Appendix to Supplement: What Determines Prices in the Futures and Options Markets?

One probably does need to be a rocket scientist to figure out the latest wrinkles in the pricing formulas used by professionals to determine the appropriate price to pay for any specific futures or options contract. It is possible, however, for any individual to understand the basic determinants of these prices and, therefore, to have at least a general idea why some options may sell at 15 percent and others at only 5 percent of the market value of the underlying stock.

Let's deal with the futures market first and see what is involved in the pricing of a silver future. Let's suppose that the spot price of silver for immediate delivery is $6.00 an ounce. Suppose further that the price of a silver future for delivery in three months time is $6.10. Is the futures market some kind of con game whereby buyers are, in effect, cheated out of an extra ten cents? Not at all. Consider the situation faced by an individual who needs a certain quantity of silver in three months time. He could buy the silver now at $6.00 per ounce and hold onto it until it was needed. This would involve two kinds of costs, however. First, he would have to store the silver for 90 days and this would involve storage costs of perhaps $0.01 per ounce. Second, buying silver in the spot market would involve an immediate payment of the $6.00 price and, thus, what economists call an opportunity cost. By paying for the silver now, one foregoes the opportunity to invest the money for 90 days in a perfectly safe investment such as a Treasury bill. If Treasury bill yields were 6 percent per year (or 1.5 percent per quarter), the spot buyer would have foregone the opportunity to earn \$0.09 (the \$6.00 times 1.5 percent). Hence, the buyer in need of silver would be indifferent between buying silver in the spot market at \$6.00 or buying it in the futures market at \$6.10. Similarly, the seller would be equally happy to receive \$6.00 now, which would enable her to save on storage costs and earn interest on the receipts, or receive \$6.10 at the end of the quarter.

From this simple illustration, the basic determinants of the spread between spot and futures prices can be made clear. Futures prices depend on the interest rate and storage costs. Moreover, one additional factor is likely to enter the equation—the so-called "convenience yield" of having the inventory directly on hand. In general, the futures price will be above the spot price because of the interest and storage factors, but it could in some circumstances be lower when the convenience yield is very high, as might happen if the commodity was in very short supply.

Similarly, we can list the factors determining option prices. The factors have to do with the characteristics of the options contract and those of the underlying stock and the market. Five factors are important:

1) The Exercise Price

Suppose Amazon.com is selling at \$40 per share. A call option exercisable at \$40 is clearly more valuable than one exercisable at \$50 per share, well "out-of-the-money." The higher the exercise price, the lower the value of the option. Of course, the value of the option can never go below zero. As long as there is some probability that the market price of the stock could exceed the exercise price in the future, the option will have some value.

2) The Stock Price

All other things being the same, the higher the stock price, the higher will be the price of the call option. Obviously, if a stock is selling at \$1 per share, the option could not possibly be worth more than \$1, since purchasing the stock directly would allow the investor to realize whatever appreciation develops while the investor's risk would be limited to the purchase price. An option premium on such a stock might run 5 or 10 cents per share. On the other hand, the value of an at-the-money call on a $100 stock could be $5 or $10 per share.

3) Expiration Date

The longer the option has to run, the greater its value. Consider two options on Amazon.com with an exercise price of $40 per share. Obviously, a six-month option is worth more than a three-month option, since it has an additional three months within which the call on the stock can be exercised. Thus, if something good happens to the company, the buyer of the longer option will have a longer period over which she can take advantage of any favorable outcome.

4) The Volatility of the Stock

This is the key factor determining the value of a stock option. The greater the volatility of the underlying stock in question, the higher the cost of a call option. An at-the-money call on Amazon.com, which is an extremely volatile stock, is likely to sell for very much more than an at-the-money call of similar length on a more stable stock such as Johnson & Johnson. For the stable stock, it is unlikely that the call buyer will make a killing since the stock characteristically doesn't fluctuate much. Similarly, on the down side, buying the stock directly does not involve great risk. On the other hand, buying Amazon, which is very volatile, does involve considerable risk which is limited when the investor buys the call option. At the same time, very favorable outcomes could lead to a big rise in Amazon's stock, making the potential worth of the option far greater than in the Johnson & Johnson case.

5) Interest Rates

Call prices are also a function of the level of interest rates. The buyer of a call option does not pay the exercise price until and unless he exercises the option. The ability to delay payment is more valuable when interest rates are high and, therefore, earnings opportunities on cash are very attractive.

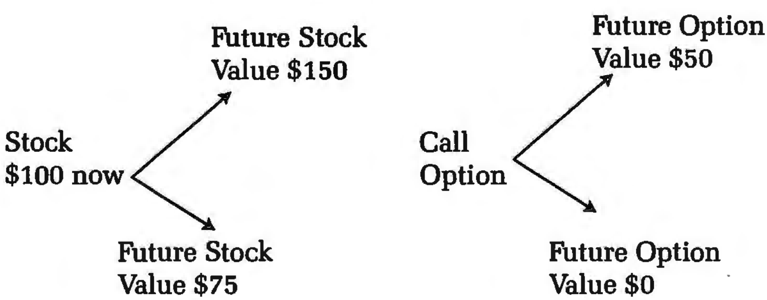
A model has been developed by Fischer Black, Myron Scholes, and Robert Merton to make quantitative estimates of options values based upon the factors just outlined. Merton and Scholes were awarded a Nobel prize for the discovery. (Black would certainly have shared in the award were it not for his untimely death.) While the mathematics of these formulas are quite forbidding, they are easily programmed on a personal computer and they are used extensively by option buyers and traders to determine the appropriate value of option premiums.

