Online Appendices for:
Dark Matter: Measuring Unobserved Productivity Growth due to Computers through its Impact on Observables

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Appendix A: Estimating Mismeasurement of Primary Factor Inputs

Above I estimate mismeasurement of what industries do with inputs (factor augmenting technical change). While this implies mismeasurement of the output of individual industries and hence, through the input output table, mismeasurement of intermediate inputs, I otherwise assume that primary factor inputs are measured correctly. As readers may question this, this appendix presents estimates of primary factor input mismeasurement.

We posit that:

$$\text{(A1)} \quad \widehat{M}^{T}_{jit} = \hat{a}^{UO}_{jit} + \widehat{M}^{M}_{jit}$$

where $\widehat{M}^{T}_{jit}$ and $\widehat{M}^{M}_{jit}$ are the true and measured growth of primary input *j* in industry *i* at time *t*, while $\hat{a}^{UO}_{jit}$ is the unobserved discrepancy between them. The discrepancy between true and measured total factor productivity growth is then:

$$\text{(A2)} \quad \hat{A}^{T}_{it} = \hat{A}^{M}_{it} - \sum_{j=N+1}^{J} \theta_{jit}\hat{a}^{UO}_{jit}.$$

The reader is reminded that the sequence of inputs 1 … J is composed of 1 … *N* industry indices and *N*+1 … *J* primary inputs. (A2) can then be plugged into the structural demand and supply system (9) or the SUR system (11) in the paper. In the latter case, assuming mismeasurement applies only to the quantity of input *j*, the SUR system becomes:

$$\text{(A3)} \quad \hat{P}^{T}_{it} = \beta_P[\hat{A}^{M}_{it} - \alpha_j\theta_{jit}] + \eta^{P}_{i} + \eta^{P}_{t} + \varepsilon^{P}_{it}$$

$$\text{and} \quad \widehat{D}^{T}_{it} = \beta_D[\hat{A}^{M}_{it} - \alpha_j\theta_{jit}] + \eta^{D}_{i} + \eta^{D}_{t} + \varepsilon^{D}_{it} \ \text{(for } D = C, X, \ldots\text{)},$$

where α_j is the economy-wide average rate of mismeasurement of the growth of input *j*, $\hat{a}^{UO}_{jit}$, and where the η denote industry and year fixed effects.

Table A1 reports the estimated α_j for all primary inputs in the BEA's total factor productivity accounts using the baseline structural and SUR models and samples of Table 5 in the paper. As noted there, the point estimates for computer hardware capital are consistently negative, but not statistically significant. Other results change in sign from one specification to another, or when consistently of one sign (structures, r&d capital and college and non-college labour) vary greatly in magnitude with the removal of own-use intermediates demand or changes in the disaggregation of demand and the sample of industries.

Table A1: Mismeasurement of the Growth of Primary Factor Inputs
(each cell a separately estimated model)

	structural model		SUR (seemingly unrelated regressions)					
variables:	PMCXRO	PMCXIGO	PMCXRO	PMCXIGO	PMCXR	PMCXIG	PQ	$PQ_{\sim O}$
industries:	44	20	44	20	44	20	61	61
computer capital	-.26 (.17)	-.44 (.38)	-.64 (.33)	-.87 (.59)	-.98 (.46)	-1.5 (.79)	-.68 (.32)	-.63 (.30)
software capital	.11 (.22)	.03 (.20)	.25 (.26)	.11 (.25)	.04 (.35)	-.01 (.34)	.01 (.24)	.00 (.22)
communi-cations capital	.68 (.57)	.38 (.50)	.20 (.67)	-.22 (.93)	.64 (.93)	.37 (1.3)	.95 (.54)	.86 (.50)
r & d capital	.04 (.08)	.11 (.08)	.12 (.15)	.58 (.17)	.07 (.21)	.78 (.25)	.19 (.14)	.19 (.13)
instruments capital	-.08 (.49)	.49 (.62)	.40 (1.0)	1.1 (1.6)	-.81 (1.4)	3.6 (2.1)	-.13 (.89)	-.03 (.83)
transport equipment	-.06 (.16)	.02 (.18)	.18 (.27)	-.44 (.31)	-.40 (.38)	-1.3 (.42)	-.37 (.22)	-.34 (.20)
other equipment	-.14 (.20)	-.26 (.32)	.02 (.25)	-1.2 (.67)	-.48 (.34)	-2.1 (.90)	-.03 (.21)	-.04 (.20)
art capital	.16 (.17)	.02 (.13)	-.14 (.31)	-.02 (.29)	.18 (.42)	.45 (.40)	.16 (.29)	.13 (.27)
structures capital	-.09 (.07)	-.09 (.07)	-.10 (.09)	-.44 (.17)	-.33 (.12)	-.73 (.24)	-.21 (.06)	-.21 (.06)
college labour	.14 (.07)	.10 (.07)	.17 (.11)	.44 (.14)	.43 (.15)	.85 (.20)	.15 (.09)	.17 (.08)
non-college labour	.08 (.06)	.18 (.10)	.14 (.10)	.54 (.19)	.34 (.14)	.94 (.26)	.05 (.08)	.05 (.07)

Notes: Mismeasurement parameters α_j, as in (A3). Otherwise, as in Table 5 in the paper.

