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Growth without Scale Effects

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An increase in the size (scale) of an economy increases the total quantity of rents that can be captured by successful innovators, which, in equilibrium, should lead to a rise in innovative activity. Conventional wisdom and the theoretical predictions of models of endogenous innovation suggest that this increased research effort should lead to more rapid growth. As noted by Charles Jones, this prediction is at odds with the postwar experience of the OECD, where the growth of the market has indeed led to an increased R & D effort that, however, has been translated into stagnant or declining growth rates. Drawing on the remarkable insights of the museum curator Seabury C. Gilfillan, this paper modifies models of endogenous innovation to allow for the possibility that a rise in the profitability of innovative activity could lead to an increased variety of differentiated solutions to similar problems. An increased variety of technologies (e.g., an increase in the number and types of contraceptives) will increase the level of utility of the average consumer. If, however, continued improvement of this increased variety of technologies requires increased research input, a rise in the scale of the market could raise the equilibrium quantity of R & D without increasing the economy's growth rate.

I. Introduction

Writing in 1935, summarizing the “social principles of invention” gleaned from his monumental study of technological change in merchant shipping, the museum curator and sociologist Seabury C. Gilfillan (1935*a*, 1935*b*) anticipated one of the central implications of

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modern growth theory: that larger economies, which devote greater resources to innovation, should grow faster. Gilfillan identified all the principal logical elements behind this conclusion:

Increasing population and/or industry stimulate invention, because they increase the absolute need for a device, and the number of potential finders, while the cost of finding it remains the same. There are more mouths to eat the innovation, so to speak, and more eyes to find it. . . . The difficulty of finding it is no more than before, the value when found is greater, and so, many more inventions are found. We shall speak . . . of the much vaster increase in the population effective for invention, thru technic and other education. . . . Invention and other kinds of innovation differ from all other pursuits of mankind, in that the proportion of overhead to operating costs is very many fold. . . . More population does not help with more portrait photographs, because more labor must go to make them. . . . But as to invention, increasing population or industry entails that each inventor's work is more widely useful than before, at the same time that there are more inventors to work. [1935*b*, pp. 58–59]

The elements of Gilfillan's argument have been explored, in a more rigorous fashion, in recent models of endogenous innovation. Early one-factor models (e.g., Romer 1988; Aghion and Howitt 1990) found that the growth rate was increasing in the size of the economy (population), for precisely the demand and supply reasons emphasized by Gilfillan. Later multifactor extensions of these same models (e.g., Romer 1990*b*; Grossman and Helpman 1991*c*; Aghion and Howitt 1992) showed that the appropriate measure of scale was the supply of factors used intensively in the innovation process, typically assumed to be skilled labor. Increases in other factors of production, such as raw labor, were found to have ambiguous effects, since they raised both the return to innovation (by expanding the total market for new products) and its opportunity cost (by raising the marginal product of skilled labor in nonresearch sectors). Further extensions to international trade (e.g., Grossman and Helpman 1991*b*) raised additional ambiguities, such as when increases in the supply of skilled labor in imitative lesser developed economies reduced world growth rates by lowering the return to innovative activity in the developed world. The robust conclusion, however, appears to be that an equiproportional increase in all factors of production or solely in the factors of production used intensively in the innova-

tion (as opposed to imitative) process increases rates of innovation and growth.

One of the crucial characteristics of inventive activity identified by Gilfillan, the existence of both fixed and variable costs, has been elegantly set forth by Romer in two recent papers (1990*a*, 1990*b*). Romer argues that the fundamental characteristic of “technology” is that it is nonrival; that is, its use in one activity does not bar its use elsewhere. Nonrivalry helps give rise to scale effects since, *ceteris paribus*, it translates any uniform increase in the scale of the economy into an unambiguously larger profit flow for each innovator. This characteristic sharply distinguishes the new models of innovation from endogenous growth models based on the accumulation of rivalrous human or physical capital (e.g., Lucas 1988; Jones and Manuelli 1990; Rebelo 1991), where the aggregate size of the economy, however measured, has no impact on growth rates.¹

Despite their role as one of the most consistent implications of models of endogenous innovation, scale effects have been the focus of only two of the many empirical studies inspired by the new growth theory. In a cross-sectional analysis, Backus, Kehoe, and Kehoe (1992), after an extensive econometric search, discover a significant link between scale and the growth of manufacturing, but they can find no significant correlation whatsoever at the level of aggregate gross domestic product. In terms of theory, Backus et al.’s cross-sectional analysis is somewhat problematic since the appropriate measure of scale is given by the aggregate market size across which goods and research knowledge flow, which, for integrated trading economies, might be poorly correlated with domestic measures of scale. Jones (1995*a*, 1995*b*) finesses the problem of measuring scale by analyzing, directly, inputs into the research and development process. He notes that while the absolute number of R & D scientists and engineers in the major OECD economies has risen several-fold over the past few decades, the total factor productivity growth rates of these economies have remained constant or declined. Thus, while the subject has yet to be analyzed exhaustively, the existing empirical evidence in favor of scale effects might best be described as inconsistent.² This suggests the importance of at least considering

¹ Models of learning by doing, in which nonrivalrous knowledge is accumulated in a serendipitous fashion, generally exhibit scale effects (e.g., Young 1991). An exception is the paper by Matsuyama (1992), where the accumulation of knowledge is divided by the number of rivalrous entrepreneurs. In linking knowledge with specific entrepreneurs, Matsuyama makes it a rivalrous input in production and thereby purges scale effects from the model.

² In an interesting paper, Kremer (1993) notes that, across the broad sweep of human history, the rate of population growth has been increasing in the level of population and argues that this is evidence of scale effects. He also notes, however,

how the endogenous process of innovation, characterized by all the features emphasized by Gilfillan and modern growth theorists, might lead to an outcome in which the aggregate resource base devoted to innovation rises, but growth rates do not, that is, an absence of scale effects.

