Alpha and Beta of Buyout Deals: A Jump CAPM for Long-Term Illiquid Investments

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PRELIMINARY

COMMENTS WELCOME

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This paper estimates the risk and performance of buyout transactions. In a typical buyout deal, a buyout fund acquires a company. The fund becomes the sole shareholder, and a large fraction of the acquisition price is financed with debt. The fund then owns and manages the company for an extended period of time, typically several years, until the company is eventually sold in an exit, either into the public market, to a corporate acquirer, or to another buyout fund (or liquidated). We evaluate the risk and risk-adjusted performance that buyout funds generate on their equity investments in these deals. We find an alpha of 8.3%-8.6% annually, and a beta of 2.2-2.4. These numbers represent the total return earned on the levered equity investment in the buyout deals, gross-of-fees, and the return is shared between the limited partners ("LP"), which provide the capital to the fund, and the general partner ("GP"), which charges carried interest and management fees for managing the fund.

Evaluating the risk and performance of buyout deals is important. When assessing the role of buyout transactions in the economy, it would be a concern if these transactions were financially underperforming the market. Moreover, the risk and performance are important for LPs, such as pension funds, university endowment, and sovereign wealth funds, which manage PE investments. Our analysis considers the return to the equity invested in individual buyout deals, but LPs are obviously mostly concerned about the aggregate risk and return of an entire fund, net of carried interest and management fees. In order to evaluate fund-level performance, however, it is useful to start from the individual deals and aggregate up to fund-level performance. Most existing studies have evaluated buyout performance using data that aggregate fund-level net-of-fees cash flows, without attributing these cash flows to the individual deals. For example, Kaplan and Schoar (2005) regress fund-level IRRs on S&P-500 and report a coefficient of 0.38-0.41. Franzoni, Nowak, and Phalippou (2011) and Driesssen, Lin, and Phalippou (2011) estimate other one-factor market models and report market betas of 0.9 and 1.3-1.5. Jegadeesh, Kraussl, and Pollet (2010) take another approach and estimate alpha and betas of publicly-traded private equity funds, and find betas of 0.7-1.0.

Betas around 1.0 are puzzling, though. A simple Modigliani-Miller (1958) calculation suggests that the beta should be 2.2-2.7. To see this, note that Axelson, Jenkinson, Stromberg, and Weisbach (2013) find an average D/E ratio around 3.0 in a sample of buyout transactions (i.e., a typical buyout deal is financed with 75% debt and 25% equity). By definition, publicly-traded companies have an average levered beta (or equity beta) of one, and with an average D/E ratio of 0.5, their unlevered beta (or asset beta) is 0.66. After matching portfolio companies in buyout deals to their publicly-traded comparable counterparts, Ljungqvist and Richardson (2003) find average betas for these comparable companies of 1.04-1.13, suggesting that portfolio companies in buyout transactions have average betas and hence also have unlevered betas of 0.66. With a D/E ratio of 3.0, Modigliani-Miller then implies a levered equity beta of 2.7. A beta of 2.7 may be too high, however, because this calculation assumes risk-free debt. With a debt beta of 0.1 (Kaplan and Stein, 1990), the equity beta becomes 2.2. This beta, however, is still far above the betas of 0.7-1.5 from previous empirical studies. These studies suggest that buyout funds can acquire regular companies with equity betas around 1.0 and then increase their leverage six-fold, yet leave their systematic risk unchanged. This “beta puzzle” challenges our understanding of the relationship between leverage and risk.

There are three potential explanations for this puzzle: First, the betas found using deal-level cash flows may be downward biased and underestimate the true value. Second, the GP's fees may reduce the beta. The Modigliani-Miller calculation applies to the equity investment in the deal, gross of fees, whereas the fund-level cash flows used in previous studies are reported net of fees. Third, Modigliani-Miller may not apply to the portfolio companies of buyout funds. The process of acquiring these companies, taking them private, and restructuring their governance may change their financial performance and reduce their risk exposure relative to publicly-traded companies. Investigating these explanations, we find some support for the first two explanations, but we find no support for the third one.

To evaluate the risk and performance of buyout transactions, we face two difficulties: First, deal-level performance data for buyout investments are highly confidential and they have been difficult to access for academic researchers. We present a new such dataset obtained from a large fund-of-funds, referred to as "The LP." The LP manages more than USD 50 billion of investments in buyout and venture capital funds, and the dataset contains the LP's internal deal-level cash-flow information for 2,075 buyout deals, managed by around 250 funds. Importantly, the LP compiles performance data gross of the GPs' "carried interest," enabling us to compare the gross- and net-of-fee risks, which helps resolve the second explanation for the "beta puzzle." Second, the long-term illiquid nature of buyout deals means that their performance is not well described by standard asset pricing models, which are more designed for traded securities with regularly quoted returns. Hence, we evaluate buyout performance using a range of models, and provide some extensions of these models to better capture the long-term illiquid nature of buyouts.

