Objects in Physics and Philosophy

J. Butterfield: *Philosophical Aspects of QFT*, Mich. 2021; Monday 29 Nov, as rescheduled!

This handout, prompted by requests for this theme, covers in order;

(i) objects in general, and particles in particular, including in quantum theory (Sections 0 to 4);

(ii) the topic of whether spacetime points are objects (briefly in Section 5: there is an Essay about the Hole Argument ...);

(iii) particle and field in QFT (Section 6: reprising the main handout for this course, *Quantization of linear dynamical systems I*).

0. Objects in Philosophy, and in Physics :

The ancient tradition and jargon:— An object is a *substance*, its properties and relations are *universals*. Much discussion of: (i) the conditions for persistence of an object over time (ii) essential ('it must have the property') vs. accidental properties.

The tradition and jargon after Frege and the rise of modern logic:— A less contentions, a "thinner" notion of object. An object is the *referent* of a *singular term*. A singular term is an individual constant a ('Plato'), or definite description 'the F' (F a predicate, e.g. 'the tallest person') , or a completed functional expression $f(a)$ ('the father of Plato'). Singular terms slot in to the argument places of *predicates*. These represent 1-place properties, $F(x)$ e.g. '... is red', or 2-place relations $H(x, y)$ e.g. '...loves...'.

The atomistic tradition: 'lumps in the void': Democritus, Lucretius, Giordano Bruno, Galileo, Hobbes, Newton ... It seems okay, but what happens in contact? The idea is refined by Euler's idea (ca 1740) of a *point-particle*. For him, it was an idealisation. After all: infinite mass density! yet it also allows distinct masses at extensionless points of space. And what should we say about collisions of two point-particles? eg one falling in the Newtonian gravitation potential well of the other.

The rise of field theory. Note the weaker and stronger senses of 'physically real' that this dveelopment prompts.

(i): The Newtonian gravitational potential (cf. Poisson equation $\nabla^2\phi = \rho$) is physically real in the sense that in a Newtonian world, a test-particle at x will accelerate at $-\nabla\phi$, under the gravitational influence of the mass density ρ. So in philosophy jargon: ϕ encodes an infinite conjunction of counterfactual conditionals about how test-particles would accelerate. (Agreed: even at this level, there are broad issues about the word 'real', about whether our scientific theories are objective or subjective ('mere models'), scientific realism etc.)

(ii): But ϕ has neither energy or momentum, unlike the electromagnetic field F. Besides, after special relativity, mass and energy are identified, so that the distinction 'ponderable matter' vs. 'field' falls away. So F is physically real in this stronger sense. .

(iii): In general relativity, even geometry, ie the metric field, is physically real in much the same sense as the electromagnetic field F. It is responsive to (its state depends on) the distribution of matter and radiation, it has energy and momentum.

1. Identity in Logic :
The indiscernibility of identicals: $\forall x \forall y \{x = y \supset (\forall \Phi)[\Phi(x) \equiv \Phi(y)]\}$.
The identity of indiscernibles ('PII')): $\forall x \forall y \{(\forall \Phi)[\Phi(x) \equiv \Phi(y)] \supset x = y\}$.
So, "Leibniz's law": $\forall x \forall y \{x = y \equiv [\Phi(x) \equiv \Phi(y)]\}$.

What about the range of the quantifier '$\forall \Phi$'?
Technical issues: second-order logic, and first-order schemas.
Philosophical issues: admitting haecceities/thisnesses like 'being a', represented by '$x = a$' threatens to make the principle of the identity of indiscernibles trivial.

