

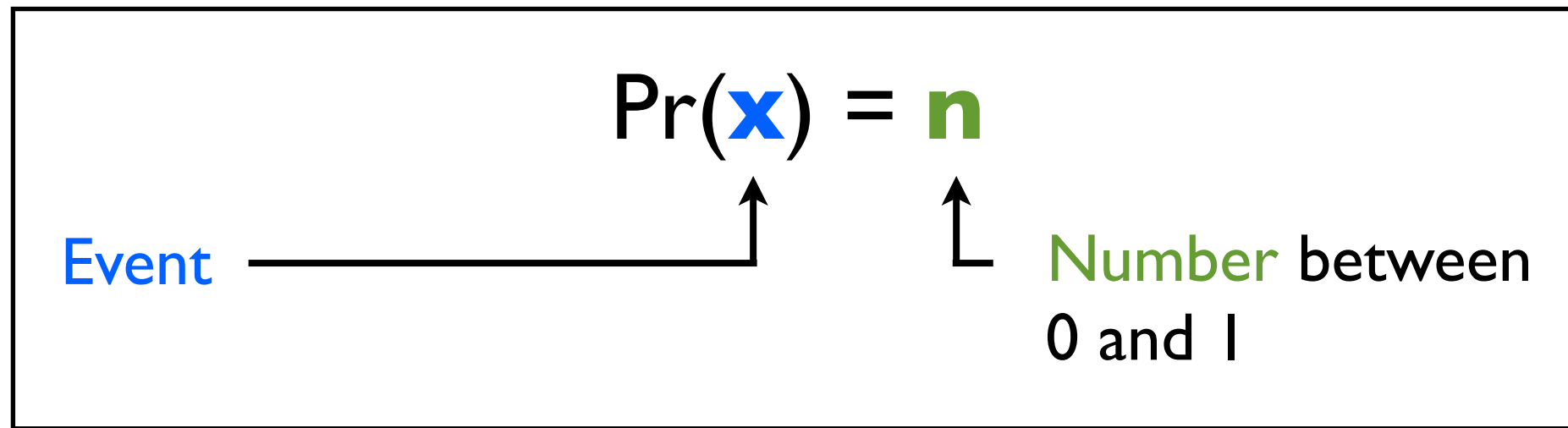
Probability in Medical Ethics

Introduction to some Essentials

**Probability Theory: a well-confirmed
theory about the frequency of events**

Language of Probability

“The probability of **x** is **n**”



Examples:

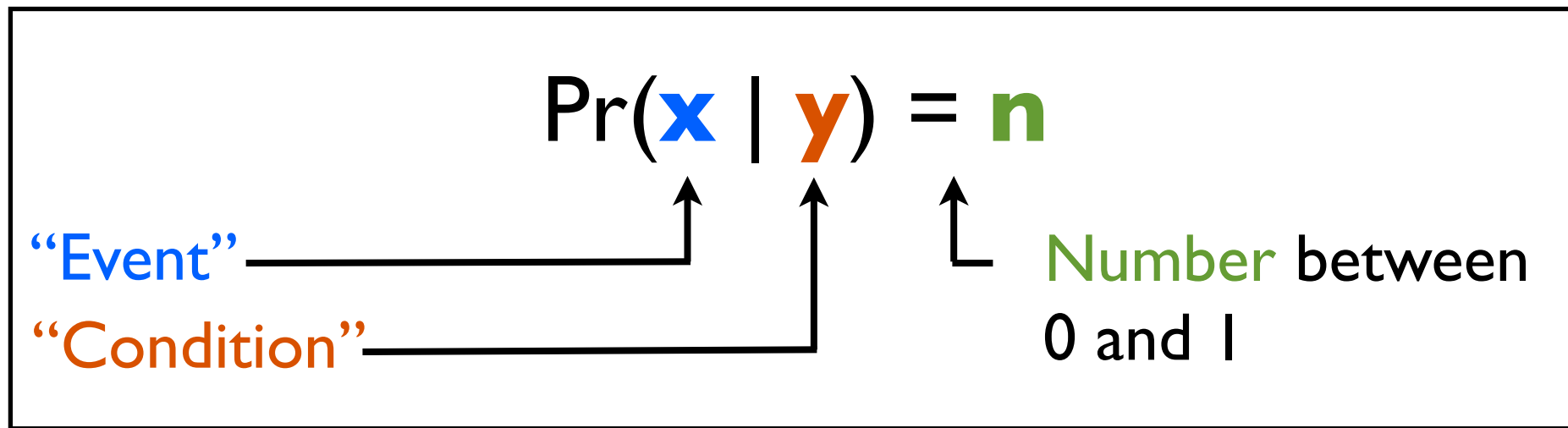
$$\Pr(\mathbf{Coin\ flip\ lands\ heads}) = \mathbf{1/2}$$

$$\Pr(\mathbf{Die\ roll\ is\ a\ 5}) = \mathbf{1/6}$$

$$\Pr(\mathbf{Dice\ roll\ snake\ eyes}) = \mathbf{1/36}$$

Conditional Probability

“The probability of **x** given that **y** is **n**”