Three related papers have estimated deal-level performance of venture capital ("VC") investments, which have structures that are similar to buyout deals, but for which data with individual transaction have been more readily accessible. Gompers and Lerner (1997) estimate the static capital asset pricing model ("CAPM") using the deals of a single VC firm (Warburg Pincus) from 1972 to 1977, and they find an alpha of 30.5% with a beta of 1.08-1.44. Cochrane (2005) and Korteweg and Sorensen (2011) argue that venture capital data suffer from a severe selection bias, because better-performing start-ups are more frequently observed in the data and underperforming start-ups tend to disappear without any observed exit or liquidation event, and they develop various dynamic sample selection corrections. This sample selection problem is a smaller concern for buyout deals, because buyout transactions involve mature companies and these companies have well-defined and observed exit events, even when this exit is bankruptcy or liquidation.

Finally, two studies have evaluated the performance of buyout investments using fund-level data without directly estimating the risks of these investments.[1] Ljungqvist and Richardson (2003) analyze data from another large LP investing during 1981-1993 (in a total of 19 VC funds and 54 buyout funds), assuming that these deals have the same risk as comparable publicly-traded companies. Kaplan and Schoar (2005) evaluate fund-

[1] Two recent surveys are Ang and Sorensen (2013) and Meyer, Cornelius, Diller, and Guennoc (2013).

level buyout performance using data from Venture Economics, of which 746 funds were fully or mostly liquidated at the time of the study, assuming a beta of one.

I. Data and Return Measures

Our data are provided by a large fund-of-funds, which we refer to as the LP. The LP is one of the world's largest investors in private equity, including both buyout and venture capital funds. The LP has collected performance data for most of the individual deals made by the funds in which it invested as an LP. Our performance data are current as of February 2013, but we limit our sample to investments made up to 2008 to give these deals time to mature. Before 2008, the LP has invested in around 250 buyout funds, and we have performance information for 2,075 deals. Stromberg (2007) report that the total PE industry, at the time, comprised around 8,000 deals. Today, the LPs overall private equity allocation (including venture capital) exceeds USD 50 billion.

The LP classifies buyout funds as either: Large cap, US mid-market, EU mid-market, or non-traditional markets ("NTM") funds. Loosely defined, large cap funds are funds with total committed capital exceeding USD 5 billion, although in the earlier part of our sample, during the 1990s, this cut-off was lower. Mid-market funds typically have fund sizes between USD 500 million and 5 billion. Funds classified as non-traditional markets ("NTM") include funds that focus on Advanced Asia (primarily Japan, and Australia), Emerging Asia (primarily China and India), Latin America, and Eastern Europe. Although not included in this study, The LP also invests in venture capital funds, which invest in early and later-stage startups, including some growth-capital funds that are managed by venture capital GPs.

B. Cash Flow Data

The LP collects cash-flow information for most of the individual buyout deals undertaken by the funds in which it has invested, including the timing and amounts of these cash flows. Typically, for buyout investments, these cash flows consist of an initial investment followed by an ultimate exit. The recorded cash flows are gross of the "carried interest," which is the performance fees charged by the PE firm that is the fund's general partner ("GP"). Moreover the cash flow data do not subtract management fees. Due to this absence of fees, we can evaluate risk and return gross-of-fees, which are comparable to the Modigliani-Miller calculation in the introduction. We then subtract the fees and evaluate their effects on the risk and return. We have performance information for 2,075 individual deals, and the timing of these deals is reported in Table I.

--- TABLE I ---

C. Return Measures

To distinguish different return measures, let the annualized arithmetic return be denoted uppercase *R*. When a deal has only two associated cash flows, an initial investment, $I(t)$, and a subsequent exit, $X(t')$, then *R* is given as:

$$R = \left(\frac{X(t')}{I(t)} \right)^{\frac{1}{t'-t}} - 1 \ . \tag{1}$$

Without intermediate cash flows, *R* equals the internal rate of return ("*IRR*"), as commonly defined. With intermediate cash flows, due to interim recapitalizations or equity injections, this return is not well defined and instead we define R as the IRR of the deal's cash flows. The total arithmetic return is $R(t',t)=(R+1)^{t'-t}-1$. We use lowercase $r(t,t')$ to denote log-returns. The total log-return is $r(t',t)=\ln\left(R(t,t')+1\right)$, and the annualized log-return is $r=\frac{r(t,t')}{t'-t}$.

To illustrate, a \$1 initial investment that turns into a \$3 exit after 5 years will have a total return of $R(0,5)$ = 200%, an annualized return of R = 24.6%, a total log-return of $r(0,5)$ = 109.9%, and an annualized log-return of r = 22.0%. For smaller values, the log-return is close to the arithmetic return. For larger values, these two returns are not directly comparable. One inherent limitation of the log-return is that the calculation fails for returns of -100%. In our data, 479 out of 2,075 buyout deals had returns of -100%. In these cases, we set the log-return to -3, corresponding to a -95% arithmetic return. We find that value of this cut-off has very little effect on our results, and we also report estimates of a Tobin model that explicitly accounts for the left-truncation of the return distribution.

