Earman (2011): The Unruh Effect for Philosophers

For Part III Essay, and Philosophy of QFT: Lent 2020: J. Butterfield (jb56@cam.ac.uk)

Textbook presentations include: Robert Wald *QFT in curved spacetime and black hole thermodynamics* (1984), Chapter 5. Cf. also:

(i) Wald *General Relativity* 1984: Sec 6.4, pp. 149-152, for a discussion of Rindler coordinates on Minkowski spacetime, used for motivating the Kruskal extension;

(ii) R. Clifton and H. Halvorson, 'Are Rindler Quanta real? Inequivalent particle concepts in QFT', *British Journal Philosophy of Science* **52**, 2001;

(iii) Crispino et al, 'The Unruh effect and its applications', *Rev. Modern Physics* 2008, volume 80)

1 Introduction

The effect is that an observer with constant linear acceleration a through the Minkowski vacuum, for say a non-interacting scalar field, will find herself immersed in a thermal bath at temperature T_U proportional to a. In fact:

$$T_U = \frac{\hbar a}{2\pi ck} \tag{1}$$

So an acceleration of 10^{24} cm/sec^2 (!) is required to achieve a temperature of 300 C. But from now on, we set $\hbar = k = c = 1$.

Earman's overall themes are:

(i) There are at least three different approaches to consider.

(ii) Discussion centres around the operationalization, and maybe demotion!, of the particle concept in QFT; cf. also Sec 1.1 of Wald (1994);

(iii) Earman favours the algebraic quantum theory, especially modular theory, rigorous approach (Sections 2 to 5); rather than the detector approach (Section 7), espoused by Unruh himself.

The overall idea of the algebraic approach is as follows. Here: (a) (b) and (d) correspond to Earman's Section 2; (c) (i-iii) corresponds to his Appendix A; (c)(iv) and (e) corresponds to his Section 3, and Appendix B.

(a) Consider KMS states as the generalization for infinite quantum systems (thermodynamic limit in quantum statistical mechanics; and quantum fields) of the Gibbs equilibrium states; a KMS state on a von Neumann algebra is defined relative to a one-parameter group of automorphisms of the algebra (i.e. a dynamics in Heisenberg picture)...*But...*

(b) The Tomita-Takesaki theorem 'reverses' this. Namely: for any faithful, normal state ϕ on a von Neumann algebra, there is a unique one-parameter group of automorphisms, $\sigma_s, s \in \mathbb{R}$, such that ϕ is a KMS state for that group.

(c) The standard practice in AQFT is to consider the GNS representations of an appropriately defined vacuum state. More specifically:

(i) for the Klein-Gordon field with mass $m \geq 0$ on Minkowski spacetime, the Weyl algebra is explicitly constructed; and this construction yields a preferred vacuum state ϕ_M (M for 'Minkowski'), which corresponds to the usual vacuum of heuristic theory;

(ii) In fact: more generally: for any algebraic state on this algebra that is quasi-free (i.e. all n-point functions are sums of products of 2-point functions), the GNS representation is (unitarily equivalent) to a Fock space representation with the GNS vector being the Fock vacuum;

(iii) Also, (i) can be generalized to curved spacetimes: in particular, there is a rigorous quantization of the Klein-Gordon field on any stationary globally hyperbolic spacetime, cf. Wald (1994, Sections 4.3 to 4.5) and Earman Appendix A ;

