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The Unruh effect for philosophers ☆

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ABSTRACT

The importance of the Unruh effect lies in the fact that, together with the related (but distinct) Hawking effect, it serves to link the three main branches of modern physics: thermal/statistical physics, relativity theory/gravitation, and quantum physics. However, different researchers can have in mind different phenomena when they speak of "the Unruh effect" in flat spacetime and its generalization to curved spacetimes. Three different approaches are reviewed here. They are shown to yield results that are sometimes concordant and sometimes discordant. The discordance is disconcerting only if one insists on taking literally the definite article in "the Unruh effect." It is argued that the role of linking different branches of physics is better served by taking "the Unruh effect" to designate a family of related phenomena. The relation between the Hawking effect and the generalized Unruh effect for curved spacetimes is briefly discussed.

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1. Introduction

One way to achieve immortality in physics is to have your name attached to an important equation or effect. By this measure William G. Unruh is numbered among the immortals by having his name attached to

$$T_U = \frac{\hbar a}{2\pi c k} \quad (1)$$

which asserts that an observer in constant linear acceleration through the Minkowski vacuum for a non-interacting scalar field will find herself immersed in a thermal bath at a temperature proportional to the magnitude a of her (proper) acceleration. (From here on set $\hbar = k = c = 1$ unless otherwise noted.) In Unruh's (1990, pp. 108–109) own colorful characterization "You could cook your steak by accelerating it". This method of cooking is not apt to replace a charcoal grill since an acceleration of $10^{24}\,\text{cm/s}^2$ is required to achieve a temperature of 300 °C (Unruh, 1990, p. 109). But it is not the *size* of the effect but its *existence* that matters: the Unruh effect and the related (but distinct) Hawking effect serve to link the three main branches of modern physics—thermal/statistical physics, relativity theory/gravitation, and quantum physics—and to my knowledge these are the only effects that currently serve this function. Together they are widely regarded as forming a valuable signpost in the search for a quantum theory of gravity (see Smolin, 2000).

The literature on the Unruh effect begins with Unruh (1976, 1977a, 1977b)[1] and it continues in a steady stream down to the present day, with the number of citations to Unruh (1976) averaging 50 or above in recent years. A related effect was obtained earlier by Davies (1975); namely, when the right Rindler wedge (see Section 3 below) is equipped with a reflecting wall to the right of the origin, an observer uniformly accelerated through the Minkowski vacuum sees the wall radiate at (what would come to be called) Unruh or Davies–Unruh temperature. The reader who wishes to get a sense of the development of the various treatments of the Unruh effect may consult the articles by Sciama, Candelas, and Deutsch (1981), Birrell and Davies (1982), Takagi (1986), Fulling and Ruijsenaars (1987), Ginsburg and Frolov (1987), and Wald (1994, Chap. 5). A recent review article by Crispino, Higuchi, and Matsas (2008) will surely become a standard reference. An overview of proposed experimental tests can be found in Rosu (2001) and Crispino et al. (2008).

The Unruh effect is not uncontroversial—some critiques can be found in Belinskii, Karnakov, Mur, and Marozknyi (1997), Fedetov, Mur, Narozhny, Belinskii, and Karnakov (1999), Narozhny, Fedotov,

☆ The "for philosophers" qualification is not supposed to signal that the discussion is less technical than treatments in the physics literature but rather that the emphasis is on issues of concern to philosophers of science. For this reason physicists may find the discussion uninteresting and/or annoying. For this I have no apology but only the explanation that, despite the focus on a common subject matter, physics and philosophy of physics necessarily differ in style and emphasis.

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[1] The results of Unruh (1977a) were presented in July 1975 at the first Marcel Grossmann Meeting on General Relativity. I take this to be the first public presentation of the Unruh effect.

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Karnakov, Mur, and Belinskii (2000, 2002), and Oriti (2000).[2] Although the naysaying is largely without merit, the fact that the naysayers can publish in main line physics journals shows that there is some confusion about what the Unruh effect is. This confusion derives in large part, I will argue, from the fact that there are at least three different approaches to the Unruh effect that yield related but not always concordant results. This does not mean that the Unruh effect does not exist (as some naysayers insinuate) but rather that "the Unruh effect" is a bit of a misnomer since it refers ambiguously to one or another of a family of effects.

Additionally, the controversy was also fueled by the fact that the Unruh effect has been enlisted in the service of two allied campaigns. The goal of one was to operationalize the particle concept in quantum field theory (QFT), the slogan being that "Particles are what particle detectors detect."[3] The other campaign had as its goal the demotion of the particle concept in QFT to second class citizenship, the argument being that the notion of particle has to be relativized to a reference frame or an observer. The Unruh effect supposedly supports this campaign as follows: the detector employed by Unruh (1976) in his initial exploration of the eponymous effect and the monopole version used by DeWitt (1979) have been labeled "particle detectors," and (allegedly) when such detectors are in constant linear acceleration through the Minkowski vacuum they register a thermal flux of particles, variously called Rindler or Fulling particles.[4] Thus, one sometimes sees references to the "Fulling-Unruh effect" (Korbakken and Leinaas, 2004) or the "Fulling-Davies–Unruh effect" (see Vanzella and Matsas, 2001; Matsas and Vanzella, 2003).[5] My own view (which I will not defend here) is that there are strong reasons for regarding particles as having the status of epiphenomena in QFT in that the best interpretation of the theory does not count them as being part of the basic ontology but rather seeks to explain how and why particle-like behavior arises under certain circumstances; but I believe that the case for this interpretational stance can be made without having to appeal either to an operationalist conception of particle or the dubious notion that a uniformly accelerating observer encounters a flux of Fulling quanta (see Section 6).

In what follows I will examine three approaches to the Unruh effect. Section 2 gives a brief review of modular theory, the mathematical tool needed for the most rigorous and model-independent approach. The application of modular theory to the Rindler wedge and other regions of Minkowski spacetime is discussed in Section 3. Section 4 reviews the generalization of the Unruh effect to curved spacetimes and the relation of this generalized effect to the Hawking effect. As with the flat spacetime case, the key concepts are drawn from modular theory. While there can be no doubt about the precision and power of the mathematical apparatus of modular theory, there are reasons to be chary about drawing physical consequences from the apparatus. Some of these reasons are aired in Section 5. This makes it desirable to explore other approaches to the Unruh effect that do not rely on modular theory. Section 6 explores a way of understanding the Unruh effect in terms of the Fulling quantization of the Klein–Gordon field on a Rindler wedge region of Minkowski spacetime. Section 7 discusses the explication of the Unruh effect in terms of the response of accelerated detectors. Summary and conclusions are presented in Section 8. Throughout the focus of the discussion will be on non-interacting scalar fields because this is the case for which a large number of precise results have been proven. Readers interested in the Unruh effect for interacting fields can start with Gibbons and Perry (1978) and Unruh and Weiss (1984). The list of references at the end represents only a small slice of the vast literature on the Unruh effect and topics directly related to it, but it is intended to be representative enough to provide guidance to readers who wish to explore various facets of the Unruh effect in more depth.

Achieving a balance between readability and rigor when expositing these topics is not easy, and I can only hope that the choices I have made do not fatally compromise either goal. As far as possible, details on operator algebras, relativistic spacetimes, etc. have been relegated to the Appendices.

2. KMS states and modular theory

For a number of leading theoretical physicists, the official version of the Unruh effect is explicated in terms of KMS states and modular theory (see, for example, Sewell, 1982; Kay and Wald, 1991; Wald, 1994, 1999, 2001; Haag, 1996, Section V.4.1). However, as far as I am aware Unruh himself has never endorsed this approach, and his own expositions of the eponymous effect emphasize the detector approach described below in Section 7. This section provides a quick and superficial review of some of the terminology and results of modular theory. Readers desiring more details are referred to Bratelli and Robinson (1987) and Emch and Liu (2001).

In quantum statistical mechanics (QSM) a *Gibbs state* at inverse temperature β ($=1/\mathrm{kT}=1/T$ in our chosen units) is expressed as a density operator $\varrho_\beta=\exp(-\beta H)/\mathrm{Tr}(\exp(-\beta H))$ acting on a Hilbert space $\mathcal{H}$, where H (the Hamiltonian) is a self-adjoint operator on $\mathcal{H}$. Such a state describes the equilibrium of, say, a box of gas in contact with a heat reservoir at temperature $1/\beta$. The density operator ϱ_β defines an algebraic state φ on the von Neumann algebra $\mathfrak{B}(\mathcal{H})$ of bounded operators on $\mathcal{H}$ by setting $\varphi_\beta(A):=\mathrm{Tr}(\varrho_\beta A)$, $A\in\mathfrak{B}(\mathcal{H})$ (see Appendix A). Further, the Hamiltonian H generates a one-parameter group of dynamical automorphisms of that algebra by $\sigma_t(A):=\exp(itH)A\exp(-itH)$, $t\in\mathbb{R}$ and $A\in\mathfrak{B}(\mathcal{H})$. As befitting of an equilibrium state, φ_β is invariant under these automorphisms, i.e. $\varphi_\beta(\sigma_t(A))=\varphi_\beta(A)$ for all $A\in\mathfrak{B}(\mathcal{H})$. Assuming that the extension of σ_t to complex values of t is such that $z\mapsto\varphi_\beta(A\sigma_z(B))$ is analytic in the strip $\{0<\mathrm{Im}(z)<\beta\}$ of the complex plane, it is easy to verify that φ_β satisfies the condition $\varphi_\beta(A\sigma_{i\beta}(B))=\varphi_\beta(BA)$, $A,B\in\mathfrak{B}(\mathcal{H})$, which will play a key role in what follows.

Now consider a case where there may be no density operator of the appropriate form—say because normalization fails, as will be the case when H has a continuous spectrum as usually happens when the thermodynamic limit is taken in which the number of particles N and the volume V of the gas go to $+\infty$ while keeping N/V constant. It is highly desirable to have an analogue of a Gibbs equilibrium state to cover such situations. Only after physicists produced the desired analogue was it realized that mathematicians had independently been developing the relevant technical apparatus. In hindsight the key concepts can be introduced as follows. At the most general level the system of interest is described by a von Neumann algebra of observables $\mathfrak{M}$ (which typically will not be

[2] At various junctures these critical papers display a bizarre quality. For a response to the Russian group and a rejoinder from them, see Fulling & Unruh (2004) and Narozhny, Fedotov, Karnakov, & Mur (2004).

[3] "What we mean by a "particle" cannot sensibly be expressed without reference to a detector. All we can predict and discuss (as far as the *physical* world is concerned) are the experiences of detectors" (Davies, 1978, p. 71).

[4] I will speak of Fulling quanta since Rindler had no hand in showing how to quantize the Klein–Gordon field from the point of view of Rindler observers (see Section 6 and Appendix C).

[5] "In 1976 Unruh found that the Minkowski vacuum, i.e. the state associated with the non-existence of particles with respect to inertial observers, corresponds to a thermal bath of particles at the temperature $T_{FDU}=a/2\pi$ ($\hbar=c=k=1$) to uniformly accelerated observers with proper acceleration a=*const*. This has clarified previous results of Davies (1975), and confirmed Fulling's conclusion that *elementary particles* are observer dependent." (Matsas & Vanzella, 2003, p. 1573) I leave it to the reader to decide whether this is an accurate characterization of what "Unruh found" in 1976.

isomorphic to the familiar $\mathfrak{B}(\mathcal{H})$ of ordinary QM) and a one-parameter group of automorphisms σ_s, $s \in \mathbb{R}$, of $\mathfrak{M}$. (I have used s rather than t as the group parameter since I do not want to beg the question as to whether s is time.[6]) A state φ on $\mathfrak{M}$ satisfying the condition that for any $A,B \in \mathfrak{M}$ there is a function $f_{A,B}(z)$ analytic on the strip $\{0 < \mathrm{Im}(z) < \beta\}$ such that $f_{A,B}(s) = \varphi(\sigma_s(A)B))$ and $f_{A,B}(s+i\beta) = \varphi(B\sigma_s(A))$ for all $s \in \mathbb{R}$ is called a (σ_s,β)*–KMS state* after Kubo, Martin, and Schwinger, who first recognized the importance of the condition for quantum statistical mechanics (see Kubo, 1957; Martin and Schwinger, 1959).[7] Such a state satisfies the condition noted above that is characteristic of the algebraic counterpart of a Gibbs state; namely, $\varphi(A\sigma_{i\beta}(B)) = \varphi(BA)$ for all A and B in a weakly dense σ_s-invariant subalgebra of $\mathfrak{M}$. Note that KMS states are guaranteed to be σ_s-invariant; thus, if σ_s can be identified with the dynamical automorphism group, KMS states exhibit stationarity, the most basic property of equilibrium states. And KMS states also possess other properties of equilibrium states, such as stability and passivity[8] (see Bratelli and Robinson, 1987; Emch and Liu, 2001). In sum, there is strong evidence that KMS states provide the correct mathematical generalization of Gibbs states. KMS states have features that Gibbs states lack, but this is typically all to the good. For example, a (σ_s,β)-Gibbs state (if it is exists) is unique; but there can be many distinct (σ_s,β)-KMS states. The latter fact allows KMS states to represent different thermodynamical phases. This feature of KMS states will not play any role here.

Note that β can be eliminated by rescaling the group parameter: φ is a (σ_s,β)-KMS state if and only if it is a $(\sigma_u,-1)$-KMS state, where $u=-\beta s$. Without any loss of generality mathematicians set $\beta=-1$ and call this form of the resulting form of the KMS condition the *modular condition.* The minus sign has no physical significance and simply reflects the fact that the mathematicians who developed modular theory used a sign convention different from the one used by the physicists who worked in QSM.

The formalism of KMS states is quite flexible, and it applies not only to states of systems obtained from taking the thermodynamic limit in QSM but also, for example, to states in relativistic QFT as will be seen in the following section. The application proceeds via the celebrated Tomita–Takasaki theorem:

> *Theorem*. Let $\mathfrak{M}$ be a von Neumann algebra acting on a Hilbert space $\mathcal{H}$ and let φ be a faithful normal state on $\mathfrak{M}$. Then there exists a unique one-parameter group of automorphisms σ_s, $s \in \mathbb{R}$, of $\mathfrak{M}$ such that φ satisfies the modular condition with respect to σ_s, i.e. φ is a $(\sigma_s,-1)$-KMS state.