Let $R_M(t,t')$ denotes the total return on the public market over the corresponding period, and similarly let R_M, r_M, and $r_M(t,t')$ denote the annualized return, annualized log-return, and total log-return on the public market. Risk-free returns are defined analogously. All market returns and risk-free rates are calculated from market data downloaded from Kenneth French's website.

II. Excess Returns

As a starting point, consider basic buyout returns without any form of risk adjustment. Across fund types and investment periods, Table II reports annualized arithmetic returns and excess returns, calculated relative to the market return and the risk-free rate over the same period as the individual deals. Column (1) shows average IRRs, and the last row shows an average return of 12.1% annually for the deals in our sample. In the early part of the sample (1994-2000) the average IRR was -1.2%, and in the late part of the sample (2001-2007), the average IRR improved to 22.6%. Public markets also performed differently in these two periods, however, and Column (2) shows the performance relative to the market, calculated as *IRR* minus R_M, where the market return is the annualized market return calculated over the period of each individual deal in our sample. Across all deals, average performance relative to the market was 9.2% annually. In the early period, relative performance was -1.4%, improving to 17.6% in the late period. This performance seems fairly consistent across Large Cap, US Mid-Market, and EU Mid-Market funds, but funds in Non-traditional Markets (NTM) may have underperformed these other fund types.

--- TABLE II ---

Column (3) presents excess returns relative to the risk-free rate. Interestingly, the returns in reported Columns (3) and (4) are very close, because the public market has not performed very well over the sample period. On average, the market return calculated over the period of each deal has largely been equal to the risk-free rate, and the excess return on on the market, calculated over the period of each deal, is close to zero. In Table

II, Column (4) shows that Large Cap and US Mid-Market funds experienced negative excess market returns, and the overall average excess return on the market is -0.2%. Recall that the beta measures how buyout returns "ride the market," and alpha measures the additional excess performance. However, for the buyout deals in our sample, there has been no market return to ride. The excess return on the market has been essentially zero, and the 9.0% excess return on the buyout deals, reported in Column (3), is all alpha. This low excess return on the market allow us to provide a "model-free," alpha, which does not require any assumptions about linearity or the distribution of the returns. Estimating the beta, however, requires more statistical modeling, which involves such assumptions.

III. Static CAPM

To evaluate risk and risk-adjusted performance, a natural starting point is the static capital asset pricing model ("CAPM"). Using ordinary least squares ("OLS"), we estimate the standard CAPM regression:

$$R - R_F = \alpha + \beta\left(R_M - R_F\right) \ . \qquad (2)$$

Here, R is the annualized return of each deal, R_F is the risk-free rate, and R_M is the annualized market return. Summary statistics for the excess returns are in Columns (3) and (4) of Table II and Table III presents the estimated coefficients. Table III shows specifications with and without investment-year fixed effects ("FEs"). With these FEs, the regression compares the performance of investments started in the same year, but with different durations, and the variation in the market returns arises due to their different durations. The CAPM is valid with and without these FEs, and the results are similar.[2] The results in Table III show very large systematic risk exposures, with betas ranging from 2.21-2.48 in the late part of the sample and increasing to 3.58-3.75 in the early part. Across all deals, the average beta is 3.01-3.14, and these betas are substantially higher than the betas around 1.0 found in earlier empirical studies. They are even above the 2.2-2.7 range that was implied by the Modigliani-Miller calculation.

--- TABLE III ---

Despite these high betas, the risk-adjusted performance remains positive. Across all deals, the average alpha is 9.53%-9.55% annually. This alpha is close to the "model-free" alpha of 9.0% inferred from the returns in Table II. The alpha ranges from 7.56%-8.14% in the early sample period and increases to 14.0%-14.65% in the late part of the sample. Perhaps surprisingly, in the early part of the sample, the alpha remains positive even though buyouts underperformed the market, as shown in Table II. This finding is due to the market dynamics over the sample period. Column (4) in Table II shows that the early period experienced negative excess market returns relative to the risk-free rate. The reason for these negative excess returns is illustrated in Figure 2, which shows the market performance over our sample period. This period experienced two market "bubbles," which peaked roughly 7 years apart in 2000 and 2007. Market has followed a pattern with two parallel cycles roughly six-years apart, which is close to the average length of a PE buyout deal, of 4.5 years. Figure 1 shows the distribution of the durations of buyout deals, and it is not uncommon for a deal to last for 6 or 7 years and hence be

[2] With year-FEs the reported intercept is calculated as the weighted average FE (see documentation for Stata's AREG command).

"in phase" with the overall market. When a deal is in phase with the market, the average market return, calculated over the period of the deal, is much closer to zero than one would expect from the large swings in the overall market performance. Hence, despite the two "bubbles" and subsequent declines, each exceeding 40%, the average annualized excess market return over the periods of the deals in our sample is just -0.2% and during the early period, this excess return is -3.5%. When the excess market return is negative, high-beta investments should severely underperform the market. In the early part of the sample, buyout deals did underperform, but only slightly, implying a positive alpha.

