Pergamon

Stud. Hist. Phil. Mod. Phys., Vol. 27, No. 4, pp. 511–524, 1996
Published by Elsevier Science Ltd. Printed in Great Britain.
1355-2198/96 $15.00 + 0.00

ESSAY REVIEW

Local Quantum Physics

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Rudolf Haag, *Local Quantum Physics* (Berlin: Springer-Verlag, 1992; corrected 2nd edn, 1993[1]), xiv + 356 pp. ISBN 3-540-53610-8 hardback £35.00; xv + 390 pp. ISBN 3-540-61049-9 softcover (revised and enlarged edition, 1996), £32.00

> Our civilization is characterized by the word 'progress'. Progress is its form rather than making progress one of its features. Typically it constructs. It is occupied with building an ever more complicated structure. And even clarity is only sought as a means to this end, not as an end in itself. For me on the contrary clarity, perspicuity are valuable in themselves. I am not interested in constructing a building, so much as in having a perspicuous view of the foundations of typical buildings
>
> Ludwig Wittgenstein

1. Introduction

Local quantum physics is understood as the synthesis of special relativity and quantum physics. According to A. S. Wightman, himself no mean contributor to this field, 'Rudolf Haag has thought as long and as deeply about the foundations of relativistic quantum mechanics as anyone alive', and the present book is in many ways his scientific autobiography. Its principal subject matter, algebraic quantum field theory, has a reputation of being rather esoteric, axiomatic and mathematical (and, some pagans would say, irrelevant to physics). To the extent that this is true, at least for Haag himself the mathematics is merely there to serve the objective of providing a conceptual analysis of quantum field theory and related matters, which is as precise and deductive as possible. Thus the book may be seen as a contribution to the foundations of modern physics; its appeal should be to mathematical physicists and philosophers of physics alike. In this review we shall concentrate on points of conceptual, rather than mathematical interest.

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[1] Page numbers refer to this edition.

S1355-2198(96)00020-2

In order to appreciate the extent to which Haag's approach to quantum field theory marks a departure from more conventional scenarios, it is useful to make a brief remark about the history of this subject (cf. Schweber, 1994). From its inception in 1927, quantum field theory has been subject to the question whether particles or fields should be primary. We find physicists of equal stature on both sides of the debate: Dirac, the middle Heisenberg, Feynman, and Wheeler thought particles should be the starting point in the formulation of the theory, whereas Pauli and the early as well as the later Heisenberg, Tomonaga, and Schwinger favoured fields. In linear field theories there is a complete duality between particles and fields, implemented by the well-known Fock space construction. Moreover, by rewriting the Tomonaga–Schwinger perturbation method as an S-matrix theory, Dyson to some extent blurred the distinction between the two pictures even in interacting theories. Thus it came to be believed that for every particle there should be a field, and *vice versa*.

In two thaumaturgic papers, published in 1955 and 1958 respectively, Haag pointed out that the alleged one-to-one correspondence between particles and fields holds, if at all, only at the asymptotic level. That is, in certain theories one may take the 'time goes to $\pm\infty$' limit (in a suitable mathematical sense) of a given field (in the Heisenberg picture); these limit fields φ_{as} obey a free field equation and canonical commutation relations, and describe freely moving particles in the usual way. However, the given field may either be among the so-called 'fundamental' or 'elementary' fields of the theory (usually understood to be the fields occurring in the classical Lagrangian), or a composite (sum of products) of such. In fact, from the point of view of scattering theory there is no distinction between elementary and composite particles. Even the distinction between elementary and composite fields is not intrinsic, affected as it is by arbitrary field redefinitions. It may well be possible that a quantum field theory does not describe any particles at all, but if it does, the associated free fields are asymptotically related to the Heisenberg fields, rather than being their lowest-order approximation; (the latter is the starting point of the usual interaction picture, which Haag in the first-mentioned paper proved not to exist in interacting field theories).

Accordingly, for Haag quantum field theory is a quantum theory of fields, in which particles are at best derived objects. The rôle of fields is to implement the principle of locality (in German, *Nahwirkungsprizip*). As will be explained shortly, quantum fields are to be seen as mere co-ordinatizations of the local field algebras or algebras of local observables.

If even the fundamental fields fail to be fundamental, what is the intrinsic content of a quantum field theory? This question was answered by Haag and his disciples between 1957 and 1964.[2] The edifice they created is based on the following ideas.

[2] There is a short historical note on p. 111 of the book. For more information, cf. Araki (1992), Kastler (1992) and Schroer (1995).

(1) For physical as well as mathematical reasons (analyzed in the classical work of Bohr and Rosenfeld, and Wightman, respectively), quantum fields $\varphi(x)$ at a point x in space–time make no good sense. One has to smear the fields as $\varphi(f) = \int d^4x\varphi(x)f(x)$, where f is a smooth 'test' function on space–time. The collection of all smeared fields whose test functions vanish outside some bounded region $\mathcal{O}$ generates an algebra $\mathcal{A}(\mathcal{O})$, which evidently does not change under polynomial redefinitions of the smeared fields. Thus the naive correspondence $x \to \varphi(x)$ should be replaced by the map $\mathcal{O} \to \mathcal{A}(\mathcal{O})$; elements of $\mathcal{A}(\mathcal{O})$ are said to be localized in $\mathcal{O}$.

