2

Asymptotic Reasoning

This chapter will introduce, via the consideration of several simple examples, the nature and importance of asymptotic reasoning. It is necessary that we also discuss an important feature of many patterns or regularities that we may wish to understand. This is their universality. "Universality," as I've noted, is the technical term for an everyday feature of the world—namely, that in certain circumstances distinct types of systems exhibit similar behaviors. (This can be as simple as the fact that pendulums of very different microstructural constitutions all have periods proportional to the square root of their length. See section 2.2.) We will begin to see why asymptotic reasoning is crucial to understanding how universality can arise. In addition, this chapter will begin to address the importance of asymptotics for understanding relations between theories, as well as for understanding the possibility of emergent properties. Later chapters will address all of these roles and features of asymptotic reasoning in more detail.

2.1 The Euler Strut

Let us suppose that we are confronted with the following physical phenomenon. A stiff ribbon of steel—a strut—is securely mounted on the floor in front of us. Someone begins to load this strut symmetrically. At some point, after a sufficient amount of weight has been added, the strut buckles to the left. See figure 2.1. How are we to understand and explain what we have just witnessed?

Here is an outline of one response. At some point in the weighting process (likely just prior to the collapse), the strut reached a state of unstable equilibrium called the "Euler critical point." This is analogous to the state of a pencil balancing on its sharpened tip. In this latter case, we can imagine a hypothetical situation in which there is nothing to interfere with the pencil—no breeze in the room, say. Then the pencil would presumably remain in its balanced state forever. Of course, in the actual world we know that it is very difficult to maintain such a balancing act for any appreciable length of time. Similarly,

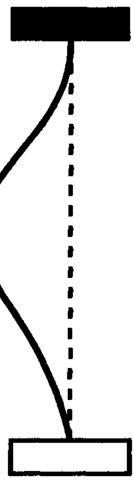

Figure 2.1: Buckling strut

molecular collisions will "cause" the strut to buckle either to the left or to the right. Either of these two buckled states is more stable than the critical state in that the addition of more weight will only cause it to sag further on the same side to which it has already collapsed.

So, in order to explain why the strut collapsed to the left, we need to give a complete causal account that (1) characterizes the details of the microstructural makeup of the particular strut, (2) refers to the fact that the strut had been weighted to the critical point, and (3) characterizes the details of the chain of molecular collisions leading up to the one water vapor molecule, the culprit, that hits the strut on its right side. If we were actually able to provide all these details, or at least some relevant portion of them, wouldn't we have an explanation of what we observed? Wouldn't we understand the phenomenon we have witnessed?

Both common sense and at least one prominent view of the nature of explanation and understanding would have it that we would now understand what we have seen. By providing this detailed causal account, we will have shown how the particular occurrence came about. We will have displayed the mechanisms which underlie the phenomenon of interest. On this view, the world is generally opaque. Providing accounts like this, however, open up "the black boxes of nature to reveal their inner workings" (Salmon, 1989, p. 182). We can call this view a causal-mechanical account.

On Peter Railton's version of the causal-mechanical account, the detailed description of the mechanisms that provides our explanation is referred to as an "ideal explanatory text."

> [A]n ideal text for the explanation of the outcome of a causal process would look something like this: an inter-connected series of

> law-based accounts of all the nodes and links in the causal network culminating in the explanandum, complete with a fully detailed description of the causal mechanisms involved and theoretical derivations of all of the covering laws involved.... It would be the whole story concerning why the explanandum occurred, relative to a correct theory of the lawful dependencies of the world. (Railton, 1981, p. 247)

For the strut, as suggested, this text will refer to its instability at the critical point, to the fact that it is made of steel with such and such atomic and molecular structure, and to the details of the collision processes among the "air molecules" leading up to the buckling.

But how satisfying, actually, is this explanation? Does it really tell us the whole story about the buckling of the strut? For instance, one part of the "whole story" is how this particular account will bear on our understanding of the buckling of an "identical" strut mounted next to the first and which buckled to the right after similar loading. Was what we just witnessed a fluke, or is the phenomenon repeatable? While we cannot experiment again with the very same strut—it buckled—we still might like to know whether similar struts behave in the same way. Going a bit further, we can ask whether our original causal-mechanical story sheds any light on similar buckling behavior in a strut made out of a different substance, say, aluminum? I think that the story we have told has virtually no bearing whatsoever on these other cases. Let me explain.