Another concern is whether the static CAPM is an appropriate model of buyout deals. The static CAPM is a single-period model. It is not dynamic, and it does not compound. Typically, the static CAPM is applied to traded securities with return that are quoted regularly, such as daily, weekly, or monthly, and the standard CAPM can then be interpreted as a repeated static model over this time interval. It is important, though, that the returns are all calculated over periods of the same length. For buyout deals, this is problematic. Without quoted market prices, buyout returns are only observed at the end of a deal, and deals have large variations in their durations, as seen in Figure 2. This variation may bias the estimates of the static CAPM, and a more appropriate model is the continuous-time or log-return version of the CAPM.

IV. Log-return CAPM

The continuous-time CAPM in log returns (Merton 1973) is a dynamic version of the CAPM. An important advantage of this version of the model is that it compounds and consistently accommodates returns calculated over periods of different lengths (see the discussion in Campbell, Lo, MacKinlay 1997, pp. 363-64). A disadvantage is that this model requires stronger assumptions about the distribution of the return process.

To fix ideas and notation, it is useful to derive the log-return CAPM from fundamentals. There are three assets: A risk-free bond, the public market portfolio, and the PE deal. These assets generate the following returns. The risk-free bond pays the (continuously-compounded) log-return r_F. The return to the public market portfolio follows the geometric Brownian motion:

$$\frac{dM(t)}{M(t)} = \mu_M dt + \sigma_M dW_M(t) \ . \tag{3}$$

The value of a company acquired in a deal follows the geometric Brownian motion:

$$\frac{dV(t)}{V(t)} - r_F dt = \alpha dt + \beta\left(\frac{dM(t)}{M(t)} - r_F dt\right) + \sigma_I dW(t) \ , \tag{4}$$

where $V(t)$ is the time-t value of the company, alpha is the excess risk-adjusted return, and sigma denotes the instantaneous volatility. By definition, the Brownian motions $dW(t)$ and $dW_M(t)$ are independent. The investment is made at time t, the exit is at time t', and the duration of the deal is $t'-t$. The return is only observed at exit. The total log-return is $r(t,t') = \ln\left(V(t')/V(t)\right)$, and a standard application of Ito's lemma gives the discrete-time dynamics of this log-return:

$$r(t,t') - r_F(t,t') = (t'-t)\delta + \beta\left(r_M(t,t') - r_F(t,t')\right) + \varepsilon(t,t') \ , \tag{5}$$

where $\delta = \alpha - \tfrac{1}{2}\sigma_I^2 + \tfrac{1}{2}\sigma_M^2\beta(1-\beta)$, and the error term is distributed as:

$$\varepsilon(t,t') \sim N\left(0,(t'-t)\sigma^2\right) \quad . \tag{6}$$

A. Empirical Implementation

The log-return CAPM model is usually estimated using feasible GLS ("FGLS"). FGLS estimation proceeds in two steps. First, equation (5) is estimated without adjusting for the heteroscedastic error term, and the residuals are used to estimate the variance structure. In the second step, equation (5) is normalized by the inverse of the predicted standard deviation, and this normalized equation is then estimated with OLS. (For a textbook description of this FGLS procedure, see Goldberger (1991), pp. 297-301.)

In the first step, we estimate the regression in equation (5) and calculate the squared residuals. These squared residuals, denoted e^2, are then regressed on the length of each deal and a constant term, using the OLS regression:

$$e^2 = s_0 + (t'-t)s_1 + \varepsilon \quad . \tag{7}$$

The coefficient s_1 captures the increase in the variance as the length of a deal increases, which estimates the instantaneous variance term σ^2 in equation (6). The estimated coefficients are in Table IV. Note that there is substantial variation in the variances estimated in the early and late parts of the sample. For the full period, the instantaneous variance is 14.1%, implying an instantaneous volatility of 37.5%. For comparison, the instantaneous volatility of the market (log-)return over the sample period is 18.9%. Hence, the instantaneous volatility of buyout deals is about twice that of the public market, which is reasonable given the higher levels of leverage of these deals.

--- TABLE IV ---

In the second step of the FGLS estimator, the regression in equation (5) is normalized by the predicted standard deviation, which is calculated using the s_0 and s_1 coefficients from equation (7). The resulting equation is estimated with OLS:

$$\frac{r(t,t') - r_F(t,t')}{\sqrt{s}} = \frac{t'-t}{\sqrt{s}}\delta + \beta\left(\frac{r_M(t,t') - r_F(t,t')}{\sqrt{s}}\right) + \varepsilon \ . \tag{8}$$

Estimates of this regression are in Column (1) in Panel A of Table V. The estimated intercept is -0.046 and the beta is 2.417. This beta can be directly interpreted as an estimate of the systematic risk of returns to buyout deals.[3] It is somewhat lower than the beta estimates from the static CAPM, but the log-return CAPM is a more appropriate model given the irregular nature of buyout returns. And a beta of 2.4 is more consistent with the initial Modigliani-Miller calculation.