(2) In ordinary quantum mechanics, the passage from an operator A to any function $f(A)$ merely corresponds to a relabelling of its eigenvalues, and does not really change the observable represented by A. In particular, it suffices to look at functions for which $f(A)$ is bounded. This justifies the claim that in quantum mechanics it is in principle sufficient to deal with bounded operators.[3]

(3) The theory ought to be defined by its collection of observables, which are localized in space–time. In particular, fermionic fields (such as the Dirac spinor field) should only enter through bilinear combinations (observable currents). More generally, fields that are not invariant under transformations without observable effects (such as gauge transformations or global internal symmetry transformations) should not directly occur in the basic formulation of the theory; only their invariant combinations should.[4]

(4) The particular way the local observables act on a specific Hilbert space of states (as in ordinary quantum field theory) turns out to be determined by the global behaviour of these states. Since all physical measurements are localized in space–time, the specification of the theory should not include this action (but only the algebraic relations between the observables, and their localization).

(5) The causal structure of space–time in special relativity is reflected by the property that two observables which are localized in regions that are spacelike separated must commute (which Haag calls 'Einstein causality').[5]

Combining these notions, Haag concludes that a quantum field theory is defined by a map (often referred to as a 'net') $\mathcal{O} \to \mathfrak{A}(\mathcal{O})$, where $\mathcal{O}$ runs through the regions of finite extension in space–time, and each $\mathfrak{A}(\mathcal{O})$ is an algebra of abstract bounded operators. The Hermitian (self-adjoint) elements of $\mathfrak{A}(\mathcal{O})$ are interpreted as the observables localized in $\mathcal{O}$. The total algebra of local

[3] Haag attributes this insight to Segal. The advantage of working with bounded operators is that they can be freely added and multiplied.

[4] This principle is reminiscent of Heisenberg's insistence in 1925 that quantum mechanics ought to be formulated in terms of observables alone.

[5] As reviewed by Summers (1990) and Butterfield (1994), Einstein causality indeed rules out the possibility of superluminal causation. The basic step in showing this is a theorem of Schlieder: if a pure state ψ collapses to a mixture ψ_M in the process of measuring an observable A in the usual way (Lüders' rule), then the expectation value of any observable B which commutes with A is the same in ψ as in ψ_M.

observables is the union $\mathfrak{A}_{loc} = \cup_{\mathcal{O}}\mathfrak{A}(\mathcal{O})$, where the union is over all bounded regions.[6] The physical interpretation of the theory is supposed to follow from the net.[7] In particular, the physical meaning of an operator is determined by its localization. The causality requirement, the single most important ingredient of Haag's approach to quantum field theory, is that the commutator $[A, B]$ vanishes for all pairs such that $A \in \mathfrak{A}(\mathcal{O}_i)$ and $B \in \mathfrak{A}(\mathcal{O}_j)$ with $\mathcal{O}_i$ and $\mathcal{O}_j$ spacelike separated (as defined in special relativity).

This is a long way from conventional quantum field theory.[8] The most extraordinary claim of algebraic quantum field theory is that notions like isospin, colour, flavour, gauge transformations and everything else that is in the front of the mind of any quantum field theorist, but is not directly related to observations in space–time, can be reconstructed from the net $\mathcal{O} \to \mathfrak{A}(\mathcal{O})$. In Haag's own words, 'ultimately all physical processes are analyzed in terms of geometrical relations of (unresolved) phenomena'. Mathematically, everything is determined by the way the various algebras $\mathfrak{A}(\mathcal{O})$ sit inside $\mathfrak{A}_{loc}$, in specific positions relative to each other.

It should be emphasized that algebraic quantum field theory is by no means the only subject of Haag's work; in addition, one finds important work on quantum statistical mechanics. Together these have led to an active and ongoing interplay between quantum physics and the mathematical theory of operator algebras. Much of this is reflected in the book.

2. A Tour Through the Book

Chapter I is a brief overview of basic quantum field theory, which will be a useful reminder for those who have already done a first course in this subject (it seems difficult to read the book without any previous exposure to quantum field theory; in addition, one should have a good working knowledge of elementary Hilbert space and operator theory). The choice of topics is not completely standard from the point of view of modern textbooks: Wigner's analysis of the irreducible unitary representations of the Poincaré group[9] and the Peierls bracket are reviewed, the path integral is not. More or less the same topics (as well as many others) are covered in the recent introductory book by Weinberg (1995), who adopts a similar point of view, but gives much more detail, as well as a rich supply of applications. (Serious students of quantum field theory will want to read both books cover to cover.)

Chapter II reviews axiomatic quantum field theory *à la* Wightman: the axioms are introduced, and some of their consequences (analyticity properties,

[6] Here one has to identify $\mathfrak{A}(\mathcal{O}_1)$ with a subalgebra of $\mathfrak{A}(\mathcal{O}_2)$ if $\mathcal{O}_1 \subset \mathcal{O}_2$.

[7] The analogue of this net in non-relativistic quantum mechanics would be the specification of the position operators and the Hamiltonian.

