Revising Claims and Resisting Ultimatums in Bargaining Problems*

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Abstract

We propose a simple mechanism which implements a unique solution to the bargaining problem with two players in subgame-perfect equilibrium. The mechanism incorporates two important features of negotiations; players can revise claims in an attempt to reach a compromise or pursue their claims in an ultimate take-it-or-leave-it offer. Players restrain their claims to avoid a weak bargaining position or their resistance to uncompromising behavior to acquire leadership. The Nash solution and the Kalai-Smorodinsky solution are implemented in the extreme cases when respectively no and all revisions are allowed.

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1 Introduction

Negotiations often share the following two features. First, players revise initial claims in order to reach a compromise. Their ability to make revisions depends on the context of the negotiations and may differ among players. Second, concessions may be induced by the threat of an ultimate take-it-or-leave-it offer. However, negotiators discourage such uncompromising behavior by adopting a firm posture - threatening to walk away from negotiations without agreement - when facing such an ultimatum. These two features are extensively discussed in the negotiation literature (Sebenius 1992, Lewicki et al. 1994) and also appear in practical guides for negotiators, as in the defense procurement and acquisition guidelines by the US Department of Defense:[1] “Aim high” but “Give yourself room to compromise” and “Be willing to walk away from or back to negotiations”.

In the bargaining literature, Harsanyi (1977) justified the solution of Nash (1950) by comparing the risk limits of players in the pursuit of their claims. A player’s risk limit is the highest probability of disagreement that he would accept in the pursuit of his claim in an ultimatum, when accepting his opponent’s claim is the alternative. The player with higher risk limit is in a weak bargaining position and is more likely to accept his opponent’s claim. Since a lower claim decreases the own risk limit and increases the opponent’s risk limit, players avoid a weak bargaining position by exhibiting restraint in the formulation of their claims. Risk limits are equalized if each player claims his payoff in the Nash solution.

Moulin (1984) justified the solution of Kalai and Smorodinsky (1975) in an auction in which each player bids a probability of disagreement when an uncompromising opponent pursues his dictatorial outcome in an ultimatum. The player with the lower bid is given the advantage to propose any feasible utility allocation as a compromise. Hence, the competition for first-mover advantage rewards restraint in the choice of resistance probabilities against uncompromising behavior. In a maxmin equilibrium of the bidding strategies, both players commit to equal resistance probabilities which eliminates first-mover advantage. They both propose the Kalai-Smorodinsky solution in which they reduce their claims in the same proportion. In particular, this solution solves the trade-off for each player between the commitment to higher resistance in order to deter uncompromising behavior and the commitment to lower resistance in order to obtain a leadership position.

In his justification of the Nash solution, Harsanyi assumed that claims cannot be revised, leaving little room to compromise. In his justification of the Kalai-Smorodinsky solution, Moulin assumed that players pursue their dictatorial outcomes in an ultimatum, excluding restraint in the formulation of claims. The two approaches motivate the

[1] The Contract Pricing Reference Guides (Vol5, Ch6) of the DPAP of the US Department of Defense, http://www.acq.osd.mil/dpap/cpf/docs/contract_pricing_finance_guide/vol5_ch6.pdf .

analysis of a mechanism with four stages showing how avoidance of a weak bargaining position and competition for first-mover advantage interact. Players start by making *claims*, as in Harsanyi. In the second stage, players bid *resistance probabilities*, as in Moulin. Leadership is acquired by the player with the lowest bid. In the third stage, the leader proposes a compromise within the set of feasible compromises which depends on his claim but remains beyond his control in all other respects. In the final stage, the follower accepts or rejects the compromise. If he rejects, then he obtains his claim in an ultimatum unless he meets resistance to which the leader is committed by the second stage; the negotiations end in disagreement with the leader's resistance probability.

The single distinguishing feature of these games is the extent to which claims can be subsequently revised. The revision procedure defines the Pareto-efficient *maximal revision* of each player's claim. The room to compromise is the gap between the maximal utility which a player can give to his opponent in the maximal revision and in the pursuit of his claim. The Nash solution and the Kalai-Smorodinsky solution are implemented in subgame-perfect equilibrium in the extreme cases excluding or admitting all revisions respectively. The main contribution of this paper is the highlighting of when and how the strategic justifications of Moulin and Harsanyi interact for intermediate revision procedures considered in the negotiation literature. The key in this interaction is the new concept of the *extended* Nash product of a player's claim, which multiplies his claim with the opponent's utility in his maximal revision. The player with the larger extended Nash product of his claim is the strong player as he needs a lower resistance probability to impose his maximal revision which avoids an ultimatum. In particular, players face a trade-off between claiming more so as to achieve more in an ultimatum and claiming less so as to obtain a strong bargaining position. This allows us to analyze how the aforementioned features in the negotiations literature play out in equilibrium. Players should not only aim high when formulating claims, but also leave sufficient room to compromise in order to obtain a strong bargaining position.

The paper shows that in equilibrium there is interaction between both strategic justifications in intermediate revision procedures, with two exceptions. Players restrain their claims which makes them equally strong, as in Harsanyi, but at the same time they restrain their resistance so that their concessions stand in the same proportion to their claims, as in Moulin. We distinguish between two cases. In the first case, maximal revisions are incompatible. Competition for the strong bargaining position induces restraint in the formulation of claims, unless one player has a claim which puts him in a strong bargaining position for all claims of the other player. The strong player imposes his maximal revision for the largest of such claims in the first exception. Otherwise, at least one of the players gains by reducing his claim, which closes the gap between the maximal revisions or makes them compatible. In the second case, maximal revisions are compatible. The *proportional solution* in which the players' utilities stand in equal proportion to their claims is a feasible compromise. The competition for leadership equalizes the players' resistance probabilities by adopting Moulin's maxmin bidding strategies. By the monotonicity of the proportional solution for strictly compatible maximal revisions, each player gains by increasing his claim, unless claims are maximal. Players agree on the Kalai-Smorodinsky solution in the second exception. Otherwise, none of the players can gain by changing his claim only when maximal revisions meet for claims with equal extended Nash products. The maximal revisions of the equilibrium claims meet in the Kalai-Smorodinsky solution for the bargaining problem with these claims as ideal points.

The mechanism underlines that room to compromise is essential for a strong bargaining position, as recommended in the negotiation literature. When a negotiator is able - for a claim below his maximal claim - to increase his opponent's payoff in his maximal revision, larger extended Nash products improve his bargaining position allowing for a better deal. A negotiator gains in equilibrium by facing fewer restrictions regarding the revisions of all claims below his maximal claim. Still, such exogenous restrictions can be important in particular contexts. For example, restrictions on revisions can be explicitly specified in the mandate given to the negotiator by his principal or arise from costs of revising initial plans. The restrictions may also arise from unfulfilled expectations raised by the initial claims or from aversion to making concessions. In these examples, one expects better agreements for negotiators who do not fear to disappoint their principals or suppress their frustration. Our analysis sheds light on this, evaluating more generally the impact of revision procedures on the bargaining outcome.

The mechanism also clarifies the role of ultimatums with endogenously chosen risk of disagreement needed for imposing a compromise. This is further illustrated for unrestricted revisions in the alternating-offer game (Rubinstein (1982)). In each round the responder can stop negotiations in an ultimatum and the proposer needs time to build resistance in order to deter such ultimatum for a better deal. The introduction of ultimatums moves the equilibrium outcome away from the Nash solution - the equilibrium solution of the alternating-offer game with equal waiting times - towards the Kalai-Smorodinsky solution - the equilibrium solution of the four-stage mechanism with unrestricted revisions.

Related Literature According to Nash (1953), the relevance of a solution concept is enhanced if one arrives at it from very different points of view. The Nash program, as reviewed in Thomson (2010), attempts to complement the axiomatic properties of solution concepts with non-cooperative foundation. While Harsanyi (1977), Moulin (1984), Binmore et al. (1986) and Howard (1992) implement the Nash program for a single bargaining solution, we achieve implementation for a family of solutions in subgame-perfect equilibrium, as Miyagawa (2002) and Anbarci and Boyd (2011). Miyagawa's mechanism implements any solution that maximizes a welfare function belonging to a set of quasi-concave functions, including the Nash and Kalai-Smorodinsky solution. The second player counters the offer of the first player, but this offer is restricted to provide the same aggregate welfare as the first offer. In the mechanism of Anbarci and Boyd, compatible utility allocations are implemented in a first stage and incompatible utility allocations are implemented with equal probability in a second stage, unless there is an exogenously imposed probability of disagreement. The Kalai-Smorodinsky solution is the unique robust solution which both players demand above a threshold. There is no general robustness ranking for other solutions. We propose a mechanism with endogenously chosen probability of disagreement which occurs only off the equilibrium path and which induces restraint in the claims depending on the revision procedure. Interestingly, we find the Nash and Kalai-Smorodinsky solution for two opposite extremes. By considering intermediate revision procedures, we are able to compare and deepen our insight in Harsanyi's and Moulin's seminal contributions to the Nash program.

Schelling (1956) discusses take-it-or-leave-it offers and commitments as strategy devices. Kahneman and Tversky (1995) show that loss aversion appears as concession aversion in the context of negotiations. The experimental literature shows that people accept losses by rejecting unfair outcomes in ultimatums (Camerer (2003)). Punishing unfair treatment is rationalized in Fehr and Schmidt (1999). We refer to this literature to justify the commitment of accepting the loss of disagreement with positive probability in an ultimatum.

The paper is organized as follows. The next section defines the bargaining problem, the four-stage mechanism and the revision procedures. Section 3 analyzes the extreme revision procedures allowing no or all revisions. Section 4 characterizes the solution for intermediate revision procedures. We provide examples of revision procedures in section 5. Before concluding, the robustness of the mechanism is analyzed in section 6. The complete description of the subgame-perfect equilibrium and proofs are given in appendix.

2 The model

In this section, we define the bargaining problem and a mechanism for selecting a solution in the bargaining set.