Side-remark: A proposed replacement for Leibniz's law is the Hilbert-Bernays(-Quine) axiom, which mentions every primitive predicate of one's language, quantifying in each argument-place:

$$\begin{aligned} \forall x \forall y \Big\{ x = y \;\; \equiv \;\; \Big[& \ldots \wedge \; (F^1_i x \equiv F^1_i y) \; \wedge \; \ldots \\ & \ldots \wedge \; \forall z \left((G^2_j xz \equiv G^2_j yz) \wedge (G^2_j zx \equiv G^2_j zy) \right) \; \wedge \; \ldots \\ & \ldots \wedge \; \forall z \forall w \Big((H^3_k xzw \equiv H^3_k yzw) \wedge (H^3_k zxw \equiv H^3_k zyw) \\ & \qquad\qquad \wedge \; (H^3_k zwx \equiv H^3_k zwy) \Big) \; \wedge \; \ldots \Big] \Big\} \end{aligned}$$

Notice how this identifiies '=' with discernibility within the language.

2. Identity in quantum theory :
The Hilbert space $\mathcal{H}$ for an assembly S consisting of N indistinguishable quantum particles is built from the N-fold tensor product $H^N := H \otimes ... \otimes H$ (N factors). We define permutations as unitary operators on $\mathcal{H}$. Let x, y be two of the particles, and P_{xy} be the corresponding transposition (flip) operator. It is usual to postulate *Permutation Invariance* as:

(PI): for any pure state ϕ of S: if x, y are indistinguishable bosons, $P_{xy}\phi = \phi$ ('symmetric'); and if x, y are indistinguishable fermions, $P_{xy}\phi = -\phi$ ('anti-symmetric').

Technical details:

(i): For $N = 2$, $\phi = \Sigma_{ij}\ c_{ij}\ u_i \otimes v_j$ is symmetric iff $c_{ij} = c_{ji}$: i.e. a symmetric matrix of coefficients. So the subspace $\mathcal{S}(H^2)$ of symmetric vectors is of dimension $\frac{1}{2}n(n+1)$.

(ii): Similarly, the subspace $\mathcal{A}(H^2)$ of anti-symmetric vectors is of dimension $\frac{1}{2}n(n-1)$.

(iii): $\mathcal{S}(H^2) \perp \mathcal{A}(H^2)$.

(iv): $H^2 = \mathcal{S}(H^2) \oplus \mathcal{A}(H^2)$. NB: This does not generalize to $N > 2$. For $N \geq 3$, there are also other subspaces, orthogonal to $\mathcal{S}(H^2) \oplus \mathcal{A}(H^2)$, and invariant under all permutations: "paraparticles".

(iv): Example: If $n := \dim(H) = 2$, and $\dim(H^2) = 4$, then $\dim(\mathcal{S}(H^2)) = 3$; and $\dim(\mathcal{S}(H^2)) = 1$.

(vi): General theory: Permutation operators as providing a representation of the symmetric group S_N on H^N. Each boson or fermion state (i.e. ray) is a 1-dimensional (sub)-representation. The state of a paraparticle is given by a (sub)-representation of dimension greater than one.

(vii): The indistinguishability of particles justifies our requiring quantities Q to be permutation-invariant; but does not justify (PI). More precisely, indistinguishability first justifies quantum expectation values being permutation-invariant, i.e. $\langle P\phi \mid Q \mid P\phi\rangle = \langle \phi \mid Q \mid \phi\rangle$. But this implies $P^{\dagger}QP = Q$, i.e. $[P, Q] = 0$. This *Indistinguishability Postulate*, (IP), allows paraparticles; and so it does not imply (PI).

Philosophical issues:

(i): *Trajectories*: There is a tradition that (a) classical particles can always be distinguished by their spatial or spacetime trajectories, saving the PII; whereas (b) quantum particles have no trajectories and so cannot be thus distinguished.

In fact, both claims can and should be questioned.

(a): Consider two indistinguishable classical particles in an otherwise empty universe (e.g. Newton's absolute space, or a relativistic spacetime), and then permute them—have you changed the physical state, or merely its mathematical representative?

(b): Consider the pilot-wave theory of de Broglie and Bohm!

(ii): *'In the same state'*: There is a tradition that symmetrization for bosons and anti-symmetrization for fermions means that (a) indistinguishable bosons can be in the same state, but (b) indistinguishable fermions cannot be; so that (c) PII holds for fermions but not bosons.

