Notes on the Unruh Effect

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1. Introduction

The Unruh effect is a collection of results for relativistic quantum field theory. The usual interpretation of these results is that an observer with uniform acceleration α will interpret the Minkowski vacuum state as having non-zero particle number, and energy described by a thermal density matrix with temperature $T = \alpha/2\pi$.

1.1. **Philosophical commentary.** The original aim of Unruh (1977) was to provide an analogy for the discussion of Hawking radiation and black hole thermodynamics. Others have drawn more radical conclusions from the apparent observer-relative status of particles:

> The Unruh effect may appear paradoxical to readers who are used to thinking that quantum field theory is, fundamentally, a theory of 'particles', and that the notion of 'particles' has objective significance. ... No paradox arises when one views quantum field theory as, fundamentally, being a theory of local field observables, with the notion of 'particles' merely being introduced as a convenient way of labelling states in certain situations. (Wald, 1994, p.118)

However, there is a good deal of controversy amongst philosophers about how to interpret the Unruh effect.

- Clifton and Halvorson (2001) supplement Wald's reading with a Bohr-inspired take on the Unruh effect, as consisting in 'complementary' descriptions of the vacuum.

- The conclusion that particles do not exist depends on there being a strong sense in which the two descriptions of particles are inequivalent; Arageorgis et al. (2003) argue that this has not yet been established.
- Earman (2011) argues, it is not obvious that all arguments purporting to derive the Unruh effect represent the same physical phenomenon.
- Baker (2009) argues that, if the reasoning against the existence of particles does go through, then the very same argumentation can be used to establish that fields do not exist either.

1.2. **Physical intuition.** The Unruh effect can be motivated in heuristic terms by appeal to Noether's theorem. Suppose we define the energy associated with a Poincaré-invariant field to be the conserved quantity associated with time-translation symmetries. In the right 'wedge' of any lightcone in Minkowski spacetime (called a 'right Rindler wedge'), there are actually two timelike symmetry groups (two timelike Killing fields) specified by, respectively:

(i) the inertial trajectories; and

(ii) the uniform acceleration trajectories corresponding to Lorentz boosts (details after eq. 3 below).

The conserved 'energy' for one will not necessarily be conserved for the other, nor can the definitions of energy be expected to reach their lowest values in the same state. In other words, they cannot be expected to agree on whether or not a given state is a stationary vacuum. As a result, the concept of 'the vacuum', and the associated concepts of particle-number and temperature, are liable to be relative to the timelike Killing field.

On this way of looking at it, the Unruh effect is an essentially quantum effect. For the argument only goes through if the lowest energies associated with these two Killing fields really are different. But, for classical fields, they are generally the same. This is because the inertial time-translation Noether charge (energy) for a classical field ϕ is the integral of the Hamiltonian density $H(\phi, \pi)$ which is minimised when $\phi = 0$ everywhere.[1]

The situation changes when we redefine energy from the perspective of quantum theory, where energy is associated with the spectrum of an operator. Here again it is possible to get an intuition for how this happens using classical field theory, by considering a different definition of energy, associated with a relativistic dispersion relation.

For example, consider the case of the classical Klein-Gordon field on two-dimensional Minkowski spacetime $(\mathbb{R}^2, \eta_{ab})$ with global Euclidean coordinates (t, x). The field is a smooth function $\phi : \mathbb{R}^2 \to \mathbb{C}$ that satisfies the Klein-Gordon equation with $m \geq 0$, in units where $c = 1$:

$$\left(\nabla_a \nabla^a - m^2\right) \phi = 0. \tag{1}$$

In a reference frame (t, x), its solutions ϕ are linear combinations of plane-waves of the form $\psi(t, x) = e^{iEt - px}$, for any $E, p \in \mathbb{R}$ satisfying the relativistic dispersion relation, $E^2 - p^2 = m^2$.

Suppose that we take the 'energy' of a plane-wave $\psi(t, x)$ to be given by its four-momentum density, $m = \sqrt{E^2 - p^2}$. To describe the 'inertial vacuum' in this reference frame, we could then consider the solution of minimum coordinate energy $E(p)$ as a function of p (i.e. with m fixed), which occurs when $p = 0$ and $E = m$. This occurs with the plane wave $\psi_0(t, x) = e^{imt}$, which might be called the inertial ground state. In all inertial coordinate systems, it is stationary (up to a phase) and has the same energy-momentum density m.

