Modelling Time in Logic and Metaphysics

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Is the flow of time—'temporal becoming', 'the movement of the now'—an objective feature of reality?

Or are past, present and future all 'on a par': with the 'now' being like the 'here'?

I believe: the flow is not objective: past, present and future are all equally real.

I will suggest it is best to make the debate precise by asking:
(i) whether 'reality grows'; and
(ii) whether time branches towards the future.

It is hard to make precise proposals along these lines.

The proposals depend on words like ‘real’, ‘actual’, ‘fundamental’, ‘concrete’ etc.: words which are vague—and, worse: disputed.

Besides: Precise proposals often seem:
(a) to fail to express ‘the moving now’.
(b) hard to reconcile with relativity.

That gives us some reason to conclude that the flow is not objective!

Plan

McTaggart's jargon (1908): the 'A' vs. 'B' theories of time.
The A-theory holds that the passage of time is objective.
The B-theory (the 'block-universe' view) denies this.

Another jargon: that there are 'tensed facts' ('tenserism') or not ('detenserism').

The B-theorist, or detenser, says: Abraham Lincoln is just as real as Bill Clinton, just as Venus is just as real as Earth: Lincoln is merely 'temporally far away from us', just as Venus is spatially far away.

Similarly for a young child's first grandchild, supposing the child will have one. There are, now and for always, myriadly many facts about the grandchild: it is just very hard to know them!

There are various kinds of A-theorist. They vary about which feature of temporal reality, they accuse the B-theorist of missing.

One main kind says: 'Much less is real than the B-theorist says'.

For example: They say:
(a) Only the present exists: i.e. only presently-existing objects, states of affairs etc. are real.

Or they say:
(b) Only the present and past exist.

To keep the debate simple, one hopes that the two sides agree on the distinction between real and unreal—in meaning (intension), though not of course in instances (extension).

Here, it is natural to relate the debate to one about modality: possibility and necessity. Thus 'unreal' is glossed as 'merely possible'.

Such an A-theorist says that the future, and maybe also the past, is not actual, but is merely possible.

For the past, this implies: Abraham Lincoln and Sherlock Holmes are on a par!

Another kind of A-theorist says:
'The block universe is all real. But it leaves out facts about what is now (and so also, about what is past and what is future): ever-changing, "dynamic", facts.'

Or: 'The block universe conflates in one alleged reality, what is truly a plethora of perspectival realities. Each moment has its own perspectival reality: e.g. the block universe, with that moment "labelled" as 'now'.

The B-theorist will reply:
'I agree that: each time is present, is now, relative to itself, and is T time-units past/future relative to times (objects, states of affairs) T time-units future/past relative to it! What more do you want?!'

The A-theorist tends to reply: 'The facts I want, the facts about what is now etc., are *irreducible* to your block-universe facts: they are *tensed facts*'.

It is hard to make this sort of debate precise. It is tempting to relate it to debates about semantics ... though we should beware of the gap between semantics and metaphysics . . .

Recall the fundamental notions of logic!

In *propositional logic*, we model the logical behaviour of 'and', 'or', 'not' and 'if-then':—

(i) syntactically, as connectives that build sentences from others, e.g. $A \wedge B, A \vee B, \neg A$: hence

(ii) semantically, as truth-functions: $A \wedge B$ is true iff A is true and B is true: hence truth-tables, with each row of a truth-table representing a possible valuation of all formulas that is defined by the values assigned to the basic letters $A, B, \ldots$; hence the notion of logical consequence, $A_1, \ldots, A_n \models B$ iff every valuation that makes true all $A_1, \ldots, A_n$ also makes true B.