2.1 The bargaining problem

Let $N = \{1, 2\}$ be the set of players. The players are male. Player $-i$ is player i's opponent for $i \in N$. The closed and convex set S is a subset of $D = [0, 1] \times [0, 1]$. The elements of S are the normalized utility allocations $u = (u_1, u_2)$ associated with feasible outcomes, which are known by each player. For $i \in N$, the concave function $u^P_{-i} : [0, 1] \to [0, 1] : u_i \mapsto \sup \{ u_{-i} | \, u \in S\}$ is assumed to be strictly decreasing. It takes the value $u^P_{-i}(0) = 1$ in the dictatorial outcome of player $-i$ and the value $u^P_{-i}(1) = 0$

in the dictatorial outcome of player i. The utility allocation in disagreement is $(0,0)$. It follows that the set S defines a strictly comprehensive bargaining problem. The set of Pareto-efficient utility allocations in S is the Pareto frontier $PO(S) \equiv \{ u \in S |\, u_2 = u_2^P(u_1)\}$.

2.2 The mechanism

The extensive form of the mechanism Γ for selecting a solution in S has four stages.

The first stage is a demand stage, inspired by Harsanyi's demand game. Both players simultaneously formulate *utility claims* $p \in D$, where p_i is player i's utility when he successfully pursues his claim in an ultimatum. The second stage is a bidding stage, inspired by Moulin's auction game. Both players simultaneously bid *resistance probabilities* $q \in D$, where q_i is the probability of disagreement when player i's opponent pursues his claim in an ultimatum. The third stage is the compromising stage. The player with the lowest resistance probability is the leader L, who makes a compromise proposal. The fourth and final stage is the approval stage. The follower F accepts L's compromise or pursues his claim risking that negotiations end in disagreement with probability q_L.

The claims p of the demand stage serve a double purpose. While player i's claim p_i defines his utility when he wins in his ultimatum, it also defines the *maximal revision* $m^i(p_i) \in PO(S)$ of his claim in the compromising stage, where $m_i^i(p_i)$ is his maximally revised claim and $m_{-i}^i(p_i)$ is his opponent's payoff in the maximal revision of his claim. We consider comprehensive revision procedures.

Definition 1. *The revision procedure $m = (m^1, m^2)$ is comprehensive iff m_{-i}^i is a non-increasing concave function on $[0,1]$ such that $u_{-i}^P(p_i) \leq m_{-i}^i(p_i) \leq 1$ for $p_i \in [0,1]$ and $i \in N$.*

The revision procedure is beyond the control of the players and the single distinguishing feature of each mechanism with extensive form Γ. All comprehensive revision procedures are collected in the set $\mathcal{M}$. We index the extensive form Γ for special comprehensive revision procedures. In Γ^H, as in Harsanyi (1977), no revisions are allowed and each player's payoff in his maximal revision is equal to his claim, that is, $m_i^i(p_i) = p_i$ for $p_i \in [0,1]$. In Γ^M, as in Moulin (1984), unrestricted maximal revisions of any claim yield the payoffs of the opponent's dictatorial outcome, that is $m_i^i(p_i) = 0$ for $p_i \in [0,1]$.

The set of player i's compromises is

$$C_i(p_i) \equiv \left\{ c \in S |\, m_i^i(p_i) \leq c_i \leq p_i \right\} \text{ for } p_i \in [0,1] \text{ and } m \in \mathcal{M}.$$

We assume for tractibility that a leader can also propose those compromises which are feasible for the follower.[2] Hence, the set of feasible compromises is

$$C(p) \equiv C_1(p_1) \cup C_2(p_2).$$

Claims are incompatible if $p \notin S$. Maximal revisions are incompatible if for incompatible claims $C_1(p_1) \cap C_2(p_2) = \emptyset$.

The resistance probabilities in the second stage serve a double purpose as well. While q_i is player i's choice of the probability of disagreement when he is leader and his opponent rejects his compromise in an ultimatum, it also rewards lower resistance with first-mover advantage. In case of equal bids, leadership is assigned to player 1 for some labeling of the players. This rule is a mapping $\mathcal{M} \rightarrow \{1,2\}$ which we define in Definition 3. Hence,

$$L(q) = \begin{cases} i & \text{if } q_i < q_{-i}, \\ 1 & \text{if } q_1 = q_2. \end{cases}$$

The rules of the mechanism can be summarized as follows:

- **Stage** 1. All $i \in N$ formulate claims $p \in D$.
- **Stage** 2. All $i \in N$ bid resistance probabilities $q \in D$.
- **Stage** 3. The leader $L(q)$ proposes the compromise $c \in C(p)$.
- **Stage** 4. The follower $F \in N \setminus \{L(q)\}$ chooses R$\in$ {Y,N}.

The payoffs for player $i \in N$ are

$$u_i = \begin{cases} c_i & \text{if R} = \text{Y}, \\ (1 - q_L)\, p_F & \text{if R} = \text{N and } i = F, \\ (1 - q_L)\, u_L^P(p_F) & \text{if R} = \text{N and } i = L. \end{cases}$$

3 Two extreme revision procedures

We start by showing that the Kalai-Smorodinsky solution and the Nash solution can be implemented in subgame-perfect equilibrium in mechanisms with the same extensive form, but with different procedures for revising claims. All revisions are allowed for the former and no revisions are allowed for the latter. The non-cooperative justifications of these solutions recast the arguments of Moulin (1984) and Harsanyi (1977) respectively.

[2] This assumption is relaxed in section 6. In all the mechanisms, we exclude upward revised claims ($m_i^i(p_i) > p_i$) and inefficient revisions ($m_{-i}^i(p_i) < u_{-i}^P(p_i)$). We also ignore comprehensive bargaining problems which are not strictly comprehensive. Such extensions would only change strategies without changing the allocation implemented in subgame-perfect equilibrium.

3.1 The Kalai-Smorodinsky solution

Kalai and Smorodinsky (1975) proposed the Pareto-efficient allocation $u^{KS}(p)$ for which $u_1^{KS}(p)/p_1 = u_2^{KS}(p)/p_2$ as a solution to the reduced bargaining problem

$$S(p) \equiv \{\, u \in S |\, u \leq p\} \text{ for } p \notin S \setminus PO\,(S)\,.$$

We call $u^{KS}(p)$ the *proportional solution* of $S\,(p)$. In this solution, the concessions are proportional to the claims and the payoffs are increasing in the own claim. The mechanism Γ^M with unrestricted revisions provides a non-cooperative justification of the Kalai-Smorodinsky solution of S. Following Moulin (1984), bidding

$$q_i^{KS}\,(p) = 1 - \frac{u_{-i}^{KS}(p)}{p_{-i}}$$

corresponds to a maxmin strategy for player $i \in N$. To see this, consider the resistance probability q_i of player i in stage 2. If player i leads the negotiations (i.e. $q_i \leq q_{-i}$), knowing that his opponent rejects any compromise with payoff below $(1 - q_i)p_{-i}$ in stage 4, he proposes the Pareto-efficient compromise

$$c^i\,(q_i) \in \arg\max_{c \in S(p)} \{\, c_i |\, c_{-i} \geq (1 - q_i)p_{-i}\}$$

in stage 3. If the opponent leads the negotiations with bid q_{-i} (i.e. $q_{-i} \leq q_i$), then $(1 - q_{-i})\,p_i$ is player i's payoff. Since $q_{-i} \leq q_i$, player i's payoff as a follower is bounded from below by $(1 - q_i)\,p_i$. Hence, $\min\left\{c_i^i(q_i), (1 - q_i)\,p_i\right\}$ is a lower bound for his payoff when bidding q_i. As higher resistance probability increases his payoff $c_i^i(q_i)$ as leader, but decreases his payoff $(1 - q_i)\,p_i$ as follower, the minimum of the two payoffs reaches a maximum when $u_i = c_i^i(q_i) = (1 - q_i)\,p_i$. Since $u_{-i} = (1 - q_i)\,p_{-i}$, the payoffs of the players stand in the same proportion to their claims in the maxmin bidding strategy of player i. The resistance probabilities are equal for $q_1^{KS}\,(p) = q_2^{KS}\,(p)$ and each player would propose the proportional solution as a leader. Hence, the proportional solution is implemented in subgame-perfect equilibrium.

The stages 2 to 4 of Γ^M recast the mechanism proposed by Moulin (1984) for the reduced bargaining problem $S\,(p)$. Since in the Kalai-Smorodinsky solution for $S(p)$, $u_i^{KS}(p)$ is monotone in p_i, nobody shows restraint in the formulation of claims in stage 1 and $p = (1, 1) = \mathbf{1}$. By augmenting Moulin's model with a demand stage, Γ^M justifies Moulin's assumption that players make maximal claims.

Proposition 1. *The Kalai-Smorodinsky solution $u^{KS}\,(\mathbf{1})$ is implemented in subgame-perfect equilibrium in the mechanism Γ^M with unrestricted maximal revisions.*

Proof. *See appendix.* ■

3.2 The Nash solution

The solution u^N of Nash (1950) to a bargaining problem maximizes the Nash product $u_1 u_2$ for $u \in S$. Harsanyi (1977) derived the Nash solution as an equilibrium for a demand game in which, according to the conjecture of Zeuthen (1930), the player with the higher risk limit is in a *weak* bargaining position and eventually makes concessions. For player i's positive claim p_i and his opponent's compromise c, his risk limit is defined as

$$r_i(c_i, p_i) \equiv \max\left\{1 - \frac{c_i}{p_i}, 0\right\}.$$

In our setting, the risk limit stands for the highest resistance probability that a follower in stage four would be willing to face in the pursuit of his claim when accepting the compromise c is the alternative. In other words, a follower accepts the compromise c only if his risk limit does not exceed the leader's resistance probability.

The mechanism Γ^H clarifies why the player with the lower risk limit is in a weak bargaining position and eventually concedes. In Γ^H, claims cannot be revised. Each player i obtains his claim p_i as a payoff in his unique Pareto-efficient compromise $m^i(p_i)$ which he prefers to his opponent's compromise $m^{-i}(p_{-i})$ for incompatible claims p. If player i's risk limit exceeds his opponent's, then player i ensures his claim by bidding a resistance probability q_i in between his and his opponent's risk limit. As leader, he is in a *strong* bargaining position. He obtains leadership for a resistance probability exceeding his opponent's risk limit $r_{-i}\left(m^i_{-i}(p_i), p_{-i}\right)$, so that his opponent accepts $m^i(p_i)$. On the contrary, his opponent is in a weak bargaining position because $m^{-i}(p_{-i})$ would be rejected when he leads for $q_{-i} \leq q_i < r_i\left(m^{-i}_i(p_{-i}), p_i\right)$. As leader, the weak player proposes $m^i(p_i)$ rather than facing an ultimatum, a lottery with $m^i(p_i)$ and the disagreement outcome as prizes. Hence, $m^i(p_i)$ is implemented.