[8] We are mainly thinking of Poincaré-invariant field theory in flat space–time, but much of the framework applies to curved space–times as well.

[9] There is a typo in the table on p. 30, where $p^0 \geq 0$ and $p^0 \leq 0$ should be replaced by $p^0 > 0$ and $p^0 < 0$, respectively.

CPT theorem, Reeh–Schlieder theorem etc.) pointed out. Hardly any proofs are given, and the material has presumably been included to bridge the gap between conventional quantum field theory and Haag's own approach, in line with the historical development of the subject. A far more detailed treatment of axiomatic field theory may be found in the classical textbooks: Streater and Wightman (1964), Jost (1965) and Bogolubov *et al.* (1990) (the latter containing an extensive discussion of algebraic quantum field theory as well). Although most of the theorems derived from these axioms appear to hold in conventional (Lagrangian) quantum field theory as well, the axioms themselves are now thought to be somewhat irrelevant to physics, for they are not satisfied by gauge field theories, which are currently believed to provide the correct description of physics up to the highest energies available in laboratories or astrophysics.

In any case, the emphasis in this chapter is on scattering theory from first principles, and we find, in particular, a detailed discussion of what is generally called the Haag–Ruelle collision theory.[10] Here one assumes that the key Wightman axioms hold,[11] and that there exists a mass gap.[12] It is then possible to construct asymptotic particle states, which are assembled into a Fock space sitting inside $\mathcal{H}$ (perhaps coinciding with it). These states are obtained by acting on the vacuum state with certain smeared polynomials in the fields, and taking a $t \to \pm\infty$ limit in a judicious way. This theory is quite beautiful, and historically provided some backing for the belief in Wightman's axioms.

Algebraic quantum field theory proper is introduced in Chapter III. The mathematical framework consists of the theory of algebras of bounded operators, specifically C^*-algebras and W^*-algebras (also called von Neumann algebras), to which Haag gives a brief introduction.[13] This theory is very elegant, yet somewhat intimidating. Haag provides most of the essential definitions, but the reader who would like to proceed at a more leisurely pace, and learn the theory in connection with its applications to quantum physics, might want to consult Sewell (1986) and Thirring (1981; 1983). For a full meal, cf. Bratteli and Robinson (1987).

Like groups, operator algebras may be abstractly defined by algebraic relations; one may subsequently study representations on Hilbert spaces. A state on an operator algebra assigns a real number to each observable (i.e. self-adjoint

[10] In quantum mechanics one constructs scattering states from the so-called Møller operators (see, e.g. Thirring, 1981). Since, as shown by Haag in 1955, these do not exist in interacting quantum field theories, one of the goals of the Haag–Ruelle theory is to develop scattering theory without those operators.

[11] Roughly speaking, one has a collection of quantum fields, which commute when their arguments are spacelike separated, acting on a Hilbert space $\mathcal{H}$ carrying a unitary representation of the Poincaré group P under which the fields transform covariantly, and containing a unique vacuum state which is invariant under this representation.

[12] In non-relativistic quantum mechanics one thinks of the free Hamiltonian H_0 and the full one H as related by $H = H_0 + V$, where V is the interaction. In quantum field theory, this is not the right point of view: here *H and H_0 are the same.*

[13] Unfortunately, it shares the feature of many older textbooks on operator algebras that practically no examples are included.

element of the algebra), interpreted as the expectation value.[14] As in the usual formalism, a state may be pure or mixed. From a state ω one may construct a representation π_ω of the algebra as operators acting on a concrete Hilbert space $\mathcal{H}_\omega$; the state is then represented by a unit vector Ω_ω in this space. There are natural notions of irreducibility,[15] multiplicity, and unitary equivalence of representations.

One problem with the structure laid out so far is the existence of far too many states on $\mathfrak{A}$. Most of these seem to have no decent physical interpretation, corresponding e.g. to infinite energy in a finite region. The way out is to somewhat 'soften the dogma' that the net $\mathcal{O} \to \mathfrak{A}(\mathcal{O})$ contains all physical information by adding the extra input of specifying a collection (folium) of physically realizable states, which do have tolerable properties. An important point is that the physicality requirement can be stated locally; that is, it concerns the restriction of a state to each local algebra $\mathfrak{A}(\mathcal{O})$. Once such a folium has been specified, one can extend the local algebras[16] to somewhat larger algebras $\mathcal{R}(\mathcal{O})$.[17]

The correspondence $\mathcal{O} \to \mathcal{R}(\mathcal{O})$ becomes much better behaved if one restricts the regions $\mathcal{O}$ to causally complete regions.[18] If we denote the spacelike complement of a region $\mathcal{O}$ by $\mathcal{O}^\perp$, then $\mathcal{O}$ is said to be causally complete if $\mathcal{O}^{\perp\perp} = \mathcal{O}$. In any case, $\mathcal{O}^{\perp\perp}$ is the causal completion of $\mathcal{O}$. The set $\mathcal{K}$ of causally complete regions forms an orthocomplemented lattice, with $\mathcal{O}_1 \vee \mathcal{O}_2 = (\mathcal{O}_1 \cup \mathcal{O}_2)^{\perp\perp}$ and $\mathcal{O}_1 \wedge \mathcal{O}_2 = \mathcal{O}_1 \cap \mathcal{O}_2$.

