THE PHILOSOPHY OF SPACE & TIME

BY

HANS REICHENBACH

Translated by

MARIA REICHENBACH
AND
JOHN FREUND

With Introductory Remarks by

RUDOLF CARNAP

DOVER PUBLICATIONS, INC.
NEW YORK

Published in Canada by General Publishing Company, Ltd., 30 Lesmill Road, Don Mills, Toronto, Ontario.
Published in the United Kingdom by Constable and Company, Ltd.

This Dover edition, first published in 1957, is a new English translation of *Philosophie der Raum-Zeit-Lehre.*

Standard Book Number: 486-60443-8

Library of Congress Catalog Card Number: 58-7082

Manufactured in the United States of America
Dover Publications, Inc.
180 Varick Street
New York, N.Y. 10014

likened to the method in elementary analytic geometry which establishes an equivalence between a formula with two or three variables and a curve or a surface. The imagination is thus given conceptual support that carries it to new discoveries. In analogy to the auxiliary concept of the curvature of a surface, which is measured by the reciprocal product of the main radii of curvature, Riemann introduced the auxiliary concept of *curvature of space*, which is a much more complicated mathematical structure. Euclidean space, then, has a curvature of degree zero in analogy to the plane, which is a surface of zero curvature. Euclidean space occupies the middle ground between the spaces of positive and negative curvatures: it can be shown that this classification corresponds to the three possible forms of the axiom of the parallels. In the space of positive curvature *no* parallel to a given straight line exists; in the space of zero curvature *one* parallel exists; in the space of negative curvature *more than one* parallel exists. In general, the curvature of space may vary from point to point in a manner similar to the point to point variation in the curvature of a surface; but the spaces of *constant curvature* have a special significance. The space of constant negative curvature is that of Bolyai-Lobatschewsky; the space of constant zero curvature is the Euclidean space; the space of constant positive curvature is called spherical, because it is the three-dimensional analogue to the surface of the sphere. The analytical method of Riemann has led to the discovery of more types of space than the synthetic method of Bolyai and Lobatschewsky, which led only to certain spaces of constant curvature. Modern mathematics treats all these types of space on equal terms and develops and manipulates their properties as easily as those of Euclidean geometry.

§ 3. THE PROBLEM OF PHYSICAL GEOMETRY

Let us now return to the question asked at the end of § 1. The geometry of physical space had to be recognized as an empirical problem; it is the task of physics to single out the *actual* space, i.e., physical space, among the *possible* types of space. It can decide this question only by empirical means: but how should it proceed?

The method for this investigation is given by Riemann's mathematical procedure: the decision must be brought about by *practical* *measurements in space.* In a similar way as the inhabitants of a spherical surface can find out its spherical character by taking measurements, just as we humans found out about the spherical shape of our earth which we cannot view from the outside, it must be possible to find out, by means of measurements, the geometry of the space in which we live. *There is a geodetic method of measuring space analogous to the method of measuring the surface of the earth.* However, it would be rash to make this assertion without further qualification. For a clearer understanding of the problem we must once more return to the example of the plane.

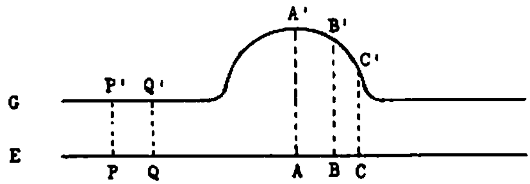

Fig. 2. Projection of a non-Euclidean geometry on a plane.

Let us imagine (Fig. 2) a big hemisphere made of glass which merges gradually into a huge glass plane; it looks like a surface *G* consisting of a plane with a hump. Human beings climbing around on this surface would be able to determine its shape by geometrical measurements. They would very soon know that their surface is plane in the outer domains but that it has a hemispherical hump in the middle; they would arrive at this knowledge by noting the differences between their measurements and two-dimensional Euclidean geometry.

An opaque plane *E* is located below the surface *G* parallel to its plane part. Vertical light rays strike it from above, casting shadows of all objects on the glass surface upon the plane. Every measuring rod which the *G*-people are using throws a shadow upon the plane; we would say that these shadows suffer deformations in the middle area. The *G*-people would measure the distances $A'B'$ and $B'C'$ as equal in length, but the corresponding distances of their shadows AB and BC would be called unequal.

Let us assume that the plane *E* is also inhabited by human beings and let us add another strange assumption. On the plane a mysterious force varies the length of all measuring rods moved about in that plane, so that they are always equal in length to the corresponding shadows projected from the surface *G*. Not only the measuring rods, however, but all objects, such as all the other measuring instruments and the bodies of the people themselves, are affected in the same way; these people, therefore, cannot directly perceive this change. What kind of measurements would the *E*-people obtain? In the outer areas of the plane nothing would be changed, since the distance *P'Q'* would be projected in equal length on *PQ*. But the middle area which lies below the glass hemisphere would not furnish the usual measurements. Obviously the same results would be obtained as those found in the middle region by the *G*-people. Assume that the two worlds do not know anything about each other, and that there is no outside observer able to look at the surface *E*—what would the *E*-people assert about the shape of their surface?