Jones (1995*a*) proposes a solution to the scale effects puzzle. Standard models of endogenous innovation assume the existence of strong intertemporal spillovers in innovative activity, wherein innovative activity today increases the absolute productivity of future innovators. Put differently, these models assume that the proportional productivity of innovators remains constant; that is, knowledge acquired from past innovations allows any innovator, at any time, to improve existing technologies by a given percentage.³ Jones weakens this assumption, allowing, for example, that the absolute productivity of innovators might be constant (i.e., uninfluenced by past innovation). If so, then as technology advances, greater and greater quantities of resources have to be devoted to innovative activity to sustain a given (proportional) growth rate. As such, the model explains the absence of intertemporal scale effects in the OECD experience, where R & D inputs have risen but growth rates have not. One difficulty with Jones's model, however, is that it does not, in general, allow for sustained growth. If the absolute productivity of innovators does not improve along with productivity in the rest of the economy, then, increasingly, labor will be drawn out of innovation into production and growth will cease.⁴ An increase in market size, expanding the pool of rents, can renew growth, but only temporarily. Thus, fundamentally, Jones's model is one that links the level of income to the level of scale, eliminating the influence of scale on growth, but only by purging all long-run growth from the model.

Gilfillan, in his time, was also puzzled by the apparent empirical absence of scale effects.⁵ As it so happens, however, he unknowingly

that this stylized fact is also completely consistent with models that lack scale effects, as it is, for that matter, with any model in which the rate of population growth is positively related to the level of income per capita (which has trended upward along with population).

³ It is interesting that the existence of strong intertemporal spillovers in innovative activity, with current innovation increasing the productivity of future innovators, was also emphasized by Gilfillan (his principle 11, "inventions persist").

⁴ It was precisely in response to this problem, as it appeared in early exploratory models of endogenous innovation (e.g., Judd 1985; Grossman and Helpman 1989), that Romer (1990*b*) first introduced the assumption of proportional spillovers.

⁵ Although Gilfillan did not have any direct measures of inputs into R & D, he found quite troubling the constancy of the patents per capita ratio between 1883 and 1933, in the face of the growth of the U.S. market and the increase in technical training. In this regard, Gilfillan's concerns foreshadow the recent academic interest in the postwar decline in the United States and other OECD economies of the ratio

provided an answer to the scale effects puzzle, in his principle of “equivalent innovation”:

inventions are not only duplicated about the same time by identical solutions . . . but also paralleled by equivalent but unlike means for reaching the same goal around the same time. . . . When Watt needed a guide for his piston rod, he made what he considered his most elegant invention, his “parallel motion.” In a book of 1901, I find 20 different ways of doing the same thing. . . . [In] contraception . . . we find 18 radically different methods indicated in a recent book, without counting minor variations. . . . In marine history we recall numerous kinds of sails, all for much the same result, the four old methods for shortening sail . . . the four ways of eliminating the stoker from a powered ship. . . . [A] new configuration of forces [can] call forth a number of independent solutions by different inventors about the same time, some identical and others unlike, even utterly unlike, yet filling the same need. [1935*b*, pp. 137–39]

Increases in market size, in the profitability of inventing a solution to a problem, might call forth a greater variety of potential solutions to that problem, raising the average level of consumer utility. If, however, the continued improvement of this increased variety of technologies requires additional research input, the equilibrium level of R & D expenditure might rise, without necessarily being associated with an increase in the rate of product quality improvement, that is, growth.

This paper seeks to develop a model that incorporates Gilfillan’s principle of equivalent innovation, while retaining many of the standard features of the new models of endogenous innovation (e.g., nonrivalry and proportional intertemporal knowledge spillovers). My approach is to modify standard “quality ladders” models of product improvement (i.e., Grossman and Helpman 1991*c*; Aghion and Howitt 1992) to allow for an endogenous degree of product variety. Increases in the pool of rents available to entrepreneurs can now be exhausted by increased innovative activity on either of two dimensions, vertical (quality) or horizontal (variety). I use a simple example, based on Dixit-Stiglitz (1977) preferences, to show how product variety can, in fact, completely dissipate the increased rents associated with an expansion in market size, making the growth rate of

of new patents to R & D scientists/expenditures, e.g., Evenson (1984), Griliches (1989), and Kortum (1993).

product quality, and the long-run growth of the economy, completely independent of the scale of the economy. In this case, policy interventions that merely expand the pool of rents, for example, proportional subsidies to innovation or the opening of an economy to trade, actually have no effect on the economy's long-run growth rate, although they have an important influence on the degree of product variety and, by extension, level of utility.

The paper proceeds as follows: Section II presents my simple model of growth without scale effects. Section III relates the model to the existing theoretical literature, arguing that its results stem from the fact that it joins a quality ladders dimension, where intertemporal knowledge spillovers can sustain growth, with a variety dimension, where intertemporal knowledge spillovers are insufficient to sustain growth. It is the existence of (at least) one innovative dimension that cannot sustain growth that allows the model to exhaust increases in rents in raising the level, rather than the growth rate, of utility. This section also establishes that the zero scale effects outcome is by no means extreme. Rent dissipation through product proliferation could actually lower the economy's growth rate, that is, give rise to negative scale effects, since the expansion of variety could reduce the incentive to improve product quality. Section IV examines the socially optimal degree of variety and quality improvement, and Section V concludes the paper.⁶

II. A Simple Model of Growth without Scale Effects

A. Model Structure

Consider an economy populated by L consumers, each of whom inelastically supplies one unit of labor at all times and seeks to