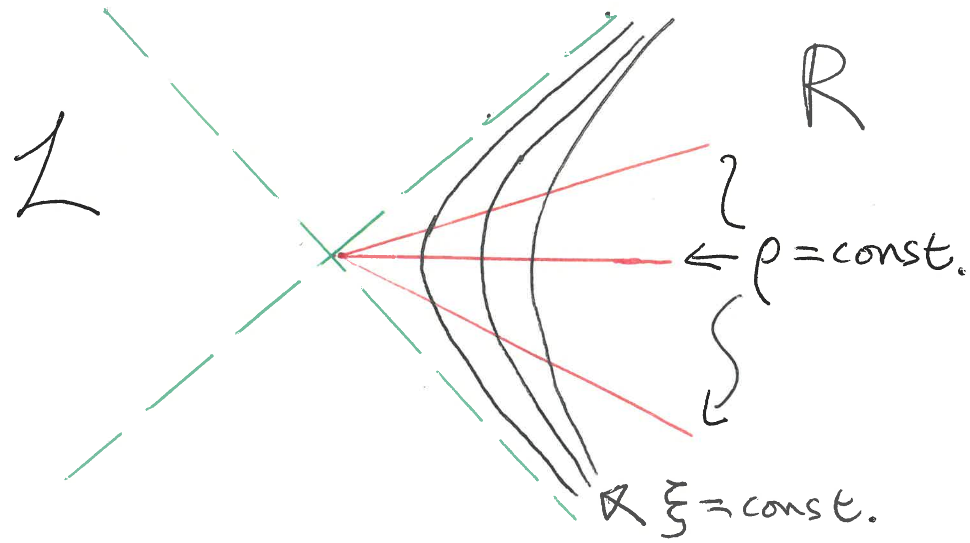
(iv) To apply (b)'s Tomita-Takesaki theorem to (c) (i), we argue that ϕ_M restricted to $\mathcal{A}(O)$, the C^*-algebra for some spacetime region O, induces a faithful, normal state, $\phi_{M,O}$ say, on the von Neumann algebra, $\mathcal{N}(O)$, affiliated to $\mathcal{A}(O)$ using the GNS representation of ϕ_M on $\mathcal{A}(O)$. (So $\mathcal{N}(O)$ is defined as the double commutant of $\pi_{\phi_{M|\mathcal{A}(O)}}(\mathcal{A}(O))$.)

(d) One would not expect the one-parameter group of automorphisms, $\sigma_s, s \in \mathbb{R}$, with respect to which $\phi_{M,O}$ is a KMS state to have any physical significance. In particular: one would not expect it to correspond to a time-evolution along a congruence of timelike lines (i.e. to dynamics as seen by 'observers' travelling those worldlines). Or that the inverse temperature β mathematically associated with the state should correspond to a measured temperature ..*But...*

(e) Bisognano and Wichmann (1975) show that: if we choose O to be the right Rindler wedge R of Minkowski spacetime, i.e. given by $x > |t|$ with (x, y, z, t) an inertial coordinate system, then the one-parameter group of automorphisms for $\phi_{M,O} \equiv \phi_{M,R}$ *does correspond* to the Rindler coordinates (ξ, y, z, ρ) related to (x, y, z, t) by

$$x = \xi \cosh \rho \ \ ; \ \ t = \xi \sinh \rho \qquad (2)$$

i.e. coordinates such that the timelike lines $\xi =$ constant are hyperbolae of linear acceleration $a = 1/\xi$ (so: acceleration constant along each line, but varying between lines), that are orthogonal to the spacelike hypersurfaces (indeed: Cauchy surfaces) ρ constant. Here, the phrase '*does correspond*' means that (with suitable rescaling): ρ can be taken as the parameter σ in $\sigma_s, s \in \mathbb{R}$. In short: the restriction of the Minkowski vacuum state to $\mathcal{N}(R)$ is a $(\sigma_\rho, 2\pi)$ KMS state.



2 KMS states and modular theory

1): Recall that in quantum statistical mechanics: a Hamiltonian H and inverse temperature $\beta = 1/T \equiv 1/kT$ define a Gibbs equilibrium state $\rho_\beta := \exp(-\beta H)/(\mathrm{Tr}(\exp(-\beta H)))$; which defines an algebraic state $\phi_\beta := \mathrm{Tr}(\rho_\beta A)$ for all $A \in \mathcal{B}(\mathcal{H})$. Also H defines a dynamics as a one-parameter groups of automorphisms of the algebra of quantities, i.e. $\sigma_t(A) := \exp(itH)A\exp(-itH)$.

2) Now assume that the extension of σ_t to complex values of t makes $z \mapsto \phi_\beta(A\sigma_z(B))$ analytic in the strip $\{0 < \mathrm{Im}(z) < \beta\}$ of the complex plane. It follows that for all $A, B \in \mathcal{B}(\mathcal{H})$:

$$\phi_\beta(A\sigma_{i\beta}(B)) = \phi_\beta(BA) \tag{3}$$

3) A state ϕ on a von Neumann algebra $\mathcal{M}$ such that: for any $A, B \in \mathcal{M}$, there is a function $f_{A,B}(z)$ analytic on the strip $\{0 < \mathrm{Im}(z) < \beta\}$ of the complex plane, such that for all $s \in \mathbb{R}$

$$f_{A,B}(s) = \phi(\sigma_s(A)B) \text{ and } f_{A,B}(s + i\beta) = \phi(B\sigma_s(A)) \tag{4}$$

is called a (σ_s, β) *KMS-state.* Such a state is σ_s-invariant, and also obeys other stability properties that make it the best generalization to infinite quantum systems of the notion of a Gibbs equilibrium state. (It also can be shown to satisfy the algebraic corollary, in 2), of a Gibbs equilibrium state, i.e. eq (3), for all A, B in a weakly dense, σ_s-invariant subalgebra of $\mathcal{M}$.)