Let's consider the case of a virtually identical strut mounted immediately next to the first. What explains why it buckled to the right after having been loaded just like the first one? On the view we are considering, we need to provide an ideal explanatory text, which, once again, will involve a detailed account of the microstructural make-up of *this* strut, reference to the fact that it has been loaded to its critical point, and, finally, a complete causal story of all the molecular collisions leading up to the striking on the left side by a particular dust particle. Most of these details will be completely different than in the first case. Even though both struts are made of steel, we can be sure that there will be differences in the microstructures of the two struts—details that may very well be causally relevant to their bucklings. For instance, the location of small defects or fractures in the struts will most likely be different. Clearly, the collision histories of the various "air molecules" are completely distinct in the two cases as well. After all, they involve different particles. The two explanatory texts, therefore, are by and large completely different. Had we been given the first, it would have no bearing on our explanation of the buckling of the second strut.

In the case of an aluminum strut, the explanatory texts are even more disjoint. For instance, the buckling load will be different since the struts are made of different materials. Why should our explanation of the behavior of a steel strut bear in any way upon our understanding of the behavior of one composed of aluminum?

At this point it seems reasonable to object: "Clearly these struts exhibit similar behavior. In fact, one can characterize this behavior by appeal to Euler's formula:[1]

$$P_c = \pi^2 \frac{EI}{L^2}.$$

How can you say the one account has nothing to do with the other? Part of understanding how the behavior of one strut can bear on the behavior of another is the recognition that Euler's formula applies to both." (Here P_c is the critical buckling load for the strut. The formula tells us that this load is a function of what the strut is made of as well as certain of its geometric properties—in particular, the ration I/L^2.)

I agree completely. However, the focus of the discussion has shifted in a natural way from the particular buckling of the steel strut in front of us to the understanding of buckling behavior of struts in general. These two foci are not entirely distinct. Nevertheless, nothing in the ideal explanatory text for a particular case can bear upon this question. "Microcausal" details might very well be required to determine a theoretical (as opposed to a measured phenomenological) value for Young's modulus E of the particular strut in front of us, but what, in all of these details, explains why what we are currently witnessing is a phenomenon to which Euler's formula applies? The causal-mechanical theorist will no doubt say that all of the microcausal details about this strut will yield an understanding of why *in this particular case* the Euler formula is applicable: These details will tell us that E is what it is, and when all the evidence is in, we will simply see that P is proportional to I/L^2.

But, so what? Do we *understand* the phenomenon of strut buckling once we have been given all of these details? Consider the following passage from a discussion of explanation and understanding of critical phenomena. (The technical details do not matter here. It is just important to get the drift of the main complaint.)

> The traditional approach of theoreticians, going back to the foundation of quantum mechanics, is to run to Schrödinger's equation when confronted by a problem in atomic, molecular, or solid state physics! One establishes the Hamiltonian, makes some (hopefully) sensible approximations and then proceeds to attempt to solve for the energy levels, eigenstates and so on.... The modern attitude is, rather, that the task of the theorist is to *understand* what is going on and to elucidate which are the crucial features of the problem. For instance, if it is asserted that the exponent α depends on the dimensionality, d, and on the symmetry number, n, but on no other factors, then the theorist's job is to explain *why* this is so and subject to what provisos. If one had a large enough computer to solve Schrödinger's equation and the answers came out that way, one would still have *no understanding* of why this was the case! (Fisher, 1983, pp. 46–47)

[1] E is Young's modulus characteristic of the material. I is the second moment of the strut's cross-sectional area. L is the length of the strut.

If the explanandum is the fact that struts buckle at loads given by Euler's formula, then this passage suggests, rightly I believe, that our causal-mechanical account fails completely to provide the understanding we seek. All of those details that may be relevant to the behavior of the particular strut don't serve to answer the question of why loaded struts in general behave the way that they do. Actually, what does the explaining is a systematic method for abstracting from these very details.

The point of this brief example and discussion is to motivate the idea that sometimes (actually, very often, as I will argue) *science requires methods that eliminate both detail and, in some sense, precision.* For reasons that will become clear, I call these methods "asymptotic methods" and the type(s) of reasoning they involve "asymptotic reasoning."