They would certainly say the same as the *G*-people, i.e., that they live on a plane having a hump in the middle. They would not notice the deformation of their measuring rods. But why would they not notice this deformation?

We can easily imagine it to be caused by a physical factor, for instance by a source of heat under the plane *E*, the effects of which are concentrated in the middle area. It expands the measuring rods so that they become too long when they approach *A*. Geometrical relations similar to those we assumed would be realized; the distances *CB* and *BA* would be covered by the same measuring rod and heat would be the mysterious force we imagined.

But could the *E*-people discover this force? Before we answer this question we have to formulate it more precisely. If the *E*-people knew that their surface is really a plane, they could, of course, notice the force by the discrepancy between their observed geometry and Euclidean plane geometry. The question, therefore, should read: how can the effect of the force be discovered if the nature of the geometry is not known? Or better still: how can the force be detected if the nature of the geometry may not be used as an indicator?

If heat were the affecting force, *direct* indications of its presence could be found which would not make use of geometry as an *indirect* method. The *E*-people would discover the heat by means of their sense of temperature. But they would be able to demonstrate the heat expansion independently of this sensation, due to the fact that heat affects different materials in different ways. Thus the *E*-people would obtain one geometry when using copper measuring rods and another when using wooden measuring rods. In this way they would notice the existence of a *force*. Indeed, direct evidence for the presence of heat is based on the fact that it affects different materials in *different* ways. The fact that the difference in temperature at the points A and C is demonstrable by the help of a thermometer is based on this phenomenon; if the mercury did not expand more than the glass tube and the scale of the thermometer, the instrument would show the same reading at all temperatures. Even the physiological effect of heat upon the human body depends upon differences in the reactions of different nerve endings to heat stimuli.

Heat as a force can thus be demonstrated directly. The forces, however, which we introduced in our example, cannot be demonstrated directly. They have two properties:

(a) They affect all materials in the same way.
(b) There are no insulating walls.

We have discussed the first property, but the second one is also necessary if the deformation is to be taken as a purely metrical one; it will be presented at greater length in § 5. For the sake of completeness the definition of the insulating wall may be added here: it is a covering made of any kind of material which does not act upon the enclosed object with forces having property *a*. Let us call the forces which have the properties *a* and *b* *universal forces*; all other forces are called *differential forces*. Then it can be said that only differential forces, but not universal forces, are directly demonstrable.

After these considerations, what can be stated about the shape of the surfaces E and G? G has been described as a surface with a hump and E as a plane which appears to have a hump. By what right do we make this assertion? The measuring results are the same on both surfaces. If we restrict ourselves to these results, we may just as well say that G is the surface with the "illusion" of the hump and E the surface with the "real" hump. Or perhaps both surfaces have a hump. In our example we assumed from the beginning that E was a plane and G a surface with a hump. By what right do we distinguish between E and G? Does E differ in any respect from G?

These considerations raise a strange question. We began by asking for the actual geometry of a real surface. We end with the question: Is it meaningful to assert geometrical differences with respect to real surfaces? This peculiar indeterminacy of the problem of physical geometry is an indication that something was omitted in the formulation of the problem. We forgot that a unique answer can only be found if the question has been stated exhaustively. Evidently some assumption is missing. Since the determination of geometry depends on the question whether or not two distances are really equal in length (the distances AB and BC in Fig. 2), we have to know beforehand what it means to say that two distances are "really equal." Is *really equal* a meaningful concept? We have seen that it is impossible to settle this question if we admit universal forces. Is it, then, permissible to ask the question?

Let us therefore inquire into the epistemological assumptions of measurement. For this purpose an indispensable concept, which has so far been overlooked by philosophy, must be introduced. The concept of a *coordinative definition* is essential for the solution of our problem.

§ 4. COORDINATIVE DEFINITIONS

Defining usually means reducing a concept to other concepts. In physics, as in all other fields of inquiry, wide use is made of this procedure. There is a second kind of definition, however, which is also employed and which derives from the fact that physics, in contradistinction to mathematics, deals with real objects. Physical knowledge is characterized by the fact that concepts are not only defined by other concepts, but are also coordinated to real objects. This coordination cannot be replaced by an explanation of meanings, it simply states that *this concept* is coordinated to *this particular thing*. In general this coordination is not arbitrary. Since the concepts are interconnected by testable relations, the coordination may be verified as true or false, if the requirement of uniqueness is added, i.e., the rule that the same concept must always denote the same object. The method of physics consists in establishing the uniqueness of this coordination, as Schlick [1] has clearly shown. But certain preliminary coordinations must be determined before the method of coordination can be carried through any further; these first coordinations are therefore definitions which we shall call *coordinative definitions*. They are *arbitrary*, like all definitions; on their choice depends the conceptual system which develops with the progress of science.

Wherever metrical relations are to be established, the use of coordinative definitions is conspicuous. If a distance is to be measured,

[1] M. Schlick, *Allgemeine Erkenntnislehre*, Springer, Berlin 1918, Ziff. 10.