In fact, these claims can and should be questioned. Under scrutiny, (a) to (c) fail: two indistinguishable bosons *or* fermions that are constituent particles of an assembly *must* be in the same quantum state. Cf. Section 3.

This result suggests that PII is pandemically *false* in quantum theory. But under scrutiny, this *also* fails! For the Hilbert-Bernays account shows us ways of distinguishing particles that outstrip the orthodox notion of quantum state. Cf. Section 4.

3. Two indistinguishable constituent particles are quantum-indistinguishable

Let x and y be two indistinguishable constituent particles of an assembly S of N particles. So $\mathcal{H}$ is a N-fold tensor product with factor H_x for x. Let Q_x represent the quantity Q on component x: $Q_x := \mathbb{I} \otimes ... \otimes \mathbb{I} \otimes Q \otimes \mathbb{I} \otimes ... \otimes \mathbb{I}$, with Q in x's place. Let Q' be a (possibly indistinct) quantity; let q be a value of Q; and let q' be a (possibly indistinct) value of the quantity Q'. Then we have, for any state ϕ obeying (PI), and any third particle z in S (possibly distinguished from x, y):

$$\begin{aligned} pr_\phi(Q_x \text{ has value } q) &= pr_\phi(Q_y \text{ has value } q); && (1)\\ pr_\phi(Q_x \text{ has value } q/Q'_y \text{ has value } q') &= pr_\phi(Q_y \text{ has value } q/Q'_x \text{ has value } q'); && (2)\\ pr_\phi(Q_z \text{ has value } q/Q'_x \text{ has value } q') &= pr_\phi(Q_z \text{ has value } q/Q'_y \text{ has value } q'). && (3) \end{aligned}$$

Proof for (2) and (3): the equations follow from readily proved properties of P_{xy}. Let Q^q_x be Q's spectral projector for value q. then the left hand side of (2) is by definition:

$$pr_\phi(Q_x \text{ has value } q/Q'_y \text{ has value } q') := \langle \phi \mid Q^q_x \; Q'^{q'}_y \mid \phi \rangle \; / \; \langle \phi \mid Q'^{q'}_y \mid \phi \rangle \; , \qquad (4)$$

while the right hand side of (2) is by definition

$$pr_\phi(Q_y \text{ has value } q/Q'_x \text{ has value } q') := \langle \phi \mid Q^q_y \; Q'^{q'}_x \mid \phi \rangle \; / \; \langle \phi \mid Q'^{q'}_x \mid \phi \rangle \; . \qquad (5)$$

(1) implies that the denominators on the right hand sides of (4) and (5) are equal. To prove the denominators equal, we use:

$$P_{xy} Q_x P_{xy} = Q_y \qquad (6)$$

to get

$$\begin{aligned} \langle \phi \mid Q^q_x \; Q'^{q'}_y \mid \phi \rangle &= \langle \phi \mid P_{xy} Q^q_y (P_{xy})^2 \; Q'^{q'}_x P_{xy} \mid \phi \rangle \\ &= \langle P_{xy}\phi \mid Q^q_y \; Q'^{q'}_x \mid P_{xy}\phi \rangle \; = \; \langle \phi \mid Q^q_y \; Q'^{q'}_x \mid \phi \rangle \qquad (7) \end{aligned}$$

where the last equation uses (PI). The proof of (3) is similar: the denominators of its two sides are equal by (1), and for the numerators, we use (6) together with the analogous result, $P_{xy} Q_z = Q_z P_{xy}$.

In words: For any assembly of indistinguishable quantum particles (whether fermions or bosons), and any state of the assembly (appropriately (anti)-symmetrized), and any two particles in the assembly: the two particles' probabilities for all single-particle quantities are equal; and so are appropriate corresponding two-particle conditional probabilities, including probabilities using conditions about a third particle.

This can also be expressed in terms of the reduced density matrices (reduced states) of the particles. namely: these states are equal.

Thus fermions not only 'can be in the same state', just as much as bosons can be (*pace* the usual slogan form of the exclusion principle). Also, a pair of indistinguishable particles of either type *must* be in the same state.

