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Experimental Black-Hole Evaporation?

W. G. Unruh

Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada

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It is shown that the same arguments which lead to black-hole evaporation also predict that a thermal spectrum of sound waves should be given out from the sonic horizon in transsonic fluid flow.

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Black-hole evaporation[1,2] is one of the most surprising discoveries of the past ten years. Black holes emit thermal radiation with a temperature given by $hc^3/8\pi kGM$, and thus seem to combine quantum mechanics and gravitation to produce thermodynamics. This theoretical result suffers, however, from certain difficulties. In particular, the result is derived under the assumptions that the quantum fields in question do not affect the gravitational field in which they propagate, that the gravitational field itself is unquantized, and that the wave equation for the quantum field is valid on *all* scales. Any breakdown of these assumptions would seem to imply the breakdown of the evaporation process. A further difficulty is that the experimental investigation of the phenomenon would seem to be virtually impossible, and would depend on the highly unlikely discovery of a small black hole (a relic of the initial stages in the life of the universe perhaps)[3] near the Earth.

However, a physical system exists which has all of the properties of a black hole as far as the quantum thermal radiation is concerned, but in which all of the basic physics is completely understood. In this system one can investigate the effect of the reaction of the quantum field on its own mode of propagation, one can see what the implications are of the breakdown of the wave equation at small scales on the evaporation process, and one might even contemplate the experimental investigation of the thermal emission process.

The model of the behavior of quantum field in a classical gravitational field is the motion of sound waves in a convergent fluid flow. The equations of motion for an irrotational fluid are given by[4]

$$\nabla \times \vec{v} = 0$$

$$\rho[\partial\vec{v}/\partial t + (\vec{v}\cdot\nabla)\vec{v}] = -\nabla p - \rho\nabla\Phi\,,$$

$$\partial\rho/\partial t + \nabla\cdot(\rho\vec{v}) = 0\,,$$

where p is the pressure which is assumed to be a function of ρ, and Φ is an external force potential. Defining

$$g(\xi) = \int^{e^\xi} (\rho')^{-1}[dp(\rho')/d\rho']\,d\rho'\,,$$

$$\xi = \ln\rho\,,$$

$$\vec{v} = \nabla\psi\,,$$

we have

$$\partial\psi/\partial t + \tfrac{1}{2}\vec{v}\cdot\vec{v} + g(\xi) + \Phi = 0$$

$$\partial\xi/\partial t + \vec{v}\cdot\nabla\xi + \nabla\cdot\vec{v} = 0\,.$$

Linearizing these equations about some solution ψ_0, $\vec{v}_0 = \nabla\psi_0$, and ξ_0, with

$$\xi = \xi_0 + \tilde{\xi}\,, \quad \psi = \psi_0 + \tilde{\psi}\,,$$

we obtain, after some manipulation,

$$\partial\tilde{\psi}/\partial t+\vec{v}_0\cdot\nabla\tilde{\psi}+g'(\xi_0)\tilde{\xi}=0,\qquad \rho_0^{-1}[\partial(\rho_0\tilde{\xi})/\partial t+\nabla\cdot(\rho_0\vec{v}\tilde{\xi})]+\rho_0^{-1}\nabla\cdot(\rho_0\nabla\tilde{\psi})=0,$$

which result in an equation for $\tilde{\psi}$,

$$\frac{1}{\rho_0}\left[\frac{\partial}{\partial t}\frac{\rho_0}{g'(\xi_0)}\frac{\partial\tilde{\psi}}{\partial t}+\frac{\partial}{\partial t}\frac{\rho_0\vec{v}_0}{g'(\xi_0)}\cdot\nabla\tilde{\psi}+\nabla\cdot\left(\frac{\rho_0\vec{v}}{g'(\xi_0)}\frac{\partial\tilde{\psi}}{\partial t}\right)-\nabla\cdot\rho_0\nabla\tilde{\psi}+\nabla\cdot\left(\vec{v}\frac{\rho_0}{g'(\xi_0)}\vec{v}\cdot\nabla\tilde{\psi}\right)\right]=0\,.$$

These are precisely the equations for a massless scalar field in a geometry with metric

$$ds^2=\frac{\rho_0}{c(\rho_0)}\{[c^2(\rho_0)-\vec{v}_0\cdot\vec{v}_0]dt^2+2dt\,\vec{v}_0\cdot d\vec{x}-d\vec{x}\cdot d\vec{x}\},$$

where $c^2(\rho_0)=g'(\ln\rho_0)$ is the local velocity of sound. For simplicity, I will assume this to be a constant.

If we assume that the background flow is a spherically symmetric, stationary, and convergent flow,[5] we can define a new time[6]

$$\tau=t+\int\frac{v_0^{\,r}(r)dr}{c^2-v_0^{\,r2}(r)}$$

in which case the metric becomes

$$ds^2=\frac{\rho_0}{c}\left((c^2-v_0^{\,r2})d\tau^2-\frac{c\,dr^2}{c^2-v_0^{\,r2}}-r^2(d\theta^2+\sin^2\theta\,d\varphi^2)\right).$$