However, it is not stationary from the perspective of a uniformly accelerated reference frame, known as *Rindler coordinates* (t', x'). These are defined in the Rindler wedge where $x > t$ and $x > -t$ by,

$$t' := \tfrac{1}{2} \ln \left(\frac{x + t}{x - t} \right) \qquad\qquad x' := (x^2 - t^2)^{1/2}, \tag{2}$$

[1]For example, the (inertial) energy density for the Klein-Gordon field is $H(\phi, \pi) = \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2$. The integral of this quantity over space is minimised when $\phi = \pi = 0$ everywhere.

from which we can infer conversely that,

$$t = x' \sinh t' \qquad\qquad x = x' \cosh t'. \tag{3}$$

So here, the timelike lines $x' =$ constant are hyperbolae of constant linear acceleration (acceleration constant along each line but varying between lines) that are orthogonal (Minkowski-orthogonal!) to the spacelike planes (indeed: Cauchy surfaces) $t' =$ constant.

Expressing the inertial ground state ψ_0 in Rindler coordinates we find,

$$\psi_0(t', x') = e^{imx' \sinh t'}. \tag{4}$$

It is easy to check that this state is not stationary in Rindler coordinates: $\frac{d}{dt'}\psi_0(t', x') = imx' \sinh t' \cosh t' \psi_0(t'x')$, which is not generally zero. So, the inertial ground state is not stationary, let alone associated with a conserved value of energy. In fact, since $\psi_0(t', x')$ is not a plane wave but a superposition of plane waves (via its Fourier decomposition), it is associated with multiple non-zero 'energies' — a kind of classical Unruh effect.

This thinking can be given a more detailed analysis that splits positive fre- quency modes in order to introduce a notion of classical particle number. This turns out to give a first-order approximation of the quantum expression of the Unruh effect; see Higuchi and Matsas (1993) and Crispino et al. (2008). However, it is less general: it depends on the mass of the Klein-Gordon field being non-zero, since $m = 0$ implies the inertial vacuum plane-wave becomes a constant scalar field again, which is the same in inertial coordinates, Rindler coordinates, and indeed all coordinates.

So, a more complete picture of the Unruh effect requires quantum field theory, to which we turn next.

1.3. **Overview of the quantum derivation.** In the next sections, we will discuss the derivation of the Unruh effect in a very general form. As mentioned at the outset, there are many derivations of the Unruh effect. We will discuss one due to

Kay and Wald (1991) that is of interest because it is much more general than Klein-Gordon fields on Minkowski spacetime. This derivation, sometimes referred to as the 'generalised Unruh effect', can be sketched as follows; the details will be filled out after that.

(1) **Classical field theory.** We consider a general class of Lorentzian manifolds defined by a geometric structure known as a 'bifurcate Killing horizon'. Examples include Minkowski spacetime, and also the Schwarzschild black hole. When our derivation is applied to the latter, we find a sense of thermal radiation for black holes, though it is not quite the same as Hawking radiation. Our classical field theory on this background spacetime will generally allow two distinct notions of time evolution, as in the heuristic argument above.

(2) **Quantise in two different ways.** Each notion of time evolution determines a complex structure for the purposes of quantisation. So, our pair of distinct time evolutions give rise to a pair of distinct quantisations, due to the presence of distinct complex structures.

(3) **Compare quantisations.** Finally, we compare quantisations. This will be a little awkward, because their states turn out to be 'disjoint', or mutually indescribable. However, we can still say some general things, and we can use some approximation results like Fell's theorem, as well as more general thermal analysis like modular theory, in order to derive a non-zero temperature.

2. Classical field theory

We will be considering an argument for the Unruh effect that applies to a general class of classical field theories, on a general class of Lorentzian manifolds. The main restriction on the Lorentzian manifolds is that they be globally hyperbolic and admit a special kind of horizon, to be discussed below. The main restriction on the field theories will be that they are associated with a hyperbolic partial differential equation.

2.1. **Bifurcate Killing horizons.** Classical field theory assigns scalar, tensor, or spinor fields to a Lorentzian manifold (M, g_{ab}), which is assumed to be *globally hyperbolic*. A Lorentzian manifold is defined to be globally hyperbolic iff the diamond regions $I^+(p) \cap I^-(q)$ are compact for all points $p, q \in M$. This is equivalent to the existence of a *Cauchy surface*: i.e. a spacelike hypersurface that every inextendible timelike and null curve intersects exactly once (Geroch, 1970). This in turn implies the existence of a foliation of the spacetime $t \mapsto \Sigma_t$ into Cauchy surfaces Σ_t parametrised by the real numbers $t \in \mathbb{R}$, i.e. the manifold M is diffeomorphic to $\Sigma_0 \times \mathbb{R}$. The idea is that the values of fields on a Cauchy surface provide the 'initial conditions' for a differential equation describing the fields' evolution, and indeed: hyperbolic partial differential equations for fields have a locally well-posed initial-value problem.