⁶ This is by no means the first paper to examine innovation in both horizontal and vertical dimensions (see, e.g., Economides 1993). In general, however, the industrial organization literature did not examine how the interaction between these two dimensions might influence the economy's response to changes in scale. Two exceptions, which have been brought to my attention, are Peretto (1995) and Smulders and van de Klundert (1995). Developed independently around the same time as this paper, these papers examine the interaction between the two dimensions and note the potential for innovation on the horizontal dimension to make the economy's growth rate a zero, or even negative, function of scale. The analysis in those papers is much more complex, considering the interaction between oligopolistic firms and allowing for a range of intra- and interfirm knowledge spillovers. This paper, while missing the richness of their analysis, focuses more directly on the fundamental role played by (1) the overall magnitude of intertemporal knowledge spillovers in each innovative dimension and (2) the response of the elasticity of demand with respect to innovation on the dimension that can sustain growth (product quality) to changes in the amount of innovation on the dimension that cannot sustain growth (variety).

maximize the present discounted value of the logarithm of consumption:

$$\max U = \sum_{t=0}^{\infty} B^t \ln[C(t)]. \quad (1)$$

Output is produced by perfectly competitive firms that combine intermediate inputs in a Dixit-Stiglitz production function:

$$Q(t) = \left\{ \int_0^{\infty} [\lambda_i(t) x_i(t)]^{\alpha} di \right\}^{1/\alpha}, \quad (2)$$

where $x_i(t)$ and $\lambda_i(t)$ denote the quantity and quality, respectively, of intermediate input i used at time t .

In this economy, intermediate inputs are produced by monopolistically competitive firms, which, in order to produce a given intermediate input at time t , must make a fixed (labor) investment in period $t - 1$, with the investment cost increasing in the quality of the good they seek to produce:

$$F[\lambda_i(t), \bar{\lambda}(t - 1)] = \begin{cases} fe^{\mu\lambda_i(t)/\bar{\lambda}(t-1)} & \text{if } \lambda_i(t) \geq \bar{\lambda}(t - 1) \\ fe^{\mu} & \text{otherwise,} \end{cases} \quad (3)$$

where f and μ are constants and $\bar{\lambda}(t - 1)$ is an index of technological opportunity, representing the intertemporal spillover of knowledge from previous generations of products. If good i has previously been produced, then $\bar{\lambda}(t - 1)$ is given by $\lambda_i(\max)$, the highest quality of product i produced up to time $t - 1$. If good i has never been produced, $\bar{\lambda}(t - 1)$ is the average of the $\lambda_i(\max)$'s, which I term $\bar{\lambda}(\max)$. The reader might think of this investment as being made up of two components: a standard production fixed cost of fe^{μ} , independent of quality, and a research cost of $fe^{\mu\lambda_i(t)/\bar{\lambda}(t-1)} - fe^{\mu}$. Following their fixed investment in period $t - 1$, firms may produce any quantity of their product in period t at a constant marginal (labor) cost of c .

B. General Equilibrium

As is well known, with logarithmic utility, aggregate consumer expenditure, $E(t)$, evolves according to

$$\frac{E(t + 1)}{E(t)} = [1 + r(t)]B, \quad (4)$$

where $r(t)$ is the interest rate between periods t and $t + 1$. For a given level of consumer expenditure, maximization on the part of

final-output producers yields the equally familiar constant elasticity of substitution demand for each intermediate input:

$$x_i^D(t) = \frac{E(t)p_i(t)^{-\epsilon}\lambda_i(t)^{\epsilon-1}}{\int_0^{N(t)} p_i(t)^{1-\epsilon}\lambda_i(t)^{\epsilon-1} di}, \quad (5)$$

where $\epsilon = 1/(1 - \alpha)$, $p_i(t)$ is the price of product i , and $N(t)$ is the number of available products, both at time t .

In each period, potential investors/firms must choose whether or not to enter (incur fixed costs) and, if they enter, must select a product and a level of product quality. As no firm can appropriate the intertemporal knowledge spillover and a fixed-cost investment must be made each period, each firm's decision horizon is only one period long. Since, regardless of market structure, firm returns will always be increasing in the level of product quality, there is no gain to incurring the fixed cost fe^u to produce less than the state-of-the-art quality. Consequently, at time $t - 1$, $\lambda_i(\max)$ in each industry will always be given by $\lambda_i(t - 1)$. Since all industries are symmetric, in equilibrium, product quality will be the same in each sector, that is, $\bar{\lambda}(\max) = \lambda_i(t - 1) = \lambda(t - 1)$. Thus firms will face the same costs whether they improve an existing product or introduce a new one, and since monopoly profits always exceed individual firm profits under duopoly, no two firms will choose to produce the same product. Finally, since all firms are infinitesimally small, there will be no strategic interactions, and intermediate input production, in each period, will take place under conditions of symmetric monopolistic competition.

In sum, the entry problem for each firm can be reduced to one of selecting, in period $t - 1$, a period t product quality and price so as to maximize net discounted monopolistically competitive profits:

$$\max_{p_i(t), \lambda_i(t)} \frac{[p_i(t) - c]x_i^D(t)}{1 + r(t - 1)} - F[\lambda_i(t), \lambda(t - 1)], \quad (6)$$

where I have taken labor as the numeraire, that is, set the wage rate equal to one. Differentiating with respect to $p_i(t)$ and $\lambda_i(t)$ provides two first-order conditions:

$$x_i^D(t) + [p_i(t) - c] \frac{\partial x_i^D(t)}{\partial p_i(t)} = 0 \quad (7)$$

and

$$\left[\frac{p_i(t) - c}{1 + r(t-1)} \right] \frac{\partial x_i^D(t)}{\partial \lambda_i(t)} - \frac{\partial F[\lambda_i(t), \bar{\lambda}(t-1)]}{\partial \lambda_i(t)} = 0, \quad (8)$$

and free entry implies that net profits will be zero:

$$F[\lambda_i(t), \bar{\lambda}(t-1)] = \frac{[p_i(t) - c]x_i^D(t)}{1 + r(t-1)}. \quad (9)$$

