39
PROBABILITY

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Historical sketch

The origin of the notion of *probability*, taken in the quantitative sense that is nowadays attached to it, is usually traced back to the decade around 1660 and associated with the work of Blaise Pascal and Pierre Fermat, followed by that of Christiaan Huygens and many others.

Since its beginnings, the notion of probability has been characterized by a peculiar duality of meaning: its *statistical* meaning concerning the stochastic laws of chance processes; and its *epistemological* meaning relating to the degree of belief that we, as agents, entertain in propositions describing uncertain events. Such a duality lies at the root of the philosophical problem of the interpretation of probability, and has nurtured various schools animated by the conviction that a specific sense of "probability" should be privileged and made the essence of its definition. After a long period in which the "doctrine of chance" and the "art of conjecture" had peacefully coexisted, this absolutist tendency became predominant around the middle of the nineteenth century and gave rise to the different interpretations of probability that will be described in the following sections.

By the turn of the eighteenth century, probability had progressed enormously, having progressively widened its scope of application. Great impulse to its development came from the application of the notion of the *arithmetic mean* first to demographic data, then to fields like medical practice and legal decisions, and finally to the physical and biological sciences.

A pivotal role in the history of probability was played by the Bernoulli family, including Jakob, who started the analysis of *direct probability*, that is, the probability to be assigned to a sample taken from a population whose *law* is known, and proved the result usually called the "weak law of large numbers." The theorem holds for binary processes, namely processes that admit of two outcomes – such as "heads" or "tails" and the "presence" or "absence" of a certain property – and says that if p is the probability of obtaining a certain outcome in a repeatable experiment, and m the number of successes obtained in n repetitions of the same experiment, the probability that the value of m/n falls within any chosen interval $p \pm \varepsilon$ increases for larger and larger values of n, and tends to 1 as n tends to infinity. Bernoulli's result is based on the concept of *stochastic independence*, which receives an unambiguous definition for the first time. Bernoulli's work also sheds light on the relationship between probability and frequency, by keeping separate the probability and the frequency with which the events of the considered dichotomy can theoretically occur in any given number n of experiments, and sets the probability distribution over possible frequencies: 0, 1, 2, …, n, usually called "binomial distribution." Bernoulli's work on direct probability was gradually generalized by other probabilists, including De Moivre, Laplace, and Poisson, to receive great impulse in the nineteenth and twentieth centuries, especially by Borel, Cantelli, and the Russian probabilists Chebyshev, Markov, Lyapunov, and Kolmogorov.

Other important members of the Bernoulli family were Nikolaus, who formulated the so-called "Saint Petersburg problem," and Daniel, who did seminal work on mathematical expectation and laid the foundations of the theory of errors, which reached its peak with the subsequent work of Gauss.

Special mention is due to Thomas Bayes, who proposed a method for assessing *inverse probability*, that is, the probability to be assigned to an hypothesis on the ground of available evidence. Whereas by direct probability one goes from the known probability of a population to the estimated frequency of its samples, by inverse probability one goes from known frequencies to estimated probabilities. Inverse probability is also called the "probability of causes," because it enables the estimation of the probabilities of the causes underlying an observed event. The method is based on the idea that the *final* or *posterior* probability $P(H|E)$ of a certain hypothesis (H), given a certain piece of evidence (E), is proportional to the product of the *initial* or *prior* probability $P(H)$ of the hypothesis calculated on the basis of background knowledge, and the so-called *likelihood* $P(E|H)$ of E given the considered hypothesis, namely on the assumption that the considered hypothesis holds. A general formulation of Bayes's rule, that takes into account a family of hypotheses $H_1 \ldots H_n$, is the following:

$$P(H_i|E) = [P(H_i) \times P(E|H_i)] / \Sigma_{i=1}^{n} [P(H_i) \times P(E|H_i)].$$