At the abstract level sketched above modular theory provides no way to choose a preferred parameterization of the automorphism group and, thus, allows the "temperature" $1/\beta$ to be set at any value in the range $(0,+\infty)$ by rescaling the group parameter. And relatedly, the abstract theory provides no connection between the group parameterization and the experienced time of an observer who is to measure the "temperature" $1/\beta$. The missing ingredients have to come not from the mathematical theory but from the details of physical applications to concrete cases. One promising class of applications occurs in QFT when the modular group has a geometric action on spacetime. Examples are discussed in the next section.

3. Modular automorphism groups with geometric actions in Minkowski spacetime and the modular time hypothesis

In the algebraic formulation of relativistic QFT (see Haag, 1996) a C^*-algebra $\mathcal{A}(\mathcal{O})$ of observables is associated with each open bounded region of $\mathcal{O} \subset \mathbb{R}^4$ of Minkowski spacetime $\mathbb{R}^4,\eta_{ab}$. This association is assumed to have the net property that if $\mathcal{O}_1 \subset \mathcal{O}_2$ then $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$. If $\mathcal{O}_1$ and $\mathcal{O}_2$ are relatively spacelike, Einstein causality (aka microcausality) demands that $[\mathcal{A}(\mathcal{O}_1),\mathcal{A}(\mathcal{O}_2)]=0$. The quasi-local algebra for the entirety of Minkowski spacetime $\mathcal{A}(\mathbb{R}^4)$ is given by $\overline{\bigcup_{\mathcal{O}}\mathcal{A}(\mathcal{O})}$, where the overbar denotes the closure with respect to the C^*-norm. The focus of most of the discussions of the Unruh effect is the Klein–Gordon field of mass $m \geq 0$ (see Appendix C). I will continue this tradition, although as will become apparent the apparatus used here has much wider applicability. The construction of the Weyl form of the canonical commutation relation (CCR) algebra for the Klein–Gordon field has been carried out and extensively investigated (see, for example, Kay and Wald, 1991; Wald, 1994). This rigorous quantization procedure yields a preferred vacuum state, φ_M, referred to as the Minkowski vacuum state. Heuristically, it corresponds to the vacuum obtained by quantizing the field using inertial time to pick out the positive frequency modes of the field (see Appendix C).[9]

In any approach to QFT that takes the basic algebras to be C^*-algebras, the von Neumann algebras that are the home of some of the physically important observables are representation-dependent objects. The standard practice is to focus on vacuum representations, and that practice will be followed here. So let φ_M be the Minkowski vacuum state for the Weyl CCR algebra $\mathcal{A}(\mathbb{R}^4)$ of the Klein–Gordon field. The von Neumann algebra affiliated with the local algebra $\mathcal{A}(\mathcal{O})$ for an open region $\mathcal{O} \subset \mathbb{R}^4$ is $\mathfrak{M}_M(\mathcal{O}) := (\pi_{\varphi_M|_{\mathcal{A}(\mathcal{O})}}(\mathcal{A}(\mathcal{O})))''$ where $\varphi_M|_{\mathcal{A}(\mathcal{O})}$ denotes the restriction of φ_M to the subalgebra $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathbb{R}^4)$, $\pi_{\varphi_M|_{\mathcal{A}(\mathcal{O})}}$ is the GNS representation determined by $\varphi_M|_{\mathcal{A}(\mathcal{O})}$, and "$''$" denotes the double commutant (see Appendix A). To apply KMS theory to $\mathfrak{M}_M(\mathcal{O})$ we need to be assured that the (unique) canonical extension of $\varphi_M|_{\mathcal{A}(\mathcal{O})}$ to $\mathfrak{M}_M(\mathcal{O})$ is a faithful normal state. (From here on I will use the same symbol for this state and its canonical extension.) To obtain this assurance for any region $\mathcal{O}$ whose causal complement $\mathcal{O}^c$ (consisting of all spacetime points that are spacelike with respect to $\mathcal{O}$) contains a non-null open set, start from the fact that the GNS representation of $\mathcal{A}(\mathbb{R}^4)$ determined by φ_M is (unitarily equivalent to) a Fock space representation in which the GNS vector is just the Minkowski vacuum vector $|0_M\rangle$. By the Reeh–Schleider theorem $|0_M\rangle$ is a cyclic vector with respect to $\mathfrak{M}_M(\mathcal{O}^c)$ and, thus, is a separating vector for $\mathfrak{M}_M(\mathcal{O}^c)'$. By Einstein causality $\mathfrak{M}_M(\mathcal{O}) \subset \mathfrak{M}_M(\mathcal{O}^c)'$ and, thus, $|0_M\rangle$ is a separating vector for any $\mathfrak{M}_M(\mathcal{O})$ where $\mathcal{O}$ has non-null causal complement. Thus, the vector state $\chi_{|0_M\rangle}$ on $\mathfrak{M}_M(\mathcal{O})$ corresponding to $|0_M\rangle$ is a faithful normal state for $\mathfrak{M}_M(\mathcal{O})$.[10] By the Tomita–Takasaki theorem $\chi_{|0_M\rangle}$ is a $(\sigma_s,-1)$-KMS state with respect to a unique

[6] The Connes–Rovelli thermal time hypothesis, crudely put, is that time—not just the direction of time but time itself—arises from statistical considerations (see Connes & Rovelli, 1994; Rovelli, 2004, Sections 3.4 and 5.5) and that in appropriate circumstances the group parameter s is to be identified with the physical time that governs macroscopic thermodynamical processes. A discussion of this fascinating proposal will be reserved for another occasion.

[7] KMS states are defined not only for von Neumann algebras but for C^*-algebras as well. I emphasize the application to von Neumann algebras because of the use made below of the Tomita–Takasaki theorem.

[8] Passivity means that energy cannot be extracted from the system by an external perturbation that is periodic in time.

[9] The use of different inertial coordinates will result in different quantizations, but they are all unitarily equivalent.

[10] Here $\chi_{|0_M\rangle}$ is the algebraic state defined by $\chi_{|0_M\rangle}(A) = \langle 0_M|A|0_M\rangle$ for all $A \in \mathfrak{M}_M(\mathcal{O})$. To see that this state is faithful note that if $\chi_{|0_M\rangle}(A^*A)=0$ then $\|A|0_M\rangle\|=0$ which means that $A|0_M\rangle=0$ since $|0_M\rangle$ and, thus, $A=0$ because $|0_M\rangle$ is a separating vector. That $\chi_{|0_M\rangle}$ is normal follows from the fact that it is a vector state. This might seem puzzling since $\chi_{|0_M\rangle}$ is a mixed state (see below). In ordinary QM the set of vector states is identical with the set of pure states. But in the setting of algebraic QM the difference between pure and mixed states corresponds to those that determine respectively irreducible and reducible GNS representations.

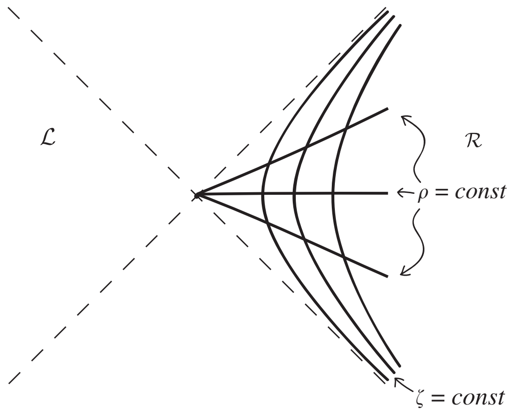

Fig. 1. Rindler wedge regions of Minkowski spacetime; dashed lines indicate null directions.

automorphism group σ_s. But $\chi_{|0_M\rangle}$ is nothing other than the canonical extension of $\varphi_M|_{\mathcal{A}(\mathcal{O})}$ to $\mathfrak{M}_M(\mathcal{O})$.

Until the work of Bisognano and Wichmann (1975, 1976) this easy result caused not the slightest stir in the mathematical physics community since there was no *a priori* reason to expect that the modular group associated with an arbitrary open region of Minkowski spacetime with non-null causal complement would have geometrical significance. But remarkably, such significance holds for Rindler wedge regions. The right Rindler wedge $\mathcal{R}$ pictured in Fig. 1 is the region $x > |t|$ where (x,y,z,t) is an inertial coordinate system. $\mathcal{R}$ is covered by Rindler coordinates (ζ,y,z,ρ) where the (ζ,ρ) are related to (x,t) by

$$x = \zeta\cosh(\rho), \quad t = \zeta\sinh(\rho) \tag{2}$$

In Rindler coordinates, the Minkowski line element becomes

$$ds^2 = d\zeta^2 + dy^2 + dz^2 - \zeta^2\, d\rho^2 \tag{3}$$

which makes it evident that (ζ,y,z,ρ) is a static coordinate system (see Appendix B). For future reference note that the Rindler coordinates "go bad" on the boundaries $x = \pm t$ of $\mathcal{R}$ where ρ takes infinite values. And also note that when considered as a spacetime in its own right, $\mathcal{R}$ is globally hyperbolic with the hypersurfaces $\rho = const$ of constant Rindler time forming a foliation of Cauchy surfaces. The orthogonal trajectories of these surfaces are timelike hyperbolae corresponding to the worldlines of observers in constant linear acceleration. The (proper) acceleration a along one of these trajectories varies from trajectory to trajectory according as $a = 1/\zeta$, and the proper time τ_a along such a trajectory is given by $\tau_a = \zeta\rho = \rho/a$. The distance between trajectories as measured along the hypersurfaces $\rho = const$ is independent ρ, giving an example of what is called Born rigid motion (see Appendix B).

The results of Bisognano and Wichmann (1975, 1976) show that the modular automorphism group for the restriction of the Minkowski vacuum state to $\mathfrak{M}_M(\mathcal{R})$ does have geometrical significance since its generators are the Lorentz boosts on $\mathcal{R}$, i.e. the orbits of the modular group are the hyperbolae of constant acceleration. When the group parameter is chosen to be the Rindler time ρ, the restriction of the Minkowski vacuum state to $\mathfrak{M}_M(\mathcal{R})$ is a $(\sigma_\rho,2\pi)$-KMS state.

The relevance of this result for the Unruh effect—which was initially explored by Unruh (1976) not on the basis of modular theory but rather by exploiting detectors (see Section 7)—was first recognized by Sewell (1982) who proposed that a natural rescaling of the modular group would give the Unruh temperature (1). The temperature $T = 1/2\pi$ from the Bisognano–Wichmann theorem, Sewell wrote

> is not the observed temperature, however, since this is based on the time [ρ], rather than the proper time τ_a. In fact, the temperature observed by O_{acc} [a uniformly accelerating observer] will take the value T_a, corresponding to a rescaling of time from [ρ] to τ_a …[i.e. $T_a = a/2\pi$]. (p. 209)

It seems fair to interpret these remarks as embodying what I will call the *modular temperature hypothesis*, which comes in a restricted and an extended form. Suppose the modular automorphism group σ_s for a (σ_s,β)-KMS state φ has geometrical significance in that the modular flow is everywhere a timelike flow on spacetime. The restricted modular temperature hypothesis (RMTH) applies to the case where the proper time τ of an observer whose worldline belongs to the said flow is such that $d\tau/ds$ is constant along the observer's worldline. It posits that the inverse temperature $\tilde{\beta}_\tau$ measured by this observer is given by $\beta \times d\tau/ds$. In the Rindler wedge case this posit (per design) associates the inverse Unruh temperature $2\pi/a$ with an observer moving with constant linear acceleration a. The extended modular temperature hypothesis (EMTH) is designed to cover cases where $d\tau(s)/ds$ varies along the worldline of an observer. It posits that the local inverse temperature $\tilde{\beta}_\tau(s)$ measured by an observer whose worldline belongs to the modular flow is given by $\beta \times d\tau(s)/ds$. The modular temperature hypothesis is rarely made explicit, but without it—or some similar hypothesis—modular theory does not associate any definite temperature with an observer. Martinetti and Rovelli (2003) explicitly advocate the EMTH in order to apply modular theory to diamond and future cone regions of Minkowski spacetime.

A diamond region $\mathcal{D}(L)$ of Minkowski spacetime, where L is the dimension of the diamond, can be specified in inertial coordinates (x, y, z, t) as a region such that $|\vec{x}| + |t| < L$. Restricted to $\mathcal{D}(L)$, the Minkowski vacuum state for an $m = 0$ Klein–Gordon field is a KMS state for the diamond algebra. The $m = 0$ field is conformally invariant and a conformal transformation can be used to map a wedge region to a diamond region. The latter can be used to transfer the modular flow on the wedge to the diamond, resulting in timelike worldlines with uniform acceleration that start at the past vertex of the diamond and terminate at the future vertex. This suggests that the modular group for $\mathcal{D}(L)$ has a geometric action and that the group orbits are identical with those given by the transfer construction. A proof that the suggestion is correct is given by Hislop and Longo (1982). Application of the EMTH yields the result that the modular temperature associated with an orbit parameterized by proper time is τ_a given by (Martinetti and Rovelli, 2003)

$$T(\tau_a) = \frac{La^2}{2\pi(\sqrt{1+a^2L^2} - \cosh(a\tau_a))}. \tag{4}$$

For observers with a large acceleration a (which is needed for an orbit that stays near the boundary of the diamond) and for large L, the associated modular temperature is approximately equal to the Unruh temperature $a/2\pi$. For a centrally located observer, who has zero acceleration, the extended modular temperature hypothesis associates a temperature of $2/\pi\mathcal{T}$, where $\mathcal{T} = 2L$ is the elapse of proper time along the observer's worldline from the bottom tip to the apex of the diamond. Such observers confined to small diamonds and, thus, having short lifetimes can console themselves with the fact that they have a high temperature associated with them. Indeed, by making their lifetimes sufficiently short, they can overcome the smallness of the (suppressed) numerical factor $\hbar/2\pi ck$ so as to make the modular temperature rise above the ordinary background room temperature.