Dividing (8) by (9) and making use of the symmetry of the problem allows one to rearrange (7) and (8) so as to provide the key equilibrium relations:

$$\frac{p - c}{p} = \frac{1}{E_{p_i}^{x_i^D}} \quad (10)$$

and

$$E_{\lambda_i}^F = E_{\lambda_i}^{x_i^D}, \quad (11)$$

where E_j^i denotes the elasticity of i with respect to j . Equation (10) is the familiar monopoly pricing relation, which states that firms set prices so as to equate the unit profit margin to the inverse of the elasticity of demand. Equation (11) indicates that firms select product quality so as to equate the elasticity of the research cost with respect to quality (the cost) to the elasticity of demand with respect to that variable (the benefit). For the constant elasticity of substitution demand of the model, these relations take the following functional forms:

$$\frac{p - c}{p} = \frac{1}{\epsilon} = 1 - \alpha \quad (10')$$

and

$$\mu \frac{\lambda(t)}{\lambda(t-1)} = \epsilon - 1 = \frac{\alpha}{1 - \alpha}, \quad (11')$$

where I have assumed that $(\epsilon - 1)/\mu > 1$, that is, that products are improved in equilibrium. With both the elasticity of demand with respect to price and the elasticity of demand with respect to quality independent of the size of the market and the number of entrants, the equilibrium markup and rate of product improvement are both completely determined by parameters of the research and production functions.

In the case of the labor market, equilibrium requires that the total labor used in manufacturing and investment/research equal the

available supply, L . The labor used in manufacturing simply equals total consumer expenditure divided by the price per unit (of each symmetric good) times the unit marginal cost, that is,

$$L_M(t) = \frac{E(t)}{p} c = \alpha E(t), \quad (12)$$

whereas the labor used in research equals the number of entrants times the research investment per firm:

$$L_R(t) = N(t+1) f e^{\mu\lambda(t+1)/\lambda(t)}. \quad (13)$$

Consequently, labor market clearing requires that

$$L = \alpha E(t) + N(t+1) f e^{\mu\lambda(t+1)/\lambda(t)}. \quad (14)$$

To determine the equilibrium number of products, I begin by noting that, in the symmetric equilibrium, the free-entry relation, (9) above, reduces to

$$f e^{\mu\lambda(t)/\lambda(t-1)} = \frac{(1-\alpha)E(t)}{N(t)[1+r(t-1)]}. \quad (15)$$

Advancing (15) one period, substituting into (14), and making use of the first-order condition for the growth of consumer expenditure, (4), allows one to solve for the value of consumer expenditure as a function of the various parameters:

$$L = \alpha E(t) + \frac{(1-\alpha)E(t+1)}{1+r(t)} = E(t)[\alpha + (1-\alpha)B] \quad (16)$$

or

$$E(t) = \frac{L}{\alpha + (1-\alpha)B}. \quad (16')$$

Thus equilibrium requires that the economy move immediately to a steady-state level of consumer expenditure, with a constant interest rate $1+r=B^{-1}$. Substituting (16) back into (15) and rearranging solves for the total number of entrants (in any period) as a function of the parameters of the model and the aggregate market size (as given by L):

$$N = \frac{(1-\alpha)E(t)}{[1+r(t-1)]f e^{\mu\lambda(t)/\lambda(t-1)}} = \frac{(1-\alpha)BL}{[\alpha + (1-\alpha)B]f e^{\epsilon-1}}. \quad (17)$$

In sum, each period N firms (as given by [17]) invest in R & D, improving the current generation of products by a factor of $(\epsilon - 1)/\mu$. In the following period these firms compete monopolistically

in the production of intermediate inputs, charging a common price $p = c/\alpha$. Consumer expenditure is constant, as is the rate of interest and the share of labor in research and manufacturing.

C. Scale Effects, in Growth Rates and Levels

To examine the impact of scale on the growth and level of income in this economy, it is useful to first derive an expression for output per capita. To this end, from equation (2) we can see that in the symmetric equilibrium the total output of the economy is given by

$$Q(t) = \lambda(t)\bar{x}(t)N(t)^{1/\alpha}, \quad (18)$$

where $\bar{x}(t)$, the demand for each intermediate input, in turn equals

$$\bar{x}(t) = \frac{E(t)}{p(t)N(t)} = \frac{\alpha L}{c[\alpha + (1 - \alpha)B]N(t)}. \quad (19)$$

Consequently, output per capita, $q(t)$, is given by

$$q(t) = \frac{\alpha\lambda(t)N(t)^{(1-\alpha)/\alpha}}{c[\alpha + (1 - \alpha)B]}. \quad (20)$$

In equilibrium the number of varieties (N) is constant. Thus long-run growth in this economy is driven by the growth of product quality. Since the elasticity of demand with respect to product quality is independent of L , changes in the scale of the economy have no implications for the long-run growth of output per capita, which is simply given by $\lambda(t)/\lambda(t-1) = (\epsilon - 1)/\mu$. The total number of product varieties, N , is, however, linearly related to the level of L . Thus the *level* of utility is related to the *level* of scale. A larger market calls forth additional differentiated solutions to general needs. If technology is nonrivalrous, as in this model, this greater variety of solutions is available to all and, consequently, unambiguously raises the level of welfare.

Since the level of utility is related to the level of scale, changes in scale will generate important transitional (i.e., temporary) dynamics in growth rates. From (20), we see that the logarithmic growth of output per capita between any two periods is given by

$$\ln\left[\frac{q(t)}{q(t-1)}\right] = \ln\left[\frac{\lambda(t)}{\lambda(t-1)}\right] + \left(\frac{1-\alpha}{\alpha}\right)\ln\left[\frac{N(t)}{N(t-1)}\right]. \quad (21)$$

Thus growth during any given period can be decomposed into a long-run component, the increase in product quality, which is independent of the scale of the economy, and a transitional element,

the growth of product variety, which is influenced by changes in scale. In a manner similar to the role of capital accumulation in the neoclassical growth model, these level effects could be of considerable quantitative significance and deserving of careful investigation. The distinction between levels and growth rates remains a useful one, however, since the underlying sustainable growth rate of economies in which measures of scale have been rising rapidly might be well below their historical experience.⁷

The absence of scale effects (in growth rates) in the model indicates that any policy action that simply enlarges or contracts the total pool of rents available to inventive entrepreneurs will influence only the level of income, without changing its long-run growth rate. Thus consider the movement of two identical economies from a position of autarky to one of free trade. In the integrated trading equilibrium, this policy change is describable as an increase in the labor force of a unified economy from L to $2L$. From (17), we see that the number of available varieties doubles. Each economy produces as many varieties as before and maintains the same output per variety. Consumption per variety falls, as each economy trades half of its intermediate output for half of the differentiated intermediate output of the other economy. The rise in intermediate variety, however, more than offsets the fall in intermediate quantity, raising the level of final-goods output in both economies.⁸ Still, the elasticity of demand with respect to quality and hence the growth rates of both economies remain unchanged.