4. Discerning objects by symmetric and irreflexive relations So should we infer that fermions as well as bosons pandemically violate the principle of the identity of indiscernibles? No: for the Hilbert-Bernays account provides ways that two objects can be discerned, which are not captured by these orthodox quantum probabilities—and yet which *are* instantiated by quantum theory.

For two objects can be discerned, even while sharing all their monadic properties and their relations to all other objects, by a symmetric irreflexive relation R between them.

Example from metaphysics (or classical physics): two leaden spheres, a mile apart in an otherwise empty universe, sharing all their monadic properties, are discerned by the symmetric irreflexive relation, $R =$ '... is a mile away from ... '.

We can use non-commuting quantities to build such a relation. In words: '... is a system whose position is compatible with the momentum of ...'.

Recall that for a single particle x, $[Q, P] = i\hbar$. So $[Q_x, P_x] = [Q_y, P_y] = i\hbar$, while $[Q_x, P_y] = 0$. Then the relation R defined by

$$R(x, y) \text{ iff } [Q_x, P_y] = 0. \tag{8}$$

is symmetric and irreflexive, and discerns the particles.

Should we worry that Q_x, P_y are not symmetrized, i.e. violate (IP)? (If a relation R is defined in terms of symmetrized operators on $\mathcal{H}$, then R is symmetric; but the converse fails.) Then we proceed as follows, in terms of probabilities and thereby states.

Introduce for two particles (so here suppressing x, y), the "average" and "half-difference" operators:

$$\bar{Q} := \frac{1}{2}(Q \otimes \mathbb{I} + \mathbb{I} \otimes Q) \ ; \ \Delta_Q := \frac{1}{2}(Q \otimes \mathbb{I} - \mathbb{I} \otimes Q) \tag{9}$$

Calculating $\overline{Q^2}$ and $\bar{Q}^2$, we get that the symmetrized operator

$$\Delta_Q{}^2 \equiv \frac{1}{4}(Q \otimes \mathbb{I} - \mathbb{I} \otimes Q)^2 = \frac{1}{2}(\overline{Q^2} - Q \otimes Q) = \overline{Q^2} - \bar{Q}^2. \tag{10}$$

So $\Delta_Q{}^2$ is a quantity that deserves the name 'variance of Q'. We now define a relation in terms of this quantity's expectation value. So the relation depends on the state ϕ, as well as Q; and is parameterized by $t \in \mathbb{R}$.

Reintroducing x, y, we will now write $\Delta_Q{}^{x,y}$, and $\Delta_Q{}^{x,x} := \frac{1}{2}(Q \otimes \mathbb{I} - Q \otimes \mathbb{I}) \equiv 0 \equiv \Delta_Q{}^{y,y}$. Thus we define

$$R(x, y; Q, \phi, t) \;\text{ iff}: \; \langle \, (\Delta_Q{}^{x,y})^2 \, \rangle_\phi \; = \; t \; . \tag{11}$$

Now choose ϕ such that $(\Delta_Q{}^{x,y})^2$'s actual expectation value in ϕ, call it t_0, is non-zero. Then we have: $R(x, y; Q, \phi, t_0)$, while not-$R(x, x; Q, \phi, t_0)$, and not-$R(y, y; Q, \phi, t_0)$.

5. Spacetime points as objects ('substantivalism')? Einstein's hole argument ... The hole argument, originally devised by Einstein in late 1913, as an argument *against* general covariance, forms a crossroads of the history, philosophy and indeed physics of general relativity.

After Einstein had in 1913 devised (with Grossmann) a theory (nowadays called 'the Entwurf Theory') satisfying all his desiderata for a relativistic theory of gravitation except general covariance, he came to doubt general covariance. He soon confirmed his doubts by inventing this argument, to the effect that *any* generally covariant theory would be radically indeterministic, since specifying all matter fields, and the metric field, throughout all of spacetime except a small region (the "hole") could not determine the fields in the region. For general covariance means that, given one model i.e. solution of the theory, a "re-painting" of the matter and metric on the region's set of points, by pushing forward with a diffeomorphism of the region, produces another model of the theory. And this model disagrees with the given model about which point has a given value of the matter and metric fields. Late in 1915, after Einstein had found the field equations of general relativity, which are of course generally covariant, he re-assessed the argument as showing only that we should not think of spacetime points as objects independent of their field values. For this history, and later debate about the meaning of general covariance, cf. [1].