2.2 Universality

The discussion of Euler struts in the context of the causal-mechanical view about explanation leads us to worry about how similar behaviors can arise in systems that are composed of different materials. For instance, we have just seen that it is reasonable to ask why Euler's formula describes the buckling load of struts made of steel as well as struts made of aluminum. In part this concern arises because we care whether such a phenomenon is repeatable. Often there are pragmatic reasons for why we care. For instance, in the case of buckling struts, we may care because we intend to use such struts or things like them in the construction of buildings. But despite (and maybe because of) such pragmatic concerns, it seems that science often concerns itself with discovering and explaining similar patterns of behavior.

As I noted in chapter 1, physicists have coined a term for this type of phenomenon: "universality." Most broadly, a claim of universality is an expression of behavioral similarity in diverse systems. In Michael Berry's words, saying that a property is a "universal feature" of a system is "the slightly pretentious way in which physicists denote identical behaviour in different systems. The most familiar example of universality from physics involves thermodynamics near critical points" (Berry, 1987, p. 185).

There are two general features characteristic of universal behavior or universality.

1. The details of the system (those details that would feature in a complete causal-mechanical explanation of the system's behavior) are largely irrelevant for describing the behavior of interest.

2. Many different systems with completely different "micro" details will exhibit the identical behavior.

The first feature is, arguably, responsible for the second. Arguments involving appeal to asymptotics in various forms enable us to see how this is, in fact, so.

It is clear that we can think of the Euler formula as expressing the existence of universality in buckling behavior. The formula has essentially two components.

First, there is the system—or material—specific value for Young's modulus. And second, there are the "formal relationships" expressed in the formula.

To see how ubiquitous the concept of universality really is, let us consider another simple example. We want to understand the behavior of pendulums. Particularly, we want to understand why pendulums with bobs of different colors and different masses, rods of different lengths, often composed of different materials, all have periods (for small oscillations) that are directly proportional to the square root of the length of the rod from which the bob is hanging. In other words, we would like to understand *why* the following relation generally holds for the periods, θ, of pendulums exhibiting small oscillations:[2]

$$\theta = 2\pi\sqrt{\frac{l}{g}}. \tag{2.1}$$

One usually obtains this equation by solving a differential equation for the pendulum system. The argument can be found near the beginning of just about every elementary text on classical mechanics. In one sense this is an entirely satisfactory account. We have a theory—a well-confirmed theory at that—which through its equations tells us that the relevant features for the behavior of pendulum systems are the gravitational acceleration and the length of the bob. In a moment, we will see how it is possible to derive this relationship without any appeal to the differential equations of motion. Before getting to this, however, it is worthwhile asking a further hypothetical question. This will help us understand better the notion of universality and give us a very broad conception of asymptotic reasoning.

Why are factors such as the color of the bob and (to a large extent) its microstructural makeup irrelevant for answering our why-question about the period of the pendulum? There are many features of the bob and rod that constitute a given pendulum that are clearly irrelevant for the behavior of interest. What allows us to set these details aside as "explanatory noise"? Suppose, hypothetically, that we did not have a theory that tells us what features are relevant for specifying the state of a pendulum system. Suppose, that is, that we were trying to *develop* such a theory to explain various observed empirical regularities "from scratch," so to speak. In such a pseudo-history would a question about the relevance of the color of the bob to its period have seemed so silly? The very *development of the theory* and the differential equation that describes the behavior of pendulums involved (the probably not so systematic) *bracketing as irrelevant many of the details and features that are characteristic of individual systems.*

Next, suppose we are in a state of knowledge where we believe or can make an educated guess that the period of the pendulum's swing depends only on the mass of the bob, the length of the pendulum, and the gravitational acceleration. In other words, we know something about classical mechanics—for instance, we have progressed beyond having to worry about color as a possible variable to be

[2]Here "l" denotes the length of the rod and "g" is the acceleration due to gravity.

considered. Can we, without solving any differential equations—that is, without appeal to the detailed theory—determine the functional relationship expressed in equation (2.1)? The answer is yes; and we proceed to do so by engaging in dimensional analysis (Barenblatt, 1996, pp. 2–5). In effect, the guess we have just made is sufficient to answer our why-question about the period of a pendulum.