As in Harsanyi's justification of the Nash solution, players compete to be in a strong bargaining position by adjusting their claims to have the higher risk limit. The player i with the higher risk limit has the higher Nash product $m^i_{-i}(p_i)\, p_i$. For $p_i = u^N_i$, player i maximizes the Nash product of his claim, so that his risk limit is never below his opponent's. This claim ensures a strong bargaining position for incompatible claims. None of the players can receive a payoff below his payoff in u^N, as he would have a profitable deviation. Hence, both players would propose u^N as a leader and u^N is implemented in subgame-perfect equilibrium in Γ^H.

Proposition 2. *The Nash solution u^N is implemented in subgame perfect equilibrium in the mechanism Γ^H without revisions of claims.*

Proof. *See appendix.* ■

3.3 Discussion

The solutions of Nash and of Kalai and Smorodinsky are implemented in mechanisms with the same extensive form to be distinguished only by the extent to which claims can subsequently be revised. In both mechanisms first-mover advantage in stage 3 disappears in equilibrium since, respectively for $c = m^1(p_1) = m^2(p_2) = u^N$ and $c = u^{KS}(\mathbf{1})$,

$$r_1(c_1, p_1) = r_2(c_2, p_2).$$

However, the reason for achieving equality of risk limits in the two solutions is different.

Moulin's justification of the Kalai-Smorodinsky solution focuses on situations with compatible maximal revisions. As long as maximal revisions are compatible, there is no reason to show restraint in claims. However, each player faces a trade-off between increasing resistance in order to block the other player's ultimatum in the pursuit of a better deal and decreasing resistance to acquire the leadership position. Competition for leadership forces players to show restraint in their resistance. For maximal claims, $q_i^{KS}(\mathbf{1}) = r_{-i}(u_{-i}^{KS}(\mathbf{1}), 1) = 1 - u_{-i}^{KS}(\mathbf{1})$. The maxmin strategy equalizes the resistance probabilities leading to the Kalai-Smorodinsky solution.

Harsanyi's justification of the Nash solution focuses on situations with incompatible maximal revisions. Each player faces a trade-off between decreasing his claim for obtaining a strong bargaining position and increasing his claim to increase his payoff in his ultimatum. Competition to be in a strong bargaining position forces players to show restraint in claims. As long as the strong player's claim exceeds his payoff in the Nash solution, his strong bargaining position can be put in jeopardy. The weak player can become strong for a claim with a larger Nash product and become leader for a lower resistance probability. Hence, both players claim their payoffs in the Nash solution. Without revisions, $m_i^i(p_i) = p_i$. Since maximal revisions are the same in equilibrium, restraint in claims drives the risk limits towards zero ($r_i(u_i^N, u_i^N) = 0$).

4 Comprehensive revision procedures

This section analyzes the interaction of Harsanyi's and Moulin's justification of a bargaining solution for any comprehensive revision procedure. For any positive claim, unlike in Γ^H, maximal revisions may increase the opponent's payoff, but, unlike in Γ^M, may remain incompatible. In order to characterize a strong bargaining position, we introduce the concept of the extended Nash product of a player's claim, the product of a player's claim and his opponent's utility in the maximal revision of his claim. The player with the larger extended Nash product is the *strong* player who imposes his maximal revision or the proportional solution, as he prefers. The solution in Γ for $m \in \mathcal{M}$ can therefore be obtained in a two-stage mechanism in which the strong player imposes his preferred option after both players formulated their claims. With two exceptions, the maximal revisions meet in the proportional solution for the equilibrium claims.

4.1 Bidding Resistance Probabilities: the Extended Nash Product

The minimal resistance probability needed by player i to impose his maximal revision $m^i(p_i)$ is

$$\rho_i(p) \equiv r_{-i}(m^i_{-i}(p_i), p_{-i}) \text{ for } i \in N. \tag{1}$$

For all concessions in $C_i(p_i)$ other than player i's maximal revision, the opponent's risk limit exceeds $\rho_i(p)$. We therefore say that the player with the higher minimal resistance probability is in a weak bargaining position because he, unlike his opponent, can no longer impose a compromise within his set of feasible compromises as a leader when his opponent bids a resistance probability strictly in between $\rho_1(p)$ and $\rho_2(p)$. From (1),

$$\rho_i(p) \geq \rho_{-i}(p) \text{ iff } p_i m^i_{-i}(p_i) \leq p_{-i} m^{-i}_i(p_{-i}) \text{ for } i \in N.$$

Definition 2. *The product $p_i m^i_{-i}(p_i)$ is the extended Nash product of player i's claim p_i.*

The characterization of the weak and the strong player by Harsanyi (1977) remains valid with revisions of claims by extending the concept of the Nash product. The extended Nash product and the Nash product coincide when no revisions are allowed. For $m \in \mathcal{M}$, the claim $\hat{p}_i$ which maximizes the unimodal extended Nash product of player i's claim is unique. A strong bargaining position is valuable in the competition for leadership. Recall that leadership is given to the player with label 1 in case both players bid equal resistance probabilities.

Definition 3. *The player with the label 1 for $m \in \mathcal{M}$ is a player for whom $\hat{p}_1 m^1_2(\hat{p}_1) \geq \hat{p}_2 m^2_1(\hat{p}_2)$ holds.*

In case of equal maximized extended Nash products, any preferential treatment can be excluded by giving the label 1 to each player with equal probability.

Definition 4. *Player s is strong and player w is weak for claims p which are not strictly compatible*

$$\begin{array}{ll} (i) & \text{if } p_s m^s_w(p_s) > p_w m^w_s(p_w), \\ (ii) & \text{if } p_s m^s_w(p_s) = p_w m^w_s(p_w) \text{ with } s = 1, w = 2. \end{array}$$

The characterization of the strong and weak player allows us to combine Harsanyi's approach with an emphasis on the importance of a strong bargaining position with Moulin's approach with an emphasis on the competition for leadership.[3] As in Harsanyi, the strong player's strategic advantage is driven by the first-mover advantage of the player bidding lower resistance probability when maximal revisions are incompatible. By bidding a resistance probability in between $\rho_s(p)$ and $\rho_w(p)$ in case (i) of Definition

[3] We always explicitly mention the claims p for which one of the players is given the label s. The player s who is strong for p may remain strong or may become weak for other claims.

4, the strong player is strong enough to impose his maximal revision as a leader. If the weak player becomes leader by underbidding the strong player, the weak player's minimal resistance probability is too high for imposing his maximal revision. As a leader, the weak player proposes a compromise within the strong player's set of feasible compromises, which he prefers to the strong player's ultimatum, a lottery with as prizes the disagreement outcome and the outcome in which the strong player obtains his unrevised claim.

In case (*ii*) of Definition 4, $\rho_1(p) = \rho_2(p)$. By the labeling of the players, $s = 1$ for p. By the rule assigning leadership, $L(q) = 1$ for $q_1 \leq q_2$ and $L(q) = 2$ for $q_2 < q_1$. If maximal revisions are incompatible for claims with equal extended Nash products, then only player 1's maximal revision can be implemented. However, player 2 can undo player 1's advantage and become strong by any small reduction of $p_2 > \hat{p}_2$ or by claiming $\hat{p}_2$, unless player 1 has a claim for which he is strong for all claims of player 2 when $\hat{p}_1 m_2^1(\hat{p}_1) > \hat{p}_2 m_1^2(\hat{p}_2)$. In that case, existence of an equilibrium in the formulation of claims requires that a tie ($q_1 = q_2$) is resolved in favor of player 1.

If the proportional solution is a feasible compromise, any player can make sure that it is implemented by Moulin's maxmin bidding strategy of resistance. Since both players propose the same compromise as a leader, leadership and the labeling of the players does not matter. The proportional solution is a feasible compromise only if it is weakly preferred by the strong player to his maximal revision. This follows from

$$q_i^{KS}(p) \geq \rho_i(p) \text{ iff } u^{KS}(p) \in C_i(p_i) \text{ for } p \notin S \setminus PO(S). \tag{2}$$

an immediate implication of (1), the monotonicity of $r_{-i}(., p_{-i})$ and $q_1^{KS}(p) = q_2^{KS}(p)$.

It follows that acquiring a strong bargaining position is valuable only if $\rho_w(p) \geq \rho_s(p) > q_s^{KS}(p)$ for incompatible maximal revisions. In that case, the strong player prefers his maximal revision to the proportional solution so that $q_s^{KS}(p) > r_s(m_s^s(p_s), p_s)$. If the weak player leads for $q_w \in [r_s(m_s^s(p_s), p_s), \rho_s(p)]$, he is strong enough to impose $m^s(p_s)$, his preferred outcome in $C_s(p_s)$. If $s = 1$ and $q_s \in [\rho_s(p), \rho_w(p)]$, the weak player leads for $q_w < q_s \leq \rho_w(p)$ and is never strong enough to impose $m^w(p_w)$ as a leader. The strong player is just strong enough to impose $m^s(p_s)$ when he leads for $q_s = \rho_s(p)$. In both cases, $m^s(p_s)$ is implemented.

Lemma 1. *If u is the solution in Γ for $m \in \mathcal{M}$ in a subgame in which the claims are not strictly compatible, then u is the proportional solution if it is a feasible compromise and the strong player's maximal revision otherwise.*

Proof. *See appendix.* ■

4.2 Formulating Claims: a Simple Demand Game

We now characterize the equilibrium claims in the first stage. By Lemma 1, the relevant set of compromises for claims which are not strictly compatible is

$$\hat{C}_i(p) \equiv \left\{m^i(p_i), u^{KS}(p)\right\} \text{ for } i \in N.$$

If claims p are strictly compatible, player 1 is sure to be leader for $q_1 = 0$ and player 2 accepts $\left(u_1^P(p_2), p_2\right)$ which gives him the payoff of his ultimatum. Hence, the four-stage mechanism Γ for the $m \in \mathcal{M}$ can be reformulated as a two-stage mechanism $\hat{\Gamma}$ à la Nash's demand game:

- **Stage** a**.** All $i \in N$ formulate claims $p \in D$.
- **Stage** b**.**
  - If claims are strictly compatible, player 1 selects $\left(u_1^P(p_2), p_2\right)$.
  - If claims are not strictly compatible, the strong player s for the claims p selects an allocation in $\hat{C}_s(p)$.