--- TABLE V ---

In the log-return CAPM the estimated intercept is not a measure of the risk-adjusted performance. It is not an alpha. The derivation of equation (5), however, also shows that the alpha can by found by adding an adjustment term to the intercept (denoted delta):

$$\alpha = \delta + \left[\tfrac{1}{2}\sigma_I^2 - \tfrac{1}{2}\sigma_M^2\beta(1-\beta)\right] \ . \tag{9}$$

[3] Formally, the instantaneous beta needs to be adjusted to be comparable to the discrete-time beta in the static CAPM. Cochrane (2005) in footnote 5, and Franzoni, Nowak, and Phalippou (2013) in equations (6) and (7) provide expressions for this adjustment. Both papers find that the adjustment is small, and that the instantaneous beta is (almost) directly comparable to the discrete-time beta.

Using the estimates in Column (1) of Panel A in Table V, this adjustment gives an alpha of 8.6%, annually $\left(= -0.046 + \left[0.141/2 - 0.189^2/2 \times 2.417 \times (1-2.417)\right]\right)$. This alpha seems reasonable. It is close to the "model-free" alpha of 9.0% from Table II. As noted, an important benefit of this log-return CAPM is that it is consistent when the returns are measured over periods of different lengths, however, it also requires stronger assumptions about the return process, and the alpha estimate is more uncertain because it requires an adjustment with one-half the instantaneous variance, and this variance is estimated with considerable uncertainty. Nevertheless, the estimates from the log-return CAPM seem to be more credible than those from the static CAPM.

A limitation of the log-return CAPM is that it may be sensitive to returns of -100%. In our data, 479 out of 2,075 buyout deals had returns of -100%, and for these deals we set the log-return to -3, corresponding to a -95% arithmetic return. To investigate the sensitivity of our results to this specification, we first vary the cut-off level (e.g., by setting the log return to -4), and we find that this has a small effect on our results. As a more formal approach, Panel B of Table V reports Tobit estimates that explicitly account for the left-truncation of the return distribution. With the Tobit specification, the beta increases to 2.934, and the intercept declines to -0.089. Despite the changes in these coefficients, the implied alpha only decreases slightly to 8.3% annually, compared to 8.6% from before.

Another limitation of the log-return CAPM is that it is not entirely consistent with the empirical variance structure reported in Table IV. The log-return model implies that the distribution of the error term in equation (6) must increase linearly with the length of a deal. Table IV, however, shows a positive and highly statistically significant intercept of 2.967, and in the limit, when the length of a deal goes to zero, the variance of its (log-)return goes to 2.967. In the standard log-return CAPM this limit should be zero.

V. Jump Specification

The finding that the volatility does not vanish as the length of a deal goes to zero suggests that the performance of all buyout deals, even short ones, is exposed to a certain amount of risk, independently of the length of the deal. Formally, such a shock can be modeled as a jump term in the valuation process. Intuitively, such a jump is a natural feature of long-term illiquid investments. For example, if a buyout deal involves initially acquiring a company at a discount (or premium), and this discount is random, it will introduce a given amount of volatility in the deal's performance, even for very short deals. Similarly, this volatility may arise at the exit if the company is sold at a random discount (or premium). Including a jump term in the valuation process captures the part of the volatility that is constant regardless of the length of a transaction. This jump may also make the model more robust to returns of -100%. These returns are problematic in the log-return CAPM, because a geometric Brownian motion can only approach -100% returns asymptotically, but a jump distribution with wide support may allow the return to "jump" to -100%.

To formally include a jump term in the log-return CAPM, we first write a specification with jumps both at the initial investment and at the exit. We then show that these two jumps collapse into a single compound jump, and that the timing of the jump(s), either at the beginning, in the middle, or at the end of the deal is immaterial. Let the initial investment be I, and let the value at exit be X. During the deal, the underlying value follows the valuation process $V(t)$ from the log-return CAPM described previously. Initially, this underlying value jumps relative to the investment, and it is given as follows:

$$V(t) = I \times \varepsilon_I \quad . \tag{10}$$

For example, if the PE firm acquires a company at a 20% discount relative to its underlying value, then $\varepsilon_I = 1.25$. Similarly, the observed exit value, X, jumps relative to the underlying valuation at time t':

$$X = V(t') \times \varepsilon_X \quad . \tag{11}$$

The initial investment, I, and the exit value, X, are observed in the data, but the underlying valuation process, $V(t)$, is unobserved. The jump terms are independent across deals and independent of each other. Moreover, because the jumps are instantaneous, they are independent of the Brownian motions $dW(t)$ and $dW_M(t)$ and there is no risk premium associated with these jumps. Let $\tilde{r}(t,t') = \ln\left[X(t')/I(t)\right]$ be the observed log return on the deal:

$$\tilde{r}(t,t') = \ln[V(t')/V(t)] + \left(\ln[\varepsilon_I] + \ln[\varepsilon_X]\right) \quad . \tag{12}$$

The last term, in parentheses, shows that the two jumps, at the beginning and end of the deal, collapse into a single compound jump term. This compound jump term is given as $\varepsilon_J = \ln[\varepsilon_I] + \ln[\varepsilon_X]$. Its mean and variance are denoted γ and σ_J^2, and we are not imposing any assumptions on the distribution of the jump term.