Let us now take a fixed reference state ω. Another orthocomplemented lattice of interest is the one consisting of (von Neumann) subalgebras of $\mathfrak{B}(\mathcal{H}_\omega)$ (i.e. the algebra of all bounded operators on $\mathcal{H}_\omega$). The orthocomplementation is given by the commutant,[19] and the join and meet are $\mathfrak{M}_1 \vee \mathfrak{M}_2 = (\mathfrak{M}_1 \cup \mathfrak{M}_2)''$ and $\mathfrak{M}_1 \wedge \mathfrak{M}_2 = \mathfrak{M}_1 \cap \mathfrak{M}_2$, respectively.

In addition, once ω, understood as lying in the physical folium, has been specified, one may identify the $\mathcal{R}(\mathcal{O})$ with their concrete realizations on $\mathcal{H}_\omega$, and extend the net $\mathcal{O} \to \mathcal{R}(\mathcal{O})$ so as to include unbounded regions $\mathcal{O}$.[20] This brings us to a truly marvellous aspect of Haag's theory. *One may postulate that*

[14] This assignment is linear, and such that positive operators have positive expectation values. The unit operator always has expectation 1.

[15] Irreducible representations correspond to pure states.

[16] Initially taken to be C^*-algebras.

[17] These are von Neumann algebras, which by definition are algebras of bounded operators on a Hilbert space which are closed in the weak operator topology. The extension in question is just the weak closure in the representation induced by any physical state.

[18] The so-called additivity axiom implies that the net $\mathcal{O} \to \mathfrak{A}(\mathcal{O})$ is completely determined by such regions; see Horuzhy (1990) and Baumgärtel and Wollenberg (1992). These books describe a number of further axioms, and derive many theorems from them. Also, the latter book, which is complementary to Haag's, contains a comprehensive introduction to and analysis of the mathematical structure of algebraic quantum field theory.

[19] The commutant $\mathfrak{M}'$ of an algebra of bounded operators $\mathfrak{M}$ on a Hilbert space is the set of all bounded operators which commute with all members of $\mathfrak{M}$.

[20] The reference state is usually taken to be a vacuum state. The extension depends on the reference state, even if one stays in the physical folium.

the map $\mathcal{O} \to \mathcal{R}(\mathcal{O})$ be a homomorphism of orthocomplemented lattices. Recall, for example, that Einstein causality merely requires that $\mathcal{R}(\mathcal{O}^{\perp}) \subseteq \mathcal{R}(\mathcal{O})'$; the postulate implies that $\mathcal{R}(\mathcal{O}^{\perp}) = \mathcal{R}(\mathcal{O})'$,[21] an equality which is central to algebraic field theory.[22] The '$\wedge$' part of the postulate is believed (or has been shown) to hold in all reasonable theories, but the '$\vee$' part excludes even free field theories. It would be an algebraic quantum field theorist's dream that the full postulate be satisfied by gauge theories, but at present there seems to be no evidence for this.

The basic structure of algebraic field theory has now been laid out.[23] What remains is a huge reconstruction programme, in which all the usual ingredients of quantum field theory and elementary particle physics (internal symmetries, unobservable charged fields, Fermi or Bose statistics, particles, antiparticles, scattering cross-sections, etc.) are to be derived from the net of observables. This programme has been completed for theories with short-range forces in four space–time dimensions by the DHR [24] analysis, which is the main topic of Chapter IV.

Haag saw that a superselection sector may be identified with an irreducible representation[25] of the algebra of observables; superselection rules arise if there exist inequivalent representations of $\mathfrak{A}$. The Hilbert space of states of the usual formalism arises as a direct sum over all physically relevant representation spaces ('sectors'). In this setting, the matrix element of any observable between vectors in different sectors identically vanishes, which fact captures the better-known definition of superselection. The essence of the DHR analysis is to firstly construct a large enough family of sectors, and secondly define charged fields as operators that map the various sectors into each other.[26] These charged fields are unobservable, and do not necessarily commute at spacelike distances: one of the results of the theory is that one can have either commutation or anticommutation. In case the field theory describes particles, these alternatives are directly related to their Bose or Fermi statistics; (also, one has the usual connection between spin and statistics). In view of the two-line textbook argument leading to the Bose–Fermi alternative, one may wonder why this part of the DHR analysis is necessary. This argument claims that a permutation of identical particles can only change the state vector by a phase, which then must be ± 1 since repeating the permutation yields the identity. This argument would

[21] With characteristic modesty, Haag refers to this as the 'duality relation'. Everybody else calls it 'Haag duality'.

[22] Haag duality is violated in certain theories, but its non-fulfilment provides interesting information about the vacuum structure of such theories.

[23] Chapter III contains a discussion of the Goldstone theorem as well, but unfortunately the arguments only apply to symmetries of the algebra of observables. For the breakdown of internal symmetries, cf. Buchholz *et al.* (1992).

[24] This algebraic household acronym stands for Doplicher–Haag–Roberts.

[25] Unitarily equivalent representations correspond to the same sector.