One can easily see how many policy interventions, which in contemporary models of endogenous innovation influence the long-run growth of the economy, will be ineffective (in growth rates) in this model. Thus the imposition of tariffs, on either a unilateral or multilateral basis, or the provision of proportional R & D subsidies (which rebate a fixed share of R & D expenditures) will change the total pool of rents available to entrepreneurs without influencing the elasticity of demand with respect to product quality. These policies will influence the level of income, without changing its long-

⁷ Jones (1994) makes this observation with regard to the OECD. I should note that if the scale of the economy grows steadily in equilibrium, say because of population growth, then the “transitional” level effects will, of course, contribute in a continuing fashion to aggregate growth. The distinction between level and growth effects nevertheless remains useful. As Jones explains, the percentage of the labor force in the OECD devoted to R & D has been rising through time. In his model and mine, this would contribute to a transitional increase in the level of output (i.e., growth) well in excess of the sustainable rate (i.e., in excess of the sustainable “transitional” component due to continuing population growth).

⁸ In terms of eq. (18), \bar{x} is halved but N is doubled, raising output by a factor of $2^{(1-\alpha)/\alpha}$.

run growth rate. Other policies exist, however, that will influence both the level and growth rate of income. Thus quotas or an R & D subsidy based on the intensity of R & D effort (one that is increasing in the degree to which the research increases the quality of current inputs) will change the elasticity of demand and the elasticity of research costs with respect to product quality, respectively, leading, in equilibrium, to changes in both the level and growth rate of output per capita.

III. Discussion of the Model

Innovation in the model described above has two dimensions, a vertical dimension and a horizontal dimension. As such, it might appear, at first blush, that the model marries the vertical “product quality” models of Grossman and Helpman (1991*c*) and Aghion and Howitt (1992) to the horizontal “product variety” model of Romer (1990*b*). This is not actually true. In both the vertical and horizontal models of innovation there exists an intertemporal spillover whereby innovation today lowers innovation costs in the future. In the model described in the preceding section, there exists an intertemporal knowledge spillover in the vertical dimension, but there is no such spillover in the horizontal dimension. To incorporate a knowledge spillover in the horizontal dimension similar to that of Romer, the model would have to specify that the costs of innovation are given by

$$F[\lambda_i(t), \bar{\lambda}(t-1), N(t-1)] = \frac{fe^{\mu\lambda_i(t)/\bar{\lambda}(t-1)}}{N(t-1)}. \quad (22)$$

In this case, the cost of innovation is inversely related to the existing degree of product variety.

It is instructive to work out the equilibrium of the model when there exists a horizontal knowledge spillover of the form described in (22). Most of the equilibrium relations of the model remain as before. In particular, (10') and (11'), which describe the equilibrium intermediate input price and rate of quality improvement, are unchanged, as is (16'), which pins down the equilibrium level of expenditure. The free-entry condition, however, is now given by

$$\begin{aligned} \frac{fe^{\mu\lambda(t)/\bar{\lambda}(t-1)}}{N(t-1)} &= \frac{[p_i(t) - c]x_i^D(t)}{1 + r(t-1)} \\ &= \frac{(1 - \alpha)E(t)}{N(t)[1 + r(t-1)]} = \frac{(1 - \alpha)BL}{N(t)[\alpha + (1 - \alpha)B]}. \end{aligned} \quad (23)$$

Rearranging, we find that

$$\frac{N(t)}{N(t-1)} = \frac{(1-\alpha)BL}{f e^{\epsilon-1} [\alpha + (1-\alpha)B]}. \quad (24)$$

Contrary to the model without a horizontal knowledge spillover, product variety now grows steadily over time.⁹ Although the growth rate of product quality, $\lambda(t)/\lambda(t-1)$, remains fixed at $(\epsilon-1)/\mu$, the growth rate of product variety is now an increasing function of the scale of the economy. The economy once again exhibits scale effects in growth rates.

Intertemporal spillovers play a crucial role in models of innovation, since they allow innovative activity to sustain growth at a constant, time-invariant, resource cost. As noted by Jones (1995*a*), when the intertemporal spillover is not sufficiently strong to allow for constant cost growth, the growth rate of the economy asymptotes to zero.¹⁰ The model of this paper combines two forms of innovative activity, one that can lead to sustained growth (vertical quality improvements) and one that cannot (horizontal product introduction). As such, the model is more accurately described as a marriage of the quality ladders models with the “level effects” models of horizontal innovation, such as Judd (1985), Grossman and Helpman (1989), and Jones (1995*a*). Scale need now no longer be positively associated with growth, since the level effects innovative dimension can completely dissipate the increased pool of rents brought about by a rise in market size, without leading to any rise in innovative activity along the dimension that leads to sustainable growth. This point can be considered as being more general than the one- or two-dimensional analysis of the model. If there are M dimensions in which innovative activity can lead to sustained growth but one additional dimension in which spillovers are insufficient to lead to growth, then increases in rents can be dissipated, generating level

⁹ I assume that the parameters are such as to ensure that N is growing, rather than contracting, over time. Specifying that the research cost relates to the once and for all cost of introducing a new product, with no additional fixed costs in subsequent periods, avoids this problem (but needlessly complicates the algebra, since the state of the economy now consists of the sequence $\{N(i), \lambda(i)\}_{i=-\infty}^t$, which summarizes the number and quality of products introduced in all previous periods and still produced at time t).