Some Lorentzian manifolds, which include Minkowski spacetime, Schwarzschild spacetime, and even 'acoustic black holes', all share a common structure that makes it possible to associate them with thermal radiation. That structure is called a 'bifurcate Killing horizon'. It can be defined in a few stages.

Let (M, g_{ab}) be an $n > 2$-dimensional Lorentzian manifold. A Killing field χ^a is vector field that is tangent to a 1-parameter group of isometries; equivalently, it is a vector field along which the Lie derivative of the metric vanishes, $\mathcal{L}_\chi(g_{ab}) = 0$. A Killing horizon (with respect to a non-zero Killing field $\chi^a a$) is a null surface H that is normal to χ^a, in that $\chi^a \eta_a = 0$ for all (null) vectors η^a tangent to H. Finally, a bifurcate Killing horizon for a 4-dimensional Lorentzian manifold is a pair of Killing horizons H_1 and H_2 with respect to the same Killing field χ^a, which intersect at a 2-dimensional surface S contained in a Cauchy surface. Some technical details in the analysis entail infrared divergences when the manifold M is 2-dimensional; to avoid this, we will stick to the realistic case in which $\dim M > 2$.

2.2. **Minkowski and Schwarzschild horizons.** A bifurcate Killing horizon appears in Minkowski spacetime $(\mathbb{R}^4, \eta_{ab})$ at the plane of intersection of any two null-surfaces.

In Euclidean coordinates (t, x, y, z) with $\eta_{ab} = (diag(-1, 1, 1, 1))_{ab}$,the group of isometries associated with Lorentz boosts (i.e. boosts to each of the various possible velocities in the x-direction) thread the Killing field $\chi^a = \left(t\frac{\partial}{\partial x}\right)^a + \left(x\frac{\partial}{c\partial t}\right)^a$. That is: the integral curves of this vector field are the hyperbolae of constant linear acceleration $x' =$ constant (cf. eq. 3). This Killing field is normal to the surface satisfying $x = t$, and to the surface $x = -t$, and so both are Killing horizons. Their plane of intersection at $x = t = 0$ is contained in the Cauchy surface $(0, x, y, z)$, and so they form a bifurcate Killing horizon. As we shall see, this turns out to be the essential geometric feature underpinning the Unruh effect.

An analogous observation can be made about the Schwarzschild black hole $(\mathbb{R}^4, g_{ab})$, by converting the metric into Kruskal coordinates. In Schwarzschild coordinates (t, r, θ, ϕ), the static vector field $\chi^a = \left(\frac{\partial}{\partial t}\right)^a$ is a Killing field. To identify the bifurcate Killing horizon associated with it, it is easiest to switch to Kruskal coordinates (T, X, θ, ϕ), where the Killing field takes the same form as the boost Killing field above, and the bifurcate Killing horizons $X = T$ and $X = -T$ define the black hole event horizon.

2.3. **Classical field solutions.** We would like to view a classical field system from a phase-space perspective that allows us to quantise it. We'll review how this works in the special case of a Klein-Gordon field on Minkowski spacetime $(\mathbb{R}^4, \eta_{ab})$, and then move on to the more general case. From the very beginning we will introduce a preferred definition of time evolution; this choice is a crucial element of the Unruh effect.

Let us first view time evolution from the perspective of a global coordinate system (t, x). The Klein-Gordon equation is derived by extremising the Klein-Gordon action $S = \int L dt$, where the Lagrangian is given by,

$$L(\phi, \dot{\phi}) = \tfrac{1}{2}\left(\dot{\phi}^2 - (\nabla\phi)^2 - m^2\right), \tag{5}$$

with $\phi : \Sigma_0 \to \mathbb{R}$ a smooth function with compact support[2] on the $t = 0$ plane Σ_0, and $\dot{\phi} = (\partial/\partial t)\phi$ denoting the coordinate derivative.

The conjugate momentum variables at $t = 0$ are defined by $\pi = \delta S/\delta\dot{\phi}$, which for the Klein-Gordon field gives $\pi = \dot{\phi}$, also assumed to be of compact support.