4) ϕ is a (σ_s, β) KMS-state iff it is a $(\sigma_u, -1)$ KMS-state, where $u = -\beta s$. So without loss of generality, we set $\beta = -1$, and call the resulting form of the KMS condition, the *modular condition.*

5) The Tomita-Takesaki theorem now states: for any faithful, normal state ϕ on a von Neumann algebra $\mathcal{M}$ acting on a Hilbert space $\mathcal{H}$, there is a unique one-parameter group of automorphisms, $\sigma_s, s \in \mathbb{R}$, such that ϕ satisfies the modular condition with respect to σ_s, i.e. ϕ is a $(\sigma_s, -1)$ KMS-state.

3 Modular automorphism groups with geometric actions on Minkowski spacetime;

1) See Appendix A for details of:

(i) *-algebras, C^*-algebras;

(ii) their representations, especially the GNS representation induced by an (algebraic) state; properties of states such being mixed, pure, faithful and normal; properties of representations such as cylic and irreducible;

(iii) von Neumann algebras as having the merit of being generated by their projections; the double commutant theorem; cyclic and separating vectors; factors and types;

(iv) unitary equivalence, quasi-equivalence and disjointness of representations;

(v): (adding some physics!): Weyl algebra, quasi-free states; there is a rigorous quantization of the Klein-Gordon field on any stationary globally hyperbolic spacetime (but this fails for the Rindler wedge—which needs Fulling quantization) .

2) Given the Minkowski vacuum state, ϕ_M, for the Weyl CCR algebra $\mathcal{A}(\mathbb{R}^4)$ for the quantized Klein-Gordon field on Minkowski spacetime. Restrict it to some $\mathcal{A}(O)$, to get: $\phi_{M|\mathcal{A}(O)}$. Take the von Neumann algebra $\mathcal{N}(O)$ affiliated with the GNS representation of $\mathcal{A}(O)$ that is defined by $\phi_{M|\mathcal{A}(O)}$.

It is straightforward to argue (p. 83, bottom right: based on (a) the Reeh-Schlieder theorem and (b) a vector is cyclic for an algebra $\mathcal{A}$ iff it is separating for $\mathcal{A}'$) that:—
the (unique) canonical extension of $\phi_{M|\mathcal{A}(O)}$ to $\mathcal{N}(O)$—written $\chi_{|0_M\rangle}$—is a faithful normal

state on $\mathcal{N}(O)$.

The Tomita-Takesaki theorem now implies: $\chi_{|0_M\rangle}$ is a $(\sigma_s, -1)$ KMS-state for a unique automorphism group σ_s .

3) The surprise is that ... *Now we repeat items (d) and (e) from the end of Section 1!*

4) Hence (Sewell 1982) what Earman calls the *modular temperature hypothesis* (MTH). It says: if

(i) the modular flow (i.e. the natural action on the spacetime of the automorphism group σ_s for a (σ_s, β) KMS-state) is everywhere timelike in direction, and

(ii) the derivative, $d\tau/ds$, of the proper time τ of an observer following a line of the flow, with respect to s, is constant along the flow (along the observer's world-line): then

then such an observer measures a temperature β_τ given by $\beta_\tau := \beta \times d\tau/ds$.

Note:

(a): this implies for the Rindler wedge case, that an observer with acceleration a measures the Unruh temperature in eq. (1);

(b): An extended MTH, which drops the assumption (ii), has been advocated in the literature (albeit not named as such): cf. Earman p. 84 right and 85 left for a review of e.g. Martinetti and Rovelli (2003) on the extended MTH, for algebras associated with a causal diamond, and with a future light cone. (One again secures a faithful normal state on the desired algebra, by using (a) the Reeh-Schlieder theorem and (b) a vector is cyclic for an algebra $\mathcal{A}$ iff it is separating for $\mathcal{A}'$.)

4 The Unruh effect in curved spacetimes, and its relation to the Hawking effect

The null hyperplanes (shown as lines in the 2-d Figure) intersecting in a spacelike 2-surface through the origin of Minkowski spacetime (shown as the origin in the 2-d Figure) are an example of a *bifurcate Killing horizon.* For a four-dimensional spacetime, the general definition of a *bifurcate Killing horizon* is: a pair of null surfaces N_A and N_B intersecting in a spacelike 2-surface S such that N_A and N_B are Killing horizons with respect to the same Killing field ξ_a.