¹⁰ In a horizontal model of product introduction, Jones specifies that new products are introduced according to the research function $\dot{N}(t) = N(t)^\phi L_R(t)$ and notes that when $\phi < 1$ the economy's growth rate goes to zero. Thus, even if there are intertemporal spillovers, if they are not large enough to allow for constant cost growth, the development of the economy grinds to a halt. As in Jones's analytical framework, the research cost in this paper might be specified as $f e^{\mu\lambda(t)/\lambda(t-1)^\gamma} / N(t-1)^\phi$. For sustained growth to occur, it is necessary that either γ or ϕ be greater than or equal to one.

changes in income without changing the long-run growth rate of the economy.

From the discussion above, the reader might conclude that the results of the model described in Section II are an extreme outcome. Increases in rents might be exhausted by greater innovative activity along a “growth” dimension or a “level” dimension, and, in general, one would expect the impact of any increase in market size to occur along both dimensions, leading to a rise in both the level of income and its growth rate. Thus the case in which increases in scale have purely level effects would seem to be an extreme outcome. This intuition is, however, incorrect. Recall the crucial first-order conditions analyzed in Section II, in particular the one governing the optimal rate of quality improvement for innovating firms:

$$E_{\lambda_i}^F = E_{\lambda_i}^{x_i^D}. \quad (25)$$

In equilibrium, firms set the elasticity of the research cost with respect to product improvement equal to the elasticity of demand with respect to that variable. With Dixit-Stiglitz preferences, the elasticity of demand with respect to quality is independent of the number of entering firms. More generally, however, the elasticity of demand with respect to quality might be an increasing or decreasing function of the number of entrants. In the latter case, an increase in innovative activity along the horizontal dimension, brought about by an increase in market size, would, in equilibrium, lead to lower rates of product improvement. In other words, increased innovative activity along the level dimension could easily have adverse effects on innovative activity in the growth dimension, *lowering* the overall growth rate of the economy. Thus growth can be a positive, neutral, or decreasing function of the scale of an economy. The results of Section II, while constituting a knife-edge case (in which the scale effect is exactly zero), are by no means extreme and serve to simply illustrate the role played by the level dimension in dissipating rents.¹¹

IV. Socially Optimal Growth and Variety

To conclude the analysis, I compare the market outcome with the socially optimal degree of quality improvement and product variety. Consider the problem facing a social planner who seeks to maximize the utility of the representative consumer in this world. This planner

¹¹ The reader interested in a more formal presentation should consult sec. 4 of Young (1995), which provides an example of preferences in which the elasticity of demand with respect to product quality is decreasing in the number of varieties. Consequently, the growth rate is a negative function of the scale of the economy.

will select a sequence of product qualities, $\lambda(t)$, and number of product varieties, $N(t)$, so as to¹²

$$\max_{\{\lambda(t), N(t)\}} \sum_{t=0}^{\infty} B^t \ln \left[\frac{L_M(t)/L}{L_1(t)} \right], \quad (26)$$

where $L_M(t)$ and $L_1(t)$ denote the total labor in manufacturing and the unit labor requirement (for output), respectively:

$$\begin{aligned} L_M(t) &= L - N(t+1) f e^{\mu\lambda(t+1)/\lambda(t)}, \\ L_1(t) &= c\lambda(t)^{-1} N(t)^{(\alpha-1)/\alpha}. \end{aligned} \quad (27)$$

Using the usual Euler variational arguments (see Stokey and Lucas 1989), one can establish that if a pair $[\lambda(t), N(t)]$ is part of the infinite sequence $\{\lambda(i), N(i)\}_{i=1}^{\infty}$ that maximizes (26), then $[\lambda(t), N(t)]$ must also maximize (26) conditional on the remaining values of the sequence, $\{\lambda(i), N(i)\}_{i=1, \neq t}^{\infty}$. Consequently, a necessary condition for a sequence to constitute an optimum is that the derivative with respect to the two choice variables equal zero at each time t :

$$B^{t-1} \frac{\partial L_M(t-1)/\partial N(t)}{L_M(t-1)} - B^t \frac{\partial L_1(t)/\partial N(t)}{L_1(t)} = 0, \quad (28)$$

$$\begin{aligned} B^{t-1} \frac{\partial L_M(t-1)/\partial \lambda(t)}{L_M(t-1)} - B^t \frac{\partial L_1(t)/\partial \lambda(t)}{L_1(t)} \\ + B^t \frac{\partial L_M(t)/\partial \lambda(t)}{L_M(t)} = 0, \end{aligned} \quad (29)$$

or, with these equations reformulated in terms of elasticities,

$$E_{N(t)}^{L_M(t-1)} = B E_{N(t)}^{L_1(t)}, \quad (28')$$

$$E_{\lambda(t)}^{L_M(t-1)} = B [E_{\lambda(t)}^{L_1(t)} + E_{\lambda(t)}^{L_M(t)}], \quad (29')$$

where, as before, E_j^i denotes the elasticity of i with respect to j . The left-hand-side terms of equations (28') and (29') represent the cost of increasing $N(t)$ and $\lambda(t)$, that is, the reduction in the labor available for manufacturing in period $t-1$. The right-hand-side terms of these equations represent the benefits of increasing these variables, which are discounted, since an investment in greater product variety or quality taking place in period $t-1$ does not yield benefits until

¹² In actual fact, the planner is free to select a separate quality $\lambda_i(t)$ for each variety i . However, under the assumption that the initial level of quality is the same across the board, the symmetry of the problem ensures that the planner will select the same level of quality for each variety i .

period t . In the case of an increase in $N(t)$, the benefit is simply the reduction in the unit labor requirement. In the case of an increase in $\lambda(t)$, there are two benefits: an increase in the efficiency of final-goods production represented by the reduction in unit labor requirements and an intertemporal knowledge spillover manifested in an increase in the labor available for period t manufacturing. The latter effect reflects the fact that, since research costs are related to the period-by-period percentage improvement in product quality, an absolute increase in period t product quality reduces the research labor required to reach any given level of period $t + 1$ quality.