To illustrate this formula, let us take a factory that has 3 machines for the production of bolts, of which it produces 60,000 pieces daily. Of these, 10,000 are produced by machine A_1, 20,000 by machine A_2, and 30,000 by machine A_3. All three machines occasionally produce faulty pieces, F. On average, the rejection rates of the 3 machines are as follows: 4 percent in the case of A_1, 2 percent in the case of A_2, 4 percent in the case of A_3. Given a defective bolt taken from the rejects, we ask for the probability that it was produced by each of the three machines. In order to calculate such a probability by means of Bayes's rule, we start from prior probabilities, obtained in this case from the information concerning the production of the machines. They are as follows:

$$P(A_1) = 10{,}000/60{,}000 = 1/6$$
$$P(A_2) = 20{,}000/60{,}000 = 1/3$$
$$P(A_3) = 30{,}000/60{,}000 = 1/2.$$

The likelihoods are provided by information on the rejection rates:

$P(F \mid A_1) = 4/100$
$P(F \mid A_2) = 2/100$
$P(F \mid A_3) = 4/100.$

Posterior probabilities are calculated as follows:

$P(A_1 \mid F) = (1/6 \times 4/100) / [(1/6 \times 4/100) + (1/3 \times 2/100) + (1/2 \times 4/100)]$
$= 1/5 = 20\%$
$P(A_2 \mid F) = (1/3 \times 2/100) / [(1/6 \times 4/100) + (1/3 \times 2/100) + (1/2 \times 4/100)]$
$= 1/5 = 20\%$
$P(A_3 \mid F) = (1/2 \times 4/100) / [(1/6 \times 4/100) + (1/3 \times 2/100) + (1/2 \times 4/100)]$
$= 3/5 = 60\%.$

We therefore have a probability of 20 percent that a defective bolt taken at random was produced by machine A_1, a probability of 20 percent that it was produced by machine A_2 and a probability of 60 percent that it was produced by machine A_3. The obtained result shows that, although the machine A_2 works twice as well as A_1, it is equally probable that the defective piece originates from A_2 as from A_1, because the second machine produces twice as many pieces. Machine A_3, which supplies half of the total production, is nevertheless assigned probability 3/5 of having produced the defective piece because one of the two other machines works more reliably.

The crucial step in the application of Bayes's rule lies with fixing prior probabilities. This is a matter of debate. By allowing for the evaluation of hypotheses in a probabilistic fashion, Bayes's method spells out a canon of inductive reasoning. It was applied in the first place by Laplace, and later on came to be regarded as the cornerstone of statistical inference by the statisticians of the Bayesian School. The place of Bayes's inductive method within the whole of statistics is the subject of a major ongoing controversy.

The eighteenth century saw a tremendous growth in the application of probability to the moral and political sciences. Important work in this connection was done by Condorcet, the pioneer of the so-called "social mathematics," meant to produce a statistical description of society instrumental for a new political economy.

Between the nineteenth and twentieth centuries the study of statistical distributions progressed enormously thanks to the work of a number of authors, including Quetelet, Galton, Karl Pearson, Weldon, Gosset, Edgeworth, and others, who shaped modern statistics, by developing the analysis of correlation and regression, and the methodology for assessing statistical hypotheses against experimental data through the so-called "significance tests." Other branches of modern statistics were started by Fisher, who prompted the analysis of variance and covariance, and the *likelihood method* for comparing hypotheses on the basis of a given body of data. Also worth mentioning are Egon Pearson and Jerzy Neyman, who extended the methodology of tests to the comparison of two alternative hypotheses.

In the nineteenth century, probability gradually entered physical science, not only in connection with errors of measurement, but more penetratingly as a component of physical theory. Such developments started with the work of Robert Brown on the motion of particles suspended in fluid, which paved the way to the analysis of physical phenomena characterized by great complexity, leading to the kinetic theory of gases and thermodynamics, developed by Maxwell, Boltzmann, and Gibbs. Around 1905–6 von Smoluchowski and Einstein brought to completion the analysis of Brownian motion in probabilistic terms. More or less in the same years, the study of radiation led Einstein and other outstanding physicists, including Planck, Schrödinger, de Broglie, Dirac, Heisenberg, Born, Bohr, and others to formulate quantum mechanics, in which probability became an ingredient of the description of the basic components of matter.