Formulating strictly compatible claims will not occur in equilibrium because player 2 has a profitable deviation. For claims which are not strictly compatible, each player faces the trade-off between increasing his claim - which increases his own payoff as the strong player - and reducing his claim - which may make him strong and which is valuable only if the proportional solution is not a feasible compromise. For p' such that $p'_i \geq p_i$ and $p'_{-i} = p_{-i}$, the following two inequalities determine the equilibrium claims,

$$\min_{c \in \hat{C}_{-i}(p)} c_i \leq \max_{c \in \hat{C}_i(p)} c_i \leq \max_{c \in \hat{C}_i(p')} c_i. \tag{3}$$

By the first inequality of (3), player i prefers selecting his preferred allocation in $\hat{C}_i(p)$ to leaving this choice to his opponent in $\hat{C}_{-i}(p)$. The first-mover advantage $\max\left\{m_i^i(p_i), u_i^{KS}(p)\right\} - \min\left\{m_i^{-i}(p_{-i}), u_i^{KS}(p)\right\}$ is strictly positive, unless the proportional solution is feasible for compatible maximal revisions and the identity of the leader does not matter. By the second inequality of (3), the strong player gains by increasing his claim as long as he remains strong, because $\max\left\{m_i^i(p_i), u_i^{KS}(p)\right\}$ is strictly increasing in p_i for a comprehensive revision procedure.

Lemma 2. *If u is the solution in Γ for $m \in \mathcal{M}$ in subgame-perfect equilibrium, then the equilibrium claims $p \geq \hat{p}$ are not strictly compatible. If $\bar{u}$ is the solution in a subgame with claims $\bar{p} \geq \hat{p}$ for which player i is strong, then $\bar{u}_i \geq \max\left\{m_i^i(p'_i), u_i^{KS}(p')\right\}$ for $p'_i \leq \bar{p}_i$ and $p'_{-i} = \bar{p}_{-i}$. Moreover, if player i is strong for any claim of player $-i$ when claiming $\bar{p}_i$, then $p_i \geq \bar{p}_i$ in subgame-perfect equilibrium.*

4.3 Implementing Bargaining Solutions

If the first weak inequality of (3) is satisfied with equality for $\hat{C}_1(p) = \hat{C}_2(p) = \{u^{KS}(p)\}$, then $u^{KS}(p) = m^1(p_1) = m^2(p_2)$ is implemented by Lemma 1. This is the case with two exceptions.

If the maximal revisions of claims are incompatible, competition for the strong bargaining position induces restraint in the formulation of claims, unless one player has a claim for which he is strong for all claims of the weak player. Hence, as in subcase (a) or (b) in case (ii) of Proposition 3, this player imposes his maximal revision for the largest of such claims. Otherwise, at least one of the players would gain by closing the gap between the maximal revisions or by making them compatible, which cannot occur in equilibrium.

If the maximal revisions of claims are compatible, the proportional solution is a feasible compromise. The competition for leadership equalizes the player's resistance probabilities by adopting Moulin's maxmin bidding strategies. By the monotonicity of the proportional solution, each player gains by increasing his claim for strictly compatible maximal revisions, unless the proportional solution is a feasible compromise for the maximal claims of the players, as in case (i) of Proposition 3. In that case, the Kalai-Smorodinsky solution is implemented.

If case (i) and (ii) do not hold, the maximal revisions meet for claims with equal extended Nash products and players are equally strong, as in Harsanyi. But the concessions stand also in the same proportion to the claims, as in Moulin. The Pareto-efficient frontier of the reduced bargaining problem $S(p)$ is the union of the Pareto-efficient frontiers of the compromise sets $C_1(p_1)$ and $C_2(p_1)$ which meet in $u^{KS}(p)$.

Proposition 3. *The solution u in Γ for $m \in \mathcal{M}$ in subgame-perfect equilibrium is unique. The solution u and the claims p are uniquely defined by*

$$u = m^1(p_1) = m^2(p_2) = u^{KS}(p),$$

with the following exceptions:
(i) *if* $u^{KS}(\mathbf{1}) \in C(\mathbf{1})$,
$$\text{then } u = u^{KS}(\mathbf{1}) \text{ and } p = \mathbf{1},$$
(ii) *if* $u^{KS}(\mathbf{1}) \notin C(\mathbf{1})$,
(a) *if* $m_2^1(1) \geq m_1^2(\hat{p}_2)\hat{p}_2$,
$$\text{then } u = m^1(1) \text{ and } m_1^1(1) \geq u_1^{KS}(p) \text{ for } p_1 = 1,$$
(b) *if* $m_2^1(1) < m_1^2(\hat{p}_2)\hat{p}_2 = m_2^1(\check{p}_1)\check{p}_1$ *and* $C_1(\check{p}_1) \cap C_2(\hat{p}_2) = \emptyset$,
$$\text{then } u = m^1(\check{p}_1) \text{ and } p = (\check{p}_1, \hat{p}_2),$$
(c) *if* $m^s(\tilde{p}_s) = m^w(\tilde{p}_w) = u^{KS}(\tilde{p})$ *and s is strong for $\tilde{p}$ and* $\tilde{p}_s = 1$,
$$\text{then } u = m^s(1) \text{ and } m_s^s(1) \geq u_s^{KS}(p) \text{ for } p_s = 1.$$

Proof. *See appendix.* ■

The equilibrium claims are uniquely defined with the exception of subcases (a) and

(c) of Proposition 3. If the strong player weakly prefers the maximal revision of his maximal claim to the proportional solution for some claim of the weak player which is not maximal, then this will also be the case for all larger claims of the weak player.

4.4 Unilateral Extension of Room to Compromise

We show that a negotiator cannot loose in equilibrium by facing fewer restrictions regarding the revisions he can make for identical maximal revision of his maximal claim. Consider the revision procedure $(\breve{m}^1, \breve{m}^2) \in \mathcal{M}$ giving larger payoffs to player 2 in the maximal revisions of player 1 than in $(m^1, m^2) \in \mathcal{M}$ for claims below player 1's maximal claim. That is, player 1 is given more room to compromise in $\breve{m}$ than in m. If $u = m^1(p_1) = m^2(p_2)$ or $u = m^1(\breve{p}_1)$ in the equilibrium with m, then player 1's extended Nash product is larger than 2's in $\breve{m}$ for the same p. Since extended Nash products are unimodal and $p \geq \hat{p}$, equality of the extended Nash products can be restored only for a higher claim of player 1 and a lower claim of player 2. In that case, the player giving himself more room to compromise is rewarded.

Corollary 1. *Assume that the solution u for p and the solution $\breve{u}$ for $\breve{p}$ are respectively implemented in subgame-perfect equilibrium for $m = (m^1, m^2) \in \mathcal{M}$ and for $\breve{m} = (\breve{m}^1, \breve{m}^2) \in \mathcal{M}$ in Γ. If $m^2 = \breve{m}^2$, $m_2^1(1) = \breve{m}_2^1(1)$ and $m_2^1(p_1) \leq \breve{m}_2^1(p_1)$ for all $p_1 \in [0,1]$, then $\breve{u}_1 \geq u_1$ and $\breve{p}_1 \geq p_1$.*

Proof. *See appendix.* ■

5 Examples

We characterize the solutions for specific revision procedures and show how these vary when the revision procedures are adjusted in Example 1 and 2. We relate revision procedures to the literature in Example 3.

Example 1: Piecewise-Linear Revision Procedure. For $c \in PO(S)$, $a \geq \underline{a}^c$ and $i \in N$, let

$$m_{-i}^{c,a,i} : [0,1] \to [0,1] : p_i \mapsto \min\left\{\frac{a}{c_i} - \frac{c_{-i}}{c_i}p_i, 1\right\}.$$

For a comprehensive revision procedure $(m^{c,a,1}, m^{c,a,2})$, $m_{-i}^{c,a,i}$ is tangent to S for $a = \underline{a}^c$ and for $i \in N$.

We show that c is the solution in Γ for $(m^{c,a,1}, m^{c,a,2})$ if the exceptions of Proposition 3 do not hold. Assume that $c_i \geq c_{-i}$ and $c_{-i} + c_1c_2 = \tilde{a}_i^c \geq \underline{a}^c$. For $a \in [\underline{a}^c, \tilde{a}_i^c]$, consider the claims

$$p^c = \left(\frac{a - c_1c_2}{c_2}, \frac{a - c_1c_2}{c_1}\right) \leq \mathbf{1}.$$

These claims p^c define maximal revisions which meet in c, that is, $c = m^{c,a,1}(p_1^c) =$

$m^{c,a,2}(p_2^c)$. Moreover, $c_1/p_1^c = c_2/p_2^c$ for $c \in PO(S)$ implies that $c = u^{KS}(p^c)$.[4] By Proposition 3, c is the solution.

The piecewise-linear revision procedure implies that the marginal loss of player $-i$ in player i's maximal revision of an increased claim is equal to the constant c_{-i}/c_i. This ratio, measuring the marginal reduction in the room to compromise of larger claims, uniquely identifies the solution for all a in $[\underline{a}^c, \tilde{a}_i^c]$. Increasing a on $[\underline{a}, \tilde{a}_i^c]$ increases $p_i - m_i^{c,a,i}(p_i)$ without changing the solution.

The Kalai-Smorodinsky solution is implemented for $c_{-i}/c_i = 1$. Player i's payoff is increased as this ratio is reduced below 1 above a threshold $\tilde{c}_{-i}^i/\tilde{c}_i^i$ for which $\tilde{a}_i^{\tilde{c}^i} = \underline{a}^{\tilde{c}^i}$. In this way, all elements belonging to $\cup_{i\in N}\left\{c \in PO(S) | \, u_i^{KS}(\mathbf{1}) \leq c_i \leq \tilde{c}_i^i\right\}$ are implementable by revision procedures which have a point of tangency with S. The Nash solution is implemented for $\underline{a}^c = 2c_1c_2$ when $p^c = (c_1, c_2) = u^N$ is a point of tangency between the revision procedure and S.[5] Myerson (1991) characterized the Nash solution by this property in Theorem 8.2.[6]

Finally, consider two revision procedures $\left(m^{c,\breve{a},1}, m^{c,a,2}\right)$ and $\left(m^{c,a,1}, m^{c,a,2}\right)$. If $\breve{a} > a$ and if the exceptions of Proposition 3 do not hold for both revision procedures, then player 1's equilibrium payoff in the former is increased and player 2's is reduced by Corollary 1. □

Example 2: Revision Procedures by Scalar Multiplication of S. For $\nu \geq 1$ and for $i \in N$, let

$$m_{-i}^{\nu,i} : [0,1] \to [0,1] : p_i \mapsto \sup\left\{p_{-i} | \, p \in (\nu S) \cap D\right\}.$$

Scalar multiplication of S yields a family of nested comprehensive revision procedures $\left(m^{\nu,1}, m^{\nu,2}\right)$, for which $m_{-i}^{\nu,i}(p_i)$ is constant or continuously increasing in ν for fixed p_i and for $i \in N$.