In these terms, a partial summary of Sections 1 to 3 would then be:—Among the quasi-free states on the Weyl CCR algebra for the Klein-Gordon field on MInkowski spacetime, there is a unique non-singular (i.e. Hadamard: a regularity condition on the stress-energy tensor: Wald 1994, Section 4.6, especially p. 85-95) state invariant under the automorphisms corresponding to the isometries generated by the wedges' horizon Killing field ξ_a.

Kay and Wald (1991) generalize this summary to (certain) curved spacetiems, as follows. They show that:

for a minimally coupled free scalar field propagating on a curved globally hyperbolic spacetime with bifurcate Killing horizon(s) whose surface S contains a Cauchy surface:

(i) on a 'large' subalgebra of the wedge algebra of observables, there is at most one quasi-free Hadamard state that is invariant under the automorphisms of the algebra generated by the Killing horizon isometries;

(ii) the restriction to the wedge algebra of this state—if it exists—is a KMS state at a

Hawking temperature $T_H := \kappa/2\pi$ (with respect to the automorphism group); where κ is the surface gravity on the horizon. (Here, $\kappa := \frac{-1}{2}(\nabla_a \xi_b)(\nabla^a \xi^b)$. But more physically: κ is the limit, as the horizon is approached, of the product of the norm V of the horizon Killing filed and the norm a of the acceleration.)

(iii) Using the modular temperature hypothesis (MTH: cf. Part 4) of the previous Section), this yields an "observed" temperature, $T = \frac{\kappa}{2\pi V}$. This is a *generalized Unruh effect*, at least in the sense that....

(iv) For Minkowski spacetime it recovers the temperature T_U in eq. 1.

(v) For various other spacetimes, the relevant wedges and (vacuum) states exist, and the temperature $T = \frac{\kappa}{2\pi V}$ can be computed. For example: for the Kruskal maximal extension of Schwarzschild, one uses the Hartle-Hawking vacuum.

But *warning!* (p. 86, left: and Wald 1999 *CQG* vol. 16, A177-190, quoted p. 87 left):— The vacuum states involved in this generalized Unruh effect (e.g. the Hartle-Hawking vacuum for Kruskal spacetime) are very different from the vacuum involved in the Hawking effect (which is confusingly! called the 'Unruh vacuum') ... as shown by the modes that appear to have propagated in from infinity: in the former vacua, these are populated, in the latter vacua, they are unpopulated ... (Cf. also Unruh in C. Callender and N. Huggett ed. *Physics Meets Philosophy at the Planck Scale* (2001: CUP).

More generally (p. 87): the Unruh and Hawking effects are distinguished by their lack/possession of links to: the finite temperature of, and evaporation of, a black hole.

5 Some qualms about the modular theory approach

There are doubts about the connection between

(i) the mathematical temperature given by modular theory and

(ii) the "reading of a thermometer subject to the acceleration", i.e. the firing statistics of a particle detector subject to the acceleration ; simply because:

(i'): the definition of (i) involves the whole right wedge, and its large algebra $\mathcal{A}(R)$ (or its affiliated von Neumann algebra $\mathcal{N}(R)$); while

(ii'): the behaviour of the detector in some finite segment, S say, of its worldline surely depends (should depend!) only on its history in some such finite open region, O say, containing S where, agreed: O can and indeed should! contain parts of the detector's worldline before (and maybe also after?) S, so as to allow for influence from the recent past ('transients' etc.). But an algebra $\mathcal{A}(O)$ is a subalgebra of $\mathcal{A}(R)$, (and similarly for the affiliated von Neumann algebras). And this means that $\mathcal{A}(O)$ can perhaps admit states that do not extend to $\mathcal{A}(R)$ (and similarly for the affiliated von Neumann algebras).

JNB: Two final comments:
Recall that the restriction of an entangled pure composite state to a component is mixed. Obviously, there is much more to the Unruh effect than this: and not just because the reduced state is thermal with a temperature with a simple dependence on the acceleration a.... But it is worth bearing in mind, even in connection with the black hole information paradox: cf the end of Section II of Unruh and Wald 'Information Loss' arxiv: 1703.02140.

(2): In the algebraic approach, the inertial/Minkowski, and Rindler-wedge, vacua lead to *disjoint* representations of their algebras, and so 'incommensurability' (cf. Clifton and Halvorson BJPS 2001); *pace* one's initial surmise (and *pace* e.g. Teller 1995 ca. p. 110).