Intransitivity without Zeno's Paradox

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Stuart Rachels and Larry Temkin have put forward a number of alleged counterexamples to the transitivity of the relation "all things considered better than".[1] Several of these cases share a common structure.[2] A representative example by Rachels is as follows:

Each outcome in [this] counterexample involves a single person's experience:

    A: 1 year of excruciating agony.

    B: 100 years of pain slightly (or somewhat) less intense than the pain in A.

    C: 10,000 years of pain slightly less intense than the pain in B.

    D: 1 million years of pain slightly less intense than the pain in C.

    . . .

    Y: 1×10^{48} years of pain slightly less intense than the pain in X.

    Z: 1×10^{50} years of pain slightly less intense than the mild pain in Y.

[…] Although A is worse than Z, the example creates a path from A to Z involving only changes for the worse. These changes are for the worse because increasing a pain's duration 100-fold offsets reducing its intensity slightly, or even somewhat. So the outcomes get worse until they are better, contradicting Tran-

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[1] Rachels 1998; 2001; Temkin 1987; 1996; 1997.

[2] In particular, this is true of the first three examples in Rachels 1998, the first of which is quoted below, and of the main example in Temkin 1996, also quoted below. Examples of roughly this structure date back at least to Harrod 1936, p. 148. (Harrod does not, however, explicitly draw the conclusion that betterness is not transitive.) A seemingly different kind of example appears in Temkin 1987; 1997; Rachels 2001. Those cases will not be discussed here.

sitivity. B is worse than A, C is worse than B, D is worse than C ... and Z is worse than Y, yet Z is better than A.[3]

Temkin provides a very similar case:

[C]ompare two lives, A and B. Suppose that both A and B are lengthy—perhaps, indeed, *very* lengthy— and that A and B are similar, except that A contains two years of excruciating torture, B four years of torture whose intensity is almost, but not quite, as bad as A's. [...] [M]ost would judge B worse than A. Next, compare B with C, where C stands to B as B stands to A. [...] [M]ost would judge C worse than B [...] Iterations of this reasoning imply that D would be worse than C, E worse than D, F worse than E, and so on, with the intensity of the unpleasant experiences slowly, but steadily, decreasing in each successive life. Eventually [...] one would be comparing two alternatives, say X and Y, such that X had an annoying hangnail for a *very* long time—perhaps thousands of years—and Y had a hangnail that was almost, though not quite, as unpleasant as X's, but that lasted twice as long. [...] [T]ransitivity implies that A is better than Y. But surely [...] A is not better than Y. Specifically [...] no matter how long one lived, the real but mild discomfort of a hangnail throughout one's life, would be preferable to two years of excruciating torture. Correspondingly, Y would be better than A, in violation of the axiom of transitivity.[4]

These examples illustrate three general claims, which, Rachels and Temkin maintain, together entail intransitivity.[5] These claims are as follows:

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[3] Rachels 1998, p. 73.

[4] Temkin 1996, p. 180. He attributes the original version of this example to Rachels.

[5] Like many other philosophers, they both speak of 'intransitive' betterness, and I shall follow this established usage. Strictly speaking, however, a relation *R* is intransitive only if it holds for *all* elements A, B, and C, in its domain, that if A*R*B and B*R*C, then not A*R*C. Of course, Rachels and Temkin do not claim that betterness is intransitive in this strict sense. What they argue is really that betterness is *non-transitive*, i.e., that there are *some* A, B, and C, such that A is better than B, and B is better than C, but A is not better than C.

Claim 1: For any unpleasant experience, no matter what its intensity and duration, it would be better to have that experience than one that was only a little less intense but lasted much longer.

Claim 2: There is a spectrum of distinguishable unpleasant experiences, ranging in intensity from extreme pain to very mild discomfort.

Claim 3: A very mild discomfort for any amount of time is preferable to extreme pain for a significant amount of time.[6]

Ken Binmore and Alex Voorhoeve claim that Rachels' and Temkin's general argument is invalid; Claims 1 to 3 do not entail intransitivity.[7] The argument is, according to Binmore and Voorhoeve, "a version of Zeno's paradox of Achilles and the tortoise".[8]

Although Binmore and Voorhoeve spot a flaw in Rachels' and Temkin's argument, this flaw is a merely technical one, which can be fixed without affecting the plausibility of the argument. Rachels and Temkin take for granted that Claims 1 and 2 together imply that there is a finite sequence of successively longer experiences, A, B, C,…, Y, Z, ranging in intensity from extreme pain to very mild discomfort, such that B is worse than A, C is worse than B, …, and Z is worse than Y. Whether this implication holds depends on how Claim 1 is interpreted. This claim is ambiguous between the following two readings, differing with respect to the scopes of the quantifiers involved:

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[6] Temkin 1996, pp. 179; 182ff; Rachels 2001, p. 215f.