Let u^ν be the solution and p^ν be the claims which are uniquely defined by $u^\nu = m^{\nu,1}(p_1^\nu) = m^{\nu,2}(p_2^\nu) = u^{KS}(p^\nu)$ when the exceptions of Proposition 3 do not hold. For $\nu = 1$, no revisions are allowed as in Γ^N and the Nash solution is implemented for $p^\nu = u^N$. By increasing ν above 1, obtain u^ν and p^ν as continuous functions of ν, until one of the exceptions of Proposition 3 holds. For $\nu \geq \bar{\nu}$, the maximal revisions for maximal claims are compatible and the Kalai-Smorodinsky solution is implemented. However, before $\bar{\nu}$ is reached, player 1 may impose his maximal revision for his maximal claim for $\nu \in [\tilde{\nu}, \bar{\nu}]$. In that case, $m_2^{\nu,1}(1)$ increases continuously in ν on $[\tilde{\nu}, \bar{\nu}]$ from

[4] For p^c, $m_{-i}^{c,a,i}(p_i^c) = \frac{a_i}{c_i} - \frac{c_{-i}}{c_i}\frac{a - c_1c_2}{c_{-i}} = c_{-i}$ and $\frac{c_1}{p_1^c} = \frac{c_2}{p_2^c} = \frac{c_1c_2}{a - c_1c_2}$.

[5] Remark that $c_{-i} + c_1c_2$ decreases when c_{-i}/c_i is reduced and player i prefers $c \in PO(S)$ to u^N.

[6] If $a \geq \bar{a}^c$, case (i) of Proposition 3 holds and $u^{KS}(\mathbf{1})$ is implemented. If $a \in [\tilde{a}_i^c, \bar{a}^c]$, subcase (a) of case (ii) of Proposition 3 holds. Player i imposes the maximal revision of his maximal claim and his payoff is gradually reduced for larger a to $u_i^{KS}(\mathbf{1})$. Increasing a on $[\tilde{a}_i^{\tilde{c}^i}, \bar{a}^c]$ recovers all solutions in $\left\{c \in PO(S) | \, u_i^{KS}(\mathbf{1}) \leq c_i \leq \tilde{c}_i^i\right\}$. Unlike the extensive form analyzed in Miyagawa (2002), only a subset of Pareto-efficient allocations are implemented in Γ for $\left(m^{c,a,1}, m^{c,a,2}\right)$.

$m_2^{\tilde{\nu},1}(1)$ to $u_2^{KS}(\mathbf{1})$. Hence, all the Pareto-efficient allocations in between the Nash and the Kalai-Smorodinsky solution are implemented for some $\nu \in [1, \bar{\nu}]$.

In a symmetric bargaining problem, $u^{\nu} = u^{KS}(\mathbf{1}) = u^N$ is the unique solution for any ν. In a non-symmetric bargaining problem, solutions outside the Nash and the Kalai-Smorodinsky solution may be implemented and the move from the former to the latter is not necessarily monotone.[7]

Finally, consider two revision procedures $\left(m^{\breve{\nu},1}, m^{\nu,2}\right)$ and $\left(m^{\nu,1}, m^{\nu,2}\right)$. If $\breve{\nu} > \nu$ and if the exceptions of Proposition 3 do not hold for both revision procedures, then player 1's equilibrium payoff in the former is increased and player 2's is reduced by Corollary 1.□

Example 3: Accountability and Concession Aversion. We conclude the examples by relating revision procedures to constraints on revisions discussed in the literature. For example, Crawford (1982) refers to costs for negotiators who retreat on a position that they have agreed to defend. If negotiators are agents defending interests of their principal, they have limited authority and are accountable to their principals. If revising targets must be justified, revisions of claims will be limited. Kahneman and Tversky (1995), however, argue that revisions are limited by concession aversion. This may arise not only because claims raise unfulfilled expectations, but also because one's opponent gains in a disproportionate way.

Hence, each restriction k on the revision of player i's claims, $k = 1, ..., K_i$, puts bounds in two ways. Either, a claim p_i bounds the player's revised payoff by $\underline{b}_i^k(p_i)$ from below, so that $p_i - \underline{b}_i^k(p_i) \geq 0$ is the maximal loss he can bear. Or, a claim p_i bounds the opponent's revised payoff by $\bar{b}_{-i}^k\left(u_{-i}^P(p_i)\right)$ from above, so that $\bar{b}_{-i}^k\left(u_{-i}^P(p_i)\right) - u_{-i}^P(p_i) \geq 0$ is the maximal gain of his opponent he can tolerate. It follows that, $m_{-i}^i(p_i) \leq b_{-i}^k(p_i)$, where $b_{-i}^k(p_i) = u_{-i}^P(\underline{b}_i^k(p_i))$ in the former or $b_{-i}^k(p_i) = \bar{b}_{-i}^k\left(u_{-i}^P(p_i)\right)$ in the latter. Hence, for non-increasing concave b_{-i}^k,[8]

$$m_{-i}^i(p_i) = \min\left\{b_{-i}^1(p_i), ..., b_{-i}^{K_i}(p_i), 1\right\} \in \mathcal{M}.$$

Again, as in Corollary 1, a negotiator never looses by facing fewer restrictions regarding the revisions of claims below his maximal claim. The more a negotiator is susceptible to feelings of frustration from unfulfilled expectations, the less proficient he will be in negotiating. If a negotiator acts as an agent of a principal, the higher his fear of disappointing his principal, the less ambitious the targets set by his principal and the

[7] The solution u^{ν} of a revision procedure with scalar multiplication belongs to $\cup_{i \in N}\left\{c \in PO(S) | u_i^{KS}(\mathbf{1}) \leq c_i \leq \tilde{c}_i^i\right\}$ because the piecewise-linear revision procedure of Example 1 which connects $(p_1^{\nu}, m_2^{\nu,1}(p_1^{\nu}))$ and $(m_1^{\nu,2}(p_2^{\nu}), p_2^{\nu})$ for $\nu > 1$ implements u^{ν}.

[8] If b_{-i}^k is a non-increasing concave function on $[0,1]$ for $k = 1, ...K$, then the pointwise infimum m_{-i}^i is also a non-increasing concave function on $[0,1]$. Since u_{-i}^P is a decreasing concave function, b_{-i}^k is a non-increasing concave function if $\bar{b}_{-i}^k$ is a non-decreasing concave function or $\underline{b}_i^k$ is a non-decreasing convex function.

less favorable the resulting agreement.□

6 Robustness

In this section, we justify some of the simplifying features of the mechanism Γ.

6.1 Player-specific revisions

We assumed that $C_1(p_1) \cup C_2(p_2)$ is the set of feasible compromises. That is, any opportunity for compromise available to one player is also available to the other player. We now consider player-specific revisions in $\tilde{\Gamma}$ when each player i must make compromises within his own set $C_i(p_i)$. This is the natural assumption in Example 3.

Player-specific revisions implement the same equilibrium allocation for the same revision procedure not only if $u^{KS}(\mathbf{1}) \in C_1(1) \cap C_2(1)$, but also when restraint in the formulation of claims for obtaining a strong bargaining position equalizes the extended Nash products. In the latter case, none of the players can impose a compromise that is better than his maximal revision. The restriction of player-specific revisions has no bite. When subcase (a) of case (ii) in Proposition 3 does not hold, the extended Nash products are equalized and $u = \tilde{u}$.[9]

When subcase (a) of case (ii) holds, the weak player can no longer propose in the strong player's set of feasible compromises and the strong player can impose compromises within his set of feasible compromises for his maximal claim. As in Nash demand games, the utility allocation implemented in subgame-perfect equilibrium is no longer unique. The solutions are all Pareto-efficient allocations $\tilde{u}$, giving utility not lower than $\hat{p}_2 m_1^2(\hat{p}_2)$ and not higher than $m_2^1(1)$.

Proposition 4. *Assume that $\tilde{u}$ is an allocation implemented in subgame-perfect equilibrium in $\tilde{\Gamma}$ with player-specific revisions for $m \in \mathcal{M}$. Then $\tilde{u} = u$, where u is the solution in Γ for m, unless $u^{KS}(\mathbf{1}) \notin C(1)$ and $m_2^1(1) > m_1^2(\hat{p}_2)\hat{p}_2$ when $\tilde{u} \in \left\{\bar{u} \in PO(S) | \, \hat{p}_2 m_1^2(\hat{p}_2) \leq \bar{u}_2 \leq m_2^1(1)\right\}$.*

Proof. *See appendix.* ■

6.2 Competition for leadership

We assumed that leadership is given to the player bidding the lower resistance probability against an uncompromising opponent who gives an ultimatum, as in Moulin's auction game. If leadership were given to the player with higher resistance probability, players could lead with maximal resistance against uncompromising followers. The follower would accept any compromise, including the leader's dictatorship, anticipating

[9] The bidding strategies may differ off the equilibrium path of the subgame-perfect equilibrium with player-specific revisions.

the disagreement outcome after rejection. Hence both players' bids and claims would be maximal in equilibrium.

Schelling (1956) discusses bargaining with ultimatums. He argues that adherence to a commitment - leaving the negotiation table empty-handed - must be motivated and communicated, so that it is recognized by the other party. In particular, "the process of commitment may be a progressive one, the commitments acquiring their firmness by a sequence of actions" (Schelling (1956), p. 296). In that case, competition of leadership with and without bidding of resistance yields the same outcome. The equivalence between bidding resistance probabilities and the gradual buildup of resistance is similar to the equivalence between the sealed-bid first-price and the Dutch auction. Assume that after making claims both players increase resistance, simultaneously and at the same pace, until one of the players stops and proposes a compromise. A player takes the lead as soon as he is confident that his resistance probability to an uncompromising opponent is sufficiently high to impose his compromise. The equilibrium strategies when players bid for leadership or when resistance is built up until one player takes the lead yield the same resistance probability.