The other instance where the modular group is known to have a geometric action is the case of the Minkowski vacuum state for an $m = 0$ Klein–Gordon field restricted to the algebra associated with interior $\mathcal{V}^+$ of a future light cone, which without loss of generality can be situated at the origin of an inertial coordinate system (i.e. $\mathcal{V}^+$ consists of those points such that $t > |\vec{x}|$). Since $\mathcal{V}^+$ does not have a non-null causal complement one cannot proceed as the case of a wedge or diamond. But advantage can be taken of the fact that in a Minkowski spacetime of even dimension the $m=0$ field propagates at exactly the speed of light. Consequently, the algebra of observables associated with $\mathcal{V}^+$ and the algebra associated with the twin past cone $\mathcal{V}^-$ commute (see Buchholz, 1978). By the Reeh–Schleider theorem $|0_M\rangle$ is cyclic with respect to the $\mathcal{V}^-$ algebra and, thus, is separating for the commutant algebra which includes the $\mathcal{V}^+$ algebra. The upshot is that the restriction of $|0_M\rangle$ to the $\mathcal{V}^+$ algebra defines a faithful normal state and, thus, by the Tomita–Takasaki theorem a KMS state. The modular group σ_s for this state acts by spacetime dilations (see Haag, 1996, Section V.4). Martinetti and Rovelli (2003) interpret the orbits of σ_s as inertial worldlines that fan out from the origin. Proper time along one of these worldlines is related to the modular parameter by $\tau(s) = \exp(-2\pi s)$. The extended modular temperature associated with these observers is non-zero at birth and then decreases exponentially.

These somewhat startling consequences of the EMTH might lead one to doubt it. But such doubters who at the same time want to apply RMTH to Rindler wedges have to provide a principled motivation for accepting RMTH while rejecting EMTH, for otherwise doubts about the latter will also infect the former. Such a motivation might start with the observation that only in cases where the RMTH applies is it guaranteed that the modular group σ_s for a (σ_s,β)-KMS state φ can be reparameterized using the proper time τ of an observer; and it continues by asserting that only when such a reparameterization is possible can the modular automorphism group be identified with the dynamical automorphism group for an observer with proper time τ. In these happy circumstances φ is a $(\sigma_\tau,\tilde{\beta}_\tau)$-KMS state with $\tilde{\beta}_\tau = \beta x\, d\tau/ds$, i.e. the RMTH is equivalent to the statement that the value of the inverse temperature associated with an observer is to be read off the reparameterization of modular group by said observer's proper time. Doubts about the EMTH can also be explored by comparing its predictions for $\mathcal{D}(L)$ and $\mathcal{V}^+$ with the responses of appropriate detectors.[11] I will not pursue this matter here and will restrict attention to cases falling under the RMTH.

4. The Unruh effect in curved spacetimes and its relation to the Hawking effect

A null surface $\mathcal{N}$ of a relativistic spacetime $\mathcal{M},g_{ab}$ is said to be a *Killing horizon* if there is a Killing field ξ^a ($\pounds_{\xi^c} g_{ab} = 0$) normal to $\mathcal{N}$ (see Appendix B). This concept is important to black hole physics since it is known that the event horizon of a stationary black hole must be a Killing horizon. A *bifurcate Killing horizon* for a four-dimensional spacetime consists of a pair of null surfaces $\mathcal{N}_A$ and $\mathcal{N}_B$ that intersect in a spacelike two-surface $\mathcal{S}$ such that $\mathcal{N}_A$ and $\mathcal{N}_B$ are both Killing horizons with respect to the same Killing field ξ^a. Provided that there is a Cauchy surface containing $\mathcal{S}$, the bifurcate horizons divide the spacetime up into four wedges $\mathfrak{F}$, $\mathfrak{P}$, $\mathfrak{L}$, $\mathfrak{R}$ as indicated schematically in Fig. 2.

[11] The rub, of course, is deciding what an appropriate detector is for such cases. Inertially moving Unruh–DeWitt detectors that have been switched on the asymptotic past give a null result (see Section 7). Switching on in the finite past will produce transients, but these effects should dissipate in a finite time leaving a null response. Such null responses would seem to clash with the predictions of the EMTH for $\mathcal{D}(L)$ and $\mathcal{V}^+$. Perhaps, however, Unruh–Dewitt detectors are not appropriate. But then what are appropriate detectors?

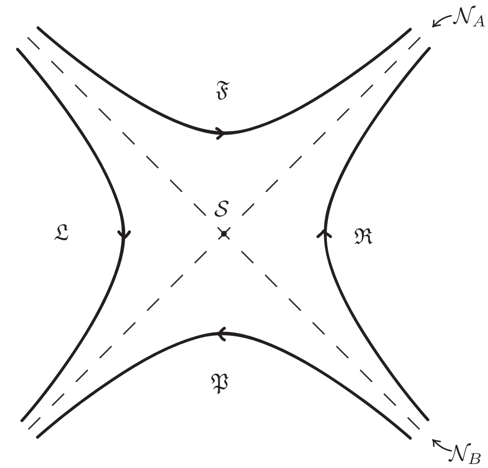

Fig. 2. Bifurcate Killing horizons; sample orbits of the horizon Killing field indicated by arrowed lines.

In the case of Minkowski spacetime, the Killing field $\xi^a = [x(\partial/\partial t)^a + t(\partial/\partial x)^a]$ has the associated bifurcate Killing horizon consisting of null planes intersecting at the origin. The wedge regions $\mathfrak{L}$ and $\mathfrak{R}$ where the Killing horizon field is timelike are, of course, the left and right Rindler wedges. Consider the quasi-free states on the Weyl CCR algebra for a Klein–Gordon field propagating on Minkowski spacetime.[12] Among these states there is a unique non-singular one that is invariant under the automorphisms of the algebra corresponding to the isometries whose generator is the horizon Killing field in the region where this field is timelike. This state is none other than the Minkowski vacuum state, and (as we already know) the restriction of this state to the right Rindler wedge algebra is a KMS state at inverse temperature 2π. A non-singular state φ is one that satisfies the Hadamard condition which guarantees that the point-splitting method of renormalization yields a finite expectation value $\langle T_{ab}\rangle_\varphi$ of the stress-energy tensor $T_{ab}(\phi) := \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}(\nabla_c\phi\nabla^c\phi + m^2\phi^2)$ for the free field ϕ (see Wald, 1994 for details).

Kay and Wald (1991) show how this situation generalizes to a minimally coupled free scalar field propagating on a curved globally hyperbolic spacetime with bifurcate Killing horizons such that the intersection $\mathcal{S}$ of $\mathcal{N}_A$ and $\mathcal{N}_B$ is contained in a Cauchy surface. More specifically, they prove that on a "large" subalgebra of a $\mathfrak{L}$ or $\mathfrak{R}$ wedge algebra of observables there is at most one quasi-free non-singular (Hadamard) state that is invariant under the automorphisms generated by the Killing horizon isometries.[13] They also show that under the further assumption of the existence of a "wedge reversal" isometry, the restriction of the unique invariant state—if it exists—to the observables of the large subalgebra of observables that are localized in one of the $\mathfrak{L}$ or $\mathfrak{R}$ wedges is a KMS state at Hawking temperature

$$T_H = \frac{\kappa}{2\pi} \tag{5}$$

with respect to the automorphism group generated by the Killing horizon isometries. Here $\kappa := -\frac{1}{2}(\nabla_a\xi_b)(\nabla^a\xi^b)$, evaluated on the horizon, is the *surface gravity*. It can be shown that $\kappa = \lim(aV)$ where $V := (-\xi^a\xi_a)^{1/2}$ is the norm of the horizon Killing field, $a := (a^b a_b)^{1/2}$ is the norm of the acceleration $a^b := \xi^c\nabla_c\xi^b/V$ of the orbit, and the limit is taken as the horizon is approached.

[12] The GNS representation induced by such a state has a natural Fock space structure (see Appendix C).

[13] Kay (1993) shows that the quasi-free assumption can be dropped.

The surface gravity κ is constant on the horizon (see Wald, 1984, Section 12.4).[14] The inevitability of the Hawking temperature for linear quantum fields is reinforced by the result of Haag, Narnhofer, and Stein (1984) showing that if there is a KMS state with respect to the bifurcate Killing horizon field and if this state satisfies a condition of local definiteness—roughly, the state must be indistinguishable from the Minkowski vacuum in the infinitesimal regime—then the temperature of the state must be $\kappa/2\pi$.

Applying the modular temperature hypothesis, the inverse temperature associated with an observer whose worldline is an orbit of the Killing horizon field is $2\pi/\kappa$ multiplied by the ratio of the observer's proper time to the modular parameter (here the Killing parameter), yielding the modular temperature

$$T = \frac{\kappa}{2\pi V}. \tag{6}$$

When appropriate vacuum states exist (6) encapsulates what can be deemed a generalized Unruh effect for curved spacetimes. The task now is to find when these states exist and to compute κ and V for these cases and, thereby, the modular temperature.

In the case of Minkowski spacetime a vacuum state for a Klein–Gordon field satisfying the conditions of the Kay and Wald (1991) theorem does exist—it is, of course, the Minkowski vacuum. As a consistency check we can compute the surface gravity $\kappa = \lim(aV)$ using the fact that in a Rindler wedge $a = 1/\zeta$ and $V = \zeta$ (recall that ζ is the Rindler spatial coordinate) giving $\kappa = 1$ and a modular temperature equal to the Unruh temperature. In the case of the Kruskal maximal extension of the Schwarzschild solution (see Fig. 3) the desired vacuum state also exists—in the literature it is called the Hartle–Hawking vacuum state $|0_H\rangle$ (see Hartle and Hawking, 1976).[15] Here

$$a = \frac{GM}{r^2\left(1-\frac{2GM}{r}\right)^{1/2}}, \quad V = \left(1-\frac{2GM}{r}\right)^{1/2}$$

where r is the Schwarzschild radial coordinate and M is the mass of the black hole, yielding a modular temperature of

$$\frac{\kappa}{2\pi V} = \frac{1}{8\pi GM\left(1-\frac{2GM}{r}\right)^{1/2}}$$

for an observer moving along the orbit $r = const$. As the horizon at $r = 2M$ is approached, $\kappa/2\pi V \to a/2\pi$, which is the same form as the Unruh effect for Minkowski spacetime. But note a key difference between this generalized Unruh effect and the Unruh effect for flat spacetime. The latter is rightly referred to as an acceleration effect, for as the Rindler spatial coordinate $\zeta \to \infty$, the acceleration of the Killing orbit $a \to 0$ and the Unruh temperature $T_U \to 0$. (Unruh himself tends to use the term "acceleration radiation" to refer to the Unruh effect in flat spacetime.) But in the case of the Hartle–Hawking vacuum for Kruskal spacetime, as $r \to \infty$, $a \to 0$ while the modular temperature $\kappa/2\pi V \to 1/8\pi GM$.[16]

Examples of non-existence results for a generalized Unruh effect are given by a minimally coupled Klein–Gordon field on either the globally hyperbolic portion of Kerr spacetime, which describes a rotating black hole, or the Schwarzschild–de Sitter spacetime, which is composed of an infinite chain of alternating Schwarzschild and de Sitter spacetimes. There is no appropriate KMS state for these cases; indeed, there is no Hadamard vacuum

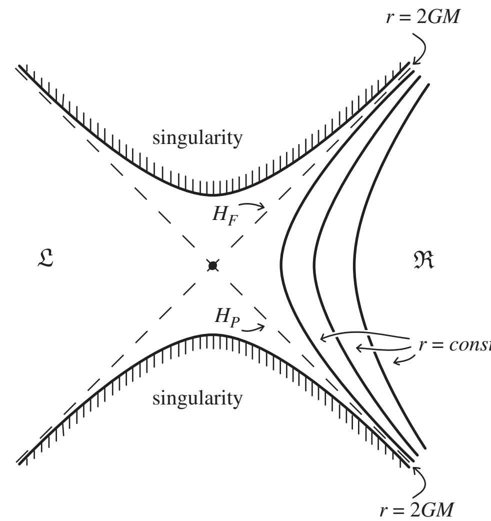

Fig. 3. Kruskal space with external Schwarzschild spacetime embedded as a wedge region $\mathfrak{R}$.

state invariant under the automorphisms generated by the Killing horizon isometries (Kay and Wald, 1991).

On a more positive note, the generalized Unruh effect does hold for de Sitter spacetime. For an $m > 0$ minimally coupled Klein–Gordon field Allen (1985) found a one (complex) parameter family vacuum states invariant under the de Sitter group of isometries.[17] But only one of these, the "Euclidean vacuum," satisfies a Hadamard-type condition. This condition is weaker than the one used in Kay and Wald (1991), so it is not apparent whether their results apply. Fortunately, there are independent proofs that the restriction of the Euclidean vacuum state to a wedge region corresponding to a bifurcate Killing field produces a KMS state at Hawking temperature $\kappa/2\pi = (1/2\pi)\sqrt{\Lambda/3}$ where $\Lambda > 0$ is the cosmological constant (see Bros and Moschella, 1996). As noted by Wald (1994, p. 127) every non-accelerated observer in de Sitter spacetime has an associated modular temperature equal to the Hawking temperature since for any timelike geodesic γ in de Sitter spacetime a bifurcate Killing field ξ^a can be chosen to be tangent to γ and normalized along γ so that $V = (-\xi^a\xi_a)^{1/2} = 1$. Here then is another example where the generalized Unruh effect is not an acceleration effect. For a conformally coupled $m = 0$ scalar field (see Appendix C) on de Sitter spacetime it has been argued that the temperature measured by an observer in uniform acceleration a through the conformal vacuum is $(1/2\pi)\sqrt{\Lambda/3 + a^2}$ (see Narnhofer, Peter, and Thirring, 1996).

All of the results reported above pertain to non-interacting scalar fields, which might occasion the worry that what is being called the generalized Unruh effect is an artifact of focusing on overly simple case. This worry is somewhat assuaged by Summers and Verch's (1996) model-independent proof that when a state on a net of local algebras is restricted to a subnet of algebras, the members of which are invariant under the automorphisms generated by the Killing horizon isometries, the result is a KMS state at inverse Hawking temperature. Of course, the question of the existence of the appropriate algebras and states is a model-dependent affair, and not surprisingly most of the positive existence results have been obtained for the simplest case of linear scalar fields.

[14] If ξ^a is a Killing field, then so is $C\xi^a$ where C is an arbitrary constant, which seems to lead to an ambiguity in the value of the surface gravity. But in the case of an asymptotically flat spacetime the ambiguity can be squelched by requiring that $V = (-\xi^a\xi_a)^{1/2} \to 1$ as $r \to \infty$.

[15] The names of the vacuum states are somewhat confusing: the Hartle–Hawking vacuum is the state relevant to the (generalized) Unruh effect while the Unruh vacuum is the state relevant to the Hawking effect (see below).

[16] For a solar mass black hole, this corresponds to a temperature of 6×10^{-8} °.