In 1933 Kolmogorov spelled out his famous axiomatization, meant to shed light on the mathematical properties of probability, and to draw a distinction between probability's formal features and the meaning it receives in practical situations. Put simply, the formal properties of probability are the following: (1) for any event A, its probability is $\geqslant 0$; (2) if A is certain, its probability equals 1; (3) probabilities are additive, that is, if two events A and B cannot both occur, $P(A \text{ or } B) = P(A) + P(B)$. Kolmogorov's axiomatization met with a wide consensus and obtained a twofold result: for one thing, it gained an equitable position for probability among other mathematical disciplines; and by tracing a clear-cut boundary between the mathematical properties of probability and its interpretations it made room for the philosophy of probability as an autonomous field of enquiry.

The classical interpretation

The "classical" interpretation is usually construed as the interpretation of probability developed at the turn of the nineteenth century by the mathematician–physicist–astronomer Pierre Simon de Laplace. Called "the Newton of France" for his work on mechanics, Laplace made a substantial contribution to probability, both technically and philosophically. His philosophy of probability is rooted in the doctrine of determinism, according to which the universe is ruled by a *principle of sufficient reason* stating that all things are brought into existence by a cause. The human mind is incapable of grasping every detail of the connections of the causal network underlying phenomena, but one can conceive of a superior intelligence able to do so. Making use of the methods of mathematical analysis and aided by probability, man can approach the all-comprehensive view of such a superior intelligence. Being made necessary by the incompleteness of human knowledge, probability is an epistemic notion, having to do with our knowledge, rather than being inherent in phenomena.

Laplace defines probability as "the ratio of the number of favorable cases to that of all possible cases," according to the statement known as the "classical" definition. This is grounded on the assumption that all cases in question are equally possible, lacking information that would lead us to believe otherwise. The stress placed on the dependence of the judgment of equal possibility on there being no reason to believe otherwise inspired the term "principle of insufficient reason" – also known in the literature as the "principle of indifference," after a terminology coined by Keynes – to refer to Laplace's assumption. In other words, for the sake of determining probability values, equally possible cases are taken as equally probable. This assumption is made for ease of analysis and is not endowed with metaphysical meaning. Laplace insists on the need to make sure that some outcomes are not more likely to happen than others, before applying his method. Moreover, Laplace's epistemic interpretation protects his definition of probability from the charge of being circular: once probability is taken as epistemic, it stands on a different ground from the possibility of the occurrence of events.

Dealing with inverse probability, Laplace enunciates a principle which amounts to Bayes's rule. Under the assumption of equally likely causes, he derives from it the method of inference called in the literature "Laplace's rule." In the case of two alternatives – like *occurrence* and *non-occurrence* – this rule allows us to infer the probability of an event from the information that it has been observed to happen in a given number of cases. If m is the number of observed positive cases, and n that of negative cases, the probability that the next case to be observed is positive equals $(m + 1) / (m + n + 2)$. If no negative cases have been observed, the formula reduces to $(m + 1) / (m + 2)$. Laplace's method is based on the assumptions of the equiprobability of priors and the independence of trials, conditional on a given parameter – like the composition of an urn, or the ratio of the number of favorable cases to that of all possible cases. The authors who later worked on probabilistic inference in the tradition of Bayes and Laplace – including Johnson, Carnap, and de Finetti – eventually turned to the weaker assumption of exchangeability.

Laplace's theory of probability was very influential. However, while it can handle a wide array of important applications, it gives rise to problems, such as the impossibility, in many situations, of determining the set of *equally likely* cases. In such situations – think for instance of the probability of a biased coin falling on either side or the probability that a given individual will die within a year – instead of looking for possible cases, we count the frequency with which events take place in order to calculate probability. Furthermore, when applied to problems involving an infinite number of possible cases, the classical interpretation generates the so-called "Bertrand's paradox," after the French mathematician Joseph Bertrand.