In *predicate logic*, we also model the logical behaviour of 'all', 'any, 'every', 'none' etc, by:—

(i) Breaking down an atomic sentence, previously represented just as $A, B\ldots$ etc, into a predicate (1-place $F()$, 2-place $G(,)$ etc., and a corresponding number of terms, i.e. arguments of the predicate: $F(a), G(b,c), H(d,e,f)\ldots$;

(ii) Allowing composition of sentences: not just by connectives, e.g. $F(a) \vee G(b,c)$ and $(F(a) \vee G(b,c)) \wedge \neg H(d,e,a)$ etc.; but also by the existential quantifier, e.g. $(\exists x)F(x)$, and the universal quantifier $(\forall x)F(x)$, and by their iterations e.g. $(\forall y)(\exists x)G(y,x)$ and $(\exists x)(\forall y)G(y,x)$.

(iii) Notice how the different orders of quantifiers, corresponding to different stage-by-stage constructions of the formula, disambiguates the two meanings: 'for everyone, there is somebody that they love' vs. 'there is somebody such that everyone loves them'.

(iv) Corresponding to these extra syntactic structures, the notion of a valuation is extended to: postulating a domain *Dom* of objects, the *domain of quantification*; and assigning:

each term $a, b, c \dots$ to an object in *Dom*: its *reference* or *denotation*

each 1-place predicate a subset of *Dom*: its *extension* or *set of instances*

each 2-place predicate a subset of $Dom \times Dom$, i.e. a set of ordered pairs of objects: its *extension* or *set of instances*; etc.

This defines the conditions for any atomic sentence e.g. $F(a)$, $G(b, c)$, and any truth-functional compound of atomic sentences to be true according to the valuation.

We also say define a formula $(\exists x)\Phi(x)$ as true according to the valuation iff there is some object in *Dom* that satisfies (in the obvious sense, by the previous definitions) the formula $\Phi()$. Similarly, $(\forall x)\Phi(x)$ is defined as true iff all objects in *Dom* satisfy $\Phi()$.

To adapt all this to modelling temporal language!

Detenserism suggests an obvious approach: a simple semantics with the two familiar truth-values.

We take a single domain of quantification containing all objects that ever exist (Lincoln, Clinton, my first grandchild if such there be...), so that the existential quantifier represents existence at some time of other, e.g. $(\exists x)(x = \text{Lincoln})$ is true absolutely, once and for all.

And the domain also includes times and time-intervals, for which the language contains terms—e.g. 'noon, 15.x.2019', 'now', 'Tuesday 15.x.2019', 'today'.

And the language contains a special predicate, E say, for temporal existence, so that e.g. $E(\text{Lincoln,1860})$ and $\neg E(\text{Lincoln,now})$ are both true.

If you want the existential quantifier to represent the present-tensed 'exists', then you can instead postulate a linear order of domains, each labelled by a time and each containing all the objects that exist at that single time.

The rest of the semantics then proceeds much as before. ...

But **tenserism** suggests more complex semantic proposals: e.g.

(i) having a third truth-value for cases where a term has no presently-existing reference; or

(ii) having times form a branching tree, to represent an 'open future'.

I shall develop only elements of the idea of a branching future: for propositional logic only, and without considering a third-truth value. My point will be that *detenserism* can also have a branching tree semantics!

Imagine, a branching tree, with points $p, q, \ldots$; we write $p < q$ for p is before q. We define at each point p, a truth-functional valuation over all sentence letters $A, B, \ldots$.

We think of A and B as present-tensed, and without any other time-indicators; cf. e.g. $F(a)$ as representing 'Albert is angry'.

We introduce non-truth-functional sentence operators F and P, so that $F(A)$ read as 'It will be the case that A' and $P(A)$ is read as 'It was the case that A. And we define semantics by:

$P(A)$ is true at p iff there is a point $q < p$ such that A is true at q.

For $F(A)$ we have a choice:

(i) at *all* points $q > p$, A is true at q: (A must forever be so!);

(ii) at *some* point $q > p$, A is true at q: (A can somewhen be so).

These two rules, (i) and (ii), agree in some of their logical behaviour. They both make valid, i.e. true at all points in all trees according to all valuations, the formula $F(F(A)) \rightarrow F(A)$. (Here a formula $\Phi \rightarrow \Psi$ is short for $\neg\Phi \wedge \Psi$.)