To estimate the "jump CAPM," note that the discrete-time dynamics of $V(t)$ are unchanged from the log-return CAPM:

$$\ln\left[V(t')/V(t)\right] - r_F(t,t') = (t'-t)\delta + \beta\left(r_M(t,t') - r_F(t,t')\right) + \varepsilon(t,t') \ . \tag{13}$$

Combining this expression with equation (12), gives the discrete-time dynamics of the jump-CAPM:

$$\tilde{r}(t,t') - r_F(t,t') = \gamma + (t'-t)\delta + \beta\left(r_M(t,t') - r_F(t,t')\right) + \tilde{\varepsilon}(t,t') \ , \tag{14}$$

Except for the gamma term, which is the mean of the jump distribution, this expression is identical to the one for the log-return CAPM. The error term is distributed:

$$\tilde{\varepsilon}(t,t') \sim N\left(0, \sigma_J^2 + (t'-t)\sigma_I^2\right) \ . \tag{15}$$

The variance now has two parts: A constant term, which is the variance of the jump term, and the standard term that increases linearly with the length of the deal.

The jump CAPM can be estimated using the same FGLS procedure, and the coefficients are reported in Columns (2)-(4) of Table V. Including the jump reduces the beta slightly, from 2.417 to 2.278, in the overall sample. Comparing the betas for the early and late parts of the sample, there is also an indication that the beta has declined, from 2.258 to 1.873, which is consistent with the lower leverage of more recent deals.

With the jump, however, the intercept increases from -0.046 to 0.041. Using the adjustment in equation (9), this increase in the intercept implies a corresponding increase in the alpha from 8.6% to 16.3%, annually. Deal performance, however, now has two parts: The standard alpha, which reflects the gradual performance accruing over the life of the deal, and the "one-time alpha" due to the jump term. Although the ongoing alpha has increased to 16.3%, the mean of the jump term is negative, given by the estimated gamma coefficient of -0.738.

A. Empirical Implementation

Estimating the magnitude of the "one-time alpha" is nontrivial. From equation (12), it follows that the total arithmetic return on the deal equals:

$$R(t,t') = V(t') / V(t) \times \exp\left[\varepsilon_J\right] \ . \tag{16}$$

Define

$$\lambda = E\left[\exp\left(\varepsilon_J\right)\right] \ . \tag{17}$$

The jump is performance neutral, on average, when $\lambda = 1$, and we define the one-time alpha as $\lambda - 1$. To recover λ from the estimated parameters, first calculate the empirical residual:

$$\hat{e} = \tilde{r}(t,t') - r_F(t,t') - (t'-t)\hat{\delta} - \hat{\beta}\left(r_M(t,t') - r_F(t,t')\right) \ . \tag{18}$$

Here, $\hat{\delta}$ and $\hat{\beta}$ are the estimated coefficients in equation (14). Note the absence of the γ term. This empirical residual is the sum of the jump error and the error in the underlying valuation process:

$$\hat{e} = \varepsilon_J + \varepsilon(t,t') \ . \tag{19}$$

Hence, $E\left[\hat{e}\right]=\gamma$ and $Var\left[\hat{e}\right]=\sigma_J^2+(t-t')\sigma_I^2$ (this variance is a conditional variance? XXXX). To recover λ, we decompose this sum of empirical residual into its two parts, using a deconvolution result for moment-generating functions of two independent random variables:

$$E\left[\exp\left(\varepsilon_J+\varepsilon(t,t')\right)\right]=E\left[\exp\left(\varepsilon_J\right)\right]E\left[\exp\left(\varepsilon(t,t')\right)\right] \; . \qquad (20)$$

Define $y=\exp[\hat{e}]$ and $x(t,t')=\exp\left[(t'-t)\left(\hat{\delta}+\tfrac{1}{2}\sigma_I^2\right)\right]$, (there should not be a delta is this past equation, I think XXXX) which equals $E\left[\exp\left(\varepsilon(t,t')\right)\right]$ under the normality assumption for the underlying valuation process. Now, λ is consistently estimated as the coefficient in the linear regression:

$$y=\lambda x(t,t')+\omega \; . \qquad (21)$$

Where ω is a residual with mean zero. Estimates of λ are reported in table XXX. Note that the standard errors are corrected for the estimation uncertainty in x().