This episode's relevance to the interpretation of general relativity was advocated in papers by Stachel, Earman and Norton in the 1980s; since when, it has been centre-stage in philosophical debates about determinism, the comparison of spacetime points in different models ("which point is which?"), and 'absolute' vs. 'relational' conceptions of spacetime [2].

In physics, the main legacy of this episode has been, broadly speaking, to recognize 'gauge-freedom' about 'which spacetime point is which' in formulations of general relativity. This applies in particular to the initial-value problem: in short, one takes the future time-development from given initial data to be unique, only up to a diffeomorphism. Cf. the expositions in [3]. But there are subtleties—*which the Essay description gives details about!*

[1] J. Norton (1984) How Einstein found his Field Equations: 1912-1915, *Historical Studies in the Physical Sciences*, 14, 253-316; reprinted in Don Howard and John Stachel (eds.) *Einstein and the History of General Relativity*: Einstein Studies, Vol. 1 Boston: Birkhauser, 1989, pp. 101-159. Norton, General covariance and foundations of general relativity: eight decades of dispute, *Reports on Progress in Physics*, **56** (1993), 791-858.

[2] For an introduction, cf. J. Norton (2019), The hole argument, in *Stanford Encyclopedia of Philosophy* (internet), https://plato.stanford.edu/entries/spacetime-holearg/. For a detailed exposition of both the history and the philosophy, cf. J. Stachel (2014), The hole argument and some physical and philosophical implications. *Living Reviews in Relativity* 17, 1-66. https://link.springer.com/article/10.12942/lrr-2014-1. The article which began the current discussion in philosophy is: J. Earman and J. Norton, (1987) What Price Spacetime Substantivalism *British Journal for the Philosophy of Science*, **38**, 515-525. One early response, discussing different definitions of determinism, is: J. Butterfield, The Hole Truth, *British Journal for the Philosophy of Science*, **40**, 1-28. A brief recent survey is: O. Pooley, The hole argument, arXiv:2009.09982; of which a more ample successor is: O. Pooley and J. Read, The mathematics and metaphysics of the hole argument, arXiv:2110.04036.

[3] R. Wald, *General Relativity*, Chap.10, University of Chicago Press, 1984; S. Hawking and G. Ellis, *The Large-scale Structure of Spacetime*, Chapter 7, Cambridge University Press; Klaas Landsman, *Foundations of General Relativity*, Chapter 8, Radboud University Press, freely downloadable at: www.radbouduniversitypress.nl; E. Gourgoulhon, 3+1 Formalism and Bases of Numerical Relativity, grqc/0703035, Section 8.

6. 'Particle' and 'field' in QFT NB: The following is adapted from Section 4.1 of the main handout for this course, *Quantization of linear dynamical systems I*—mostly by deleting the discussion of *one particle structure*: which, although central to quantization theory, is confusingly named ...

(1): First, we note that 'particle' vs. 'wave' was a crucial contrast from 1900 to about 1930, i.e. for quantum theory *before QFT*. Think of Bohr's complementarity interpretation (Lake Como, 1927) ... which transmuted in to the Copenhagen interpretation in the 1950s... Recall in particular the "time-honoured" (i.e. historically significant) "equivalence" of matrix mechanics with wave mechanics": that is, the Hilbert space isomorphism from $l^2(\mathbb{N})$ ('discrete', 'particle-ish') to $L^2()$ ("fieldy-wavy"). NB: the equivalence is conceptually, and historically subtler than the textbooks suggest. Cf. F. Muller, The Equivalence Myth, Parts I and II, *Studies in History and Philosophy of Modern Physics* 1995.