This formula is intuitively valid. Indeed, with rule (i): requiring the formula to be valid *implies* the transitivity of the relation $<$ between points, that for all p, q, r, if $p < q$ and $q < r$, then $p < r$.

So there are many choices to explore ... but the main point is ...

We can readily combine a branching tree with the idea of an actual future. (Cf. William of Ockham, Luis de Molina).

We require that through each point p, one of the maximal paths (whose past part is of course determined just by p alone) is distinguished as the actual history. We write it as $Hist(p)$. We then give the semantics of $F(A)$ in terms of this selected path:

$F(A)$ is true at p iff there is a point $q \in Hist(p)$ with $q > p$, and such that A is true at q.

This proposal is viable, though a bit subtle. For example: to make the formula $F(F(A)) \rightarrow F(A)$ valid, it *seems* necessary to require that the actual histories through different points 'mesh' in the sense that:

If $p < q$, then $Hist(q) = Hist(p)$.

So you might impose that. But then $p < q$ implies that $q \in Hist(p)$, and there is, after all, only one history!

No worries! We can require that the actual histories through different points 'mesh' in a weaker sense. Namely:

If $p < q$, and $q \in Hist(p)$ then $Hist(q) = Hist(p)$.

This semantic rule will also make the formula $F(F(A)) \rightarrow F(A)$ valid.

So much by way of glancing at logics for temporal language.

But we should beware of the gap between (a) logic and semantics and (b) metaphysics. Do the tenserist's semantic rules really express 'the moving now'?

And once we consider space: can tenserist proposals be reconciled with relativity?

I will from now on focus on the proposals that:

(i) *Time grows*. That is (roughly speaking!):
The present and past are real, but the future is unreal; so the sum-total of reality increases with time as the future becomes present (Broad 1924, Earman 2008);

(ii) *Spacetime branches towards the future*. That is (roughly speaking!):
The spacetime manifold contains many equally real alternatives about the future (indeterminism!), but only one past; (Belnap 1992, Müller 2015).

The precise proposals will often seem:
(a) to fail to express 'the moving now';
(b) hard to reconcile with relativity.

A Newtonian *Blockhead* model comprises a four-dimensional space-time M which is equipped with geometric structures G_i and physical fields P_j.
Formally, it is given by: $\mathcal{N} := \langle M \equiv \mathbb{R}^4, G_i, P_j \rangle$.
One of the G_i is an absolute time function $T : M \to (-\infty, +\infty)$.

We first construct a *Broadhead* model as the chips off a given Newtonian block.

Thus the future-truncated model $\mathcal{N}_{T \leq \Delta}$ represents the sum total of existence at time $T = \Delta$. And we define:

$\mathcal{B}(\mathcal{N}) := \langle \{\mathcal{N}_{T \leq \Delta} : -\infty < \Delta < +\infty\}, \preceq \rangle$ with $\preceq$ defined by:
$\mathcal{N}_{T \leq \Delta} \preceq \mathcal{N}_{T \leq \Delta'}$ iff $\Delta \leq \Delta'$.

Then $\preceq$ inherits the order-structure of $\mathbb{R}$. It is total (reflexive, transitive, antisymmetric, connected), linear, dense and continuous.

That seems very parasitic on the Blockhead model! Perhaps it would be less "cheating" to appeal to the class of all Blockhead models. That is, we might define:

$\mathcal{B} := \langle \mathsf{N}, \preceq \rangle$ where each element n of N is isomorphic to a chip $\mathcal{N}_{T \leq \Delta}$ of some Newtonian block model or other; and $\mathsf{n} \preceq \mathsf{n}'$ iff n can be embedded in n'.

This $\preceq$ is a preorder (reflexive and transitive). But $\preceq$ need not be antisymmetric; suppose the elements n are chips off some stationary (no B-series change) or strictly periodic Newtonian block $\mathcal{N}$.

And $\preceq$ need not be connected, or dense, or continuous.