Binomial Open Pricing

While the mathematics behind the Black-Scholes model will be inaccessible for most readers, a related model—the binomial model—can be easily understood by anyone familiar with high school algebra. Moreover, the general insights that lie behind Black-Scholes can all be found in the binomial model. Indeed, a multiperiod binomial model produces equivalent results to Black-Scholes.

In describing the binomial model we assume that only two future equally likely outcomes are possible. The stock in question can either go up or go down. A very volatile stock would go up a great deal. A more stable stock would have a narrower range of potential outcomes. We'll do our illustration of the binomial model with a volatile stock initially selling at $100 a share. We will assume that at the end of a year the stock could either go up to $150 or could fall to $75. We will show how to price a one-year call option. The one-year riskless interest rate (on Treasury bills) is taken to be 10 percent. We diagram the present and future prices of the stock and the call option below.

Illustration of Possible Future Stock and Call Option Values



Note that if the stock rises to $150 at the end of one year, the call option will be worth $50, its intrinsic value. If the stock declines, however, the option will expire worthless. Obviously, an option to buy something at $100 when it can be purchased on the market at $75 is not worth anything. Now suppose an investor forms a perfectly hedged portfolio that has exactly the same terminal value whether the market rises or falls. Such a perfectly hedged portfolio can be formed if the investor buys ⅔ of a share of stock and sells (writes) one call option with a striking price of $100, the initial stock price. Two-thirds is called the hedge ratio, and we will see in a moment how to find the correct ratio. Let's first confirm that the investor is perfectly hedged.

The illustration below shows that the investor who buys ⅔ of a share of stock and sells one call is indeed perfectly hedged. He obtains a payoff after one year of $50, whether the stock goes up or down.

Illustration of Investor Payoffs

Stock rises to $150

	Payoff
Option exercised, investor receives	$100
Investor purchases ⅓ share	−50
Net payoff	$50

Stock falls to $75

	Payoff
Option not exercised	
Investor owns ⅔ share	$50

If the stock rises, the call buyer exercises her option and presents $100 to buy a share of stock. The writer is obligated to deliver a share but he only owns ⅔ of a share. Hence the writer must purchase ⅓ share at the then-current price of $150. This costs him $50 for a net payoff of $50. If the stock falls to $75, the call is not exercised and the investor is left holding ⅔ of a share of a $75 stock, which is also worth $50.

Thus, the investor has a final payoff of $50, irrespective of whether the stock rises or falls, and is thus perfectly hedged.

How does one determine the hedge ratio? Easy. The hedge ratio is simply the ratio shown in the illustration below.

Determination of Hedge Ratio

$$\frac{\textit{Value of Call if Stock Up} - \textit{Value of Call if Stock Down}}{\textit{Value of Stock if Up} - \textit{Value of Stock if Down}}$$

$$\frac{\text{Call (Up)} - \text{Call (Down)}}{\text{Stock (Up)} - \text{Stock (Down)}} = \frac{50-0}{150-75} = \frac{50}{75} = \frac{2}{3}$$

If the stock goes up, the call has an intrinsic value of $50. (The right to buy a share of stock selling at $150 for only $100 is worth $50.) If the stock goes down, the call is worthless. (The right to buy a share of stock for $100 while it is selling in the market at only $75 is worthless.)

Just one more insight is needed to determine the value of the call option. This insight lies behind the Black-Scholes as well as the binomial model. If an investor makes a riskless investment (that is, one that is perfectly hedged and has the same payoff whether the market goes up or down), that investment should earn the riskless rate of interest. The riskless interest rate is taken to be the rate of interest on perfectly safe Treasury bills. With this insight, we can easily determine the selling price of a call option, which we will designate as C.

We saw that the investor is perfectly hedged if he buys ⅔ of a share of stock and writes one call option. Such a purchase will cost $66.67 (the cost of ⅔ share of a $100 stock) minus C, the value of the call option. (Remember, the buyer of the call pays the writer the option premium.) Such an investment, since it is perfectly hedged, should earn the one-year rate of interest, which we assumed was 10 percent. Hence we can determine the price of the call option by solving the equation shown in the illustration below.

Illustration of the Determination of the Value of a Call Option

$$\text{Amount Invested (1+ Risk-Free Interest Rate)} = \text{Certain Payoff}$$
$$[\text{Hedge Ratio (Initial Stock Price)} - C]\ (1+ \text{Interest Rate}) = \text{Certain Payoff}$$
$$[2/3(100) - C]\ (1 + .10) = 50$$
$$(66.67 - C)\ (1.1) = 50$$
$$C = 21.22$$

The call option will be priced at $21.22.

The Black-Scholes option pricing model uses precisely the same insight we just used in the binomial illustration above. Black-Scholes can be considered a multistage binomial model. We had assumed that only two outcomes were possible after one year. But suppose we consider a two-stage (six-month) binomial model. After the first six months, the stock can go up or down. After the second six months, again two outcomes are possible. This leads to three possible outcomes at the end of one year. The stock can go up in both periods or down in both. Alternatively the stock could rise (fall) in the first period and fall (rise) in the second. Thus, three outcomes are possible. We could also do a four-stage binomial model where each quarter the stock price could rise or fall and several different outcomes would be possible. Or we could do a daily binomial or even an hourly or minute-by-minute model. As we break the binomial model into finer and finer subperiods, the binomial model and the Black-Scholes model converge. The basic insights remain the same. We form perfectly hedged portfolios of holding stock and writing call options. Such riskless portfolios should earn only the risk-free interest rate in an efficiently functioning market. It turns out that actual option premiums fluctuate reasonably closely around the values suggested by the Black-Scholes model. The model is also used extensively to determine the appropriate accounting charge against earnings from the granting of executive stock options.