[17] For $m = 0$ no such vacuum state exists.

Returning to Kruskal spacetime, in addition to the Hartle–Hawking vacuum two other candidate vacuum states are available: the Boulware vacuum (see Boulware, 1975) and the Unruh vacuum (see Unruh, 1976).[18] The Boulware vacuum $|0_B\rangle$ can be seen as analogue of the Fulling vacuum $|0_F\rangle$ for a Rindler wedge algebra in Minkowski spacetime. The Fulling quantization will be discussed in Section 6 (see also Appendix C); but for present purposes suffice it to say that for both $|0_F\rangle$ and $|0_B\rangle$ the positive frequency modes are defined relative to the Killing horizon trajectories. Like $|0_F\rangle$, $|0_B\rangle$ is a non-thermal, indeed, pure state on a $\mathfrak{L}$ or $\mathfrak{R}$ wedge algebra. And just as $|0_F\rangle$ is singular (non-Hadamard) at the edges of the Rindler wedge (see Section 6), so $|0_B\rangle$ is singular on both the past (or white hole) horizon and the future (or black hole) horizon (labeled respectively H_P and H_F in Fig. 3).

The invocation of the "Hawking temperature" in the above exposition should not be taken as an invitation to conflate the generalized Unruh effect with the Hawking effect which asserts that at sufficiently late times a black hole radiates with a thermal spectrum at Hawking temperature (6) as seen by an observer near infinity (Hawking, 1975; Wald, 1975). Sometimes a derivation of the generalized Unruh effect is taken *eo ipso* as a demonstration of the Hawking effect (see, for example, Sewell, 1980). But as emphasized by Wald (1994, 1999), the Hawking effect and the generalized Unruh effect are quite distinct since they refer to different states of the quantum field—in the case of Kruskal spacetime, they refer respectively to the Unruh and Hartle–Hawking vacua.

> In the Hawking effect, the asymptotic final state of the quantum field is a state in which the modes of the quantum field that appear to a distant observer to have propagated from the black hole region of the spacetime are thermally populated at temperature $[T_H]$, but the modes which appear to have propagated in from infinity are unpopulated. This state (usually referred to as the 'Unruh vacuum') would be singular [non-Hadamard] on the white hole horizon in the analytically continued spacetime containing a bifurcate Killing horizon. On the other hand, in the Unruh effect and its generalization to curved spacetimes, the state in question (usually referred to as the 'Hartle–Hawking vacuum') is globally non-singular and *all* modes in the quantum field in the 'left and right wedges' are thermally populated.[19] (Wald, 1999, pp. A182–A183)

The difference between the generalized Unruh effect and the Hawking effect is at its starkest in the case of a Kerr black hole. As already noted there is no analogue of the Unruh effect, at least not on the approach that takes the Unruh effect and its generalization to curved spacetime to be a statement about KMS states; but the derivation of the Hawking effect in the form of particle creation in the formation of a Kerr black hole goes through (Wald, 1994, p. 129).

The importance of the Hawking effect is at least two-fold. First, it provides the physical grounding for black hole thermodynamics by showing that the Hawking temperature is the physical temperature of a black hole and, thus, that the expression for black hole entropy, originally developed by Birkenstein (1973) on the basis of a formal analogies, is truly the thermodynamic entropy of a black hole.[20] Second, when backreaction effects of the Hawking radiation are taken into account, it is found that a black hole loses mass at a rate that leads to the evaporation of a black hole in a finite time.[21] The Unruh effect does not occupy such a fundamental role in black hole physics. Nor was it intended for such a role; indeed, in its original incarnation was supposed to apply just to flat spacetime, and it was only as an afterthought that it was generalized to black hole and other curved spacetimes.

5. Some qualms about the modular theory approach

Although never made explicit, the following attitude is implicit in a significant fraction of the literature on the Unruh effect: 'The Unruh effect in flat spacetime and its generalization to curved spacetimes is no more and no less than what is given in the theorems (reviewed in Sections 3 and 4) that flow from QFT by way of modular theory. The modular temperature hypothesis need not be regarded as an extra empirical hypothesis but as part of an implicit definition of the concept of modular temperature. Since there is no arguing with theorems, case closed with no further need for discussion.' While this is a respectable attitude for mathematicians to adopt, it spells shortsightedness for physicists and an outright dereliction of duty for philosophers of physics.

A KMS state is an analogue of a Gibbs state, and as with all analogical reasoning one can wonder how reliable the analogy is and what inferences it supports; in particular, when it is safe to infer that the analogical "temperature" $1/\beta$ appearing in a KMS state has something like its ordinary thermodynamical meaning?[22] In the applications originally envisioned by Kubo, Martin, and Schwinger—KMS states for systems obtained by taking the thermodynamic limit of ordinary thermodynamical systems—doubts are readily assuaged. For example, in some model cases it can be shown that the limiting system acts as athermal reservoir at temperature $1/\beta$ in that if it is coupled to a finite system, the latter will be driven to thermal equilibrium at temperature $1/\beta$ (see Sewell, 1974). The need for reassurance is more pressing in instances where the KMS state characterizes a system that is not the thermodynamic limit of an ordinary thermodynamical system but a system that, *a priori*, does not lend itself to thermodynamical description, such as a quantum field in a vacuum state.

In the flat spacetime case the needed reassurance would take the form of comparing the deliverances of modular theory with the responses of physical objects accelerated through the Minkowski vacuum. But there is an apparent obstacle to even getting started on making such comparisons. Consider the question

[18] A overview of the properties of the three candidate vacuum states for Kruskal spacetime can be found in Novikov & Frolov (1989, Chap. 10). Rigorous algebraic versions of these states can be found in Dimock & Kay (1987).

[19] The fact that the Unruh vacuum state is singular on the white hole horizon might seem to call into question the basis of the Hawking effect. But note that Kruskal spacetime is not a good model for the formation of a spherically symmetric black hole through the process of gravitational collapse. A more realistic model would eschew the Kruskal white hole and with it the past horizon on which the Unruh vacuum state becomes singular.

[20] For a survey of black hole thermodynamics, see Wald (1998, 2001).

[21] For a solar mass black hole the evaporation time is finite but very long—on the order of 10^{67} years. What happens when a black hole evaporates is a matter of controversy, e.g. is information lost in the process? For a very readable (but somewhat biased) overview of the controversy, see Susskind (2008).

[22] A salutary example where a formal "temperature" has no connection to statistical–thermodynamical temperature is given by Srinivasan, Sriamkumar, & Padmanabhan (1997). Consider a plane wave mode $\phi(\mathbf{x},t)=\cos(\omega t-\mathbf{k}\cdot\mathbf{x})$ of a zero-mass Klein–Gordon field traveling along the x-axis in Minkowski spacetime, i.e. $\mathbf{k}=(k,0,0)$. The Fourier transform with respect to the proper time τ of an observer is given by $\tilde{\phi}(\Omega)=\int_{-\infty}^{+\infty}\cos(\omega t(\tau)-\mathbf{k}\cdot\mathbf{x}(\tau))\exp(-i\Omega\tau)$. For an observer with uniform acceleration a along the x-axis, the power spectrum $\mathcal{P}(\Omega):=\Omega/|\tilde{\phi}(\Omega)|^2$ has three terms, one of which is $N(\Omega)=1/(\exp(2\pi\Omega/a)-1)$, i.e. a Planck distribution in Ω at Unruh temperature. But as the authors note, "The system we are considering has no fluctuations or temperature in the sense of statistical physics" (p. 6693). Formal results of this type are sometimes billed as demonstrating that the Unruh effect has "classical roots" (see Pauri & Vallisneri, 1999). On the contrary, I take them as raising a caution flag—in both classical and quantum settings—when drawing physical conclusions from formal temperature expressions.

> Q1: My steak is currently accelerating through the Minkowski vacuum, and the wordlines of the points of the steak constitute a Born rigid motion described in Section 3. What modular temperature does a point undergoing an acceleration of magnitude a experience?

The answer to Q1 is *not* necessarily given by plugging the value of the acceleration into the formula $T = \hbar a/2\pi ck$. In fact without the help of further information modular theory does not supply an answer to Q1; indeed, from the point of view of modular theory whether or not it makes sense to give a definite answer to Q1 depends not only on the current state of motion of the steak but on its entire history. In brief, the point is this: the philosophy behind the modular theory approach to the Unruh effect is that a Rindler observer experiences a thermal state because she can access only a limited subalgebra of the full algebra of observables of the quantum field; but that such limited access obtains is not assured by the facts about any proper portion of the observer's history.

To develop the point, start with a humble example from ordinary QM, say, a system of two spin 1/2 particles in a singlet state which is, of course, a pure state. For an observer who has access to only one of the particles the relevant state is the restriction of the singlet state to the algebra of observables associated with said particle. Because the singlet state encodes correlations between the two particles, tracing out the degrees of freedom associated with the unobserved particle produces a mixed state on the algebra of observables associated with the particle to which the observer has access. And so it is in the case of QFT. Suppose that the system is a Klein–Gordon field on Minkowski spacetime and that the state is the Minkowski vacuum state. An observer who always has been and always will be in constant linear acceleration has access only to those observables associated with the Rindler wedge to which her worldline is confined. For her the relevant state is the Minkowski vacuum state restricted to her Rindler wedge algebra. Because the Minkowski vacuum state encodes correlations between relatively spacelike regions, the restriction of this pure state to a Rindler wedge region is a mixed state.[23] And—this is the surprise that comes out of relativistic QFT—the resulting state is not a garden variety mixed state but one with very special thermal properties, at least by the lights of modular theory. Now consider an observer who maintains constant linear acceleration for a finite stretch of (proper) time as long as you like but who is unaccelerated either in the asymptotic past or the asymptotic future. Such an observer has access to observables associated with regions outside any Rindler wedge region and, consequently, the argument that for her the relevant state is a mixed, thermal (KMS) state no longer applies. This makes it mysterious how to mesh the deliverances of modular theory with the registrations of laboratory instruments. For an instrument that registers only at $\tau = +\infty$ is useless for theory testing; and, barring backwards causation, the registration at any finite τ_0 cannot depend on whether for times $\tau > \tau_0$ the instrument is in constant linear acceleration.[24]

It is tempting to try to overcome this conundrum by pointing to the need to employ idealizations in applying theoretical physics to actual situations. This truism is certainly borne out in the part of the literature on the Unruh effect dealing with detectors where the analysis focuses on highly idealized point-like systems that serve as stand-ins for actual laboratory instruments. But the idea that modular theory can be used to make predictions about the registration of such an idealized detector during some finite period Υ of proper time during which it is undergoing constant linear acceleration through the vacuum by further idealizing the situation to one where Υ extends to $\pm\infty$ seems fundamentally mistaken. The most straightforward way to use local algebraic QFT to make predictions is to restrict the vacuum state to the local algebra of observables associated with a local neighborhood of the portion of the worldline of the detector corresponding to Υ. The result will be a KMS state, but typically the associated automorphism group will not have a geometrical significance that allows the automorphisms to be linked to the worldlines of the detector.

Ignoring this conundrum only opens the way to another worry embodied in a second question.

> Q2: Suppose that my steak is forever and always in Born rigid motion through the Minkowski vacuum and that the mean modular temperature associated with the worldlines of the steak is 300°C. Will it be charred?

Modular theory can *suggest* an answer through the modular temperature hypothesis which assigns temperatures to special families of worldlines. But to derive from QFT the response of a physical object—our steak, for example—the worldlines of the points of which form one of these special families, requires assumptions about the constitution of the object and about how the constituents couple to the quantum field. One might hope that for a wide range of constitutions and couplings the resulting response is fairly generic and is characterized by familiar thermal effects, such as the charring of our steak. The enterprise of showing that the hope is fulfilled would need to call on empirical hypotheses and mathematical techniques that are not part of modular theory, and if the enterprise is successful it would establish a version of the Unruh effect that is independent of the modular theory version. It would be surprising if there were not some concordance of the two versions; but it would be equally surprising if they were in complete agreement.

In sum, while modular theory strongly suggests that in QFT thermal effects arise from acceleration, other ways are needed to substantiate these effects, at least if "thermal" is to have a meaning that goes beyond the incestuous sense that is implicitly defined by the theorems of modular theory. Two such ways are discussed in the following sections. Section 6 takes up the idea that the thermal effects detected by an observer uniformly accelerating through the Minkowski vacuum arise from the fact that said observer encounters a thermal bath of quanta. This is an idea that might seem to be stillborn by definition since the vacuum state is devoid of quanta. However, a common response in the literature is that although the Minkowski vacuum is, of course, devoid of Minkowski quanta, said observer encounters a thermal flux of quanta of a different species—Fulling quanta. This response will be found wanting. Section 7 explores a way of rationalizing the Unruh effect in terms of the response of detectors to acceleration, an approach that is quite independent of and somewhat discordant with modular theory.

[23] *Lemma*: Let $\mathcal{A}$ be a C^*-algebra and let $\mathcal{B}$ be a C^*-algebra subalgebra of $\mathcal{A}$. If the restriction $\varphi|_{\mathcal{B}}$ to $\mathcal{B}$ of a state on $\mathcal{A}$ is a pure state, then $\varphi(XY) = \varphi(X)\varphi(Y)$ for all $X \in \mathcal{B}$ and all $Y \in \mathcal{A}$ such that $[Y,\mathcal{B}] = 0$. To apply this lemma let $\mathcal{B}$ and $\mathcal{C}$ be subalgebras of $\mathcal{A}$ associated with relatively spacelike regions. By Einstein causality $[\mathcal{B},\mathcal{C}] = 0$. To say that a state φ encodes correlations between observables belonging to $\mathcal{B}$ and $\mathcal{C}$ is to say that there are $X \in \mathcal{B}$ and all $Y \in \mathcal{C}$ such that $\varphi(XY) \neq \varphi(X)\varphi(Y)$. Hence, by the Lemma it follows that $\varphi|_{\mathcal{B}}$ is not pure.

[24] Misgivings about the use of modular theory to explicate the Unruh effect can be read between the lines of some of the physics literature. But the first explicit expression of misgiving I could find is in Schlicht (2004), which comes rather late in the game.