Examples:

$$\text{Pr}(\mathbf{\text{successful transplant}}) = \mathbf{.92}$$

$$\text{Pr}(\mathbf{\text{successful transplant}} \mid \mathbf{\text{it's your third one}}) = \mathbf{.77} \text{ (down)}$$

$$\text{Pr}(\mathbf{\text{successful transplant}} \mid \mathbf{\text{you don't smoke}}) = \mathbf{.95} \text{ (up)}$$

Why This Matters for You

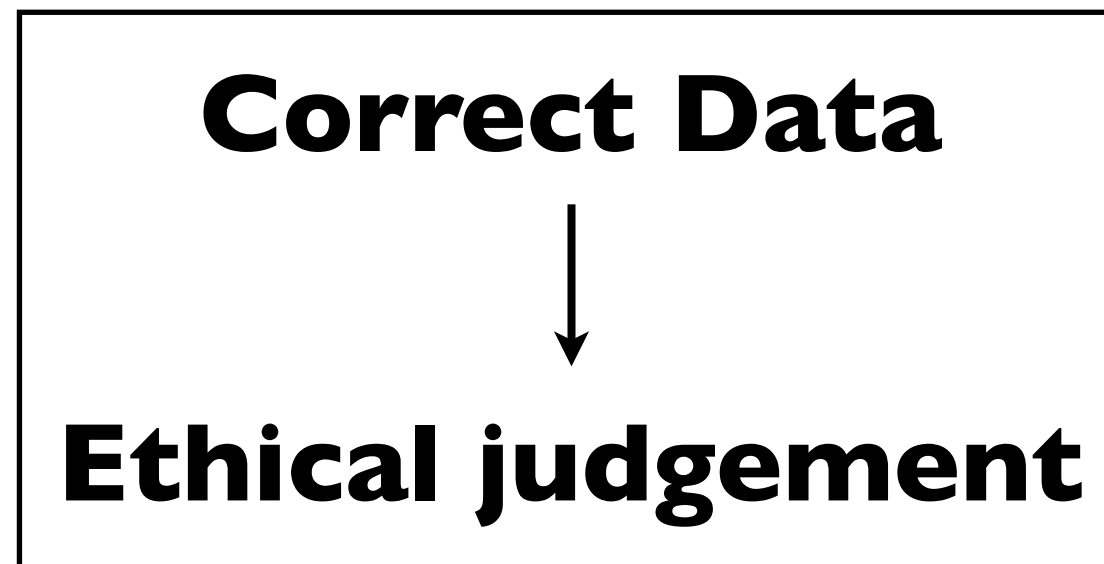
In Medicine: numerous medical reports have been shown to commit a **probabistic fallacy** (the Base Rate fallacy)

- Consequences for patient care?

In psychology: a prominent theory of cognitive dissonance may rest on a **probabilistic fallacy** (the Monty Hall fallacy) <http://www.nytimes.com/2008/04/08/science/08tier.html>

- Consequences for psychological evaluations?

We must thoroughly understand these pitfalls, in order to provide **informed ethical judgements**.



Accuracy of a Medical Test

(Sensitivity & Specificity)

$$\text{Pr}(\text{positive test} \mid \text{have disease}) = n$$

For accurate tests, this is high

$$\text{Pr}(\text{positive test} \mid \text{no disease}) = m$$

For accurate tests, this is low

Accuracy of a Pregnancy Test

(Example)

$$\text{Pr}(\text{positive} \mid \text{pregnant}) = .97$$

$$\text{Pr}(\text{positive} \mid \text{not pregnant}) = .03$$

The **Base Rate Fallacy** happens when someone assumes that test accuracy is all you need to know to make an accurate prediction. **This is false.**

- **An Accurate Pregnancy Test.**

$$\Pr(\text{positive} \mid \text{pregnant}) = .97$$

$$\Pr(\text{positive} \mid \text{not pregnant}) = .03$$

- **Suppose Pat tests positive.** Pat wants to know:

$$\Pr(\text{Pat is pregnant} \mid \text{positive}) = \text{high/low?}$$

- **Relevant missing information:** Pat is a man.

$$\Pr(\text{Pat is pregnant}) = \text{very low}$$

$$\Pr(\text{Pat is pregnant} \mid \text{positive}) = \text{very low}$$

Take-Home Moral:

Whenever you want to know

$$\Pr(\text{disease} \mid \text{positive test}) = n$$

you must always check the **base rate**:

$$\Pr(\text{disease}) = m$$

If the base-rate is low enough, then *even the most accurate tests can make incorrect predictions.*

**How can you tell when the base-rate
is “low enough” to cause problems?**

Bayes' Theorem:

$$\Pr(\text{disease} \mid \text{positive}) =$$

$$\Pr(\text{disease}) \cdot \Pr(\text{positive} \mid \text{disease})$$

$$\Pr(\text{disease}) \cdot \Pr(\text{positive} \mid \text{disease}) + \Pr(\text{no disease}) \cdot \Pr(\text{positive} \mid \text{no disease})$$

Practice: Suppose we have a fairly accurate test for ebola. In particular, suppose:

$$\Pr(\text{positive}|\text{ebola}) = 0.999$$

$$\Pr(\text{positive}|\text{no ebola}) = 0.005$$

Now suppose you administer the test to a patient in the United States, where the base-rate chance of someone having ebola is roughly one in a million (0.000001). The test reports positive for ebola. Use Bayes' Theorem and a calculator to calculate the chances that your patient actually has ebola, given the positive test.