The simplest general way to secure these is presumably to require that all the elements n of N are chips off *the same* Newtonian block $\mathcal{N}$ ...

Broadheads can hope to state (mild, honest) conditions on $\langle N, \preceq \rangle$ such that they can prove:

(i): for any $\langle N, \preceq \rangle$, there is a $\mathcal{N}$, into which each $n \in N$ can be embedded;

(ii) assuming an appropriate determinism, this $\mathcal{N}$ is unique up to isomorphism.

Thus: the Blockhead model as an ideal completion.

Assume a relativistic Blockhead model, given by a four-dimensional space-time M equipped with geometric structures G_i and physical fields P_j, that is: $\mathcal{R} = \langle M, G_i, P_j \rangle \equiv \langle M, g, P_j \rangle$

admits a global time function $t : M \to \mathbb{R}$ with bounds $l < u \in \mathbb{R}$.

Then the cheating construction of a Broadhead model $\mathcal{B}(\mathcal{R}, t)$ carries over, with t replacing Newtonian absolute time T; as does the less cheating construction, and most of the assessment of these. But ...

(1): Having a global time function is not generic among GR models. Should a Broadhead cut down the set of acceptable models, or modestly claim Becoming only for the actual cosmos?

(2): A spacetime with a global time function has continuously many such. And two such functions can match up to some hypersurface. This threatens, like the hole argument, radical indeterminism of the facts of Becoming.

Various natural ways to privilege a function apply only to a limited class of models: e.g. foliation by hypersurfaces of constant mean curvature.

And some natural limitations do not yield a unique function (or even foliation).

E.g.: Suppose you require global hyperbolicity ($\equiv$ there is a Cauchy surface, and so the spacetime is a canvas appropriate for Laplacian determinism).

But if there is one Cauchy time function, there are continuously many.

(3): A radical response is to take actual history to be given by the *class*:

$$\{\mathcal{B}(\mathcal{R}, t) : t \text{ a global time function of } \mathcal{R}\}$$

A minimal objectivity of Becoming is secured by all the elements $\mathcal{B}(\mathcal{R}, t)$ having the same $\mathcal{R}$ as their ideal completion.

Again, one can envisage representation theorems, whose cheating, theft or honest toil can be assessed ....

Recall from Section II: Logicians have long studied models of formal languages with a future-branching time.

Happily for detensers: these models can include an actual future, i.e. at each point in the tree, one of its future branches is privileged.

Belnap (1992 et seq.) rejects the actual future, and advocates models with a branching *spacetime*.
The idea is that causal-temporal precedence $\leq$ is a dense connected partial order on a single vast set W (for 'world') of *point events*, with each lower bounded chain having an infimum and with:

(1) (*Branching*): each set h of points that is maximal in being:
(i) downward-closed, i.e.
$[x \in h$ and $y < x] \Rightarrow y \in h$, and:
(ii) upward-directed, i.e.
$x, y \in h \Rightarrow \exists z \in h$ with $x, y \leq z$,

represents a possible history.

(2) (*Choice*): if C is a lower-bounded chain in h, disjoint from h', then there is a *choice event* $x \in h \cap h'$ that is maximal in $h \cap h'$ and less than every element of C: $x < C$.

Example: Take L copies of (vacuum) Minkowski spacetime, and for any two copies, choose a point x and identify the copies throughout the past Light-cone and Elsewhere of x. This makes each copy a history.

The Belnap (1992) BST axioms are logically weak; but they sustain formal investigations of irreducibly stochastic events, and even of e.g. violations of Bell inequalities.

To strengthen them to make them more physical, we need to:

[1]: Beware that indeterminism in physics and its philosophy is usually analysed in terms of, not branching of a single solution (spacetime model, possible world), but: non-isomorphic pairs of solutions having isomorphic initial segments: called *divergence*.

[2]: Choose a physical theory to guide how we strengthen; e.g. SR or GR, or better, a stochastic classical field theory within them.

[3]: Bear in mind the theory's formal obstacles to branching. In particular, GR has various theorems that branching spacetimes are unphysical (Earman 2008, Section 3).