6. Fulling and other non-Minkowskian quanta

The following are typical sentiments found in the literature on the Unruh effect:

> Unruh's observation was that the theory that is thereby constructed [quantizing using Fulling modes] is not unitarily equivalent to the usual free field theory on Minkowski spacetime. Of even greater surprise was the subsequently discovered fact that the usual Poincaré invariant vacuum state appropriate to Minkowski space …contains a thermal distribution with respect to the Fulling Fock space. (Sciama et al., 1981, p. 343)

> The Minkowski vacuum is full of Rindler [Fulling] photons, although it is devoid of Minkowski photons. (DeWitt, 1979, p. 694)[25]

Sometimes the Unruh effect is characterized as "the equivalence between the Minkowski vacuum and a thermal bath of Rindler [Fulling] particles" (Crispino et al., 2008, p. 2). The main goal of this section is to explain why I think this characterization is misleading.

As noted above, if a right Rindler wedge $\mathcal{R}$ of Minkowski spacetime is considered a spacetime in its own right it is a globally hyperbolic spacetime that is covered by the static Rindler coordinates. One can follow the quantization procedure outlined in Appendix C to quantize the Klein–Gordon field on this spacetime using the Rindler time coordinate ρ rather than the inertial time t to identify the positive frequency modes of the field, i.e. using $-i\partial/\partial\rho$ rather than $-i\partial/\partial t$ as the energy operator. This procedure was first carried out by Fulling (1972, 1973) who showed that it yields a notion of particle (or better quantum) distinct from that of the standard Minkowski quantization.

Now the idea that a number of physicists have had is that we can get a handle on what is experienced by an observer in constant linear acceleration through the Minkowski vacuum by expressing the Minkowski vacuum state $|0_M\rangle$ in the Fock space of the Fulling representation. One encounters in the literature formulae in which $|0_M\rangle$ is written as a superposition of the (tensor) products of positive frequency Fulling modes for the left $\mathcal{L}$ and right $\mathcal{R}$ Rindler wedges. Tracing out over the degrees of freedom in, say, $\mathcal{L}$ produces a mixed state for the wedge algebra $\mathcal{A}(\mathcal{R})$ in the form of a thermal density operator at Unruh temperature (see, for example, Sciama et al., 1981; Unruh and Wald, 1984; Lee, 1986; Takagi, 1986, Section 2.8; Ginsburg and Frolov, 1987). Such expressions supposedly justify the idea that an observer uniformly accelerated through the Minkowski vacuum encounters a thermal flux of Fulling quanta. These results, together with analogous results for curved spacetimes, have sometimes been advertised as "thermalization theorems" (see Israel, 1976; Lee, 1986; Takagi, 1986, Section 2.8; Ginsburg and Frolov, 1987).

Strictly speaking, however, the crucial formulae involved are mathematically ill-defined, and genuine thermalization results that establish an Unruh effect for Minkowski spacetime and a generalized effect for curved spacetime cannot bypass the need to prove the satisfaction of an appropriate KMS condition.[26] To see why the ambition of expressing the Minkowski vacuum state as a density operator in the Fulling representation is unfulfillable, begin by noting that the Fulling vacuum state φ_F is a pure state on $\mathcal{A}(\mathcal{R})$ and, thus, the Fulling representation $\pi_{\varphi_F}(\mathcal{A}(\mathcal{R}))$ is irreducible; by contrast the Minkowski representation $\pi_{\varphi_M|_{\mathcal{A}(\mathcal{R})}}(\mathcal{A}(\mathcal{R}))$ is reducible since $\varphi_M|_{\mathcal{A}(\mathcal{R})}$ is a mixed state. Thus, trivially, the Minkowski and Fulling representations are not unitarily equivalent, a fact noted in Fulling (1972). What Fulling (1972, 1973) did not establish is the stronger conclusion that these representations are disjoint, a result that follows from the fact that the von Neumann algebra $\mathfrak{M}_M(\mathcal{R}) := \pi_{\varphi_M|_{\mathcal{A}(\mathcal{R})}}(\mathcal{A}(\mathcal{R}))''$ affiliated with the Minkowski representation is a Type III factor (see Araki, 1964) while that associated with the Fulling representation $\mathfrak{M}_F(\mathcal{R}) := \pi_{\varphi_F}(\mathcal{A}(\mathcal{R}))''$ is a Type I factor (see Appendix A). In outline, the argument is that factor representations are either quasi-equivalent or disjoint; but quasi-equivalent representations must have affiliated von Neumann algebras which are *-isomorphic, which $\mathfrak{M}_M(\mathcal{R})$ and $\mathfrak{M}_F(\mathcal{R})$ are not since they are of different types. (The alert reader will have noted that this disjointness result has nothing to do with the specific nature of the Fulling representation *per se*; it relies only on the fact the representation arises from a pure state on the algebra at issue and, thus, is irreducible. Hence, the disjointness from the Minkowski representation would hold for any other candidate vacuum state on $\mathcal{A}(\mathcal{R})$.) The importance of this result is that disjointness of the representations implies that no normal state of one representation (i.e. no state expressible as a density operator in that representation) is a normal state of the other.[27]

Of course, the fact that formulae expressing $\varphi_M|_{\mathcal{A}(\mathcal{R})}$ as a density operator in the Fulling representation are mathematically ill-defined does not imply that they do not express approximately correct truths in that they yield approximately correct results when used to calculate quantities of interest. To justify this approximation idea one might appeal to Fell's theorem which shows that any non-degenerate representations π_1 and π_2 of a Weyl algebra $\mathcal{W}$ are weakly equivalent in the sense that for any normal state φ for one representation, any finite list of observables $O_1, O_2, \ldots, O_n \in \mathcal{W}$, and any error tolerances $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n > 0$, there is a normal state φ' for the other representation such that $|\varphi(O_j) - \varphi'(O_j)| < \varepsilon_j$ for all $j = 1, 2, \ldots, n$. The theorem applies in the present case since the $\mathcal{A}(\mathcal{R})$ at the center of attention is a Weyl algebra. The trouble is that appeals to Fell's theorem cannot help with the key observables of interest in the present case because they are representation-dependent in that they live not in the C^*-algebra $\mathcal{A}(\mathcal{R})$ but in the von Neumann algebras $\mathfrak{M}_M(\mathcal{R})$ and $\mathfrak{M}_F(\mathcal{R})$ affiliated with the Minkowski and Fulling representations respectively. In particular, this is true of the total Fulling particle number operator $N^F_{\mathcal{R}}$ and the number operators $N^F_{k,\mathcal{R}}$ for the individual Fulling modes (labeled by k) for $\mathcal{R}$. Nevertheless, one can try to calculate the values of these operators for the Minkowski vacuum state, and the answers one obtains in both cases is "∞" (see Letaw and Pfautsch, 1981, p. 1495). These results can be glossed as "The Minkowski vacuum is filled with an infinity of Fulling quanta of every mode," but strictly speaking what they mean is that the Minkowski vacuum vector is not in the domain of either $N^F_{\mathcal{R}}$ or $N^F_{k,\mathcal{R}}$.

Just as there are attempts to understand the Unruh effect for Minkowski spacetime in terms of a thermal flux of Fulling quanta, so there are attempts to understand the generalized Unruh effect in Kruskal spacetime (recall Section 4) in terms of a thermal flux of Boulware quanta encountered by an observer who is accelerating through the Hartle–Hawking vacuum, the Boulware and Hartle–Hawking vacua being respectively the analogues of the Fulling and Minkowski vacua (see Israel, 1976; Sciama et al., 1981; Ginsburg and Frolov, 1987). The objections to this construal of the Unruh effect for the Minkowski case apply equally to the Kruskal case on the

[25] DeWitt refers to photons because, for reasons that will become clear in the following section, he is working with the case of a $m=0$ Klein–Gordon field.

[26] If this is the point that the Russian school which naysays the Unruh effect (Belinskii et al., 1997; Fedetov et al., 1999; Narozhny et al., 2000, 2002) intends to make, then they are correct. But it hardly follows that the Unruh effect does not exist.

[27] See Bratelli & Robinson (1987, Theorem 2.4.26). Physicists sometimes express this disjointness property by saying things like "every element of the Hilbert space that contains the Rindler [Fulling] vacuum is perpendicular to the Hilbert space containing the Minkowski vacuum" and vice versa (see, for example, Gerlach, 1989).

supposition that the von Neumann algebra affiliated with the Hartle–Hawking representation of a wedge algebra for Kruskal spacetime is a non-Type I factor, for then it would follow that the Hartle–Hawking representation is disjoint from the Boulware representation. Although I believe that this supposition holds, I know of no formal proof.

Returning to the Unruh effect in Minkowski spacetime, there are excellent mathematical physicists who acknowledge the technical difficulties reviewed above but who, nevertheless, find it heuristically useful to think of the Minkowski vacuum state as defining a thermal density operator in the Fulling representation. But heuristics, no matter how useful and suggestive, cannot be trusted to adjudicate foundational issues, especially if the heuristics do not have a well-founded basis. And it is for just this reason that Wald (1994, pp. 117–118), who advocates the heuristic, points to the need for a characterization of what it means for $\varphi_M|_{\mathcal{A}(\mathcal{R})}$ to be a thermal equilibrium state without presuming (per impossible) that $\varphi_M|_{\mathcal{A}(\mathcal{R})}$ exists as a density matrix in the Fulling representation. A formal characterization that fits the bill is provided by the KMS condition. But as argued in the preceding section, that formal characterization needs to be supplemented in order to give thermodynamic content to the notion of temperature it embodies. The idea that this temperature can be understood in terms of a thermal bath of Fulling quanta was one way of trying to provide such content. Under scrutiny, however, that idea turns out to have only heuristic value. To try to add real value by appeal to KMS theory leads to a circle, and multiple loops around the circle will not provide the sought after content.

Leaving aside mathematical niceties, there is also reason to doubt that appealing to the Fulling representation is a good heuristic for getting a handle on what is experienced by an observer in constant linear acceleration through the Minkowski vacuum. It comes from the claim that a necessary condition for a state φ on the wedge algebra $\mathcal{A}(\mathcal{R})$ (or the canonical extension of φ to its affiliated von Neumann algebra $\pi_\varphi(\mathcal{R})''$) to be physically realizable is not simply that it satisfy for $\mathcal{A}(\mathcal{R})$ (or $\pi_\varphi(\mathcal{R})''$) whatever adequacy conditions are demanded for physically acceptable states but also that φ can be extended to state on the global algebra for Minkowski spacetime that also satisfies the adequacy conditions. If these adequacy conditions include the Hadamard condition then φ_F is condemned as physically unrealizable, for it cannot be extended beyond $\mathcal{R}$ as a Hadamard state since $\langle T_{ab} \rangle_{\varphi_F}$ diverges as the edges of the wedge are approached—in fact, the energy density approaches $-\infty$.

In rejoinder it might be claimed that the uniformly accelerated observers who are confined to the Rindler wedge $\mathcal{R}$ have every right to treat $\mathcal{R}$ as the entirety of spacetime and, thus, every right to quantize a la Fulling and so arrive at φ_F as the vacuum state of the field; that φ_F becomes singular when extended to a state on a larger spacetime into which $\mathcal{R}$ is embedded is irrelevant to observers confined to $\mathcal{R}$. Although this claim has some initial plausibility, its plausibility vanishes when applied to analogous situations where it leads to patently unacceptable consequences.

Consider, for example, a family of observers who are born at the origin of the inertial coordinates (x, y, z, t) of Minkowski spacetime and who disperse by moving with uniform speeds ($<c$) along the x-axis. A coordinate system (X, Y, Z, T) adapted to the motion of these observers and covering the future cone $\mathcal{V}^+$ to which they are confined is given by

$$x = a^{-1}\exp(aT)\sinh(aX), \quad t = a^{-1}\exp(aT)\cosh(aX)$$

$$y = Y, \quad z = Z \tag{7}$$

The capitalized coordinates are known as Milne coordinates. In them the Minkowski line element takes the form

$$ds^2 = \exp(2aT)(dX^2 - dT^2) + dY^2 + dZ^2 \tag{8}$$

The T=*const* hypersurfaces, which are orthogonal to the trajectories of the X=*const* worldlines of the observers, give a foliation of $\mathcal{V}^+$ by

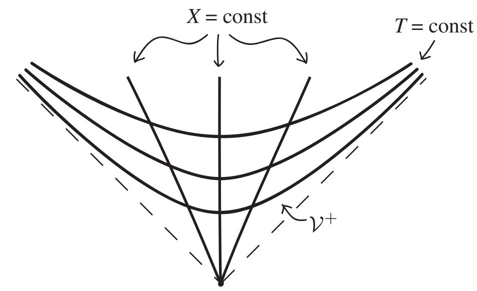

Fig. 4. Milne coordinates for a future cone region of Minkowski spacetime.

Cauchy surfaces for $\mathcal{V}^+$ considered as a spacetime in its own right (see Fig. 4). Parroting the rejoinder under consideration into the present case, the claim would be that observers confined to $\mathcal{V}^+$ have every right to consider $\mathcal{V}^+$ as the entirety of spacetime and, thus, every right to quantize appropriately to this consideration. Since the metric components in the (X, Y, Z, T) coordinate system are T-dependent, the quantization procedure outlined in Appendix C cannot be applied. But one can proceed in the spirit of the quantization applied to a Klein–Gordon field propagating in an expanding Friedman–Walker–Robertson cosmological model. That is, one can construct an *In* one-particle Hilbert space and thence a Fock space from the positive frequency modes in the asymptotic past ($T \to -\infty$) and an *Out* one-particle Hilbert space and corresponding Fock space from the positive frequency modes in the asymptotic future ($T \to +\infty$). One then finds that in this scheme there is "particle creation" because the $T \to -\infty$ positive frequency modes evolve to a linear combination of $T \to +\infty$ positive *and* negative frequency modes (see Padmanabhan, 1990). Interestingly, in the $m>0$ case the Unruh temperature makes an appearance in the formula for the number density n_k of created particles

$$n_k = \frac{1}{\exp(2\pi|k_x|/a) - 1} \tag{9}$$

where k_x is x-component of the linear momentum. But the point is that no serious physicist believes that genuine particle creation takes place in a flat spacetime.