The frequency interpretation

According to the frequency interpretation, probability is defined as the limit of the relative frequency of a given attribute, observed in the initial part of an indefinitely long sequence of repeatable events. In other words, given that the attribute A has been observed with frequency m/n in the initial part of sequence B, its probability equals $\lim_{n\to\infty} F^n (A,B) = m/n$. The frequency interpretation is empirical and objective: probability is a characteristic of phenomena that can be empirically analyzed by observing frequencies. Probability values are in general unknown, but can be approached by means of frequencies. The frequency interpretation is fully compatible with indeterminism.

Started by Robert Leslie Ellis and John Venn, frequentism reached its climax with Richard von Mises, member of the Berlin Society for Empirical Philosophy and later professor at Istanbul and Harvard. Central to von Mises's theory is the notion of a *collective*, referring to the sequence of observations of a mass phenomenon or a repetitive event. Collectives are indefinitely long and exhibit frequencies that tend to a limit. Their distinctive feature is randomness, operationally defined as "insensitivity to place selection." It obtains when the limiting values of the relative frequencies in a given collective are not affected by any of all the possible selections that can be performed on it. In addition, the limiting values of the relative frequencies, in the sub-sequences obtained by place selection, equal those of the original sequence. This randomness condition is also called the "principle of the impossibility of a gambling system" because it reflects the impossibility of devising a system leading to a certain win in any hypothetical game. The theory of probability is restated by von Mises in terms of collectives, by means of the operations of *selection*, *mixing*, *partition*, and *combination*. This conceptual machinery is meant to give probability an empirical and objective foundation. Because probability, according to this perspective, can refer only to *collectives*, it makes no sense to talk of the probability of single occurrences.

A slightly different version of frequentism was developed by Hans Reichenbach, another member of the Berlin Society for Empirical Philosophy and co-editor of *Erkenntnis* together with Rudolf Carnap, later professor at the University of California at Los Angeles. Reichenbach made an attempt to extend the frequency notion of probability to the single case. Any probability attribution is a *posit* by which we infer that the relative frequencies detected in the past will persist when sequences of observations are prolonged. A posit regarding a single occurrence of an event receives a *weight* from the probabilities attached to the reference class to which the event has been assigned. Such a reference class must obey a criterion of homogeneity guaranteeing that all the properties relevant to the event under study have been taken into account. This obviously gives rise to a problem of applicability, because one can never be absolutely sure that the reference class is homogeneous. Reichenbach distinguishes between *primitive knowledge*, where no previous knowledge of frequencies is available so that *blind* posits are made on the basis of the sole observed frequencies, and *advanced knowledge* where *appraised* posits are obtained by combining known probabilities by means of the laws of probability, particularly Bayes's rule. There emerges a view of knowledge as a self-correcting procedure grounded on posits. Reichenbach's theory includes a pragmatic justification of induction, appealing to the success of probability evaluations based on frequencies.

The propensity interpretation

Anticipated by Charles Sanders Peirce, the *propensity theory* was proposed in the 1950s by Karl Raimund Popper to solve the problem of single-case probabilities arising in quantum mechanics. Probability as propensity is a property of the experimental arrangement, apt to be reproduced over and over again to form a sequence. This is the kernel of the so-called "long-run propensity interpretation." Popper regards propensities as physically real and metaphysical (they are non-observable properties), and this gives the propensity theory a strongly objective character. In the 1980s Popper resumed the propensity theory to make it the focus of a wider program meant to account for all sorts of causal tendencies operating in the world. He then saw propensities as *weighted possibilities*, or expressions of the tendency of a given experimental set-up to realize itself upon repetition, emphasizing single experimental arrangements rather than sequences of generating conditions. In so doing, he laid down the so-called "single-case propensity interpretation." Of crucial importance in this connection is the distinction between *probability statements* expressing propensities, which are statements about frequencies in virtual sequences of experiments, and *statistical statements* expressing relative frequencies observed in actual sequences of experiments, which are used to test probability statements. Popper's propensity theory goes hand in hand with indeterminism.