--- TABLE VI ---

The generality of the non-parametric distribution of the jump term is important for the estimates of the one-time alpha. A natural distributional assumption is that the compound jump is log-normal distributed. In this case, the mean jump is $E[\exp(\varepsilon_J)]=\exp(\hat{\gamma}+\tfrac{1}{2}\sigma_J^2)$, which equals 2.75, implying a one-time alpha of 175%, which is unreasonable when compared to the non-parametric estimate above. This suggests that particular distributional features, such as a large negative jump that captures the negative 100% returns, are important components of the jump distribution.

The estimates suggest an interesting performance pattern. The large positive alpha and negative gamma may indicate that buyout funds initially acquire companies at a premium and realize a negative performance shock. This is consistent with well-known evidence from public markets where acquirers tend to pay an acquisition premium and where acquisitions are associated with negative announcement returns. Over the life of the deal, the buyout fund then generates the alpha and eventually this excess performance makes up for the loss in the acquisition. The estimates in Table V also suggest that this pattern has changed over the sample period. The early part of the sample had a higher intercept but a more negative gamma, suggesting that buyout funds paid higher premiums but subsequently earned higher returns. In the late part of the sample, the jump term is less negative, but the alpha is also smaller. The results in Tables I and II, however, suggest that these two effects have led to better combined performance of buyout deals in the later part of the sample.

V. Resolving the "Beta Puzzle"

To summarize our empirical results up to this point, gross of fees, Table II shows that buyouts have outperformed the market, and it shows a "model-free" alpha of 9.0%, annually. Using the static CAPM, Table III reports betas of 2.2-3.8 and alphas of 7.6%-14.7%, annually. Using the log-return and jump CAPMs, which may be more appropriate given the irregular and long-term illiquid nature of buyout deals, across all deals, Table V shows an average beta of 2.3-2.4 and alphas of 8.3%-8.6%. These betas are consistent with the initial Modigliani-Miller calculation, which implies a beta of 2.2-2.7, and alphas are consistent with the "model-free" alpha. With the low excess return on the market over the past decades, a positive alpha is not inconsistent with a high level of systematic risk. The final remaining issue is how to reconcile our findings, using deal-level gross-of-fees performance data, with the low betas reported by previous empirical studies, which have mostly estimated performance from fund-level net-of-fees data.

To address the "beta puzzle," we use simulated data in an iterative approach. In each iteration we simulate the performance of 3,000 individual deals, using the log-CAPM, for different values of alpha and beta, and fixing the remaining parameters at their estimated values ($\gamma = -0.5$; $\sigma^2 = 0.141$; and $\sigma_J^2 = 2.967$). For the market return and risk free rate, we use the actual (not simulated) returns over the period from 1980 to 2010. To evaluate fund-level performance, we combine these individual deals into 150 funds, with 20 deals in each fund. Each fund lasts ten years. The funds' vintage years are drawn uniformly over the sample period. The lengths of the deals are drawn from empirical distribution of the lengths of the actual deals in our sample, as plotted in Figure 2 (conditional on the deal exiting before the fund is liquidated). The aggregate fund-level cash flows are then calculated by summing the cash flows of the individual deals in the fund. These fund-level cash flows are gross of fees. We also calculate net-of-fees fund-level cash flows by subtracting management fees and carried interest, depending of the fund's profit. Using these simulated deal- and fund-level data, we estimate a range of performance models.

For each choice of alpha and beta, we repeat this simulation and estimation procedure in 100 iterations (simulating 3,000 new deals for 150 funds in each iteration). These iterations result in 100 beta estimates for each model, and Table VI reports the mean and standard deviation of these 100 estimates. The rows represent different parameter choices of alpha and beta, and the columns correspond to the different models. For each parameter choice and for each model, the table shows the average beta estimate and its standard deviation calculated across the 100 iterations.

The results for the jump-CAPM shows that this model accurately recovers the true beta parameter. In all cases, the beta estimate is well within one standard deviation from its true value. Although not reported in the table, the jump-CAPM models also accurately recovers the alpha, gamma, and sigma parameters. This is unsurprising, since the deal performance is simulated using this model, and it confirms that our simulation and estimation procedures are consistent. The results for the log-return CAPM show that model also accurately recovers the true beta parameter, despite this model not capturing the jump term and having a misspecified variance structure. The static-CAPM, however, does not always recover the true beta. This model is inconsistent when the returns are measured over periods of different lengths, and Table VI shows that this generates a bias in the beta estimate from this model, and the bias increases with the alpha. For example, with a true beta of 2.0, the estimated betas increase from 1.8 to 2.3 as the alpha increases from 0% to 20%. The specification that is closest to our empirical results (marked in bold) has an alpha of 10% and a beta of 2.0. For this specification, the bias is small, and the static CAPM gives a beta that is reasonably close to the true value, in our simulations.

In each iteration, we aggregate the deal-level performance into fund-level cash flows, we calculate the IRR of each fund. The Gross IRR is calculated from the fund-level cash flows without subtracting management fees and carried interest. The Net IRR is calculated from the fund-level cash flows after subtracting a 2% annual management fees and 20% carried interest on the potential profits of each fund, as implied by the timing and performance of the individual deals. We estimate fund-level betas by regressing the simulated IRRs on the 10-year market return over the life of each fund.