[4]: Bear in mind that if we add a topology to the BST framework, overlapping histories generally make the topology non-Hausdorff. Cf. a copy of $\mathbb{R}$ that branches *at and* above 0: then the two copies of 0 are non-Hausdorff twins.

Agreed: a copy of $\mathbb{R}$ that branches just above 0 is Hausdorff. Besides, this is like the *Choice* principle. Idea: "the splitting event does not show its colours".

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A past- and future-endless worldline $\gamma \in \mathcal{R}$; maybe of some privileged sort, eg a geodesic.

The "cheating construction" of a worldline-relativized Becoming model is then:– $\mathcal{B}(\mathcal{R}, \gamma)$ is defined as:

$$\langle \{\mathcal{R}_{J^-(p)} : p \in \gamma\}; \preceq \rangle$$

where (i) each $\mathcal{R}_{J^-(p)}$ is obtained by deleting from $\mathcal{R}$ all points not in $J^-(p)$ and then restricting the fields; and

(ii) $\mathcal{R}_{J^-(q)} \preceq \mathcal{R}_{J^-(r)}$ iff $J^-(q) \subseteq J^-(r)$.

Generalizing over γ, it is natural to require that if $q << r$, then $J^{-}(q) \subset J^{-}(r)$.

This is entailed by the spacetime being causal-past distinguishing, i.e. by

$J^{-}(q) = J^{-}(r) \quad \Rightarrow q = r$; which is much weaker than stable causality.

So the Broadhead here eliminates fewer models of orthodox GR than in Section 2.
One envisages that one could carry over from Section 1 the ideas for:

(i) a less cheating construction of $\mathcal{B}(\mathcal{R}, \gamma)$or $\mathcal{B}(\mathcal{R})$;

(ii) its need for extra conditions; and

(iii) the assessment of these.

But in some spacetimes: for some γ:

$J^-(\gamma) := \cup_{p\in\gamma} J^-(p) \subset M$; i.e. $J^-(\gamma) \neq M$.

Or even: for all γ; e.g. de Sitter spacetime.

Say that a Blockhead relativistic model $\mathcal{R}$ is *observationally indistinguishable* from another $\mathcal{R}'$ iff:

for all γ in $\mathcal{R}$, there is a γ' in $\mathcal{R}'$ such that $J^-(\gamma)$ and $J^-(\gamma')$ are isomorphic.

Then the gist of Manchak's theorems (2009,2011) is that almost every $\mathcal{R}$ is observationally indistinguishable from a non-isomorphic model $\mathcal{R}'$.

And $\mathcal{R}'$ can lack any or all of four global properties that $\mathcal{R}$ may have: being globally hyperbolic, being inextendible, being hole-free, and being spatially isotropic.

Blockheads read this as epistemic underdetermination: even an ideal observer in $\mathcal{R}$ who lives forever and observes the entire field-content of their past light-cone $J^{-}(\gamma)$ cannot know much about the global structure of her spacetime.

Presumably, Broadheads will read this as ontic underdetermination. They envisage that they have some Becoming model $\mathcal{B}$ specified in some way they consider non-cheating.

So they expect Manchak's theorems to imply that a (or any, or a generic) $\mathcal{B}$ can have as ideal completions two Blockhead models $\mathcal{R}$ and $\mathcal{R}'$ that differ on such global properties. So for the Broadhead, there is no fact of the matter about such properties ...

Cf. a fragment of constructivist mathematics embeddable into two different classical, or less-constructivist, fragments.

And Placek & Belnap (2011) show that:

(a): given the diamond topology on W, the subspace topology on any history h is Hausdorff;

(b): if there is indeterminism without choice, the diamond topology is not Hausdorff.

But Mueller (2012) shows there may be good reason to sacrifice the *Choice* principle, and so Hausdorffness for W; in favour of "the splitting event showing its colours".

For under natural definitions, *Choice* conflicts with another strong desideratum: the topology on W being locally Euclidean.