After Popper's work the propensity theory of probability enjoyed a considerable popularity among philosophers of science. Some authors, such as Donald Gillies, embrace a long-run perspective, while others, including Hugh Mellor, Ronald Giere, and David Miller, prefer a single-case propensity approach. Propensity theory has been accused of giving rise to a variety of problems. For one thing, the propensity theory faces a reference-class problem broadly similar to that affecting frequentism. Moreover, Paul Humphreys has claimed that it is unable to interpret inverse probabilities, because it would be odd to talk of the propensity of a defective bolt to have been produced by a certain machine. The notion of propensity exhibits an asymmetry that goes in the opposite direction to that characterizing inverse probability. For this reason various authors, including Wesley Salmon, appealed to the notion of propensity to represent (probabilistic) causal tendencies, rather than probabilities.

Other authors value the notion of propensity as an ingredient of the description of chance phenomena, without committing themselves to a propensity interpretation of probability. Among them is Patrick Suppes, who holds the view that propensities do not express probabilities, but can play a useful role in the description of certain phenomena, conferring an objective meaning on the probabilities involved.

The logical interpretation

According to the logical interpretation, the theory of probability belongs to logic, and probability is a logical relation between propositions, more precisely one proposition describing a given body of evidence and another proposition stating a hypothesis. The logical interpretation of probability is a natural development of the idea that probability is an epistemic notion, pertaining to our knowledge of facts, rather than to facts themselves. With respect to Laplace's classical interpretation, this approach stresses the logical aspect of probability, which is meant to give it an intrinsic objectivity.

Anticipated by Leibniz, the logical interpretation was embraced by the Czech mathematician and logician Bernard Bolzano and developed by a number of British authors, including Augustus De Morgan, George Boole, William Stanley Jevons, and John Maynard Keynes, the latter best-known for his contribution to economic theory.

For all of these authors the logical character of probability goes hand in hand with its rational character. In other words, they aimed to develop a theory of the reasonableness of degrees of belief on logical grounds. Keynes adopted a moderate form of logicism, permeated by a deeply felt need not to lose sight of ordinary speech and practice. Keynes assigned an important role to intuition and individual judgment, and was suspicious of a purely formal treatment of probability and the adoption of mechanical rules for its evaluation. He also attributed an important role to analogy, and held that similarities and dissimilarities among events must be carefully considered before quantitative methods can be applied.

Another supporter of logicism was the Cambridge logician William Ernest Johnson, who is remembered for having introduced the property of exchangeability under the name of "permutation postulate." According to that property, probability is invariant with respect to permutation of individuals, to the effect that exchangeable probability functions assign probability in a way that depends on the number of experienced cases, irrespective of the order in which they have been observed.

Logicism counts also among its followers the Viennese philosophers Ludwig Wittgenstein and Friedrich Waismann. Wittgenstein held that probability is a logical relation between propositions, which can be established pretty much as a deductive relation, on the basis of the truth-values of propositions. An active member of the Vienna Circle, Waismann saw the logical notion of probability as a generalization of the concept of deductive entailment to the case in which the scope of one proposition (premise) partially overlaps with that of another (conclusion), instead of including it. The measure of such a logical relation is defined on the basis of the scope of propositions. He also pointed out that in addition to its logical aspect, probability has an empirical side, having to do with frequency.

Waismann's conception of probability directly influenced the work of Rudolf Carnap, one of the prominent representatives of philosophy of science in the twentieth century. Starting from the admission that there are two concepts of probability – probability$_1$, or degree of confirmation, and probability$_2$, or probability as frequency, Carnap set himself the task of developing the former notion as the object of *inductive logic*. Inductive logic is developed as an axiomatic system, formalized within a first-order predicate calculus with identity, which applies to measures of confirmation defined on the semantic content of statements. Since it allows for making the best estimates based on the given evidence, inductive logic can be seen as a rational basis for decisions. Unlike probability$_2$, which has only one value that is usually unknown, logical probability may be unknown only in the sense that the logico-mathematical procedure leading to it is not figured out. Logical probability is analytic and objective: in the light of the same evidence, there is only one rational (correct) probability assignment. Carnap devised a *continuum of inductive methods*, characterized as a blend of a purely logical component and a purely empirical element, among which the so-called "symmetric" functions, having the property of exchangeability, occupy a privileged position. Carnap's methods belong to the broader family of Bayesian methods. When addressing the problem of the justification of induction, Carnap appealed to *inductive intuition*, in an attempt to keep inductive logic totally within an *aprioristic* domain, while dispensing with the pragmatic criterion of successfulness.