[26] The final step from the charged fields of DHR to quantum fields in the conventional sense requires a procedure to pass from nets to quantum fields and back. This difficult issue is not discussed in Haag's book; see Horuzhy (1990), Baumgärtel and Wollenberg (1992), and Borchers and Yngvason (1992).

be correct if all operators on the state space were observable, but it misses the fact that by assumption observables are invariant under permutations. This opens the possibility of having arbitrary unitary representations of the permutation group.[27]

A physically relevant sector is defined by DHR as a representation which corresponds to states ω that do not appreciably differ from the vacuum state ω_0 at spatial infinity[28] A weaker requirement replaces 'infinity' by 'infinity minus a spacelike cone'; the DHR analysis (whose main input is Haag duality in the vacuum sector) largely goes through in that case as well. In either case[29] one concludes after arduous mathematical reasoning that the superselection sectors correspond to the unitary representations of a compact group (playing the rôle of the internal symmetry group of the theory). Roughly speaking, the existence of antiparticles is then equivalent to the existence of a conjugate to any such representation. In the non-abelian case the internal symmetry group plays yet another rôle in the DHR analysis: it converts particles with parastatistics into particles with Bose or Fermi statistics having an additional 'colour' charge.[30]

Given the depth and complexity of the matter, Haag's discussion of the superselection structure is rather concise. For example, since a discussion of multiplicities in the case of non-abelian symmetry groups is missing,[31] a conventional quantum field theorist will still be unable to see how the charged fields of the DHR analysis relate to the usual charged fields acting on a Fock space of (asymptotic) particle states.

Also, the original DHR analysis could have given rise to awkward night thoughts concerning the local nature of the theory. For superselection sectors are initially defined as inequivalent representations of the algebra of observables, yet the inequivalence disappears the moment one restricts the corresponding states to any local algebra. Hence one would think that these sectors are only globally determined, contradicting both the basic locality idea of algebraic field theory and the explicit criterion DHR used to select physically relevant sectors. Such worries are soothed by the fact that the DHR analysis is performed

[27] Unless they are one-dimensional (the Bose or Fermi case), such representations describe so-called parastatistics.

[28] The technical criterion is that ω and ω_0 induce unitarily equivalent representations of $\mathfrak{A}(\mathcal{O}^{\perp})$ for all sufficiently large bounded regions $\mathcal{O}$. Haag does not say how $\mathfrak{A}(\mathcal{O}^{\perp})$ is defined (the net $\mathcal{O} \to \mathfrak{A}(\mathcal{O})$ was initially only given for bounded $\mathcal{O}$); it is the C^*-algebra generated by all $\mathfrak{A}(\tilde{\mathcal{O}})$ for which $\tilde{\mathcal{O}}$ is spacelike to $\mathcal{O}$. All states on $\mathfrak{A}$ are automatically defined on this larger net.

[29] Neither case covers gauge theories. While these do fit into the basic structure of algebraic field theory, tools for analyzing their superselection structure largely remain to be developed (cf. Section VI.3 of the book, and ongoing work by D. Buchholz and R. Verch).

[30] In space–times of dimension lower than 4 these conclusions do not hold; neither does the Fermi–Bose alternative. This is because the set of ordered pairs of double cones (in $d = 2$) or spacelike cones ($d = 3$) is no longer connected. Apart from the brief comments in the book, and references therein to the work of Fredenhagen–Rehren–Schroer and Fröhlich *et al.*, cf. Kastler (1990) and Schomerus (1995).

[31] See in part Doplicher *et al.* (1969), who turn things around by starting from the charged fields and investigating how the observables emerge; this is much closer to the standard approach to internal symmetries.

through the study of so-called localized morphisms of the algebra of observables, but a discussion of this point would not have been out of place. Fortunately, more recent reformulations and mathematical re-interpretations of the theory of charged superselection sectors largely clarify and demonstrate the local nature of these sectors. For a fuller picture, the reader is referred to Buchholz *et al.* (1986), Roberts (1990; 1995), Baumgärtel and Wollenberg (1992) and Baumgärtel (1995).

Chapter V, the longest in the book by a fair margin, involves a mathematical theme which is central to practically all modern developments in algebraic quantum field theory and quantum statistical mechanics in infinite volume. This is the Tomita–Takesaki theory of von Neumann algebras (which was developed in the late 60s, partly in interplay with work on quantum statistical mechanics by Haag, Hugenholtz and Winnink). This theory describes the situation of a von Neumann algebra $\mathfrak{M}$ acting on a Hilbert space $\mathcal{H}$ which contains a vector Ω which is cyclic and separating for $\mathfrak{M}$: this means that (the closure of) $\mathfrak{M}\Omega$ equals $\mathcal{H}$, while $A\Omega = 0$ only holds for $A \in \mathfrak{M}$ if $A = 0$.

