Probability in Medical Ethics

Introduction to some Essentials

Probability Theory: a well-confirmed theory about the frequency of events

Language of Probability

"The probability of **x** is **n**"

$$\Pr(\mathbf{x}) = \mathbf{n}$$

Event

Number between 0 and 1

Examples:

Pr(**Coin flip lands heads**) = **1/2**

Pr(**Die roll is a 5**) = **1/6**

Pr(**Dice roll snake eyes**) = **1/36**

Conditional Probability

"The probability of **x** given that **y** is **n**"

$$\Pr(\mathbf{x} \mid \mathbf{y}) = \mathbf{n}$$

"Event" → **x**

"Condition" → **y**

Number between 0 and 1 → **n**

Examples:

Pr(**successful transplant**) = **.92**

Pr(**successful transplant** | **it's your third one**) = **.77** (down)

Pr(**successful transplant** | **you don't smoke**) = **.95** (up)

Why This Matters for You

In Medicine: numerous medical reports have been shown to commit a **probabistic fallacy** (the Base Rate fallacy)

- Consequences for patient care?

In psychology: a prominent theory of cognitive dissonance may rest on a **probabilistic fallacy** (the Monty Hall fallacy) http://www.nytimes.com/2008/04/08/science/08tier.html

- Consequences for psychological evaluations?

We must thoroughly understand these pitfalls, in order to provide **informed ethical judgements**.

Correct Data

↓

Ethical judgement

Accuracy of a Medical Test

(Sensitivity & Specificity)

For accurate tests, this is high

Pr(**positive test** | **have disease**) = **n**

For accurate tests, this is low

Pr(**positive test** | **no disease**) = **m**

Accuracy of a Pregnancy Test

(Example)

Pr(**positive** | **pregnant**) = **.97**

Pr(**positive** | **not pregnant**) = **.03**

The **Base Rate Fallacy** happens when someone assumes that test accuracy is all you need to know to make an accurate prediction. **This is false.**

- **An Accurate Pregnancy Test.**

  Pr(**positive** | **pregnant**) = **.97**

  Pr(**positive** | **not pregnant**) = **.03**

- **Suppose Pat tests positive.** Pat wants to know:

  Pr(**Pat is pregnant** | **positive**) = **high/low?**

- **Relevant missing information:** Pat is a man.

  Pr(**Pat is pregnant**) = **very low**

  Pr(**Pat is pregnant** | **positive**) = **very low**

Take-Home Moral:

Whenever you want to know

$$\Pr(\textbf{disease} \mid \textbf{positive test}) = \textbf{n}$$

you must always check the **base rate**:

$$\Pr(\textbf{disease}) = \textbf{m}$$

If the base-rate is low enough, then *even the most accurate tests can make incorrect predictions*.

How can you tell when the base-rate is “low enough” to cause problems?

Bayes' Theorem:

$$\Pr(\textbf{disease} \mid \textbf{positive}) = \frac{\Pr(\textbf{disease})\cdot\Pr(\textbf{positive}\mid\textbf{disease})}{\Pr(\textbf{disease})\cdot\Pr(\textbf{positive}\mid\textbf{disease}) + \Pr(\textbf{no disease})\cdot\Pr(\textbf{positive}\mid\textbf{no disease})}$$

Practice: Suppose we have a fairly accurate test for ebola. In particular, suppose:

Pr(positive|ebola) = 0.999
Pr(positive|no ebola) = 0.005

Now suppose you administer the test to a patient in the United States, where the base-rate chance of someone having ebola is roughly one in a million (0.0000001). The test reports positive for ebola. Use Bayes' Theorem and a calculator to calculate the chances that your patient actually has ebola, given the positive test.