A further version of logicism was developed by the geophysicist Harold Jeffreys, who built on it a probabilistic epistemology having a strongly constructivist flavor, which shares some features of the subjective approach.

The subjective interpretation

According to the subjective interpretation probability is the *degree of belief* entertained by a person, in a state of uncertainty regarding the occurrence of an event, on the basis of the information available. The notion of degree of belief is taken as a primitive notion, which has to be given an operative definition, specifying a way of measuring it. A first option in achieving this goal is the method of bets, endowed with a long-standing tradition dating back to the seventeenth century. Accordingly, one's degree of belief in the occurrence of an event can be expressed by means of the odds at which one would be ready to bet. For instance, a degree of belief of 1/6 in the proposition that an unbiased die will turn up 3 can be expressed by the willingness to bet at odds 1:5 – namely, pay 1 if the die does not turn up 3, and gain 5 if it does. The general idea is to value the probability of an event as equal to the price to be paid by a player to obtain a unitary gain in case the event occurs. This method gives rise to some problems, like that of the diminishing marginal utility of money, in view of which various alternative methods have been devised.

Anticipated by the British astronomer William Donkin and the French mathematician Émile Borel, the subjective approach was given a sound basis by the multifarious genius of Frank Plumpton Ramsey. He adopted a definition of degree of belief based on preferences determined on the basis of the expectation of an individual of obtaining certain goods, not necessarily of a monetary kind, and specified a set of axioms fixing a criterion of *coherence*. In the terminology of the betting scheme, coherence ensures that, if used as the basis of betting ratios, degrees of belief should not lead to a sure loss. Ramsey stated that coherent degrees of belief satisfy the laws of probability. Thereby coherence became the cornerstone of the subjective interpretation of probability, the only condition of acceptability that needs to be imposed on degrees of belief. Once degrees of belief are coherent, there is no further demand of rationality to be met.

The decisive step towards a fully developed subjective notion of probability was made by Bruno de Finetti whose "representation theorem" shows that the adoption of Bayes's method, taken in conjunction with the property of exchangeability, leads to a convergence between degrees of belief and frequencies. This makes subjective probability applicable to statistical inference, which according to de Finetti can be entirely based on it – a conviction shared by the neo-Bayesian statisticians. For the subjectivist de Finetti *objective* probability, namely the idea that probability should be uniquely determined, is a useless notion. Instead, one should be aware that probability evaluations depend on both subjective and objective elements, and refine probability appraisals by means of calibration methods.

Concluding remarks

Of the various interpretations of probability outlined above, the classical interpretation is by and large outdated, especially in view of its commitment to determinism. Though the same cannot be said for the logical interpretation, its formalism, especially in connection with Carnap's work, has made it unpalatable to scientists. It should be added that philosophers of science of Bayesian orientation seem on the whole prone to embrace the more flexible approach based on subjective probability.

The frequency interpretation, due to its empirical and objective character, has long been considered the natural candidate for the notion of probability occurring within the natural sciences. But while it matches the uses of probability in areas like population genetics and statistical mechanics, it faces insurmountable problems within quantum mechanics, where probability assignments to the single case need to be made. The propensity interpretation was put forward precisely to solve that difficulty. In the debate that followed Popper's proposal, propensity theory gained increasing popularity, but also elicited several objections.

Subjective probability has an undisputable role to play in the realm of the social sciences, where personal opinions and expectations enter directly into the information used to support forecasts, forge hypotheses, and build models. Various attempts are being made to extend the use of subjective probability to the natural sciences, including quantum mechanics.

While the controversy on the interpretation of probability is far from settled, the pluralistic approach, which avoids the temptation to force all uses of probability into a single scheme, is gaining ground.

See also Bayesianism; Confirmation; Determinism.

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