The points $v = (\phi, \pi)$ of this kind define a real linear manifold V, with addition and scalar multiplication of real numbers defined in the obvious way. We then define the symplectic structure $\Omega : M \times M \to \mathbb{R}$ for each pair of points $v_1 = (\phi_1, \pi_1)$ and $v_2 = (\phi_2, \pi_2)$ to be,

$$\Omega(v_1, v_2) = \int_{\Sigma_0} (\pi_1\phi_2 - \pi_2\phi_1). \tag{6}$$

Since V is a vector space, it can be naturally identified with the tangent space at each point. When Ω is lifted in this way to a map on pairs of tangent vectors, it becomes a symplectic form Ω_{ab}, which given the Hamiltonian density $H(\phi, \pi)$ generates the time evolution of the field as a symplectic flow. The structure (V, Ω), equipped with H, is thus a symplectic vector space that characterises our classical field system—and it is nearly ready to be quantised following the Segal prescription.

Generalisation: Nothing about this construction is unique to Minkowski spacetime, or even the Klein-Gordon field. To do it on a general curved Lorentzian manifold (M, g_{ab}), we only need a preferred notion of time evolution, and a corresponding preferred decomposition into spacelike hypersurfaces, that will allow us to identify a configuration space and so a phase space.

Since we have assumed global hyperbolicity, we can foliate the Lorentzian manifold into a one-parameter set of Cauchy surfaces $t \mapsto \Sigma_t$. Let τ^a be a timelike vector field such that $\tau^a \nabla_a t = 1$. Then we can take our local coordinate system to be (t, x^1, x^2, x^3) with $\tau^a \nabla_a x^i = 0$ for $i = 1, 2, 3$, and so $t^a = (\partial/\partial t)^a$. The rest of the construction then proceeds exactly as before: define our configuration variables to be the smooth functions ϕ on the Cauchy surface Σ_0 that have compact support. Given

[2]The restriction of compact support guarantees that each field is locally definable, and will prove mathematically useful below.

a Lagrangian $L(\phi, \dot{\phi})$ and action function $S = \int L dt$, define the conjugate momentum and symplectic structure as before. The result is again a symplectic vector space (V, Ω) that is (nearly) ready to be quantised.

3. Quantising in two ways

3.1. **How time-evolution determines a complex structure.** *Review:* The Segal quantisation of a linear symplectic space (V, Ω) begins with the choice of a complex structure J (meaning: an automorphism of V that squares to the negative identity map, $J^2 = -I$) that is compatible with Ω, in that $\Omega(Jv, Jw) = (v, w)$ for all $v, w \in V$. Intuitively, such a J 'flips ϕ to $-\pi$' in canonical coordinates. This J allows us to do everything we needed to quantise (V, Ω). In short:

(i) we view V as a vector space over $\mathbb{C}$;

(ii) we define a metric structure g by the prescription (the *Kähler condition*) $g(v, w) := \Omega(v, Jw)$, and

(iii) we define a complex inner product $\langle v, w\rangle := g(v, w) + i\Omega(v, w)$: whose Cauchy-completion is a Hilbert space.

Unfortunately, the complex structure J that underpins this construction is not unique. This may be viewed as arising from the fact that (V, Ω) has no built-in notion of length. As a result, any J that 'flips ϕ to $-a\pi$' for some $a > 0$ will be a complex structure, but which—for example—stretches the eccentricities in the ellipses of a harmonic oscillator. However, J does turn out to be uniquely determined once we choose a notion of time evolution that is 'appropriate' for quantisation in a certain sense.

Time evolution on (V, Ω) is determined by a smooth function $h : V \to \mathbb{R}$, which generates a symplectic flow: it determines a vector field $\xi^b := \Omega^{ab} d_a h$ that is assumed to be complete, and whose integral curves are given by a continuous one-parameter group of symplectomorphisms $t \mapsto \sigma_t$. An 'appropriate' J for quantisation should lead to the "quantum image" of $t \mapsto \sigma_t$ being a unitary representation. This is in order to retain the assumption of time-translation invariance in our classical system: each map σ_t is an automorphism of (V, Ω), and so its quantisation should be an automorphism of the Hilbert space. This turns out to hold for the Hilbert space we constructed (two paragraphs above) if and only if $\sigma_t J = J\sigma_t$ for all $t \in \mathbb{R}$. (Exercise: check this.)