[7] Binmore and Voorhoeve 2003, p. 274ff.

[8] Binmore and Voorhoeve 2003, p. 272.

Claim 1*: For any unpleasant experience A of intensity i and duration d, no matter how great i and d are, there is a small difference in intensity ε, such that it would be better to have A than an experience of intensity $j = i - \varepsilon$ and duration nd, for some number n.

Claim 1**: There is a small difference in intensity ε, such that, for any unpleasant experience A of intensity i and duration d, no matter how great i and d are, it would be better to have A than an experience of intensity $j = i - \varepsilon$ and duration nd, for some number n.

If Claim 1 is interpreted in either of these ways, the following is a suitable formulation of Claim 2:

Claim 2*: For any intensity level i (except the lowest level, if there is one), there is a distinguishable intensity level $j = i - \varepsilon$.

Claims 1* and 2* do not together entail that there is a sequence of the required kind. To see this, suppose that there are numbers x and y, $x > 2y$, such that x represents the intensity level of an experience of extreme pain, while y represents the intensity level of a very mild discomfort. For every positive integer $n > 1$, there might be a distinct intensity level $z = y + (1/n)x$. In this sequence, obviously, $z > y$, for any n. Thus, Claims 1* and 2* could be satisfied although the sequence never reaches an experience of intensity y. Transitivity may then hold even if Claim 3 is true.

Binmore and Voorhoeve apparently read Claim 1 as Claim 1*. There is, however, nothing in Rachels' or Temkin's discussion that excludes the alternative, Claim 1** interpretation. Concerning Temkin, there is even some slight evidence in its favour. With respect to the quoted example, he writes that "the intensity of the unpleasant ex-

periences [is] slowly, but *steadily*, decreasing in each successive life".[9] Describing the decrease as "steady" seems to suggest that it is Claim 1**, rather than Claim 1*, that he has in mind.

In any case, we may leave exegetical questions to one side, and decide to understand Claim 1 as Claim 1**. This makes Rachels' and Temkin's argument valid. Claims 1** and 2* do together imply the existence of a sequence of the required kind. Claim 1** allows us to assume that the difference in intensity is the same between any two adjacent experiences in the sequence. This ensures that we will get from an intensity x to an intensity y experience in a finite number of steps. Although Claim 1** is logically stronger than Claim 1*, its intuitive plausibility is, as far as I can see, equally great.

There is also another problem with Binmore's and Voorhoeve's objection. Interpreting Claim 1 as Claim 1* does not block the inference to intransitivity, unless we presuppose an infinite number of distinguishable intensity levels. But it is implausible to assume that humans have sufficiently fine powers of discrimination to ensure the existence of such an infinity of levels.

Rachels and Temkin could therefore rebut the objection either by interpreting Claim 1 as Claim 1**, or by arguing that there are only finitely many distinguishable levels of intensity. Yet another option would be to simply rely on the intuitive plausibility of the assumption that there is at least one sequence of the required type. Temkin is, in fact, attracted to this move:

[E]ven if there is reason to question one or more of [Claims 1 to 3], I think *some* version of [the quoted] counterexample is almost undeniable. I find it compelling that there is some sequence from A to Y such

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[9] Temkin 1996, p. 180; my emphasis.

that A is better than B, B better than C, C better than D, and so on. But I *cannot* believe that A is better than Y.[10]

This intuitive argument will, of course, gain in strength if one is able to give a plausible example of such a sequence. Rachels and Temkin maintain that they have, indeed, provided such examples.

In conclusion, Rachels' and Temkin's argument against the transitivity of betterness cannot be dismissed as a version of Zeno's paradox. There are several responses available to this charge. Whether some version of the argument is actually sound is another matter.[11]

References

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[10] Temkin 1996, p. 181; emphasis in the original. Also Rachels can be interpreted as employing this strategy in his (1998), where he does not explicitly rely on any general principles, corresponding to Claims 1 to 3. However, such principles are introduced in his (2001), where he indicates (p. 233, n. 14) that his argument is the same as in the earlier paper.

[11] My (2003) contains two far from conclusive counterarguments.

Rachels, Stuart, 2001, "A Set of Solutions to Parfit's Problems", *Noûs* 35, pp. 214-238.

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