (iii) The difference between these two mathematical formalisms is substantial enough to warrant them being interpreted in terms of different ontologies since they are defined based upon a different set field variables (and therefore different fundamental degrees of freedom); (iv) In this context we can then *define* the ontology of the low energy effective theory as being emergent from that of the high-energy theory.

Two crucial connections can be understood as necessary towards establishing this framework for defining emergence. The first connection is between the scaling of the relevant parameter (energy relative to the cutoff) and the numerical suppression of particular aspects of the formalism (terms in the Lagrangian density): as the energy relative to the cutoff becomes low, the contribution of certain terms within the Lagrangian density become relatively much smaller than other terms. The second connection is between the smallness of those terms and their eliminablity from the formalism (Lagrangian density): the terms which are relatively much smaller are removed from the Lagrangian density because they are less empirically significant.[5] Whereas, the first connection is established purely by the form of the relevant equations, the second relies on our *interpretation* of small parameter values of terms within the Lagrangian density as implying the negligible physical relevance of whatever ontological structures can be associated with those terms. Given this interpretational step leading to the second condition, we can then give a precise meaning to the statement that 'the ontology of the EFT is emergent'; but without it there is a gap in our chain of reasoning.

We thus again run into the subtle, yet crucial, issue of needing a physical basis for neglecting small parameter values: we have to establish that the magnitude of a term within the Lagrangian density is related to its effect on the measurement to be performed. Now, for the case of EFTs we of course have a fairly straightforward basis for making the necessary connection between the formal and the empirical. Terms which make a very small contribution to the Lagrangian density will make a correspondingly small contribution to the relevant S-matrix. Given an interpretation of this matrix in probabilistic terms (via its connection to the relevant scattering amplitudes), we can then establish that neglecting very small terms in the Lagrangian density will have a negligible effect to any measurement performed on the system. Thus, we have a solid basis for the relevant interpretational step, and can consider the ontology of the low level theory as a 'robust but emergent structure' in a sense that has the necessary empirically grounding.

Clearly, there is much more that could be said on these issue, all we have here is a rather simplistic scheme for characterising both emergence and EFTs. However, given this analysis we can at least provide a substantive idea of what it means to say that 'the ontology of the EFT is emergent'. It would seem reasonable to require the notion of emergence deployed in the defence of an Everettian ontology to be as least as substantive as this basic sketch.

Let us then distill the key elements of this notion of emergence, as we have defined it in the context of effective field theory, into a more general schema applicable to the many worlds/decoherence case. In essence what we had in the EFT case was: (i) a clear notion of the the parameter that is being scaled between the two regimes; (ii) two distinct mathematical formalisms, one relevant to each regime with the formal distinction established (at least partially) via the scaling of the parameter; and (iii) an explicit basis for connecting the difference between the two formalisms to an ontological distinction. Together these three aspects were taken as sufficient to establish an emergent ontology in the EFT case—we should therefore accept something similar as establishing a discrete branching structure as an emergent ontology.

[5] This crucial feature can be seen explicitly within the 'Wilsonian EFT' scheme where the *irrelevant* terms are neglected on precisely the basis of scaling in powers of E/Λ (where E is the energy scale and Λ is the cutoff). It is also implicit within the 'continuum EFT' scheme since a similar discarding of terms due to scaling behaviour is needed to calculate the matching correction. See Bain (2012).

We may now once more return to the case of decoherence and try to use a concept of emergence similar to the one deployed above. Environment induced decoherence is a process with effects that scale with both time and complexity: on longer timescales and in more complex systems decoherence effects become greater. It is natural, then, to think of the two regimes that we are considering as being: a short time scale, low complexity regime; and a longer time scale high complexity regime. For simplicity we shall call them the 'macro-regime' and 'micro-regime', and consider a basic model of decoherence (along the lines of Zurek (2003, pp. 12–14)) within which the time parameter controls the suppression of the relevant off-diagonal terms. We thus have that in the micro-regime the relevant mathematical formalism is a density matrix of the form ρ^C above, and in the macro-regime the relevant mathematical formalism is a density matrix of the form ρ^R. There is an explicit basis for considering this formal difference as grounding an ontological difference since the states that can be represented through the formalism relevant to the micro-regime include the discrete and coherent states, and those that can be represented by the formalism relevant to the micro-regime include only discrete states.

We are therefore in a position to call the ontology of the macro-regime a robustly emergent structure, provided the two crucial connections discussed above hold. Firstly, we need to connect the scaling of the relevant parameter (time) and the significance of particular aspects of the formalism (off-diagonal elements of the density matrix). In Zurek's simple model (Zurek 2003, pp. 12–14) this connection obtains explicitly since the 'decoherence term' in his Eq. 17 leads to a strong decay (i.e. reduction with respect to time) in the off-diagonal elements, but has negligible effect on the on-diagonal elements. Thus, the first connection holds. The second connection required is between the relative smallness of these off-diagonal density matrix elements and their eliminablity from the formalism. Above we saw that in order for this connection to hold we need a *physical basis* for the *interpretational step* of neglecting small values. In order to justify neglecting very small entries within a density matrix we need to connect this formal structure to some empirical structure: we must argue that neglecting low Born weighted states has a negligible impact upon measurements of the system while neglecting high Born weighted states has a significant impact. Thus we must apply some version of the Born rule as a measure of probability of experimental outcomes to the comparison of different components of the wave function. This means, however, that we arrive at precisely the conceptual dilemma that was detailed in Sect. 3. Our two incoherence arguments may thus again be brought to bare, and the position for the proponent of a DWE type scheme becomes just as fraught as before.

5 Conclusion

As it stands, the DWE approach to quantum theory is conceptually incoherent. As has been argued above, the approach relies upon decoherence arguments that conflict with its own fundamental precepts regarding probabilistic reasoning in two different respects, corresponding to our two incoherence charges **I1** and **I2**. This conceptual conflict obtains even if the decoherence arguments deployed are aimed merely towards the establishment of certain 'emergent' or 'robust' structures within the wave function. As we have seen, to be relevant to physical science notions such as robustness must be empirically grounded, and, on our analysis, this grounding can only plausibly be done in precisely the probabilistic terms that lead to the identified conceptual conflict. Thus, the incoherence problems outlined here necessitate either the provision of a new, non-probabilistic empirical grounding for the notions of robustness and emergence in the context of decoherence, or the abandonment of the Deutsch–Wallace–Everett programme for quantum theory.

Some ideas are too attractive to be considered false; and some are too ugly to be considered true. Still others, despite their apparent beauty, turn out simply inconsistent, no matter how seductive they may be at first sight. This third category of elegant incoherence carries the highest risk of leading astray, and it is there, alas, that the seemingly revolutionary Deutsch–Wallace approach to Everettian quantum theory must, as it stands, be situated.

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