6.3 Alternating offers

We finally discuss ultimatums when the uncompromising follower has the option of continuing negotiations with a counteroffer, as in the alternating-offer game of Rubinstein (1982). Even if leadership alternates exogenously, adding ultimatums as an option of stopping negotiations induces restraint in the resistance probabilities as in the four-stage mechanism with competition for leadership. After formulating claims in the first stage in the extensive form $\breve{\Gamma}$, players take turns in making proposals in their compromise set until one player accepts his opponent's proposal or pursues his claim in an ultimatum. In line with Rubinstein's game, we focus on the revision procedure for which all revisions are feasible in the mechanism $\breve{\Gamma}^M$. Hence players start with formulating maximal claims in order to maximize their payoffs as followers in an ultimatum. The progressive process of commitment to a resistance probability in the follower's ultimatum is assumed to be time-consuming. The higher the probability of disagreement, the longer it takes to convince one's opponent of one's firmness.[10] Player i's discounted payoff of the compromise c for the resistance probability q_i is $\exp(-\varepsilon q_i)c_i$ for some positive ε. Since delay is costly, the leader proposes as soon as he is confident that he can block an ultimatum.

For each element of a decreasing sequence of small positive ε, consider a compromise proposal $c^i(\varepsilon) \in PO(S)$ and resistance probability $q_i(\varepsilon)$ for each player $i \in N$. For

[10] "Be willing to walk away from or back to negotiations", the guideline for the negotiator referred to in the introduction, can be viewed as costly signalling of one's firmness to the follower. The commitment to a higher resistance probability also takes more time in persuading one's principal of the need to be firm. Remark that the leader's payoff in an ultimatum will be decreasing in the resistance probability only when restrictions on the revision of claims induce restraint in the formulation of claims.

the compromises to be proposed and accepted in equilibrium, one needs

$$c_i^{-i}(\varepsilon) = \exp(-\varepsilon q_i(\varepsilon))c_i^i(\varepsilon) = (1 - q_{-i}(\varepsilon)) \text{ for all } i \in N. \tag{4}$$

By the first equality in (4), accepting the opponent's offer $c^{-i}(\varepsilon)$ is as good as waiting for a time $\varepsilon q_i(\varepsilon)$ before proposing $c^i(\varepsilon)$ for all $i \in N$. For equal waiting times $\varepsilon q_i(\varepsilon) = \varepsilon q_{-i}(\varepsilon)$, the Nash products of the proposals are equal, as in Rubinstein's game. By the second equality in (4), accepting the opponent's compromise $c^{-i}(\varepsilon)$ is as good as stopping with an ultimatum, in which case the initial claim is obtained with probability $1 - q_{-i}(\varepsilon)$. Before player i can respond with a counterproposal or an ultimatum to the proposal $c^{-i}(\varepsilon)$, player $-i$ has built up a resistance probability $q_{-i}(\varepsilon) = r_i\left(c_i^{-i}(\varepsilon), 1\right)$ which deters an ultimatum, as in Γ^M.

By combining the equalities in (4) for both $i \in N$, it follows that

$$\frac{\ln c_2^2(\varepsilon) - \ln c_2^1(\varepsilon)}{\ln c_1^1(\varepsilon) - \ln c_1^2(\varepsilon)} = \frac{r_1\left(c_1^2(\varepsilon), 1\right)}{r_2\left(c_2^1(\varepsilon), 1\right)} = \frac{1 - c_1^2(\varepsilon)}{1 - c_2^1(\varepsilon)}.$$

For $\varepsilon \to 0$, $c^1(\varepsilon)$ and $c^2(\varepsilon)$ converge to c^*, which by l'Hopital's rule satisfies

$$-\left.\frac{d \ln c_2}{d \ln c_1}\right|_{c_2^* = u_2^P(c_1^*)} = \frac{1 - c_1^*}{1 - c_2^*}.$$

The lefthand side is increasing in c_1^* and equal to 1 for $c^* = u^N$. The righthand side is decreasing in c_1^* and equal to 1 for $c^* = u^{KS}(\mathbf{1})$. Hence, the compromise c^* lies strictly in between the Nash and the Kalai-Smorodinsky solution, unless both solutions coincide.

Assume that $u_i^N > u_i^{KS}(\mathbf{1})$. The introduction of ultimatums in Rubinstein's alternating offer game moves the equilibrium solution away from the Nash solution towards the proportional solution. When the Nash solution is proposed as a compromise, player $-i$'s risk limit when pursuing his maximal claim is greater than player i's risk limit since $u_i^N > u_{-i}^N$. Player i needs more time to build up the necessary resistance, so that his higher impatience inhibits him to obtain a compromise as good as u^N. In Rubinstein's game, the player's impatience is exogenously determined by the waiting time for making a counterproposal. In $\breve{\Gamma}^M$, the impatience of the players is endogenized by the choice of resistance. A player's impatience thus increases with his own payoff in his compromise proposal, as he requires a higher resistance to make this compromise acceptable.

Similarly, the introduction of alternating offers in an extension of the four-stage mechanism with unrestricted revisions moves the equilibrium solution away from the proportional solution towards the Nash solution. Since the solution to Γ^M implies equal risk limits, its implementation in $\breve{\Gamma}^M$ would imply equal waiting times between alternating offers equal to $\varepsilon q^{KS}(\mathbf{1})$. However, for short equal waiting times, a compromise is proposed and accepted only if the payoffs are close to those in the Nash solution. Proposing an offer which deters ultimatums is necessary but not sufficient for its acceptance with an option to counteroffers. The anticipation of counteroffers with ultimatums results in unequal waiting times. The player preferring the proportional solution to the Nash solution will make a proposal which his opponent prefers to the proportional solution. As this reduces the opponent's risk limit, he can reduce his resistance needed to block an ultimatum below $q^{KS}(\mathbf{1})$ and thus the time he must wait before making his proposal.

7 Conclusion

We analyzed a simple, intuitive mechanism that implements a unique solution to the bargaining problem with two players. The mechanism introduces ultimatums and the need to build resistance or to revise claims in a compromise in order to discourage negotiators to give ultimatums. We generate a whole family of solutions by varying the extent to which claims can be revised during the negotiations. The Nash solution is the unique equilibrium solution, if negotiators cannot revise claims. The ability to revise claims was assumed to be beyond the control of the negotiators in the course of negotiations. If a player has a claim for which he is strong for all claims of his opponent, then he gains by reducing the room for compromise for his maximal claim without jeopardizing his strong bargaining position. However in all other cases, if a negotiator were to suppress his feelings of frustration or if he did not fear to disappoint his principal by making large concessions, he would achieve better deals. In the evaluation of the performance of a negotiator, results loom larger than circumstances under which his results were achieved. Hence, it seems plausible that professional negotiators will strive for more room to maneuver. Similarly, principals will learn by experience to give discretionary power to their negotiators as to decide which concessions have to be made. If restrictions on revisions of claims other than maximal claims are loosened in conflicts between experienced negotiators, the predicted allocation would be the Kalai-Smorodinsky solution.

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8 Appendix

8.1 The subgame-perfect equilibrium

Let $\sigma = (\sigma_1, \sigma_2)$ be a strategy profile in Γ for $m \in \mathcal{M}$. The history $h^{\tau-1} \in \mathcal{H}^{\tau-1}$ at stage $\tau = 1, ..., 4$ is recursively defined by $h^\tau = \left(a^\tau, h^{\tau-1}\right)$ and $h^0 \in \emptyset$, where $a^1 = p \in D$, $a^2 = q \in D$, $a_L^3 = c \in C\left(p\right)$ and $a_F^4 \in$\{Y,N\}. The strategy of player i at stage τ in the subgame for the history $h^{\tau-1}$ in σ is denoted by $a_i^{\tau,\sigma}(h^{\tau-1})$. We denote by σ the strategy profile in subgame-perfect equilibrium in Γ.

Assuming that F accepts a compromise in a tie when $c_F = (1 - q_L)p_F$, by the definition of the risk limit for $h^3 \in \mathcal{H}^3$,

$$a_F^{4,\sigma}(h^3) = \begin{cases} \text{Y if } q_L \geq r_F(c_F, p_F) \text{ and } p_F > 0, \\ \text{N otherwise.} \end{cases}$$

The leader L proposes the compromise $c \in C\left(p\right)$, which is accepted by F and gives L the largest payoff, so that for $h^2 \in \mathcal{H}^2$,

$$a_L^{3,\sigma}\left(h^2\right) \in \arg \max_{u \in C(p)} \left\{ c_L | \, a_F^\sigma(c, h^2) = \text{Y} \right\}.$$

Since $r_F(., p_F)$ is decreasing, $c = a_L^{3,\sigma}\left(h^2\right) \in PO\left(S\right)$ for $q_L = r_F(c_F, p_F)$. The choices $a^{2,\sigma}(h^1)$ for $h^1 \in \mathcal{H}^1$ and $a^{1,\sigma}(h^0)$ in subgame-perfect equilibrium for Γ^R are given in Lemma 1, Lemma 2 and Proposition 3.

8.2 Proofs of Propositions

Proof of Proposition 1.

The mechanism Γ^M belongs to the class of mechanisms considered in Proposition 3. In Γ^M, $C_1\left(1\right) = C_2\left(1\right) = D$, so that $u^{KS}\left(\mathbf{1}\right) \in C\left(1\right)$ for $i \in N$ and that case (i) of Proposition 3 holds. We refer to the proof of the first case of Lemma 1 to show that players make the bids $q = q^{KS}\left(p\right)$ and the first case in Proposition 3 showing that $u^{KS}\left(\mathbf{1}\right)$ is implemented in subgame-perfect equilibrium.■

Proof of Proposition 2.

The mechanism Γ^N belongs to the class of mechanisms considered in Proposition 3. In Γ^N, $m_{-i}^i\left(p_i\right) = u_{-i}^P\left(p_i\right)$ for $i \in N$. The extended Nash product of a claim of a player in Proposition 3 is equal to the Nash product in that case. It is maximized for $\hat{p}_i = u_i^N$ for $i \in N$ and the maximized values are equal for the two players. For $p = \hat{p} = u^N$, $u^N = m^1(p_1) = m^2(p_2) = u^{KS}(p)$. We refer to the proof of Lemma 1 for the bidding strategies with $m_1^1 = p_1$ and to the fourth case in Proposition 3 showing that u^N is implemented in subgame-perfect equilibrium.■

Proof of Proposition 3.