Padmanabhan (1990) worried that this example suggests that alleged instances of particle creation in curved spacetime might be spurious effects due to an unfortunate choice of coordinate system and urged that "We have to produce a sensible criterion which will distinguish particle creation due to spacetime curvature effects from effects due to choice of coordinates" (1990, p. 2473). A different but equally effective response to the challenge raised by this example is to develop a criterion of physically realizable states and to show that it excludes the *In* and *Out* vacuum states for $\mathcal{V}^+$ obtained from quantizing in the Milne coordinates. Such a criterion is already at hand, viz. a physically realizable state on the algebra of observables associated with $\mathcal{V}^+$ must be extendible to a non-singular state on the full algebra of observables associated with Minkowski spacetime. This criterion has been deployed by Winters-Hilt, Redmount, and Parker (1999), who motivate it by the idea that physically admissible states are those that arise from the evolution of regular initial data in the remote past. And they show that it excludes the Milne *In* and *Out* vacuum states on $\mathcal{V}^+$ because such states are ill-behaved at the boundary of $\mathcal{V}^+$.

7. Detectors

To give operational content to the idea that an observer accelerated through the Minkowski vacuum experiences thermal effects the most natural move is to equip her with an appropriate detector that is coupled to field, and because the Unruh temperature varies with the acceleration and, thus, from worldline to worldline, it is natural to try to design a point-like detector. The DeWitt (1979) detector obliges by using a point-like particle initially in its ground state. A transition of the detector particle to an excited state counts as the detection event, although what the detector is detecting is not immediately obvious. The coupling of the DeWitt detector to the Klein–Gordon field ϕ is of the form $\lambda m(\tilde{x}(\tau))\phi(\tilde{x}(\tau))$, where $\tilde{x}(\tau)$ is the worldline of the detector parameterized by proper time τ, $\lambda>0$ is the coupling constant, and $m(\tilde{x}(\tau))$ is the monopole charge. The Unruh (1976) detector, consisting of a particle in a box, is somewhat more realistic in that it is spatially extended. In the Rindler wedge case its components follow the Lorentz boost trajectories with mean acceleration a. The DeWitt monopole detector can be viewed as a long wavelength or low-energy approximation of the box detector (see Grove and Ottewill, 1983). I will discuss results concerning the monopole detector, and will follow standard practice in the literature by referring to it as the Unruh–DeWitt detector.

Consider an Unruh–DeWitt detector which has been switched on in the asymptotic past and which is moving through the Minkowski vacuum. To first order in perturbation theory the probability that at proper time τ_0 the detector will be found in an excited state at energy E above its ground state energy E_0 is given by

$$C(\mathcal{E})\int_{-\infty}^{\tau_0} d\tau \int_{-\infty}^{\tau_0} d\tau' \exp(-i\mathcal{E}(\tau-\tau'))\langle 0_M|\phi(\tilde{x}(\tau))\phi(\tilde{x}(\tau')|0_M\rangle \tag{10}$$

where $\mathcal{E}:=E-E_0$. The coefficient $C(\mathcal{E})$, which expresses the sensitivity of the detector and depends on the internal details of the detector, will be ignored. The focus will be on the response function $F_{\tau_0}(\mathcal{E}):=\int_{-\infty}^{\tau_0} d\tau \int_{-\infty}^{\tau_0} d\tau' \exp(-i\mathcal{E}(\tau-\tau'))\langle 0_M| \phi(\tilde{x}(\tau))\phi(\tilde{x}(\tau')|0_M\rangle$ and more particularly on the time derivative of $F_{\tau_0}(\mathcal{E})$ with respect τ_0, which determines the detector transition rate and which is independent of internal details of the detector. A little manipulation shows that

$$\dot{F}_{\tau_0}(\mathcal{E}) = 2\int_{-\infty}^{0} ds\, \mathrm{Re}[\exp(-i\mathcal{E}s)]\langle 0_M|\phi(\tilde{x}(\tau_0))\phi(\tilde{x}(\tau_0+s)|0_M\rangle \tag{11}$$

$\dot{F}_{\tau_0}(\mathcal{E})$ is independent of τ_0 when the worldline $\tilde{x}(\tau)$ of the detector is such that the geodesic distance between two points $\tilde{x}(\tau_1)$ and $\tilde{x}(\tau_2)$ depends only on $|\tau_1-\tau_2|$, which is characteristic of a stationary motion (Letaw, 1981). In such cases the subscript on $\dot{F}_{\tau_0}(\mathcal{E})$ will be dropped. There has been some debate about how to perform the regularization needed to get a finite answer from the computation of the two-point function $\langle 0_M|\phi(\tilde{x}(\tau))\phi(\tilde{x}(\tau')|0_M\rangle$, but recently general agreement on this matter has crystallized (see Schlicht, 2004; Langlois, 2006; Louko and Satz, 2006).

For inertial motion the response of the detector is null in the sense that $\dot{F}(\mathcal{E})=0$ for $\mathcal{E}>0$ although, of course, $\dot{F}(\mathcal{E})$ is non-zero for $\mathcal{E}<0$ due to the possibility of spontaneous emission from the detector.[28] For a detector in hyperbolic motion with acceleration a through the Minkowski vacuum of an $m=0$ field on Minkowski spacetime of dimension $d=4$ the response is thermal (Planckian) at Unruh temperature:

$$\dot{F}(\mathcal{E}) = \frac{\mathcal{E}}{2\pi}\frac{1}{\exp(\mathcal{E}/T)-1}, \quad T=\frac{a}{2\pi} \tag{12}$$

While this result fits well the modular theory approach to the Unruh effect, a discordance between the modular theory approach and the detector approach becomes apparent in other cases. In particular, the modular theory derivation of the Unruh effect sketched in Sections 2 and 3 applies to $m>0$ Klein–Gordon fields as well as the $m=0$ case, and it holds for all spacetime dimension d. But the response of an Unruh–DeWitt detector in hyperbolic motion in $d=4$ Minkowski spacetime is far from Planckian for large $m>0$ (Takagi, 1986, Section 4.5). This discrepancy can be traced to the fact that the KMS condition ensures thermal equilibrium in the sense of detailed balance: for $\mathcal{E}>0$, $\dot{F}(\mathcal{E})=\dot{F}(-\mathcal{E})\exp(-\mathcal{E}/T)$, i.e. the upward transition rate for $E_0\to E$ is equal to the probability of the downward transition for $E\to E_0$ multiplied by the equilibrium probability of the excited state. But this balancing is not sufficient to guarantee that the spectrum of Unruh–DeWitt detector excitations is thermal (Takagi, 1986, Section 4.3).[29] Second, for $d>4$ the expression for $\dot{F}(\mathcal{E})$ acquires an additional numerical factor that depends on $\mathcal{E}$. Third, and most surprisingly, for an $m=0$ field in a Minkowski spacetime of odd dimension d, a "statistics reversal" takes place in which the Bose factor $1/(\exp(\mathcal{E}/T)-1)$ is replaced by a Fermi factor $1/(\exp(\mathcal{E}/T)+1)$ (Takagi, 1986, Section 4.2)[30]

How does the detector approach square with the idea that the modular temperature associated with an observer in constant linear acceleration through the Minkowski vacuum can be interpreted as the temperature of a thermal bath of non-Minkowskian particles/quanta? The response of an Unruh–DeWitt detector can be analyzed in terms of the absorption of Fulling quanta and the emission of Minkowski quanta for the case of an $m=0$ Klein–Gordon field (see Unruh and Wald, 1984).[31] But such an analysis is subject to the general qualms about Fulling quanta reviewed in Section 6. Moreover, insofar as explanation consists of deduction from general principles,[32] an explanation of detector responses can be carried out entirely within the Minkowski representation, without any need to use or mention the Fulling representation: just substitute an explicit expression for the worldline $\tilde{x}(\tau)$ of the detector into the expression for $\dot{F}_{\tau_0}(\mathcal{E})$ and start calculating. Note that this explanation works for *all* $m\geq 0$ fields, *all* spacetime dimensions, and *all* motions of the detector—even those that are not associated with a stationary frame of reference and, thus, are not subject to a particle/quanta explanation since there is no natural way to associate a non-Minkowskian particle/quanta concept with such a frame, at least not one that does not involve spurious particle creation in flat spacetime. In sum, if uniformity of explanation is a virtue, the

[28] An interesting side issue is whether an inertially moving Unruh–DeWitt detector that explores the field only in a Rindler wedge region can tell the difference between the Minkowski and Fulling vacua. The question is a little delicate since an inertial detector can be confined to a Rindler wedge region for only a finite amount of proper time. But however the question is parsed the answer is certainly positive since the autocorrelation functions $\langle 0_M|\phi(\tilde{x}(\tau))\phi(\tilde{x}(\tau')|0_M\rangle$ and $\langle 0_F|\phi(\tilde{x}(\tau))\phi(\tilde{x}(\tau')|0_F\rangle$ that determine the detector responses in the Minkowski and Fulling vacua are different for any two distinct points $\tilde{x}(\tau)$ and $\tilde{x}(\tau')$ on the detector's worldline; see Unruh (1992) and compare to Grove (1988) and Candelas & Sciama (1983), the latter of whom seem to be claiming that an inertial detector cannot distinguish between $|0_M\rangle$ and $|0_F\rangle$.

[29] In the literature on detectors "thermal" is used ambiguously to characterize a response where $\dot{F}(\mathcal{E})$ satisfies detailed balance vs. a response where $\dot{F}(\mathcal{E})$ has a Planckian spectrum. I use it here in the latter sense.

[30] Some insight into the origin of this statistics reversal is given by Unruh (1986). If the autocorrelation function in (11) is computed using a complete set of Fulling modes, it is found that each mode is thermally populated. It is the integration over all the modes to which the detector is sensitive that produces the statistics reversal for odd spacetime dimension.

[31] There is an ongoing controversy about whether a uniformly accelerated oscillator will radiate at Unruh temperature (Unruh radiation); see Raine, Sciama, & Grove (1991), Unruh (1992), Ford & O'Connell (2006), and Smolyaninov (2008).

[32] In the philosophical literature this conception of scientific explanation is discussed under the label of Hempel's DN (for Deductive-Nomological) model; see Hempel (1970). There are many criticisms of this model, but for present purposes it serves as good first-order approximation to what quantum field theorists mean by explanation. Below I will consider a demand that the explanation of the detector response take the form of a deduction exhibiting particular features.

explanation of detector responses that sticks to the Minkowski representation and eschews non-Minkowskian particles/quanta is to be preferred.

A case that illustrates how the various concepts discussed above fail to meld is provided by the analysis of an Unruh–DeWitt detector at rest in a frame rotating with constant angular velocity $\tilde{\omega}$ about the axis of rotation. This frame is stationary but, of course, not static. Since the detector cannot move with a speed greater than c $(=1)$, the domain of the frame consists of those spacetime points at a distance R from the axis of rotation such that $\tilde{\omega}R<1$. Considered as a spacetime in its own right, this domain is not globally hyperbolic and, thus, rigorous quantization procedures that rely on global hyperbolicity do not apply. There is no natural notion of event horizon for the family of uniformly rotating observers, as there is for the Rindler observers in constant linear acceleration, and thus a non-null detector response for the former case cannot be classified as "horizon effect." The spacetime domain for which the rotating frame is defined does not have non-null causal complement, and so the restriction of the Minkowski vacuum state to this domain is not a KMS state (or at least the kind of argument given in Section 3 does not suffice to show that it is a KMS state). An Unruh–DeWitt detector at rest in the uniformly rotating frame shows a non-null response. $\dot{F}(\mathcal{E})$ depends on R and $\tilde{\omega}$, and although for any given R and $\tilde{\omega}$ the transition rate for the case of $m=0$ and $d=4$ has a Planck-like form it is not identically Planckian (Letaw, 1981). The fact that the detector becomes excited cannot be attributed to the work being done to maintain it in orbit because this work is zero. Nor can the response of the detector be explained as the detection of particles/quanta, at least not if one ignores the fact that the relevant portion of Minkowski spacetime is not globally hyperbolic and naively applies the canonical quantization procedure[33] to coordinates adapted to the rotating frame. For the "rotating vacuum state" that results from this procedure coincides with the Minkowski vacuum state; in this sense, an observer at rest in the rotating frame "sees" no particles/quanta, whether Minkowskian or "rotating" (Letaw, 1981; Letaw and Pfautsch, 1981; Padmanabhan, 1981).[34] This discordance disappears when the rotating frame is contained within a limiting surface at radius $R<1/\tilde{\omega}$ and Dirichlet boundary conditions are imposed on the field ($\phi(r=R,t)=0$), for then the response of a rotating Unruh–DeWitt detector is null (Levin, Peleg, and Peres, 1993; Davies, Dray, and Manogue, 1996).

These last points call for a bit of explication. In canonical quantization it is assumed that a complete set of orthonormal modes of the field ϕ appropriate to a stationary frame, the worldlines of whose points are the trajectories of a timelike Killing field ξ^a, can be obtained from solutions to $\pounds_{\xi^a}\phi=-i\omega\phi$. For a static frame, i.e. ξ^a is non-rotating as well as stationary (as assumed in the case treated in Appendix C), ω can always be chosen to be positive so that the condition of a positive norm is equivalent to the condition of positive frequency. But when ξ^a is not static—as with a uniformly rotating frame—some positive norm modes may have negative frequency. In canonical quantization the creation and annihilation operators and, thus, the particle number operators are defined by a decomposition of the field operator obtained from positive norm modes.[35] By contrast, an analysis of the response of the Unruh–DeWitt detector shows that excitation depends on the presence of negative frequency modes. Thus, in principle, excitation of the detector at rest in a rotating frame can take place even in the absence of "rotating particles". And detailed calculations show that this possibility is in fact realized when the detector is not contained within a limiting surface on which Dirichlet boundary conditions are imposed (Letaw, 1981; Letaw and Pfautsch, 1981; Padmanabhan, 1981; Sriamkumar and Padmanabhan, 2002).

Part of attraction of a particle/quanta explanation of the response of an accelerated detector is surely due to the tendency to think that a satisfactory answer to "Why is the detector showing a non-null response (even after being adjusted, if necessary, for transient effects of switching the detector on)?" must take the form "Because it is detecting X," coupled with a tendency to search for a thing-like entity to play the role of X. I think that both tendencies should be resisted. But suppose that we give in to the first; and suppose that we agree that, because of the examples just discussed, "X = non-Minkowskian quanta" is not in general a suitable filling. It then remains to say what a generally satisfactory filling *is*. The best general answer was first suggested by Candelas and Sciama (1977) who noted that the response function for an Unruh–Dewitt detector is determined by the Fourier transform of the autocorrelation function of the quantum field evaluated at points on the detector worldline, and this transform is what statisticians recognize as the power spectrum of noise or fluctuations of a stochastic process.[36] As Candelas (1980) puts it, an Unruh–DeWitt detector is a "'fluctuometer' rather than a particle detector and, therefore, contains information both about the fluctuations of the field *and* the motion of the box [detector]" (p. 2198). Thus, insofar as an Unruh–DeWitt detector can be said to be detecting X, a generally applicable (i.e. for arbitrary detector motion $\tilde{x}(\tau)$) X is "the noise or fluctuations in the Minkowski vacuum."