This setting is physically relevant in various cases where one possesses incomplete information, either beacuse one truncates the observables or because one starts from a mixed state.[32] In quantum field theory,[33] Ω is a vacuum state (vector) and $\mathfrak{M} = \mathcal{R}(\mathcal{O})$ for some (causally complete) region $\mathcal{O}$ of special interest.[34] Alternatively, $\Omega = \Omega_\omega$ could be a thermal equilibrium state, and $\mathfrak{M} = \pi_\omega(\mathfrak{A})''$ the (weak closure of the) corresponding representation of the algebra of observables. In the algebraic formalism, equilibrium states are characterized by the so-called KMS condition, which is the generalization to infinite volume of the canonical ensemble.[35] The mathematical analogy between these two cases has the surprising consequence that even a vacuum state may look like a thermal equilibrium state if it is restricted to certain local algebras $\mathcal{R}(\mathcal{O})$, and if one uses a suitable notion of time evolution.[36]

While much of the chapter is of a rather technical nature,[37] two elements should be of special interest to the foundations of physics. Firstly, one finds two[38] derivations of the KMS condition (hence of equilibrium quantum statistical mechanics) from first principles. In the first derivation, it is shown

[32] These are related, since the restriction of a pure state to a subalgebra is often mixed.

[33] In which the cyclic and separating nature of Ω usually follows from the Reeh–Schlieder theorem.

[34] Such as a wedge-shaped region $x^1 > |x^0|$. See Borchers (1995) for an overview of applications to quantum field theory in Minkowski space–time. More generally, in curved space–time one takes $\mathcal{O}$ to be a region bounded by a so-called bifurcate Killing horizon, cf. Wald (1994).

[35] In addition to Haag's treatment, see Sewell (1986) and especially Bratteli and Robinson (1981) for an exhaustive discussion of the KMS condition and its ramifications. KMS stands for Kubo–Martin–Schwinger.

[36] This is a mathematical explanation of the phenomenon of Hawking radiation of black holes, and of the related Unruh effect in flat space-time, cf. Wald (1994).

[37] For example, the Tomita–Takesaki theory is essential in showing that in quantum field theory the local von Neumann algebras $\mathcal{R}(\mathcal{O})$ are all isomorphic to each other if $\mathcal{O}$ is a bounded contractible causally complete region, and are even the same for all well-behaved theories.

[38] More derivations and greater detail may be found in Bratteli and Robinson (1981); cf. also Sewell (1986) for a more concise presentation of these ideas.

that KMS states[39] are characterized by their time-independence in conjunction with a condition of robustness under local perturbations of the dynamics. In the second, the thermal equilibrium states at positive temperature are proved to be precisely those in which no energy can be extracted by switching external forces on and off.

Secondly, Haag discusses the notion of 'statistical independence' of two local algebras of observables: this notion sharpens the causality requirement (i.e. commutativity for spacelike separated localizations), and is defined by the property that for any pair $\{\varphi_i, \varphi_j\}$ of (normal) states on $\mathcal{R}(\mathcal{O}_i)$ and $\mathcal{R}(\mathcal{O}_j)$, respectively (with $\mathcal{O}_i$ and $\mathcal{O}_j$ spacelike separated), there exists a (normal) state φ on the von Neumann algebra generated by $\mathcal{R}(\mathcal{O}_i)$ and $\mathcal{R}(\mathcal{O}_j)$ which extends φ_i and φ_j, and in which these algebras are uncorrelated (i.e. φ is a product state).[40] This property is not implied by the general setting of algebraic quantum field theory, but follows if the so-called nuclearity condition is imposed. Nuclearity is of interest in itself: it says, loosely speaking, that the collection of states that are approximately[41] localized in a bounded space–time region spans a Hilbert space that is 'almost' finite-dimensional; this is closely related to the theory having physically reasonable thermodynamic behaviour.

Untypically, Haag does not discuss the physical and conceptual ramifications of this notion of independence of spacelike separated regions, although there are interesting applications to the EPR argument and Bell's inequalities, discussed by Summers (1990), whose treatment is mainly mathematical, and Butterfield (1994), who emphasizes conceptual and philosophical aspects. Moreover, apart from Einstein causality and statistical independence a number of other 'autonomy' conditions on local regions deserve to be imposed or at least mentioned. These are reviewed by Summers (1990) and Horuzhy (1990).

The passage from nets of observables to particles as seen in the laboratory is the subject of Chapter VI. In relativistic quantum field theory, a particle is usually defined as a certain irreducible unitary representation of the Poincaré group which occupies a proper subspace of the total Hilbert space of states. This definition is too narrow. For example, in quantum electrodynamics the electron states are never on their mass shell because of the photon cloud that by Gauss' law has to accompany a charged particle.[42]. Quarks and gluons are generally construed as particles, but fail to show up in the spectrum of the theory which describes them (i.e. quantum chromodynamics) for reasons of 'confinement'. A similar comment applies to unstable 'particles' like the W and Z mediating the electroweak interaction. It turns out that algebraic quantum field theory naturally leads to a more general particle concept than Wigner's. Rather than basing itself on the energy–momentum spectrum of the theory, it is built on localization. An additional advantage is that (generalized) particle states can be

[39] Here including those defined for negative temperatures, as well as ground states.

[40] There is a disturbing typo in eq. (V.5.23), in which $\hat{\mathcal{R}}'$ should be replaced by $\hat{\mathcal{R}}$.

[41] Exact localization is not possible in view of the Reeh–Schlieder theorem; one has to allow non-local exponential tails.