We distinguish between four solutions in subgame-perfect equilibrium implemented in one of the following four cases.

In the first case, exception (i) of Proposition 3 holds. The proportional solution is a feasible compromise for $p = \mathbf{1}$ and, by Lemma 1, $u^{KS}(\mathbf{1})$ is implemented. Let s be the strong player for $p = \mathbf{1}$. By Lemma 2, $u_s^{KS}(\mathbf{1})$ for $p_w = 1$ is a lower bound for s's payoff. By its monotonicity, the proportional solution would remain feasible and would be implemented by Lemma 1 for a lower claim of player w, but would reduce w's payoff. For $p_w = 1$, the payoff of player s is bounded from above by $u_s^{KS}(\mathbf{1})$. Hence, $u = u^{KS}(\mathbf{1})$ is the unique solution for $p = \mathbf{1}$ in the first case. In the remaining cases, the proportional solution is not feasible for $p = \mathbf{1}$.

In the second case, subcase (a) or (c) of (ii) of Proposition 3 holds. In subcase (a), player 1 is strong for his maximal claim and obtains the payoff $\max\left\{u_1^{KS}(1, p_2), m_1^1(1)\right\}$ by Lemma 1. Since he is strong for $p_1 = 1$ and for all claims of player 2, his claim is maximal in equilibrium by Lemma 2. In subcase (c), player s is strong for $\tilde{p}_s = 1$ and the claim $\tilde{p}_w$. He obtains the payoff $\max\left\{u_s^{KS}(\tilde{p}), m_s^s(1)\right\}$ by Lemma 1, which is a lower bound of his payoff for $\tilde{p}_w$ by Lemma 2. Remark that the conditions $m^s(1) = m^w(\tilde{p}_w) = u^{KS}(\tilde{p})$ uniquely define $\tilde{p} \geq \hat{p}$ by the properties of the proportional solution. In both subcases, $m^s(1)$ is implemented iff $m_s^s(1) \geq u_s^{KS}(p)$ for $p_s = 1$. The proportional solution remains feasible and would be implemented for claims below p_w of the weak player, but would reduce his payoff below $m_w^s(1)$. Hence $u = m^s(1)$ is the unique solution implemented when the maximal revisions are incompatible or meet for $p_s = 1$ in the second case.

In the third case, condition (b) of (ii) of Proposition 3 holds. The extended Nash products are equal for $p = (\check{p}_1, \hat{p}_2)$, so that player 1 is strong for p. Since the proportional solution is not feasible for p, $m^1(\check{p}_1)$ is implemented by Lemma 1. Since $p_2 = \hat{p}_2$, player 1 remains strong for $\check{p}_1$ and all claims of player 2, so that player 1 never claims less than $\check{p}_1$ by Lemma 2. Remark that player 2 becomes strong for $\hat{p}_2$ and any claim of player 1 exceeding $\check{p}_1$. Player 1's payoff cannot be improved upon for the claim $\hat{p}_2$. Hence, $u = m^1(\check{p}_1)$ is the unique solution for $\check{p}_1$ and $\hat{p}_2$ in the third case.

In the fourth case, the exceptions of Proposition 3 do not hold and there exists $(\check{p}_1, \hat{p}_2)$ defining equal extended Nash products for which $C_1(\check{p}_1) \cap C_2(\hat{p}_2) \neq \emptyset$. If $C_1(p_1) \cap C_2(p_2) \subseteq C_1(\check{p}_1) \cap C_2(\hat{p}_2)$ and player s is strong for $p \geq \hat{p}$, his payoff is equal to $\max\left\{u_s^{KS}(p), m_s^s(p_s)\right\}$ by Lemma 1, which is a lower bound for the payoff of player s for p_w by Lemma 2. This lower bound is strictly decreasing in p_w if $u_s^{KS}(p) > m_s^s(p_s)$ by the monotonicity of the proportional solution. This lower bound cannot be reduced and player w cannot gain by increasing his claim as the weak player iff $C_1(p_1) \cap C_2(p_2) = \{u^{KS}(p)\}$, implying that in the solution $m^1(p_1) = m^2(p_2) = u^{KS}(p)$ for claims defining equal extended Nash products. These conditions uniquely identify $p \geq \hat{p}$ by the properties of the proportional solution. None of the players can gain by changing his claim. For a larger claim, the other player is strong and implements his maximal revision without changing the utility allocation. For a lower claim, his payoff is reduced in the proportional solution, which remains feasible and would be implemented. Hence, $u = m^1(p_1) = m^2(p_2) = u^{KS}(p)$ is the unique solution in the fourth case.■

Proof of Proposition 4.

Assume that condition (a) of (ii) of Proposition 3 holds. In the subgame for the claims $p = (1, \hat{p}_2)$, both players bid equal resistance probabilities in $[\rho_1(p), \rho_2(p)]$ and player 1 proposes $c \in C_1(1)$ such that $c_2 = (1 - q_1)\hat{p}_2$ which is as good as player 2's ultimatum. If $q_1 = \rho_2(p) = 1 - m_1^2(\hat{p}_2)/p_1$, then $c_2 = m_1^2(\hat{p}_2)\hat{p}_2$ bounds player 2's payoff from below. If $q_1 = \rho_1(p)$, $m^1(1)$ is imposed, which bounds player 1's payoff from below. There are no profitable deviations. Player 1 remains leader and proposes the same compromise for a higher resistance probability of player 2. Player 2 would lead if he lowers his or if player 1 increases his resistance probability. Either $q_2 = q_1 = \rho_2(p)$, $m^2(\hat{p}_2)$ is implemented and player 1 looses by increasing q_1. Or player 2 is unable to impose a compromise in $C_2(\hat{p}_2)$ and player 1 gives his ultimatum, giving a zero payoff to player 2 and, by concavity of u_1^P, $(1 - q_1)u_1^P(\hat{p}_2) \leq u_1^P((1 - q_1)\hat{p}_2) = u_1^P(c_2)$ as payoff to player 1. Player 1 remains leader by lowering q_1, but can only impose compromises with higher payoff for player 2. Hence for the claims p, all proposals $\tilde{u}$ for which $\tilde{u}_2 \in [m_1^2(\hat{p}_2)\hat{p}_2, m_2^1(1)]$ can be implemented in subgame-perfect equilibrium. Any claim $p_1' < 1$ of player 1 would reduce the lower bound $m_1^1(1)$ on his payoff for $q_1 = q_2 = \rho_2(p_1', \hat{p}_2)$. Any other claim than $\hat{p}_2$ of player 2 would reduce the lower bound $m_1^2(\hat{p}_2)\hat{p}_2$ of player 2. Hence, p are the equilibrium claims when condition (a) of (ii) of Proposition 3 holds.

Assume that condition (a) of (ii) of Proposition 3 does not apply. Then either $u^{KS}(\mathbf{1}) \in C_1(1) \cap C_1(1)$ for $p = \mathbf{1}$ or $\rho_2(p) = \rho_1(p)$ in Γ for $m \in \mathcal{M}$ and $\tilde{u} = u$ can be implemented in $\tilde{\Gamma}$ for m. Since $\tilde{u} \in PO(S)$, these lower bounds cannot be improved upon.■

8.3 Proof of the Lemma's

Proof of Lemma 1.

For any subgame with claims p which are not strictly compatible, $u^{KS}(p)$ is well defined. By definition, $q_1^{KS}(p) = q_2^{KS}(p) = r_i(u_i^{KS}(p), p_i)$ and $u_i^{KS}(p)/p_i = 1 - q_i^{KS}(p)$ for $i \in N$. A proposal c of L is proposed and accepted for q_L if and only if $q_L \geq r_F(c_F, p_F)$. By the monotonicity of $r_F(., p_F)$, F rejects c' if he strictly prefers c

to c'. We derive the bids in subgame-perfect equilibrium for any subgame for p. We distinguish between two cases when s is strong for p.

In the first case, $u_s^{KS}(p) \geq m_s^s(p_s)$, so that $u^{KS}(p) \in C_s(p_s) \subseteq C(p)$ and the proportional solution is feasible. We distinguish between two subcases.

In the first subcase, $u_s^{KS}(p) < p_s$, so that $r_i\left(u_i^{KS}(p), p_i\right) > 0$ and $u_i^P(p_{-i}) < u_i^{KS}(p) < p_i$ for $i \in N$. For the bidding $q = q^{KS}(p)$, $q_1 = q_2$. The allocation would remain unchanged for a higher bid of player $i \in N$, since player $-i$ would be the leader for q_{-i} and would propose $u^{KS}(p)$ which would be accepted by player i. The utility of a lower bidder i would be reduced. As a leader, either he proposes an acceptable offer which reduces his payoff by the monotonicity of $r_{-i}(., p_{-i})$ or he proposes an unacceptable offer yielding $(1-q_i)\, u_i^P(p_{-i}) \leq u_i^P(p_{-i}) < u_i^{KS}(p)$. Since no player has a profitable deviation, $q = q^{KS}(p)$ is an equilibrium for p. Moreover, player $i \in N$ ensures a payoff which is bounded below by $u_i^{KS}(p)$ for the bid $q_i = q_i^{KS}(p)$. Since $u^{KS}(p) \in PO(S)$, the lower bound for one player sets an upper bound on the payoff for the other player. Hence L proposes $u^{KS}(p)$, F accepts and both players bid $q_i^{KS}(p)$ in equilibrium for the claims p.

In the second subcase, $u_s^{KS}(p) = p_s$, so that $u_i^{KS}(p) = p_i$ and $r_i\left(u_i^{KS}(p), p_i\right) = 0$ for $i \in N$. If $u_w^{KS}(p) > m_w^w(p_s)$, $q_w = 0$ is the only way for w to avoid that s acquires leadership for $q_s > 0$ and makes a proposal in $C_w(p_w)$ which s would prefer to $u^{KS}(p)$ and which w would accept as a follower. Hence, $u^{KS}(p)$ is implemented for $q_w = 0$ and $q_s \in [0,1]$. If $u_w^{KS}(p) = m_w^w(p_s)$, that is $C(p) = \left\{u^{KS}(p)\right\}$, L has no other option than to propose $u^{KS}(p)$ and leadership is valuable for none of the players. Hence, $q_i \in [0,1]$ for $i \in N$ implements $u^{KS}(p)$. It follows that $u^{KS}(p)$ is implemented in equilibrium. Conclude that in the first case, the proportional solution is the unique solution implemented in equilibrium whenever it is feasible for claims p.