Returning to the case of an Unruh–DeWitt detector at rest in a uniformly rotating frame, two quite different attitudes towards what to count as the Unruh effect are possible. Insofar as the Unruh effect is identified with the existence of an appropriate KMS state, there is no Unruh effect in the offing since, as noted above, the restriction of the Minkowski vacuum to the spacetime domain on which the rotating frame is defined is (presumably) not a KMS state. On the other hand, those who take detector response to be the key feature of the Unruh effect may want to follow Bell and Leinaas (1987) in speaking of a "circular Unruh effect" (see also Leinaas, 1999; Levin et al., 1993). Although the transition rate function $\dot{F}(\mathcal{E})$ for the case of an $m=0$ field and spacetime dimension $d=4$ does not have exactly Planckian form, Bell and Leinaas (1987, p. 488) conclude that "a physical system in circular motion is heated by the vacuum fluctuations".

The logical extreme of the Bell–Leinaas line is to recognize a "____ Unruh effect" where the blank is filled in with any type of non-inertial motion since in every such case $\dot{F}_{\tau_0}(\mathcal{E})>0$ for $\mathcal{E}>0$. A more conservative attitude would be to limit the filling for the blank to cases for which $\dot{F}_{\tau_0}(\mathcal{E})$ has the "nearly" thermal form, where some principled criterion of nearness must be supplied. The most conservative attitude would be to limit the filling for the blank to cases where $\dot{F}_{\tau_0}(\mathcal{E})$ has exactly thermal form. Presumably, this conservative stance requires that the allowed detector motions are stationary (and thus $\dot{F}_{\tau_0}(\mathcal{E})$ is independent of τ_0).

[33] In so-called canonical quantization, the equal time CCR are imposed on the field operator Φ and the canonically conjugate momentum operator. It is assumed that the Φ can be expressed as a linear combination of the positive norm modes ϕ_i as $\Phi=\sum_i(a_i\phi_i+a_i^\dagger\phi_i^*)$. As a result the creation and annihilation operators satisfy the familiar CCR $[a_i,a_j]=[a_i^\dagger,a_j^\dagger]=0$ and $[a_i,a_j^\dagger]=\delta_{ij}$. The vacuum state is defined by the condition $a_i|0\rangle=0$ for all i.

[34] Rejecting the naive application of canonical quantization as too naive to be trusted in this case does not resolve the discordance between the detector approach and the particle approach but only makes it worse since then no particle/quanta content can be associated with the rotating frame.

[35] The absence of "rotating particles" is no surprise since the positive norm rotating modes are just the positive norm Minkowski modes transformed to rotating coordinates.

[36] In the statistics literature this is called the Wiener–Khinchin theorem; see Papoulis (1991, Section 10. 3).

Letaw and Pfautsch (1982) have classified the stationary motions for Minkowski spacetime and they found six distinct classes: the three obvious cases consisting of inertial worldlines, hyperbolic worldlines, and helical worldlines; and three less obvious cases consisting of worldlines whose spatial projections are "semi-cubical parabolas" containing a cusp at which the direction of motion is reversed, worldlines whose spatial projections are catenaries, and rotating worldlines uniformly accelerated normal to their plane of rotation. Only in the case of hyperbolic motion through the Minkowski vacuum of an $m=0$ field and spacetime dimension $d=4$ does $\dot{F}(\mathcal{E})$ have an exactly thermal form (Letaw, 1981; Rosu, 2002). Thus, on the most conservative version of the detector approach, the Unruh effect in Minkowski spacetime does not generalize to non-hyperbolic motions.

The above discussion focused exclusively on the Unruh–DeWitt detector. But accepting the detector approach as the principal means of understanding the Unruh effect carries a responsibility to broaden the discussion beyond the simple monopole detector. In the first place, there is a need to consider extended detectors. The idealization involved in the monopole detector contains an inherent tension: the probe of the field is supposed to trace out a classical worldline; but by the uncertainty principle this idealization is inconsistent with a thoroughgoing quantum treatment. Some of the complications that can arise for detectors consisting of multi-level atoms and heavy ions are discussed respectively in Marzlin and Audretsch (1998) and Mur and Karnakov (1998). And even for idealized infinitesimal point detectors there is a need to investigate couplings to the field that go beyond the simple linear coupling of the Unruh–DeWitt detector. An example of what such investigations hold in store is given by Hinton's (1983, 1984) analysis of detectors that couple to an $m=0$ field through derivatives of the field. He found that in $d=2$ Minkowski spacetime the spectrum of excitations for a derivatively coupled detector in constant linear acceleration is the same as for an Unruh–DeWitt detector. But he also found that the concurrence vanishes for $d=4$ Minkowski spacetime where the derivatively coupled detectors can give non-Planckian responses. Sriamkumar (2002) studied the response of monopole detectors that are non-linearly coupled to the field through the n-th power of the field. When d is even, the response of a such a detector in constant linear acceleration through the Minkowski vacuum is characterized by a Bose–Einstein factor for all n; but when d is odd, the response is characterized by a Bose–Einstein factor for n even and a Fermi–Dirac factor when n is odd.

In sum, if detector response is to hold the key to the Unruh effect, then either an argument has to be mounted to show that only a privileged class of detectors is appropriate for probing the thermal properties of quantum fields, or else it has to be concluded that "the Unruh effect" stands for different effects in different types of detectors.

Thus far the discussion of the detector approach to the Unruh effect has been confined to Minkowski spacetime, but obviously the response of accelerated detectors to vacuum states of quantum fields on curved spacetimes can be studied. The interested reader is referred to Candelas (1980), Sciama et al. (1981), Birrell and Davies (1982), and Langlois (2006) for relevant results. I will only mention two illustrative examples. First, for the Hartle–Hawking vacuum state in Kruskal spacetime, the spectrum of excitations of an Unruh–DeWitt detector whose worldline coincides with an orbit of the Killing horizon field tends to a thermal spectrum at Hawking temperature $1/8\pi GM$ as $r\to\infty$, which is in accord with the modular temperature assignment. Second, the response of an Unruh–DeWitt detector moving with uniform acceleration a in the vacuum of a conformally coupled $m=0$ field on $d=4$ de Sitter spacetime is thermal at the temperature $(1/2\pi)\sqrt{\Lambda/3+a^2}$ (Langlois, 2006). Attempts have been made to shortcut the details of the detector analysis for de Sitter spacetime with a clever argument (Deser and Levin, 1997, 1999; Jacobson, 1998). The idea is to exploit the conformal invariance of the field and the fact that d-dim de Sitter spacetime can be embedded as a hyperboloid in $(d+1)$-dim Minkowski spacetime. The two-point functions which determine the response of an Unruh–DeWitt monopole detector for the conformal vacuum for $d=4$ de Sitter spacetime are same as those induced by the Minkowski vacuum state of the $d=5$ embedding Minkowski spacetime. An observer who has uniform acceleration a_4 in $d=4$ de Sitter spacetime is seen to have uniform acceleration a_5 in the $d=5$ embedding spacetime. The response of a detector carried by the latter observer will be thermal (so the argument goes) at Unruh temperature $T=a_5/2\pi$. The desired result then follows from the fact that $a_5=\sqrt{\Lambda/3+a_4^2}$. This clever argument is undercut by the phenomenon of statistics reversal in Minkowski spacetimes of odd dimension.

8. Conclusion

Of the three approaches to the Unruh effect discussed above, the least helpful is what was called the particle approach which tries to rationalize the Unruh temperature in terms of a thermal flux of Fulling quanta.[37] While this approach retains some heuristic value, its value in settling foundations issues is undercut by two fatal flaws in the idea that what is seen by an observer in constant linear acceleration through the Minkowski vacuum can be addressed by expressing the Minkowski vacuum vector in the Fulling Fock space: first, because the Minkowski and Fulling representations are disjoint so that no normal state (i.e. no state expressible as a density operator) of one representation is a normal state of the other; and second because the Fulling vacuum state is arguably not a physically realizable state.

The particle approach can be seen as a flawed attempt to prove a thermalization result of the form: when the Minkowski vacuum state is restricted to a Rindler wedge algebra it is a thermal state at Unruh temperature. Where the particle approach fails the modular theory approach succeeds in proving rigorous results that do not have to rely dubious appeals to the Fulling representation. Or at least it succeeds if "thermal state" is identified with a KMS state with respect to the automorphism group generated by the Rindler wedge isometries. But it is not easy to square the philosophy behind the modular theory approach with experimental measurements: the assignment of a modular temperature to an observer requires that the observer can access only a limited subalgebra of observables, which requires in turn a knowledge of the entire past and future history of the observer; but barring backwards causation, no laboratory registration taken at a finite time is sensitive to what the instrument does in the future. Moreover, the modular theory approach does not always mesh with the deliverances of detectors. To mention two of the discordances: the modular theory approach assigns a temperature equal to Unruh temperature for an observer in constant linear acceleration through the Minkowski vacuum of an $m>0$ scalar field, but the noise of the vacuum recorded by an Unruh–DeWitt detector is far from thermal (Planckian) for large m; in the other direction, for an $m=0$ scalar field, the spectrum of excitations of an Unruh–DeWitt detector at rest in a uniformly rotating frame is nearly thermal, but the modular theory approach does not apply to this case.

The strength and weaknesses of the modular theory approach and the detector approach are complementary. The former yields general, model-independent results; but these results need to be related to experimental measurements, something that modular

[37] Or a thermal flux of Boulware quanta in the case of the generalized Unruh effect for Kruskal spacetime.

theory by itself cannot accomplish. The detector approach is attractive in that it ties the Unruh effect directly to measurable quantities; but it is highly model-dependent, with different types of detectors with different couplings to the quantum field yielding divergent verdicts about the response to various accelerated motions though the vacuum.

The shortcoming and weaknesses of the three approaches mean that a forced choice of one of them would result in a trilemma, each of whose horns gores in a different way. But such a choice is forced only if one takes the definite article in "the Unruh effect" too seriously by insisting that the phrase designates a single phenomenon characterizable in 20 words or less. In fulfilling its role of linking thermal physics, relativity theory, and quantum theory, it seems preferable to take "the Unruh effect" as designating a family of related but distinct effects. The differences are less important than the similarities. The fact that similar results emerge from different approaches applied to a number of circumstances is a strong indication that a fundamental principle of nature has been identified, even if not completely understood.

Needless to say, this eclectic reading of "the Unruh effect" makes more complicated a discussion of experimental tests of the Unruh effect. I will not attempt to provide details of the discussion here and simply refer the interested reader to Rosu (2001) and Crispino et al. (2008) for overviews of various experimental proposals. However, I do want to comment on the overall shape of the discussion. Crispino et al. (2008, p. 788) take the attitude that "the Unruh effect itself does not need experimental confirmation any more than free quantum field theory does". This would be a bit over the top even if it were true that the Unruh effect were a theorem of QFT since every extant physical theory—even those as well tested as QFT—can always use additional confirmation, especially as concerns novel effects. Second, the review given here of the various approaches to the Unruh effect should make one leery of the notion that the effect can be straightforwardly derived from QFT without the use of additional physical hypotheses or novel interpretative moves. Nevertheless, I do think the main point of Crispino et al. (2008) regarding experimental tests is exactly on the mark; namely, typical proposals for "experimental tests" of the Unruh effect are misnamed since they consist of showing how the effect can be used to rationalize experimental data. As an example, Bell and Leinaas (1987) were the first to suggest that the observed depolarization of electrons in accelerator storage rings can be understood by treating spin as a thermometer that measures the Unruh effect (for discussions of this point of view see Akhmedov and Singleton, to appear-a, to appear; Jackson, 1999; Leinaas, 1999; McDonald, 1999; Unruh, 1998). The extreme version of this way of looking at the experimental status of the Unruh effect would consist in showing that the Unruh effect is required to maintain the consistency of a well-tested sector of QFT. Just such an argument has been given by Vanzella and Matsas (2001) and Matsas and Vanzella (2003), who claim that without the "Fulling–Davies–Unruh effect" inertial and accelerating observers would reach different conclusions about the stability of protons. While I am not persuaded by their reasoning,[38] the general line of argumentation is important.

Finally, it is worth commenting on the role of the Minkowski vacuum in understanding the Unruh effect for flat spacetime. On the modular theory approach it is a remarkable feature of the vacuum—namely, that the restriction of the vacuum state to a wedge region is a KMS state whose automorphism group has a geometrical interpretation—that underwrites the Unruh effect. There is a sense on which only the vacuum state has this feature; indeed, Buchholz and Summers (1986) propose to use this property to give an intrinsic characterization of the vacuum in Minkowski spacetime. Thus, from the point of view of modular theory the Unruh effect is purely a vacuum effect. On the other hand, one might expect that the detector approach would continue to reveal thermal effects for acceleration through non-vacua; but as far as I am aware this matter has not been investigated.

Although it is disappointing to have to close while unable to supply simple and definite answers to the questions "What is the Unruh effect?" and "What are the prospects of experimental detection?", I hope that the above discussion shows, first, why simple and definitive answers are hard to come by and, second, how the investigation of these questions casts light on some of the most fundamental foundations issues at the interfaces of the main branches of modern physics.

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Appendix A. C^*-algebras and von Neumann algebras[39]

A **-algebra* is an algebra closed with respect to an involution $\mathcal{A} \ni A \mapsto A^* \in \mathcal{A}$ satisfying: $(A^*)^* = A$, $(A+B)^* = A^* + B^*$, $(cA)^* = \overline{c}A^*$ and $(AB)^* = B^*A^*$ for all $A, B \in \mathcal{A}$ and all complex c (where the overbar denotes the complex conjugate). A *C^*-algebra* is a *-algebra equipped with a norm, satisfying $\|A^*A\| = \|A\|^2$ and $\|AB\| \le \|A\|\|B\|$ for all $A, B \in \mathcal{A}$, and is complete in the topology induced by that norm. It will be assumed that all the algebras of interest contain the identity I.