[42] Overlooking this leads to the infrared problem in perturbation theory, cf. Weinberg (1995)

constructed directly from the algebra of observables $\mathfrak{A}$ (unlike the Haag–Ruelle scattering theory, where one needs unobservable charged fields).

To begin with, a particle detector is mathematically represented by a positive element C of $\mathfrak{A}$ which is 'almost'[43] localized in a region $\mathcal{O}$ and has zero vacuum expectation value. An n-particle state is then defined by its response to operators of the form $C(x_1)\ldots C(x_n)$ in the limit where the times x_i^0 approach $\pm\infty$. It turns out that the limiting procedure necessitates the use of so-called weights, which are states on a subalgebra of $\mathfrak{A}$ (which has to be a left-ideal; in the present case, the subalgebra is generated by the detector operators) that do not extend to states on all of $\mathfrak{A}$. Instead, they are represented by improper vectors in certain representations of $\mathfrak{A}$. Relating to a previous example, such single-particle weights describe pure electrons, whereas the corresponding proper states would incorporate the photon cloud. This novel approach[44] has a wide range of applications: apart from gauge theories, it is relevant to quantum field theory at finite temperature, and perhaps in curved space–time.

The final chapter contains some comments on a variety of topics, of which quantum field theory in curved space–time stands out in the present context. To the extent that no use of Poincaré symmetry and vacuum states is made, algebraic quantum field theory applies to curved space–times; indeed, it may form the only coherent framework[45] for describing quantum physics in the presence of a (classical) gravitational field. This, however, deserves a book in its own right; Wald (1994) is recommended.

The revised edition contains a new chapter, entitled *Principles and Lessons of Quantum Physics. A Review of Interpretations, Mathematical Formalism, and Perspectives.* This starts with a review of Bohr's epistemological considerations; perhaps because Haag knew Bohr personally, his discussion is particularly enlightening. Brief coverage of Einstein's critique, the measurement problem, decoherence, the arrow of time in quantum mechanics, and of axioms (allegedly) leading to the Hilbert space formalism, prepare the way for Haag's own current vision of quantum mechanics (also cf. Haag, 1990; Haag, 1996).

Still in its infancy, this is an observer-independent formulation in which, compared with the orthodox interpretation, measurements are replaced by 'events'. Here one should think of interactions (such as collisions) which are localized in space and time;[46] measurements are seen as particular examples. Events are Haag's candidate for Einstein's 'elements of reality'. A particle is seen as a causal link between two events: the creation of the particle in a source, and

[43] Since the vacuum expectation value of any positive operator in a local algebra $\mathcal{R}(\mathcal{O})$ is non-zero by the Reeh–Schlieder theorem, strictly localized operators cannot represent a particle detector.

[44] Which, following earlier ideas of Haag himself, was (and is) mainly (being) developed by D. Buchholz and collaborators.

[45] Here it seems advantageous to modify the technical machinery somewhat, employing the so-called Borchers–Uhlmann algebras (Horuzhy, 1990) rather than the Haag–Kastler nets.

[46] In the initial discussion space–time is assumed to be ontologically primary, as in algebraic quantum field theory, where observables are predicated on regions in space–time. Later on, Haag says he would prefer space–time to be a derived concept, bearing the stamp of the ordering relations between events.

its disappearance in a sink (think of a photon hitting a photographic plate). Such causal links become real only after both endpoints have been realized. A point Haag stresses again and again is that a particle does not have a position (even when its momentum is infinitely unsharp); what has a position is an event involving the particle (such as its detection).

Which events happen, and where, is governed by the probabilistic laws of quantum mechanics. What is normally seen as the collapse of the wavefunction following a measurement is now regarded as a change of an objective state after an event has occurred. Amplification is not necessary for such factualization. This leads to an evolutionary picture of physics: the past consists of a collection of events (linked by causal ties), whereas the future is open, in that there are no deterministic laws predicting which possibilities transform into facts. Irreversibility is a consequence of stochasticity; hence the arrow of time is seen to be quantum-mechanical in origin.

Taken by itself, none of the above ideas is particularly new; most modern authors treat measurements as particular kinds of interactions, and equally many are keen to abandon the so-called eigenvector–eigenvalue link (which Haag implicitly does in his discussion of particle localization). Also, the idea of a relational definition of factualization is common to most versions of the many-worlds interpretation and the modal interpretation. Combining all these with the insistence on space–time localization seems typical of Haag, however. Much remains to be elaborated, but the seeds for an attractive realist interpretation of quantum field theory appear to be there.

3. Discussion

In the community of mathematical physicists there exists no doubt that the body of work reported in Haag's book (much of which was initiated by Haag himself) is of the highest quality and depth. While experts may criticize certain details of some arguments, or even the general style of the book (which is a compromise, admittedly at times somewhat peculiar, between an overview and a textbook), the reviewer found the book pleasant and highly rewarding reading. Philosophers of physics will be daunted by the mathematics, but will eventually appreciate the precision and clarity in addressing the foundations of quantum field theory (and, to a lesser extent, quantum statistical mechanics).