We have a set of objects that represent units of length, mass, and time. These are standards everyone agrees upon—one gram, for instance, is 1/1000 of the mass of a special standard mass in a vault in the Bureau of Weights and Measures in Paris. Given these standards we will have a system of units for length, mass, and time (where L is the dimension of length, M is the dimension of mass, and T is the dimension of time). For our pendulum problem we have guessed that only the length l of the pendulum, its mass m, and the gravitational acceleration g should be relevant to its period θ. Note that l, m, and g are numbers holding for a particular choice of a system of units of measurement (e.g., centimeters, grams, and seconds). But in some sense that choice is arbitrary. Dimensional analysis exploits this fundamental fact—namely, that *the physics should be invariant across a change of fundamental units of measurement.*

The dimensions for the quantities involved in our problem are the following:[3]

$$[\theta] = T; \;\; [l] = L; \;\; [m] = M; \;\; [g] = LT^{-2}.$$

Now, consider the quantity l/g. If the unit of length is decreased by a factor of a, and the unit of time is decreased by a factor of b, then the numerical value of length in the numerator increases by a factor of a and the numerical value of acceleration in the denominator increases by a factor ab^{-2}. This implies that the value of the ratio l/g increases by a factor of b^2. Hence, the numerical value of $\sqrt{l/g}$ increases by a factor of b. Since the numerical value for the period, θ, would also increase by a factor of b under this scenario (decreasing the unit of time by a factor of b), we know the quantity

$$\Pi = \frac{\theta}{\sqrt{l/g}} \tag{2.2}$$

remains invariant under a change in the fundamental units. This quantity Π is dimensionless. In the jargon of dimensional analysis, we have "nondimensionalized" the problem.

In principle, Π depends (just like θ under our guess) upon the quantities l, m, and g: $\Pi = \Pi(l, m, g)$. If we decrease the unit of mass by some factor c, of course, the numerical value for mass will increase by that same factor c. But, in so doing, neither Π nor l nor g will change in value. In particular, $\Pi(l, m, g)$ is independent of m. What happens to Π if we decrease the unit of length by some factor a leaving the unit of time unchanged? While the value for length will increase by a factor of a, the quantity Π, as it is dimensionless, remains unchanged. Hence, $\Pi(l, m, g)$ is independent of l. Finally, what happens to Π

[3] "[$\bullet$]" $\stackrel{\text{def}}{=}$ "the dimension of $\bullet$".

if we decrease the unit of time by a factor of b while leaving the unit of length invariant. We have seen that this results in the numerical value for acceleration g increasing by a factor of b^{-2}. However, Π and l, and m remain unchanged.

This establishes the fact that $\Pi(l, m, g)$ is independent of all of its parameters. This is possible only if Π is a constant:

$$\Pi = \frac{\theta}{\sqrt{l/g}} = constant. \tag{2.3}$$

Hence,

$$\theta = constant \sqrt{\frac{l}{g}}, \tag{2.4}$$

which, apart from a constant, just is equation (2.1). The constant in (2.4) can be easily determined by a single measurement of the period of oscillation of a simple pendulum.

This is indeed a remarkable result. To quote Barenblatt: "[I]t would seem that we have succeeded in obtaining an answer to an interesting problem from nothing—or, more precisely, only from a list of the quantities on which the period of oscillation of the pendulum is expected to depend, and a comparison (analysis) of their dimensions" (Barenblatt, 1996, p. 5). No details whatsoever about the nature of individual pendulums, what they are made of, and so on, played any role in obtaining the solution.

This example is really a special (degenerate) case. In most problems, the equation for the dimensionless quantity of interest, Π, that results from the analysis will not equal a constant, but rather will be a function of some other dimensionless parameters:

$$\Pi = \Phi(\Pi_1, \ldots, \Pi_m).$$

In such cases, the analysis proceeds by trying to motivate the possibility that one or more of the Π_i's can be considered extremely small or extremely large. Then one can further reduce the problem by taking a limit so that the Π_i can be replaced by a constant: $\Pi_i(0) = C$ or $\Pi_i(\infty) = C$. This would yield an equation

$$\Pi = \Phi(\Pi_1, \ldots, \Pi_{i-1}, C, \Pi_{i+1}, \Pi_m), \tag{2.5}$$

which, one hopes, can be more easily solved. This recipe, however, involves a strong assumption—one that is most often false. This is the assumption that the limits $\Pi_i(0)$ or $\Pi_i(\infty)$ actually exist. As we will see, when they do not, dimensional analysis fails and interesting physics and mathematics often come into play.