A *representation* of a C^*-algebra $\mathcal{A}$ is a *-morphism $\pi : \mathcal{A} \to \mathfrak{B}(\mathcal{H})$ where $\mathfrak{B}(\mathcal{H})$ is the algebra of bounded linear operators on a Hilbert space $\mathcal{H}$. The representation π is said to be *cyclic* if there is a vector $|\psi\rangle \in \mathcal{H}$ such that $\{\pi(\mathcal{A})|\psi\rangle\}$ is a dense set. π is *irreducible* if there is no non-trivial subspace of $\mathcal{H}$ that is invariant under $\pi(\mathcal{A})$. Two representations π and π' are *unitarily equivalent* iff there is a unitary map $V : \mathfrak{B}(\mathcal{H}_\pi) \to \mathfrak{B}(\mathcal{H}_{\pi'})$ such that $\pi'(A) = V\pi(A)V^{-1}$ for all $A \in \mathcal{A}$. A weaker notion of equivalence of representations will be defined below once von Neumann algebras have been introduced.

A *state* on an algebra $\mathcal{A}$ is a positive linear functional $\varphi : \mathcal{A} \to \mathbb{C}$ such that $\varphi(I) = 1$. A state is said to be *mixed* if it can be written as a non-trivial convex combination of other states; otherwise it is said to be *pure*. φ is *faithful* to $\mathcal{A}$ if $\varphi(A^*A) = 0$ for $A \in \mathcal{A}$ implies that $A = 0$. The basic result on representations, called the Gelfand–Naimark–Segal (GNS) theorem, shows that each state on a C^*-algebra determines a cyclic representation: if $\mathcal{A}$ is a C^*-algebra and φ is a state on $\mathcal{A}$, then there is a Hilbert space $\mathcal{H}_\varphi$, a *-morphism $\pi_\varphi : \mathcal{A} \to \mathcal{B}(\mathcal{H}_\varphi)$, and a cyclic vector $|\Omega_\varphi\rangle \in \mathcal{H}_\varphi$ such that $\varphi(A) = \langle \Omega_\varphi | \pi_\varphi(A) | \Omega_\varphi \rangle$ for all $A \in \mathcal{A}$. The GNS representation is the unique (up to unitary equivalence) cyclic representation. The GNS representation π_φ determined by a state φ is irreducible just in case φ is pure.

A *von Neumann algebra* $\mathfrak{M}$ is a C^*-algebra of bounded linear operators acting on a Hilbert space $\mathcal{H}$ such that $\mathfrak{M}$ is closed in the weak operator topology or, equivalently, $(\mathfrak{M}')' := \mathfrak{M}'' = \mathfrak{M}$, where '′' indicates the commutant. (A sequence of bounded operators O_1, $O_2, \ldots$ acting on a Hilbert space $\mathcal{H}$ converges in the weak topology

[38] In part because their argument relies heavily on the reality of Fulling quanta. But perhaps the argument can be recast so as to avoid this feature.

[39] A comprehensive treatment of these topics can be found in Bratelli & Robinson (1987, 1997) and Kadison & Ringrose (1991). Definitions of the concepts used in the body of the paper are recapitulated here.

to O just in case $\langle \psi_1 | O_j \psi_2 \rangle$ converges to $\langle \psi_1 | O \psi_2 \rangle$ for all $|\psi_1\rangle, |\psi_2\rangle \in \mathcal{H}$.) If $\pi: \mathcal{A} \to \mathcal{B}(\mathcal{H})$ is a representation of the C^*-algebra $\mathcal{A}$, the affiliated von Neumann algebra is the weak closure of $\pi(\mathcal{A})$ or, equivalently, the double commutant $\pi(\mathcal{A})''$. A *normal state* φ of a von Neumann algebra $\mathfrak{M}$ acting on a Hilbert space $\mathcal{H}$ is a completely additive state or, equivalently, a state that can be expressed as a density matrix ϱ on $\mathcal{H}$ via the trace prescription, i.e. $\varphi(A) = \mathrm{Tr}(\varrho A)$ for all $A \in \mathfrak{M}$. A *separating vector* $|\psi\rangle \in \mathcal{H}$ for a von Neumann algebra acting on $\mathcal{H}$ has the property that $A|\psi\rangle = 0$ implies that $A=0$ for any $A \in \mathfrak{M}$. In parallel with the definition of a cyclic vector for a representation of a C^*-algebra, a vector $|\psi\rangle \in \mathcal{H}$ for a von Neumann algebra acting on $\mathcal{H}$ is cyclic just in case $\{\mathfrak{M}|\psi\rangle\}$ is dense in $\mathcal{H}$. A basic result is that a vector is cyclic for $\mathfrak{M}$ just in case it is separating for $\mathfrak{M}'$. A *factor* algebra $\mathfrak{M}$ is one whose *center* $\mathfrak{M} \cap \mathfrak{M}'$ consists of multiples of the identity. The characteristic feature of a Type I factor is that it contains minimal projectors. A Type III factor contains no finite projectors.

Quasi-equivalence of representations of C^*-algebras is the relevant generalization of the concept of unitary equivalence to reducible representations; it means that the representations are unitarily equivalent up to multiplicity. A basic result is that the quasi-equivalence of representations π_1 and π_2 of $\mathcal{A}$ is equivalent to each of the following: (a) there is a *-isomorphism $\iota: \pi_1(\mathcal{A})'' \to \pi_2(\mathcal{A})''$ such that $\iota(\pi_1(A)) = \pi_2(A)$ for all $A \in \mathcal{A}$ (Bratelli and Robinson, 1987, Theorem 2.4.26), and (b) every π_1-normal state is a π_2-normal state and vice versa. π_1 and π_2 are said to be *disjoint* just in case no π_1-normal state is a π_2-normal state and vice versa.

A *Weyl algebra* is a C^*-algebra that encodes an exponentiated form of the canonical commutation relations. For a construction of the Weyl algebra for the Klein–Gordon field on a globally hyperbolic spacetime (defined in Appendix B), see Kay and Wald (1991) and Wald (1994). A *quasi-free state* on this algebra has n-point functions that are sums of products of two-point functions. The GNS representation of such a state is (unitarily equivalent to) a Fock space representation with the GNS vector playing the role of the Fock vacuum state. A rigorous algebraic procedure is available for quantizing the Klein–Gordon field on a stationary globally hyperbolic spacetime (see Wald, 1994). Unfortunately, this procedure does not apply to Rindler spacetime since it requires a (nowhere vanishing) timelike Killing field whose norm is bounded away from zero. Thus, one must resort to the procedure described below in Appendix C.

Appendix B. Relativistic spacetimes[40]

A *relativistic spacetime* $\mathcal{M}, g_{ab}$ consists of a differentiable manifold $\mathcal{M}$ (assumed for convenience to be C^∞) together with an everywhere defined Lorentzian metric g_{ab}. The signature convention $(+++-)$ is in effect. This definition includes, of course, Minkowski spacetime where $\mathcal{M} = \mathbb{R}^4$ and $g_{ab} = \eta_{ab}$ (the Minkowski metric). A spacetime is said to be *stationary* if there exits a (nowhere vanishing) timelike vector field ξ^c satisfying the Killing condition $\pounds_{\xi^c} g_{ab} = 2\nabla_{(a}\xi_{b)} = 0$, where $\pounds_{\xi^c}$ is the Lie derivative with respect to ξ^c and ∇_a is the covariant derivative operator determined by g_{ab}. A timelike vector field ξ^c can be thought of as defining a reference frame, the trajectories of the field being the worldlines of the points of the frame. An observer whose worldline coincides with a trajectory of a stationary frame does not see any change in the metric of spacetime as her "now" sweeps up her worldline. A spacetime is said to be *static* if it admits a frame ξ^c that is both stationary and irrotational, i.e. $\xi_{[a}\nabla_b \xi_{c]} = 0$. In a static spacetime it is always possible to choose (locally) a coordinate system (x^α, t) such that the line element takes the form $ds^2 = g_{\alpha\beta}(x^\gamma)\, dx^\alpha\, dx^\beta - g_{44}(x^\gamma)\, dt^2$, where α, β, γ run over the spatial coordinates.

A variety of causality conditions can be imposed on relativistic spacetimes (see Wald, 1984, Chap. 8). One of the strongest conditions is called *global hyperbolicity*, which is equivalent to the condition that the spacetime admit a *Cauchy surface*, i.e. a spacelike hypersurface that intersects every maximally extended causal curve exactly once. An equation for a field propagating on a globally hyperbolic spacetime is said to admit an initial value formulation if appropriate initial data on a Cauchy surface fixes a unique solution of the field equation. This is the case for a Klein–Gordon field whose quantization is discussed below.

Let u^c be the normed $(u^b u_b = -1)$ tangent field of a worldline. The acceleration of the worldline is given by $a^c := \dot{u}^c = u^b \nabla_b u^c$ where the dot denotes differentiation with respect to proper time. *Constant linear acceleration* means that $\dot{a}^c := u^b \nabla_b a^c = a^d a_d u^c$. Differentiating $u^b u_b = -1$ gives $a^b u_b = 0$. Using this fact, the condition of constant linear acceleration is seen to imply that $\dot{a}^d a_d = 0$ and that the magnitude of acceleration $a := (a^d a_d)^{1/2}$ is constant. When the acceleration is along, say, the x-axis of an inertial coordinate system (x, y, z, t) of Minkowski spacetime, the worldline of an observer in constant linear acceleration has the form of a hyperbola, i.e. $x^2 - t^2 = C > 0$.

The reference frame defined by a timelike vector field ξ^c is said to be *Born rigid* if the expansion and shear of the field both vanish (see Wald, 1994 for definitions). This is equivalent to the condition that the distance between infinitesimally neighboring trajectories of ξ^c, as measured at some instant in the spacelike hyperplane orthogonal to one of them, is independent of the instant chosen. Any stationary frame is Born rigid; but the converse need not hold.

Appendix C. The Fulling quantization

In general covariant form the *minimally coupled* Klein–Gordon equation reads

$$g^{ab}\nabla_a \nabla_b \phi - m^2 \phi = 0 \tag{C.1}$$

where m is the mass of the field. A *conformally coupled* field obeys the equation

$$g^{ab}\nabla_a \nabla_b \phi - \frac{d-2}{4(d-1)} R\phi - m^2 \phi = 0 \tag{C.2}$$

where R is the Ricci scalar and d is the dimension of the spacetime. For the case $m=0$ and $d=4$, the resulting field equation $g^{ab}\nabla_a \nabla_b \phi - (R/6)\phi = 0$ is conformally invariant. In spacetimes (such as de Sitter spacetime) where R is a non-negative constant the conformally invariant equation is equivalent to the equation for a minimally coupled field with mass $m = \sqrt{R/6}$. The quantization for the minimally coupled field will be discussed here.

For a globally hyperbolic spacetime (C1) has a well-posed initial value problem. Suppose now that the spacetime is not only globally hyperbolic but static as well, and for the sake of convenience suppose that there is globally defined static coordinate system (x^α, t). In such coordinates (C1) becomes

$$-\frac{\partial^2 \phi}{\partial t^2} = g_{44}\left[\frac{1}{\sqrt{-g}}\partial_\alpha(\sqrt{-g}g^{\alpha\beta}\partial_\beta \phi) - m^2 \phi)\right] \equiv K\phi \tag{C.3}$$

Since the differential operator K contains only spatial derivatives, (C.3) can be solved by the separation of variables ansatz

[40] The recommended reference here is Wald (1984).

$\phi(\mathbf{x},t)=\psi(\mathbf{x})\chi(t)$ to give

$$\frac{d^2\chi(t)}{dt^2}+\omega^2\chi(t)=0 \tag{C.4a}$$

$$K\psi(\mathbf{x})=\omega^2\psi(\mathbf{x}) \tag{C.4b}$$

Since the spacetime is assumed to be globally hyperbolic it can be foliated by a family $\Sigma(t)$ of Cauchy surfaces. The operator K is formally symmetric and positive on the Hilbert space $L^2(\Sigma,\mu\, d^3x)$ of complex valued square integrable functions on a Cauchy surface $\Sigma\in\Sigma(t)$, with the inner product given by

$$\langle f,g\rangle := \int_\Sigma \bar{f}(\mathbf{x})g(\mathbf{x})\mu(\mathbf{x})\, d^3x \tag{C.5}$$

where $\mu=-g^{44}\sqrt{-g}$. Because it is independent of the choice of Σ from the family $\Sigma(t)$ this inner product is not indexed with Σ. Assuming that K has a unique self-adjoint extension, the square root of this extension is a positive linear operator, which serves as the single particle Hamiltonian relative to the time t. One can choose an orthonormal basis $\{\psi_k\}$ for $L^2(\Sigma,\mu\, d^3x)$ consisting of solutions to

$$K\psi_k=\omega_k^2\psi_k \tag{C.6}$$

The functions $u_k(\mathbf{x},t)=(2\omega_k^2)^{-1/2}\psi_k(\mathbf{x})\exp(-i\omega_k t)$ and their complex conjugates u_k^* (called respectively the *positive* and *negative frequency modes*) constitute a complete set of mode solutions in that a general solution to (C.3) can be written in the form

$$\phi(\mathbf{x},t)=\int_{\hat{\Sigma}}[a_k\psi_k(\mathbf{x})\exp(-i\omega_k t)+a_k^\dagger\psi_k^*(\mathbf{x})\exp(i\omega_k t)]\frac{d\mu(k)}{\sqrt{2\omega_k}} \tag{C.7}$$

The space of positive frequency solutions can be equipped with an inner product, and the completion in this inner product gives the "one-particle" Hilbert space $\mathcal{H}$ for the field. The state space for the field is constructed as the symmetric Fock space $\mathcal{F}$ over $\mathcal{H}$. That is, $\mathcal{F}$ is the completed direct sum $\oplus_{i=0}^{\infty}(S[\otimes_i\mathcal{H}])$, where $S[\otimes_n\mathcal{H}]$ denotes the symmetrized n-fold tensor product of $\mathcal{H}$ and $\otimes_0\mathcal{H}$ is stipulated to be $\mathbb{C}$.

Needless to say, this procedure applies when the static coordinate system is chosen to be an inertial coordinate system for Minkowski spacetime, and the application of the procedure leads to the familiar Minkowski Fock space $\mathcal{F}_M$ and its vacuum vector $|0_M\rangle$. (Different inertial systems produce unitarily equivalent quantizations.) The procedure also applies to the Rindler coordinates for the right Rindler wedge, leading to the Fulling Fock space $\mathcal{F}_F$ and its vacuum vector $|0_F\rangle$.

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