If we assume that at some value of $r=R$ we have the background fluid smoothly exceeding the velocity of sound,[7]

$$v_0^{\,r}=-c+\alpha(r-R)+O((r-R)^2),$$

the above metric assumes just the form it has for a Schwarzschild metric near the horizon. Dropping the angular part of the metric we have

$$ds^2\approx\frac{\rho_0(R)}{c}\left(2c\alpha(r-R)d\tau^2-\frac{dr^2}{2\alpha(r-R)}\right),$$

which compares with

$$ds\approx[(\hat{r}-2M)/2M]\,d\hat{t}^2-2M\,d\hat{r}^2/(\hat{r}-2M)$$

near the horizon of a black hole. We can now ask what happens when we quantize the sound field $\tilde{\psi}$. Outside the sonic horizon the normal modes of the $\tilde{\psi}$ field which are purely outgoing as $r\to\infty$ go as

$$\tilde{\psi}_\omega\propto\begin{cases}\exp[i\omega(\tau-r^*)]+B_\omega\exp[i\omega(\tau+r^*)], & \text{near horizon}\\ A_\omega\exp[i\omega(\tau-r^*)], & \text{near }\infty,\end{cases}$$

where we define

$$r^*=\int\frac{c\,dr}{c^2-v_0^{\,r2}}\approx\begin{cases}\dfrac{r}{c}, & r\to\infty\\ \dfrac{1}{2\alpha}\ln(r-R), & r\to R.\end{cases}$$

The operator $\tilde{\psi}$ can be expanded in terms of these modes in the region outside the horizon.

$$\tilde{\psi}=\sum_\omega(a_\omega\tilde{\psi}_\omega+a_\omega^\dagger\tilde{\psi}_\omega^*)+\text{ingoing parts of }\tilde{\psi}.$$

To determine what an observer at large r will see, we must decide on the state of the field $\tilde{\psi}$. I will assume that an observer traveling with the fluid as it flows through the sonic horizon will see the state of the field $\tilde{\psi}$ as being essentially the vacuum state. The appropriate physical time for this observer is t, and positive frequency with respect to t will define the quantum state for the fluid near the horizon. Defining the co-moving radial coordinate

$$\tilde{r}\approx(r-R)-c(t-t_0),$$

we find that the modes ψ_ω have a time dependence to the co-moving observer near the sonic horizon of

$$\psi_\omega\approx(t-t_0+\tilde{r}/c)^{i\omega/\alpha}\times(\text{a smooth function of }t,\tilde{r}).$$

This corresponds exactly to the behavior of the normal modes of a scalar field in Schwarzschild t,r coordinates as seen by a freely falling observer where the relation is

$$\exp[i\omega(\hat{t}-\hat{r}^*)]$$
$$=(\hat{T}-\hat{T}_0)^{i4GM\omega/c^3}\times(\text{a smooth function}),$$

where $\hat{T}$ is the time as measured by a freely falling observer.

In the sonic case, this behavior of the normal modes near the sonic horizon implies that this sonic black hole will emit sound waves with a thermal spectrum (multiplied by a albedo function $|A_\omega|^2$ just as for the black-hole case) where the temperature is given by[8]

$$T = \frac{\hbar}{2\pi k} \frac{\partial v^r}{\partial r}\bigg|_{\text{horizon}}.$$

This compares with

$$T = \hbar c^3/8\pi k G M$$

for a black hole where c in this case is the velocity of light rather than that of sound. In both cases the derivation uses the behavior of external modes near the horizon, which go as $(t-t_0)^{h\omega/kT}$ in the relevant physical time, for arbitrarily small $t-t_0$, to conclude that thermal emission takes place.

This system forms an excellent theoretical laboratory where many of the unknown effects that quantum gravity could exert on black-hole evaporation can be modelled. The low-energy fluid equations which have led to quantum thermal sonic emission by a transonic background flow break down at high frequencies because of the atomic nature of the fluid. At distances of 10^{-8} cm, the assumptions which I use of a smooth background flow are no longer valid just as in gravity one expects the concept of a smooth space-time on which the various relativistic fields propagate to breakdown at scales of 10^{-33} cm. Furthermore, the phonons emitted are quantum fluctuations of the fluid flow and thus affect their own propagation in exactly the same way that graviton emission affects the space-time on which the various relativistic fields propagate.

This model also admits the possibility of experimental testing, although this is an extremely slim possibility. Assuming $\partial v^r/\partial r \approx c/R$, where R is the horizon radius, we obtain a temperature of

$$T = (3\times 10^{-7}\ \mathrm{K})[c/(300\ \mathrm{m/sec})](1\ \mathrm{mm}/R).$$

This is a rather low temperature, and is probably undetectable in the presence of turbulent instabilities, etc., which would arise in trying to drive the fluid transsonically through a small nozzle. It is, however, a much simpler experimental task than creating a 10^{-8}-cm black hole.

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[1]S. W. Hawking, Nature (London) 248, 30 (1974), and Commun. Math. Phys. 43, 199 (1975).

[2]W. G. Unruh, Phys. Rev. D 14, 870 (1976).

[3]S. Hawking and B. Carr, Mon. Not. Roy. Astron. Soc. 168, 399 (1974).

[4]See, for example, L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, London, 1959).

[5]The conclusions of this paper do not depend on these assumptions but depend only on the existence of a sonic horizon.

[6]The "time" τ has no physical significance except for the small-amplitude sonic motions of the medium.

[7]In general, a shock will develop in transsonic flow, thus violating this assumption. However, by applying suitable external forces to the fluid, or by forcing it through a suitably shaped nozzle, a smooth supersonic transition can be obtained.

[8]The spherical symmetry is not essential to this result. A model of transsonic flow through a nozzle gives exactly the same results.