The appeal to limiting cases, whether regular (where the limits $\Pi_i(0)$ or $\Pi_i(\infty)$ do exist) or singular (where those limits fail to exist), constitutes paradigm instances of asymptotic reasoning. We will see many examples of such reasoning later. The important point to note here has to do with the relationship between universality and asymptotic reasoning of this sort. It is often the case that the result of this kind of reasoning about a given problem is the discovery of some relationship like (2.3) or, more generally, like (2.5). In other words, asymptotic analysis often leads to equations describing universal features of systems. This happens because *these methods systematically eliminate irrelevant details about individual systems.*

Before leaving this section on universality, let me try to forestall a particular misunderstanding of what universality is supposed to be. As the discussion of the "pseudo history" of the pendulum theory is meant to show, it is not only a feature of highly technical physical phenomena. Universality as expressed in 1 and 2 holds of everyday phenomena in science. While many everyday patterns exhibit universality, they are not, therefore, mundane. The simple observable fact that systems at different temperatures tend toward a common temperature when allowed to interact with each other is an everyday occurrence. As an instance, just think of a glass of ice water coming to room temperature. Surely this regularity is universal—whether we consider ice or a rock at a cooler temperature interacting with the warmer room, the same pattern is observed. One should not be misled by the everyday nature of this pattern into thinking that the explanation for the pattern is at all trivial. Deep results in statistical mechanics involving asymptotics are necessary to explain this phenomenon. (This particular explanation will be discussed in section 8.3.) I think that despite the everyday occurrences of universal behavior, philosophers of science have not, by and large, understood how such patterns and regularities are to be explained. I will have much more to say about this type of explanation in chapter 4 and elsewhere throughout the book.

Let's turn now to a brief discussion of what may seem to be a completely different topic—theoretical reduction. As we will see, however, there are intimate connections between questions about the explanation of universality and the various ways different theories may be related to one another.

2.3 Intertheoretic Relations

Philosophers of science have always been concerned with how theories of one "domain" fit together with theories of some other. Paradigm examples from physics are the relations obtaining between classical thermodynamics and statistical mechanics; between the Newtonian physics of space and time and the "theory" of relativity; between classical mechanics and quantum mechanics; and between the ray theory of light and the wave theory. Most philosophical discussions of these interrelations have been framed in the context of questions about reduction. Do (and if so how) the former members of these pairs reduce to the latter members? "Reduction," here, is typically understood as some variant of the following prescription:

> A theory T' reduces to a theory T if the *laws* of T' are derivable (in some sense) from those of T.

This conception of reduction may require the identification (or nomic correlation) of properties in the reduced theory (T') with those in the reducing (T). With this requirement may come all sorts of difficulties, both conceptual and technical in nature. Much also will depend upon what sort of derivability is required and how strict *its* requirements are. These are all details that I think we can safely avoid at this point in the discussion. Most philosophical accounts of reduction along these lines also require that the reducing theory *explain* the reduced theory—or at least explain why it works as well as it does in its domain of applicability.

For example, the special theory of relativity is supposed to reduce Newtonian space and time. This reduction is accomplished by "deriving" Newtonian laws where the derivation involves some kind of limiting procedure. In particular, one shows that the Newtonian theory "results" when velocities are slow compared with the speed of light. In so doing the reduction also is supposed to explain the (approximate) truth of the Newtonian conception of space and time. It explains why, for instance, we can think of space and time as divided unambiguously into spaces at distinct times, even though according to special relativity the conception of absolute simultaneity required for this conception does not exist. Clearly questions about the relationships between properties and concepts in the two theories will immediately come to the fore.

Physicists, typically, use the term "reduction" in a different way than do philosophers.[4] For each of the theory pairs mentioned in the earlier paragraph, the physicist would speak of the second member of the pair reducing to the first. Reduced and reducing theories are inverted in comparison to the philosophers' way of understanding reduction. For example, physicists refer to the "fact" that quantum mechanics reduces to classical mechanics in some kind of correspondence limit—say where we let Planck's constant $\hbar \rightarrow 0$.[5] Another example is the reduction of special relativity to Newtonian space and time in the limit $(v/c)^2 \rightarrow 0$.