It should also lead to a notion of positive (quantum) energy: when $t \mapsto U_t$ is a weakly continuous unitary group, Stone's theorem guarantees that there is a unique self-adjoint H such that $U_t = e^{-itH}$ for all $t \in \mathbb{R}$; we say U_t has 'positive energy' if and only if H has a positive spectrum.

These conditions turn out to be enough to determine a unique complex structure. Let $K : V \to \mathcal{H}$ be a linear embedding of vectors in (V, Ω) into a real, dense subspace of a Hilbert space $\mathcal{H}$, that is compatible with Ω, and with the time evolutions $t \mapsto \sigma_t$, $t \mapsto U_t$ by satisfying:

(i) $K\sigma_t = U_t K$ for all t; and

(ii) σ_t has positive energy.

Then this K determines our complex structure J on V. For let $K^{-1} : \operatorname{ran} K \to V$ be the associated inverse, which extends by density to a map on $\mathcal{H}$; then J is given by $J(v) := K^{-1}(iK(v))$. And, it is easy to check that condition (i) $K\sigma_t = U_t K$ is equivalent to the condition that $J\sigma_t = \sigma_t J$.

Kay (1979) proved that given these conditions, the linear map K is unique (up to unitary equivalence). As a result, the complex structure J is uniquely determined as well, by a dynamics that is 'appropriate' in the sense above. This means that the crucial element in determining a quantisation is really the choice of a notion of time-translation. And, as suggested in the Introduction, different notions of time-translation will be seen to give rise to the Unruh effect.

3.2. **Prescription 1: Inertial.** As before, let us first consider the case of the Klein-Gordon field system (S, Ω) before turning to the more general case. We have adopted an inertial notion of time-translations in the description of this system. We now turn to 'splitting the frequencies', sometimes known as 'first quantisation'.

We noted that, given an inertial notion of time-translation associated with Euclidean coordinates, a solution $\phi(t)$ to the Klein-Gordon equation can be viewed as a linear sum of plane waves $\psi = e^{iEt - p\cdot x}$ satisfying the dispersion relation, $E^2 - p^2 = m^2$.

Those with positive values of E are called 'positive frequency' plane waves; those with negative values are called 'negative frequency'. One commonly avoids the unfortunate problem of negative frequencies by splitting each solution $\phi(t)$ into its positive and negative frequency modes, $\phi(t) = \psi^+(t) + \psi^-(t)$, and quantising only the positive modes. However, speaking in this way hides an essential ingredient: our notion of time evolution defines a complex structure, which is required in order to determine the coefficients E and p. In our discussion, we will make its role a little more explicit.

Begin with our phase space (S, Ω), with an 'appropriate' notion of time-translation $t \mapsto \sigma_t$ defined as above by the inertial notion of time-evolution for the Klein-Gordon field. This inertial perspective on time determines a complex structure, which in turn will allow us to construct a *Fock space* and a representation of the *Weyl-algebra* Here is a sketch of that procedure.

Let $\mathcal{H}$ be the Hilbert space constructed from (S, Ω) and $t \mapsto \sigma_t$, associated with the complex structure J (which, according to the discussion above, is unique). Let $\mathcal{H}^n$ denote its n-fold symmetric tensor product (i.e., projecting onto the symmetric subspace). Then the Fock space for $\mathcal{H}$ is given by,

$$\mathcal{F}(\mathcal{H}) := \mathbb{C} \oplus \mathcal{H}^1 \oplus \mathcal{H}^2 \oplus \mathcal{H}^3 \oplus \cdots . \tag{7}$$

The *vacuum* for this Fock space is $\omega := 1 \oplus 0 \oplus 0 \oplus 0 \oplus \cdots$. The creation a^* and annihilation a operators can then be defined following a well-known prescription. To avoid cumbersome notation in these definitions, we will suppress the map K, i.e. treat K as the identity embedding like we did at the start of Section 3.1; so we write as if these operators are defined on S^{n-1} and S^n, i.e. on products of classical solutions rather than on products of their images under K (and then Cauchy-completion).

So first, for each element $v \in S$, we define a map $a_n^* : S^{n-1} \to S^n$ and $a_n : S^n \to S^{n-1}$ by $a^*(v) : (v_1, \dots, v_{n-1}) \mapsto (v, v_1, \dots, v_{n-1})$ and $a(v) : (v_1, \dots, v_n) \mapsto \langle v, v_1 \rangle (v_2, \dots, v_n)$, where $\langle v, w \rangle$ is the Hilbert space inner product defined in (iii) at the start of Section 3.1.