(2): Second, a warning remark about the words 'first quantization' and 'second quantization' (the latter usually meaning the Fock space construction). 'quantization'. Beware: Some books say: a): first quantization is about particles behaving like waves; (b) Second quantization is about waves behaving like particles (e.g. Blundell and Lancaster *Quantum field theory for the Gifted Amateur*, p. 20).

We have no quarrel with (a). After all: think of deBroglie (1924), Schroedinger's December 1925 fundamental idea, to replace the classical Hamiltonian state space T^*Q for a finite classical system with configuration space Q, by $L^2(Q)$; and at the experimental level: electron diffraction, and the two-slit experiment.

But (b) is wrong, or at least misleading. For it suggests we need an infinite classical system ("waves"), or a Fock space construction, in order that a quantum theory (or a quantization) yield us particle like features such as: (i) an operator whose spectrum is the non-negative integers, i.e. what we can call a number operator $\hat{N}$; (ii) a position operator, so that we can talk about localization.

But we don't! We have (ii) in elementary wave mechanics (Schroedinger picture/position representation). And as to (i) any orthobasis of any denumerable-dimensional Hilbert space can of course have the non-negative integers attached to its elements as eigenvalues, and ladder operators defined with respect to that basis. (And if you demand also that these eigenvalues count energy, then again ... the simple harmonic oscillator is a finite system that delivers (i).)

(3): More positively, there is a unitary equivalence of:

(i) a particle (Fock-space- or number-operator-based) "way of thinking" of a quantum field theory; (in effect: the quantities in the *commutant of the number operator*); and:

(ii) a field or wave way of thinking; (in effect: the quantities in the *commutant of the field operator*).

This unitary equivalence is made precise and proven for free bosonic fields, that are obtained by complexifyig the solution space of a classical linear system, in Theorem 1.10 of Baez et al (1992: p. 49, and Section 1.8, 1.9). (Schweber (1962, Chapter 7d and 7e is a heuristic discussion of this equivalence.). Cf. Section 4.4 of the main handout: which defines properly what Baez et al call the particle and real wave *pictures* (or *representations*). They also discuss *coherent states*, calling them the *complex wave* picture/representation.

(4): In the rest of this Section, we give a general discussion of 'particle' and 'field'. There are two overall themes.

(A): The first (and longer) theme will to beware of a false dichotomy between particle and field, in quantum field theory. For the basic object (individual, "thing") in such a theory is the quantum system itself: which behaves in some regards (especially: in some states) like a particle or a collection of them, and in other regards or states like a field.

(B): The second theme will be that, after all, fields are primary in that the system is defined by an operator-valued field on space or spacetime, i.e. an assignment to each point of space or spacetime of a linear operator on a Hilbert space (whose vectors and density matrices then give states)—and this is a quantum analogue of, for example, the electric field assigning to each point of space or spacetime an electric field vector.

(A): *Beware of a false dichotomy*:— What, after all, do we mean by 'particle' and 'field'? Clearly, the concepts get changed as we pass from classical physics, to elementary ('first quantized') quantum theory, to quantum field theory. So interpreting these theories, especially the last, is in part a matter of plotting those changes. And even the most cursory attempt to do that shows there are many different particle-like, and many different field-like, attributes that one can consider; as follows.

We might list, as attributes of classical particles that quantum particles lack in elementary quantum theory: a continuous spacetime trajectory, impenetrability. We can similarly list new attributes of quantum particles: the Fourier-transformation between position and momentum, quantum statistics. In the transition to quantum field theory, definiteness and conservation of particle number go; creation and annihilation come in.

Turning to fields, we might list, as attributes of classical fields that the wave-functions of elementary quantum theory lack: energy-momentum, being real. Yet a wave-function is like a classical field in that it represents the state ('configuration') of the system concerned, albeit in a mathematically particular representation, namely the position representation; instantaneously or throughout time, depending on one's definitions of wave-function and field. (Here, and in what follows, we use 'representation' to mean an orthonormal basis of state-vectors.) In quantum field theory, this attribute goes: the state of the system is *not* represented by the quantum field, i.e. by the assignment of operators to each (spatial or spacetime) point. The state is represented, as always in quantum theories, by a state-vector.