In the second case $m_s^s(p_s) > u_s^{KS}(p)$, so that $u^{KS}(p) \notin C_s(p_s)$ and $r_s(m_s^s(p_s), p_s) < q_1^{KS}(p) = q_2^{KS}(p)$. By (2), it follows that $\rho_w(p) \geq \rho_s(p) > q_s^{KS}(p)$, so that $r_s(m_s^s(p_s), p_s) < \rho_s(p)$ and $u^{KS}(p) \notin C_w(p_w)$. It follows that $u^{KS}(p) \notin C(p)$, so that the proportional solution is not feasible. We show that $m^s(p_s)$ is implemented for the equilibrium bids

$$
\begin{aligned}
q_w &\in [r_s(m_s^s(p_s), p_s), \rho_s(p)], \\
q_s &\in \begin{cases} [\rho_s(p), \rho_w(p)] & \text{if } s = 1, \\ [\rho_s(p), \rho_w(p)) & \text{if } s = 2. \end{cases}
\end{aligned}
$$

If $w = L$, then $q_w \leq q_s < \rho_w(p)$ or $q_2 < q_1 = \rho_2(p)$ and s rejects proposals in $C_w(p_w)$. As a result, w cannot do better than by proposing $m^s(p_s)$ in $C_s(p_s)$ which is accepted by s for $q_w \geq r_s(m_s^s(p_s), p_s)$. If $s = L$, then $q_s = \rho_s(p)$ for $q_w \leq \rho_s(p)$ implies that $m^s(p_s)$ is accepted by w and that any better proposal for s in $C_s(p_s) \setminus \{m^s(p_s)\}$, if any, is rejected by w. The payoff of player $i \in N$ is bounded below by $m_i^s(p)$ for these bids. Since $m^s(p) \in PO(S)$, the lower bound for one player sets an upper bound on the payoff for the other player. Hence, $m^s(p_s)$ is implemented for the bids q in equilibrium.

Remark that $m^s(p_s)$ would also be implemented for $q_w \in [0,1]$ if $C_s(p_s) = \{m^s(p_s)\}$ and s has no other choice than to propose $m^s(p_s)$ as a leader. We show that some player has a profitable deviation for all other bidding strategies. For $q_w < r_s(m_s^s(p_s), p_s)$, $w = L$ and proposes $u \in PO(C_s(p_s))$ for which $u_s = (1-q_w)p_s > m_s^s(p_s)$. For $q_w > \rho_s(p)$ and $C_s(p_s) \neq \{m^s(p_s)\}$, $s = L$ for $q_w > q_s > \rho_s(p)$ and can impose a preferred compromise in $C_s(p_s) \setminus \{m^s(p_s)\}$. For $q_s < \rho_s(p)$, $s = L$ when player w chooses $q_w = \rho_s(p)$ and player s must propose in $C_w(p_w)$. Finally, for $q_s > \rho_w(p)$ if $s = 1$ and $q_s \geq \rho_w(p)$ if $s = 2$, $w = L$ for $q_w = \rho_w(p)$ and can propose in $C_w(p_w)$. Hence, if any player were to change his bidding strategy, his payoff would be lower than the one in $m^s(p_s)$. We conclude that $m^s(p_s)$ is implemented in equilibrium when the proportional solution is not feasible for p.■

Proof of Lemma 2.

For strictly compatible claims p, $m_1^1(p_1) \leq p_1 < u_1^P(p_2) \leq m_1^2(p_2)$. Player 1 is leader by bidding $q_1 = 0$. For this bid, player 2's ultimatum and player 1's proposal $\left(u_1^P(p_2), p_2\right)$ are equivalent. Let player 2 bid $q_2 \in [0,1]$ if $m_2^2(p_2) = p_2$.and $q_2 = 0$ if $m_2^2(p_2) < p_2$. In the former, player 2 accepts $\left(u_1^P(p_2), p_2\right)$, player 1's preferred outcome in $C(p)$. In the latter, player 2 would be leader for $q_1 > 0$ and $\left(p_1, u_2^P(p_1)\right)$ would be implemented, reducing player 1's payoff. Hence, $\left(u_1^P(p_2), p_2\right)$ is implemented in a subgame with strictly compatible claims p. Formulating strictly compatible claims cannot occur in subgame-perfect equilibrium, since the strictly compatible claims p', $p_1' = p_1$ and $p_2' > p_2$ increase player 2's payoff and player 2 has a profitable deviation.

Assume that s is strong in the subgame for claims p which are not strictly compatible. By Lemma 1, u is the strong player's preferred option in $\hat{C}_s(p)$ for the claims p. By the monotonicity of the proportional solution and the comprehensiveness of the revision procedure, $\max\left\{m_i^i(p_i), u_i^{KS}(p)\right\}$ is strictly increasing in p_i. If $p_s < \hat{p}_s$, then by claiming $\hat{p}_s$, player s would remain strong and increase his payoff for given p_w. Since profitable deviations of one player are excluded, $p_s \geq \hat{p}_s$ in subgame-perfect equilibrium. If $p_w < \hat{p}_w$ and $u_s^{KS}(p) \geq m_s^s(p_s)$, then the proportional solution is implemented and player w could increase his payoff for a larger claim for given p_s. If $p_w < \hat{p}_w$ and $u_s^{KS}(p) < m_s^s(p_s)$, then $m^s(p_s)$ is implemented. By claiming $\hat{p}_w$, either player w becomes strong for p', $p_w' = \hat{p}_w$ and $p_s' = p_s$ and would obtain $\max\left\{m_w^w(\hat{p}_w), u_w^{KS}(p')\right\} \geq u_w^{KS}(p') > u_w^{KS}(p)$. Or player s remains strong for $\hat{p}_w$, $m^s(p_s)$ is implemented and $m_w^s(p_s)p_s \geq m_s^w(\hat{p}_w)\hat{p}_w > m_s^w(p_w)p_w$. Player s would remain strong and would gain for a claim larger than $p_s \geq \hat{p}_s$ for given p_w. Since profitable deviations of one player are excluded, $p_w \geq \hat{p}_w$ in subgame-perfect equilibrium.

Consider any subgame with claims $\bar{p} \geq \hat{p}$ implementing $\bar{u}$. If player s is strong for $\bar{p}$, he remains strong for $p_s' \in [\hat{p}_s, \bar{p}_s]$ and for $p_w' = \bar{p}_w$. Hence, $\bar{u}_s = \max\left\{m_s^s(\bar{p}_s), u_s^{KS}(\bar{p})\right\} \geq \max\left\{m_s^s(p_s'), u_s^{KS}(p')\right\}$. Moreover, if s is strong for $\bar{p}$ and for all claims of player w, then he will never claim less than $\bar{p}_s$ in subgame-perfect equilibrium.■

8.4 Proof of the Corollary

Proof of Corollary 1.

We distinguish between two cases. In the first case, condition (i) of Proposition 3 holds for m, so that $u = u^{KS}(\mathbf{1}) \in C(\mathbf{1})$. Since $C(\mathbf{1}) \subseteq \breve{C}(\mathbf{1})$, $u^{KS}(\mathbf{1}) \in \breve{C}(\mathbf{1})$ for $\breve{m}$. It follows that $\breve{u} = u$ and $\breve{p} = p = \mathbf{1}$. In the second case $u^{KS}(\mathbf{1}) \notin \breve{C}(\mathbf{1})$, so that $u^{KS}(\mathbf{1}) \notin C(\mathbf{1})$ and condition (i) of Proposition 3 does not hold for m as well as for $\breve{m}$. It suffices to consider three subcases. Remark that $\max_{p_1} \breve{m}_2^1(p_1)\, p_1 \geq m_2^1(\hat{p}_1)\, \hat{p}_1 \geq m_1^2(\hat{p}_2)\, \hat{p}_2$, so that the labeling of the players is the same for $\breve{m}$ as for m.

In the first subcase, subcase (a) of Proposition 3 holds for m. Since $m^1(1) = \breve{m}^1(1)$ and $m^2 = \breve{m}^2$, this subcase also holds for $\breve{m}$, so that $\breve{u}_1 = u_1$ and $\breve{p}_1 = p_1 = 1$.

In the second subcase, subcase (b) of Proposition 3 holds for m. Since $\breve{m}_2^1(\breve{p}_1)\, \breve{p}_1 \geq m_2^1(\breve{p}_1)\, \breve{p}_1 = m_1^2(\hat{p}_2)\, \hat{p}_2$, player 1 is strong for any claim of player 2 when claiming $\breve{p}_1$ for $\breve{m}$, so that $\breve{p}_1 \geq \breve{p}_1$ by Lemma 2 and $\breve{u}_1 \geq u_1$ when $\breve{m}^1(\breve{p}_1)$ is implemented

In the third subcase, the exceptions of Proposition do not hold or subcase (c) of (ii) holds. There exists p such that $m^1(p_1) = m^2(p_2) = u^{KS}(p)$ for m. Since the extended Nash products are equal, player 1 is the strong player. Since $\breve{m}_2^1(p_1) \geq m_2^1(p_1)$ and $\breve{m}^2 = m^2$, player 1 remains strong for p and for $\breve{m}$. By Lemma 2 for $\breve{m}$, player 1's payoff is not smaller than $\max\left\{\breve{m}_1^1(p_1), u_1^{KS}(p)\right\}$ for p_2. If $\breve{m}^1(p_1) = m^1(p_1)$, $\breve{u}_1 = u_1$ and there exist a claim for player 1 for which $\breve{p}_1 = p_1$. If $\breve{m}^1(p_1) > m^1(p_1)$, then $\breve{m}_1^1(p_1) > u_1^{KS}(p)$ and $\breve{m}_1^1(p_1)$ is implemented for p by Lemma 1. Since player 2 remains weak for larger claims than p_2, $\breve{u}_1 > u_1$. For $\breve{p}$, the equality of the extended Nash products must be restored, either for $\breve{p}_2 = \hat{p}_2$, as in the second subcase or for $\breve{m}^1(\breve{p}_1) = \breve{m}^2(\breve{p}_2) = u^{KS}(\breve{p})$. In both cases, $\breve{p}_1 > p_1$.■