Then the *creation* and *annihilation* operators are,

$$a^*(v) := a_1^*(v) \oplus \sqrt{2}E_+a_2^*(v) \oplus \sqrt{3}E_+a_3^*(v) \oplus \cdots, \tag{8}$$

$$a(v) := 0 \oplus a_1(v) \oplus \sqrt{2}a_2(v) \oplus \sqrt{3}a_3(v) \oplus \cdots, \tag{9}$$

where E^+ is the projection of S^n onto the symmetric subspace. These creation and annihilation operators are self-adjoint. They make essential use of the complex structure J, and therefore of our definition of time evolution.

These operators allow us to define a self-adjoint particle number operator $N(v) = a^*(v)a(v)$ for each $v \in S$, as well as a field operator,

$$\Phi(v) := \frac{1}{\sqrt{2}}\left(a^*(v) + a(v)\right). \tag{10}$$

Then finally, we define the mapping $v \mapsto W(v) = e^{i\Phi(v)}$. The set of unitary operators defined in this way form an irreducible unitary representation of the Weyl algebra.

Thus we have begun with a classical system (V, Ω) with $t \mapsto \sigma_t$, from this produced a Fock space, together with an algebra of observables, which together determine the statistical properties of a quantum field system. This is our first approach to quantisation. One can of course apply it to the case of the Klein-Gordon field on Minkowski spacetime. However, this reasoning applies equally to a much larger class of classical field systems on an (in general) curved spacetime.

3.3. **Prescription 2: Rindler.** The Klein-Gordon field in Minkowski spacetime has two distinct notions of time evolution in the right-Rindler wedge. The first is the inertial (or more generally 'affine') notion discused above; the second is given by the group of Lorentz boosts, which determine the trajectory of a uniformly accelerating observer in the wedge. This situation occurs whenever there is a classical field theory on a curved spacetime that admits a bifurcate Killing horizon in the sense defined above; then the Lorentz boosts are replaced with the isometries that thread the Killing field.

Let S_R be the restriction to the functions with support in the right-Rindler wedge; otherwise, our phase space (S_R, Ω) will be the same. However, there is now an alternative symplectic flow $t \mapsto \sigma'_t$ for this system. It turns out to give rise to a complex structure J that leads to a representation of the Weyl algebra that is unitarily inequivalent to the first. Thus we can proceed to construct a representation $v \mapsto W(v)$ of the Weyl algebra using exactly the same procedure described above; but it is unitarily inequivalent to our first one. And due to the restriction of S_R to functions on the right-Rindler wedge, it is best considered to be a quantum field theory associated with the right-Rindler wedge.

4. Comparing quantisations

To compare the inertial and Rindler quantisations, one needs a way of translating between them. This is made awkward by the fact that the two representations turn out to be unitarily inequivalent. In fact, the situation is worse: they are 'disjoint', meaning that no density matrix state in one is a density matrix state in the other.

However, *Fell's theorem* allows one way to do it. For in two (possibly inequivalent or disjoint) representations π_1, π_2 of the Weyl algebra, every state ρ_1 in π_1 can be *weakly approximated* by a state ρ_2 in π_2 in the sense that:
for any finite sequence of elements A_i $(i = 1, ..., n)$ of the algebra, and any sequence of n real numbers $\epsilon_i > 0$ $(i = 1, ..., n)$, there is a state ρ_2 in π_2 with

$$|\rho_1(\pi_1(A_i)) - \rho_2(\pi_2(A_i))| < \epsilon_i.$$

It is then possible to apply a Bogoljubov transformation U to the vacuum in the inertial representation, in order to view it from the perspective of the Rindler representation — the result is not strictly speaking in the Rindler representation, since U is unitary, but it is "approximate to" the Rindler vacuum state. Wald (1994) finds in particular that it gives rise to particle-content in the Minkowski vacuum.

See Clifton and Halvorson (2001) for a further sense in which, for any two distinct complex structures J_1, J_2 used to quantise (S, Ω), the Fock-vacuum in one predicts dispersion in the particle-number operator associated with the other.

A further approach is to use the modular theory of Tomita and Takesaki. On this approach, it is possible to apply the Bisognano-Wichman theorem to conclude that the Rindler automorphisms of the Weyl algebra give rise to a unique KMS-state (a generalisation of the Gibbs state) for the Minkowski vacuum, which is non-zero. Cf. Earman (2011: especially Sections 2, 3 and 5).

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