With this great variety of attributes (and no doubt more) to be considered, and related to one another, there is certainly plenty to do in plotting the changes in the two notions of particle and field. But this variety should also make one wary of loose talk about a conflict between particle and field interpretations of quantum field theory. There is probably no essence in each of these two notions, one essence contradicting the other, allowing one to then try and judge which has the upper hand in interpreting quantum field theory.

This point is supported by a long-standing (and apparently 'essentialist') proposal for how to understand 'particle' within quantum field theory: namely that a quantity is 'particle-like' iff it commutes with all particle number operators (is in the commutants of all the number operators). This proposal is certainly attractive: for instance, it makes position, momentum and spin particle-like. It also suggests that we call a representation that diagonalizes these number operators 'a particle representation'. Such a representation will then be invaluable for describing phenomena in which particle-like quantities are important. Typically, the phenomenon will involve simultaneous eigenstates of number and some particle-like quantity, and we then choose the representation by simultaneously diagonalizing number and that quantity. The paradigm case is of course scattering theory's use of eigenstates of number and momentum.

But of course, there are other (mutually non-commuting) representations, or more generally sets of states, that are invaluable for describing different phenomena, where other quantities, not number, are important (in particular: definite in value). There is the field-operator itself; and related quantities like phase and associated ideas like coherent states; both of which are invaluable for the quantum description of the electromagnetic field; (cf. e.g. Loudon (1973, Chapter 7)). Besides, since as we noted there are various connotations of 'field', there need be no unique best choice amongst these non-commuting representations (sets of states), for a corresponding definition of 'field-like quantity' and 'field representation'—.

Whether or not there is such a choice, the important point is this: it is wrong to ask which of particle and field has the upper hand in interpreting quantum field theory. For the individual described by the theory is the underlying quantum system, with its Hilbert space of states. 'Particle' and 'field' are now both matters of a representation, of a selected set of states. The glory of quantum field theory is that it allows and uses all these representations, variously appropriate for describing particle-like or field-like phenomena—where the 'like' signals due allowance for ambiguities and changes in the concepts, as sketchily plotted above. Surely this is what Dirac (taciturn as always) meant by his brief remark that his quantization of the electromagnetic field gave 'a complete harmony between the wave and light-quantum descriptions'; (Dirac 'The Quantum Theory of the Emission and Absorption of Radiation', *Proceedings of the Royal Society of London* **A** 114 (1927), 243-265, see p. 245.)

(B): *... And yet...*:— Fields are primary in that the system has infinitely many degrees of freedom, and is defined by an operator-valued field on space or spacetime. More precisely: its algebra of quantities is generated by an assignment to each point (or allowing for smearing: to each small region) of space or spacetime, of a linear operator, or a set of linear operators, on a Hilbert space: whose vectors and density matrices then give states of the system. (We will see this in more detail for algebraic quantum field theory.) So this is a quantum analogue of, for example, the classical electric field assigning to each point of space or spacetime an electric field vector—though with the difference, mentioned in (A) above, that classically this assignment is a *state* of the field concerned (or part of the state, since time derivatives may also be needed to get sufficient initial data); while here the assignment presents only the system, not a specific state..

There is a further point here, that is well made by Wald (1994, p. 2-6)—and we believe that he speaks for the majority of physicists. Wald notes that a curved spacetime lacks the symmetries of a flat spacetime (Galilean or Poincaré) that underpin the construction of a representation of the canonical commutation relations, and its uniqueness up to unitary equivalence (the Stone-von Neumann theorem)—so that we must consider various representations. The benefit of the algebraic approach (which he advocates) is that it allows one to consider all these representations on an equal footing. This reinforces the idea that fields that are primary, and that particle notions are derived, and often approximate or phenomenological. (Of course, this is not to deny the interest of foundational studies of eg rival schemes for localization within quantum field theory: cf. for example, halvorson2001nw, which uses one particle structures and the Fock spaces built from them.)