In both the social optimum and the market equilibrium, variety and growth are determined by two equations, which can be usefully compared. In the social optimum, the optimal degree of variety, for a given research cost, can be thought of as being determined by the first-order condition with respect to N , (28'). Evaluating this relation at the steady state,¹³ we get

$$\frac{N_{so} f e^{\mu g_{so}}}{L - N_{so} f e^{\mu g_{so}}} = B \frac{1 - \alpha}{\alpha}, \quad (30)$$

where g represents the growth of product quality. In the market equilibrium, for given pricing and rates of product quality improvement, the number of varieties is determined by the free-entry condition (eq. [9]), which is given by

$$f e^{\mu g_{ME}} = B \frac{[(1 - \alpha)/\alpha](L - N_{ME} f e^{\mu g_{ME}})}{N_{ME}}. \quad (31)$$

Equations (30) and (31) are clearly identical and are drawn as such in figure 1. The equilibrium product quality improvement selected by free-market firms was given by the first-order condition (8). When this is divided by the free-entry condition, that is, conditioned on the determination of the equilibrium degree of variety, it reduces to the elasticity relation, (11), whose functional expression is

$$\mu g_{ME} = \frac{\alpha}{1 - \alpha}. \quad (32)$$

The equivalent relation for the social optimum is given by the first-order condition with respect to product quality, equation (29'),

¹³ Non-steady-state values for $\lambda(t)$ and $N(t)$ lead to divergent paths. Since, from the point of view of the social planner, there are no meaningful state variables at time 0 (the levels of $\lambda(0)$ and $N(0)$ influence the initial level of utility but have no intertemporal impact), the socially optimal plan involves an immediate movement to a steady-state path.

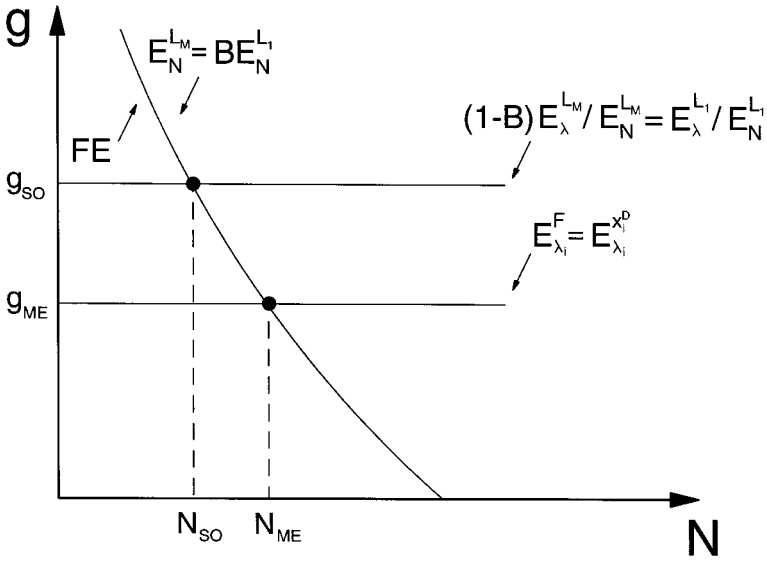


FIG. 1.—Variety and growth

which, when divided by the first-order condition with respect to variety, that is, conditioned on the determination of the optimal degree of variety, reduces to

$$(1 - B)\mu g_{so} = \frac{\alpha}{1 - \alpha}. \quad (33)$$

Equations (32) and (33) are horizontal lines, with (33) lying above (32), as drawn in figure 1.

As is well known (see Tirole 1988; Grossman and Helpman 1991*a*; Aghion and Howitt 1992), in models of endogenous innovation the socially optimal level of innovation may generally be greater or less than that achieved in the market equilibrium. In innovating, private firms both increase consumer surplus and steal the rents of other producers. The failure of firms to capture all the consumer surplus generated by their innovations, as well as their inability to internalize the intertemporal spillover of knowledge creation, leads to insufficient private innovation. The ability of firms to steal the rents of other producers leads, however, to socially excessive innovation. It is also well known that with Dixit-Stiglitz preferences the consumer surplus and rent stealing effects exactly cancel, leading, in the absence of an intertemporal spillover, to an exact equality between the

degree of innovation present in the social optimum and the free-market equilibrium.¹⁴

In the model of this paper, there is no intertemporal spillover on the horizontal dimension. Consequently, on the horizontal dimension, that is, for a given degree of product quality improvement, rates of innovation in the social optimum and market equilibrium coincide, as indicated by the equality of equations (30) and (31). On the vertical dimension, however, there is an intertemporal spillover. Thus, for a given degree of product variety (i.e., as one moves up from the x axis in fig. 1), the socially optimal degree of innovation is greater than that achieved in the market equilibrium.¹⁵ Combining both relations, we see that in equilibrium the socially optimal degree of variety lies below that achieved by the free market, whereas the socially optimal growth rate exceeds its market counterpart, with the overall resources devoted to innovation being equal in both equilibria. The social optimum can be achieved in a market setting by combining a graduated R & D subsidy (one that is rising in the degree of product quality improvement) with an overall proportional tax on R & D activity.¹⁶

The relations determining the rate of quality improvement in the social optimum and the market equilibrium, the horizontal lines in figure 1, are both independent of the scale of the economy. An increase in L , however, shifts out the free-entry/optimal variety relation. Consequently, as can be inferred from the figure, a rise in the scale of the economy increases the market equilibrium and socially

¹⁴ See Grossman and Helpman (1991*a*, pp. 82–83). This result is conditional on the lack of a perfectly competitive sector in the economy. When such a sector exists, the socially optimal plan involves removing the monopoly pricing distortion and expanding the total output of the (formerly) monopolistic competitive industry (see Dixit and Stiglitz 1977). In a model of innovation, this requires increasing innovative activity (in that sector) above the levels achieved by the market equilibrium.