In general this other sense of reduction has it that a "more refined," more encompassing (and typically more recent) theory, T_f, corresponds to a "coarser," less encompassing (and typically earlier) theory, T_c, as some fundamental parameter (call it ϵ) in the finer theory approaches a limiting value. Schematically, the physicists' sense of reduction can be represented as follows:

$$\lim_{\epsilon \rightarrow 0} T_f = T_c. \tag{2.6}$$

The equality in (2.6) can hold only if the limit is "regular." In that case, on my view, it is appropriate to call the limiting relation a "reduction." If the limit in (2.6) is singular, however, the schema fails and I think it is best to talk simply about intertheoretic relations. Let me give a brief explication of these notions.

If the solutions of the relevant formulas or equations of the theory T_f are such that for small values of ϵ they *smoothly* approach the solutions of the

[4]See (Nickles, 1973) for an important discussion of these different senses.

[5]There are questions about what this can really mean: How can a constant change its value? There are also questions about whether the claim of reduction is true, even once we have decided what to say about the varying "constant" problem.

corresponding formulas in T_c, then schema (2.6) will hold. For these cases we can say that the "limiting behavior" as $\epsilon \to 0$ equals the "behavior in the limit" where $\epsilon = 0$. On the other hand, if the behavior in the limit is of a *fundamentally different character* than the nearby solutions one obtains as $\epsilon \to 0$, then the schema will fail.

A nice example illustrating this distinction is the following: Consider the quadratic equation

$$x^2 + x - \epsilon 9 = 0.$$

Think of ϵ as a small expansion or perturbation parameter. The equation has two roots for any value of ϵ as $\epsilon \to 0$. In a well-defined sense, the solutions to this quadratic equation as $\epsilon \to 0$ smoothly approach the solutions to the "unperturbed" ($\epsilon = 0$) equation

$$x^2 + x = 0;$$

namely, $x = 0, -1$. On the other hand, the equation

$$\epsilon x^2 + x - 9 = 0$$

has two roots for any value of $\epsilon > 0$ but has for its "unperturbed" solution only one root; namely, $x = 9$. The equation suffers a reduction in order when $\epsilon = 0$. Thus, the character of the behavior in the limit $\epsilon = 0$ differs fundamentally from the character of its limiting behavior. Not all singular limits result from reductions in order of the equations, however. Nevertheless, these latter singular cases are much more prevalent than the former.

The distinction between regular and singular asymptotic relations is the same as that discussed in the last section between problems for which dimensional analysis works, and those for which it does not. The singular cases are generally much more interesting, both from a physical and a philosophical perspective, in that it is often the case that new physics emerges in the asymptotic regime in which the limiting value is being approached.

From this brief discussion, we can see that asymptotic reasoning plays a major role in our understanding of how various theories "fit" together to describe and explain the workings of the world. In fact, the study of asymptotic limits is part and parcel of intertheoretic relations. One can learn much more about the nature of various theories by studying these asymptotic limits than by investigating reductive relations according to standard philosophical models. This point of view will be defended in chapters 6 and 7.

2.4 Emergence

Questions about reduction and the (im)possibility of identifying or otherwise correlating properties in one theory with those in another are often related to questions about the possibility of emergent properties. In this section I will briefly characterize what I take to be a widely held account of the nature of emergence and indicate how I think it will need to be amended once one takes asymptotic limiting relations into account.

It is a presupposition of the "received" account of emergence that the world is organized somehow into levels. In particular, it is presupposed that entities at some one level have properties that "depend" in some sense on properties of the entities' constituent parts. Jaegwon Kim (1999, pp. 19–20) has expressed the "central doctrines of emergentism" in the following four main claims or tenets:

1. *Emergence of complex higher-level entities*: Systems with a higher-level of complexity emerge from the coming together of lower-level entities in new structural configurations.

2. *Emergence of higher-level properties*: All properties of higher-level entities arise out of the properties and relations that characterize their constituent parts. Some properties of these higher, complex systems are "emergent," and the rest merely "resultant."

3. *The unpredictability of emergent properties*: Emergent properties are not predictable from exhaustive information concerning their "basal conditions." In contrast, resultant properties are predictable from lower-level information.

4. *The unexplainability/irreducibility of emergent properties*: Emergent properties, unlike those that are merely resultant, are neither explainable nor reducible in terms of their basal conditions.

Kim also notes a fifth tenet having to do with what sort of causal role emergent properties can play in the world. Most emergentists hold that they must (to be genuinely emergent) play some novel causal role.

5. *The causal efficacy of the emergents*: Emergent properties have causal powers of their own—novel causal powers irreducible to the causal powers of their basal constituents (1999, p. 21).