However, as far as the traditional foundational questions in quantum physics are concerned,[47] Haag's book leaves the reader with questions. For one thing, both the orthodox Copenhagen spirit and Haag's increasingly critical attitude towards it are reflected in the writing. One often finds an operationalistic attitude: physical relevance seems equated with measurement and observation, and even special relativity is motivated by the need for a convention for the synchronization of clocks. Haag stresses the need for the 'Heisenberg cut' between a system and an observer with his instruments, and quotes Bohr in

[47] Cf. Saunders (1988) for a philosophical critique of algebraic quantum field theory in general.

saying that the observer plus instruments side is described by classical physics (p. 3). Mathematically, though, the system side is described by states, which are physically identified with an equivalence class of preparation procedures, whereas instruments are represented by operators in a non-commutative algebra (ideal detectors corresponding to projection operators). Few quantum field theorists would follow Haag in believing that (the observable combinations of) their quark and photon field operators describe the detectors placed in a collider (or, worse, the experimentalists' manuals) rather than the fields themselves.

Haag's attempt to motivate the formalism of quantum mechanics by appealing to the theory of self-dual cones and Jordan algebras is not really carried through, and would be fishy if it were. For infinite-dimensional state spaces do not emerge in that fashion: to arrive at those, one needs the additional axiom of facial homogeneity, whose physical relevance is agreed even by the proponents of this approach to be obscure; cf. Ajupov *et al.* (1990). Moreover, the step from Jordan algebras to C^*-algebras is non-trivial.

The physical meaning of localization is that elements of $\mathfrak{A}(\mathcal{O})$ are 'observables which can be measured in the region $\mathcal{O}$' (p. 110). However, as we saw in the discussion of Chapter VI, particle detectors can only be localized up to infinite tails. To those worrying about such epsilonics, Haag would presumably say that: 'we do not want to bother and we need not know in such detail' (p. 273). It is not clear, however, that this cavalier attitude is justified. For example, in Chapter VI (esp. pp. 281–282) we learn that (in an infinite system) the velocity of an electron defines a superselection rule, excluding quantum interference. As Haag is quick to point out, this conclusion is in manifest contradiction with experiment; tongue in cheek, he adds 'the reader is encouraged to work out how the quantum mechanical description of an electron interference experiment can be justified in the field theoretic setting' (p. 282)!

A related point is that all physically admissible (i.e. locally normal) states happen to be mixed on each local algebra $\mathcal{R}(\mathcal{O})$; their possible purity on $\mathfrak{A}$ derives from operators that are infinitely delocalized. Thus, contrary to the locality principle, one might come to believe that physically relevant things are happening at infinity. The resolution of this difficulty is given in Buchholz *et al.* (1986), and is based on the so-called split property (which is discussed by Haag, though not quite in the present context). This property, which follows from the nuclearity condition discussed earlier, says that for any pair of bounded regions $\mathcal{O}_1 \subset \mathcal{O}_2$, with associated local (von Neumann) algebras $\mathcal{R}(\mathcal{O}_1) \subset \mathcal{R}(\mathcal{O}_2)$, there is an intermediate algebra[48] $\mathcal{N}$, such that $\mathcal{R}(\mathcal{O}_1) \subset \mathcal{N} \subset \mathcal{R}(\mathcal{O}_2)$, and physical states exist which are pure on $\mathcal{N}$. Hence one can doctor the region between $\mathcal{O}_1$ and $\mathcal{O}_2$ so as to screen off all external correlations. The fact that all physical states are mixed on a region with a sharp boundary remains.

In any case, it would be welcome to have a theory describing how superselection rules in the most general sense of inequivalent representations are approximated in very large but finite regions, so that one eliminates the dilemma that $\mathfrak{A}$ (describing an infinite system) has sharp superselection rules

[48] Technically, a type I factor.

whereas none of its approximants $\mathfrak{A}(\mathcal{O})$ (for a bounded region $\mathcal{O}$) admits (non-pathological) inequivalent representations. Such a theory would be as appealing as the theory of phase transitions in classical statistical mechanics, where one can see the onset of a phase transition in a finite system by its behaviour at the boundary.

Haag expresses the feeling that it is to the credit of algebraic quantum field theory that it stands on its own, in the sense that it is not the quantization of any classical theory (unlike conventional quantum field theory). Nonetheless, it would have been useful to see what the classical counterparts of some its concepts and ingredients are. For example, does Einstein causality (whose classical analogue would be the property that certain Poisson brackets vanish) really express the mutual non-disturbance of particular measurements, or is it a statement about (the absence of) transformations generated by the observables? And what is the classical analogue of a superselection sector? Even if one does not believe that quantization is a fundamental procedure, one may see such questions in the context of the classical limit of quantum field theory, which may shed considerable light on the quantum theory itself.[49]

Despite these somewhat critical remarks, the reviewer would like to emphasize that for those interested in the foundations of quantum field theory Haag's book is second to none. Indeed, everyone interested in modern physics should read it.

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[49] For example, the classical analogue of an operator algebra is a so-called Poisson algebra, see Marsden and Ratiu (1994) and references therein. The classical counterpart of a representation is called a realization (Weinstein, 1983). Even the Tomita–Takesaki theory has a classical lookalike.