Appendix B: Similarity of FIML & 3SLS Estimates

In the paper I use full information maximum likelihood methods, as this allows me to implement instrumental variables for both the SUR and structural models in a common framework. As noted therein, results for the SUR regressions are almost identical to using 3SLS and hence do not depend upon the assumption of a normal likelihood. Table B1 shows this by reporting the FIML and 3SLS results for all instrumented specifications reported in Section IV of the paper. The 3SLS results are arrived at by regressing $\bar{\theta}$ and $\bar{\Omega}$ on the initial values θ and Ω and all other exogenous variables on the righthand side of the non-linear SUR system (14) in the paper (including lagged dependent variables and additional controls in some specifications) and then using the predicted values to estimate the non-linear SUR coefficients. Specifications are as in Tables 8 and 9 in the paper. As Stata does not have a ready-made package for non-linear 3SLS, and my interest is in point estimates rather than standard errors, the standard errors reported for 3SLS are for the non-linear SUR using the predicted values, i.e. do not account for the first stage procedure.

Table B1: Comparing FIML and 3SLS Results for
Computer & Electronics Intermediates Specifications Reported in the Paper

		no lags	1 lag	2 lags	3 lags	no lags	1 lag	2 lags	3 lags
		(A) baseline specification				(B) capital utilization adjusted TFP growth			
PMC XR	FIML	-.50	-.42	-.56	-.40	-.46	-.38	-.51	-.37
		(.10)	(.11)	(.12)	(.14)	(.10)	(.12)	(.13)	(.15)
	3SLS	-.50	-.42	-.56	-.40	-.46	-.38	-.51	-.37
		(.11)	(.12)	(.13)	(.14)	(.10)	(.12)	(.13)	(.15)
PMC XIG	FIML	-.59	-.49	-.47	-.43	-.55	-.46	-.43	-.44
		(.08)	(.09)	(.11)	(.12)	(.07)	(.09)	(.11)	(.11)
	3SLS	-.59	-.49	-.47	-.43	-.55	-.46	-.43	-.44
		(.08)	(.09)	(.11)	(.12)	(.08)	(.09)	(.11)	(.11)
$PQ_{\sim O}$	FIML	-.45	-.33	-.52	-.26	-.37	-.28	-.40	-.16
		(.17)	(.19)	(.19)	(.20)	(.18)	(.19)	(.18)	(.21)
	3SLS	-.45	-.33	-.52	-.26	-.37	-.28	-.41	-.16
		(.18)	(.19)	(.19)	(.20)	(.19)	(.20)	(.19)	(.21)
		(C) unemployment level controls				(D) unemployment change controls			
PMC XR	FIML	-.46	-.42	-.56	-.38	-.51	-.45	-.52	-.33
		(.10)	(.12)	(.13)	(.14)	(.09)	(.10)	(.11)	(.13)
	3SLS	-.46	-.42	-.56	-.38	-.51	-.45	-.52	-.33
		(.10)	(.12)	(.13)	(.14)	(.09)	(.11)	(.12)	(.13)
PMC XIG	FIML	-.58	-.49	-.45	-.39	-.60	-.47	-.42	-.37
		(.08)	(.09)	(.11)	(.12)	(.07)	(.08)	(.10)	(.11)
	3SLS	-.58	-.49	-.46	-.39	-.60	-.47	-.42	-.37
		(.08)	(.09)	(.11)	(.12)	(.08)	(.09)	(.11)	(.11)
$PQ_{\sim O}$	FIML	-.41	-.34	-.56	-.26	-.49	-.39	-.48	-.17
		(.17)	(.19)	(.19)	(.20)	(.16)	(.17)	(.18)	(.18)
	3SLS	-.41	-.34	-.56	-.26	-.49	-.39	-.48	-.17
		(.18)	(.19)	(.19)	(.20)	(.16)	(.17)	(.18)	(.18)

(E) estimation using industry means (# of observations = # of industries)

PMCXR: 44 industries		PMCXIG: 20 industries		$PQ_{\sim O}$: 61 industries	
average	instrumented	average	instrumented	average	instrumented
-.83	-.82	-.40	-.40	-.71	-.71
(.26)	(.27)	(.36)	(.35)	(.15)	(.16)

Notes: Heteroskedasticity robust standard errors in (). Specifications are as in Tables 8 and 9 in the paper.