¹⁵ If the intertemporal spillover were absent from the model, the $-B$ term would disappear from the left-hand side of (33) and the socially optimal and market degree of innovation would coincide along the vertical dimension as well.

¹⁶ Specifically, let τ be the proportion of the fixed cost of innovation (the left-hand side of [31]) borne by innovators. The socially optimal plan involves making

$$\tau = \exp \mu B \left[\frac{\alpha}{(1 - B)\mu(1 - \alpha)} - \frac{\lambda_i(t)}{\lambda(t - 1)} \right].$$

The first term within the brackets represents a proportional tax, and the second term a graduated subsidy. The proportional tax is necessary to offset the impact of the graduated subsidy on the position of the free-entry relation. In general, since the social optimum involves two objectives (variety and growth) as determined by two free-market firm optimality conditions (free entry and the elasticity relation), achievement of the social optimum requires two instruments, a proportional R & D tax/subsidy and a graduated (quality-dependent) R & D tax/subsidy.

optimal degree of variety, raising the level of utility under either regime without changing the corresponding growth rates of product quality. The social optimum, like the market equilibrium, exhibits scale effects in the level of utility, but not in its growth rate.

The intuition for the absence of scale effects in the determination of the socially optimal growth rate is best developed by stepping back slightly from the specifics of the model. Returning to equations (28') and (29'), substitute for the steady-state values of the elasticities but keep the right-hand side of (28') in its general form:

$$\frac{Nfe^{\mu g}}{L - Nfe^{\mu g}} = BE_N^{L_1}, \quad (28'')$$

$$\frac{\mu g Nfe^{\mu g}}{L - Nfe^{\mu g}} = B \left(1 + \frac{\mu g Nfe^{\mu g}}{L - Nfe^{\mu g}} \right). \quad (29'')$$

Substituting in (29'') using (28''), as was done before, yields the relation

$$(1 - B)\mu g = \frac{1}{E_N^{L_1}}. \quad (34)$$

Thus, given the other elasticities, in particular the specification of an exponential research technology, the socially optimal growth rate is seen to be inversely related to the elasticity of the unit labor requirement with respect to product variety.

The social optimum, like the market equilibrium, may exhibit positive, negative, or neutral scale effects (in growth rates). If the elasticity of the unit labor requirements is decreasing in the degree of variety, the social planner faces diminishing returns in this dimension. Consequently, when endowed with a larger resource base, she devotes resources to increasing both the degree of variety and the rate of quality improvement. When the elasticity of the unit labor requirement is increasing in the degree of variety, the social planner faces a rising marginal efficiency of investment in product variety. As the resource base of the economy expands, the social planner exploits these increasing returns to a greater degree, raising the level of utility (N) at the expense of its growth rate (g). In this case, the social optimum exhibits negative scale effects. Finally, when the elasticity of the unit labor requirement is independent of the number of varieties, the social planner faces a constant returns to scale investment technology that allows her to achieve the same proportionate increase in the level of utility for the same proportionate increase in investment. In this (knife-edge) case, the socially optimal growth rate is independent of the scale of the economy, as all increases in

the resource base are absorbed in raising the level of utility. This is precisely what happens with the Dixit-Stiglitz production function, which, consequently, mimics the zero scale effects (in growth rates) of the market equilibrium. More generally, however, since the partials with respect to variety of $E_{N_i}^{x^p}$ and $E_N^{L_1}$ depend on different derivatives of the production function, there is no hard and fast relation between the type of scale effects present in the market equilibrium and those present in the social optimum.

V. Summary and Conclusion

This paper modifies conventional “quality ladders” models of product improvement to allow for the endogenous determination of the degree of product variety. This apparently minor theoretical modification eliminates one of the most consistent predictions of models of endogenous innovation: that larger economies should grow faster. If new technologies are nonrival, increases in market size will raise the profitability of inventive activity. In standard models of endogenous innovation, increased rents can be dissipated only through faster rates of product improvement, which destroy the rents of past innovators more rapidly. The model of this paper adds an additional degree of freedom, allowing rents to be dissipated by either increased quality improvement or increased entry. It is immediately apparent that circumstances could arise in which increased entry dissipates all increases in rents, that is, in which the rate of product quality improvement is invariant with respect to the scale of the economy. In this case, changes in the profitability of innovation—brought about by changes in the scale of an economy, by the provision of proportional subsidies to R & D, or by the opening of an economy to trade—will have important effects on the level of consumer utility, without, however, changing its growth rate. This zero scale effects outcome is by no means an extreme result. As the paper has argued, when product quality and variety are jointly determined, changes in the total pool of rents can lead to any number of outcomes, including cases in which increased entry leads to a *decline* in the long-run growth rate, since a rise in the number of available varieties lowers the return to product improvement.

In the postwar OECD economy, most of the variables identified by growth theorists as benefiting growth have trended upward. Educational levels have risen, trade barriers have been eliminated, markets have grown, and, in precisely the theoretically predicted fashion, the total expenditure and effort devoted to R & D have risen dramatically. This increase in research effort has not, however, manifested itself in higher growth rates. While it is possible to explain

this puzzle in terms of measurement problems, either of the total R & D effort or of the quality and variety adjusted growth rate, it is also possible to seek a theoretical explanation. This paper brings the dynamic intertemporal models of endogenous growth back to their roots in static general equilibrium models of increasing returns (e.g., Krugman 1979), which emphasized the important impact of scale and trade on the level of utility, as determined by the variety of products available. The size of a market and the level of R & D expenditure might have as much (if not more) to do with the level of utility as with its growth rate.

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