It is evident from tenets 1–4 that the relation of the whole to its parts is a major component of the contemporary philosophical account of emergentism. Paul Teller (1992, p. 139), in fact, holds that this part/whole relationship is fundamental to the emergentist position: "I take the naked emergentist intuition to be that an emergent property of a whole somehow 'transcends' the properties of the parts." Paul Humphreys in "How Properties Emerge" (1997) discusses a "fusion" operation whereby property instances of components at one level "combine" to yield a property instance of a whole at a distinct higher level. While Humphreys speaks of fusion of property instances at lower levels, he rejects the idea that emergents supervene on property instances at the lower levels.

The part/whole aspects of the emergentist doctrine are clearly related to the conception of the world as dividing into distinct levels. I have no quarrel with the claim that the world divides into levels, though I think most philosophers are too simplistic in their characterization of the hierarchy.[6] However, I do disagree with the view that the part/whole aspects stressed in tenets 1 and 2 are essential for the characterization of all types of emergence and emergent properties. There are many cases of what I take to be genuine emergence for which one would be hard-pressed to find part/whole relationships playing any role whatsoever. I also think that many of these cases of emergence do not involve different levels of organization. It will come as no surprise, perhaps, that these examples arise when one considers asymptotic limits between different theories.

The third and fourth tenets of emergentism refer to the unpredictability and the irreducibility/unexplainability of genuinely emergent properties. Much of the contemporary literature is devoted to explicating these features of emergentism.[7] By considering various examples of properties or structures that emerge in asymptotic limiting situations, we will see in chapter 8 that these tenets of emergentism also require emendation. In the course of this discussion we will also come to see that the close connections between reduction and explanation must be severed. In many instances, it is possible to explain the presence of some emergent property or structure in terms of the base or underlying theory; yet the property or structure remains irreducible.

Furthermore, attention to asymptotic limits will reveal that there are borderlands between various theories in which these structures and properties play *essential explanatory roles.* Recognition of this fact may take us some distance toward interpreting (or re-interpreting) the fifth tenet of emergentism noted by Kim. This is the claim that emergent properties must possess novel causal powers—powers that in some sense are not reducible to the causal powers of their basal constituents. Instead of speaking of novel causal efficacy, I think it makes more sense to talk of playing novel explanatory roles. Reference to these emergent structures is essential for understanding the phenomenon of interest; and, furthermore, no explanation can be provided by appealing only to properties of the "basal constituents," if there even are such things in these cases.

One example I will discuss in detail is that of the rainbow. There are certain structural features of this everyday phenomenon that I believe must be treated as emergent. Such features are not reducible to the wave theory of light. The full explanation of what we observe in the rainbow *cannot* be given without reference to structures that exist only in an asymptotic domain between the wave and ray theories of light in which the wavelength of the light $\lambda \to 0$. This is a singular limiting domain, and only attention to the details of this asymptotic domain will allow for a proper understanding of the emergent structures.

Furthermore, we will see that the emergent structures of interest—those that dominate the observable phenomena—typically satisfy the requirements of universality. In other words, the emergent structures are by and large detail-

[6] A notable exception is William Wimsatt. See his article Wimsatt (1994) for an extended discussion of the idea of levels of organization.

[7] Kim (1999, pp. 2–3) argues, persuasively I think, that so-called nonreductive physicalism discussed in the philosophy of mind/psychology literature is the modern brand of emergentism. This literature focuses on questions of reduction and explanation.

independent and are such that many distinct systems—distinct in terms of their fundamental details—will have the same emergent features.

2.5 Conclusion

This chapter has suggested that our philosophical views about explanation, reduction, and emergence would better represent what actually takes place in the sciences were they to recognize the importance of asymptotic reasoning. The explanation of universal phenomena, as much of the rest of the book will try to argue, requires principled means for the elimination of irrelevant details. Asymptotic methods developed by physicists and mathematicans provide just such means. Furthermore, much of interest concerning intertheoretic relations can be understood only by looking at the asymptotic limiting domains between theory pairs. This is something upon which virtually no philosophical account of theory reduction focuses. Chapters 6 and 7 discuss in detail the importance of this view of intertheoretic relations. Finally, chapter 8 will expand on the claims made here to the effect that genuinely emergent properties can be found in the singular asymptotic domains that exist between certain pairs of theories.

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