Table VI shows that the beta estimated obtained using fund-level data are much less precisely estimated than those using deal-level data. Their standard deviations are often more that ten times greater than the corresponding standard deviation using deal-level data. This is unsurprising. Fund-level betas are estimated from just 150 funds, whereas deal-level betas are estimated from 3,000 deals. Table VI also shows that fund-level betas exhibit significant biases, which also appear to be increasing in alpha. The biases are especially problematic when the alpha equals zero. Nevertheless, focusing on the case that is closest to our empirical results (marked in bold), the Gross-IRR column reports an average beta estimate of 1.800, and the Net-IRR column shows that subtracting fees reduces this beta estimate to 1.336. Overall, these results go some way toward addressing the beta puzzle. It appears that subtracting the GP's management fees and carried interest reduces the estimated beta by around 0.5. However, the results also suggest that beta estimates obtained by regressing fund-level IRRs on market returns are somewhat suspect.

These biases may be due to several methodological problems that arise when estimating betas from fund-level IRRs. First, IRR is not a return. For a given cash flow, the IRR may not exist and it may not be unique. In some cases, the IRR is excessively sensitive to the cash flows early in the life of a fund. Moreover, when regressing fund-level IRRs on ten-year market returns, it may be difficult to construct the appropriate market returns. The number of companies managed by a fund varies over the life of the fund. Funds tend to have more companies under management during their middle years, and hence the fund's performance will be more sensitive to the market returns during those years. Consequently, the ten-year market return may need to be weighted by the fund's assets under management over time. Using un-weighted ten-year returns may introduce measurement error in the mature return, which would lead to a downward bias in the estimated betas using this approach.

V. Conclusion

This study estimates the alpha and beta of the equity investment in buyout deals. Our estimates differ from those from previous studies, because our data have performance information gross of fees for individual buyout deals. Gross-of-fees, we find betas of 2.2-2.4 and alphas of 8.3%-8.6% annually.

Our results help us address the "beta puzzle." Previous studies have found betas around 1.0 for buyout returns, mostly using data with fund-level net-of-fee performance. In other words, these studies suggest that buyout funds can acquire public-traded companies, which have average equity betas of 1.0 by definition, and then increase their leverage several times over, yet leave their equity beta unchanged around 1.0. This surprising. A standard Modigliani-Miller (1958) calculation that adjusts for the leverage of these deals implies that the beta should increase to 2.2-2.7. In contrast to previous studies, using deal-level gross-of-fees data and accounting for the long-term illiquid nature of buyout deals, we find that Modigliani-Miller does, in fact, hold. This is an important finding. It shows that this fundamental relationship between leverage and risk extends to privately-held and highly-levered companies.

Note also that, although not entirely comparable, the shares (units) of the publicly-traded buyout firms Blackstone Group (NYSE:BX), Fortress Investment Group (NYSE:FIG), and KKR & Co. (NYSE:KKR) have betas of 2.30, 2.42, and 2.10, respectively.

We find two explanations for the lower betas found by existing studies. The Modigliani-Miller calculation holds gross-of-fees but previous studies have used net-of-fees data. Our simulations suggest that subtracting fees may reduce the betas by about 0.5. Moreover, our simulations also indicate that betas that are estimated using aggregate fund-level cash flows, which do not attribute cash flows to individual deals, such as betas estimated by regressing fund-level IRRs on the ten-year market returns, appear to be downward biased. This bias may also contribute to the beta puzzle. In contrast, in our simulations, beta estimates using deal-level data are unbiased.

Some limitations of our analysis and results should be noted. First, our data may not be representative of buyout performance more broadly. While our sample extends across hundreds of buyout funds and thousands of individual deals, our data only cover a fraction of the overall buyout universe, and we cannot know whether our results reflect the average risk and return for buyout deals more broadly. Second, our risk and return estimates are gross of fees. Hence, they do not reflect an LP's net-of-fee performance. While we find a positive alpha, our analysis does not investigate how this alpha is shared between the GP and LP, and this performance may be partially or fully extracted by the GP in the form of management fees and carried interest. Some suggestive evidence is provided by Sorensen, Wang, and Yang (2013) who solve an LP's portfolio choice problem with private equity while accounting for the costs of illiquidity and the GP's fees. They find that the GP must generate an (unlevered) alpha of 2.06% for the LP to break even. With a D/E ratio of 3, this unlevered alpha roughly corresponds to a required alpha on the levered equity (as estimated here) of 8.24%. This required alpha is close to our estimates, indicating that LP's may, in fact, just break even, on average.

An interesting avenue for future research is to further investigate the implications of the jump-CAPM. This model may also apply to other long-term illiquid assets, such as real estate or municipal bonds, and the decomposition of performance into the standard alpha and the "one-time" jump term may be important when evaluating the risks and returns of these assets as well.

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Figure 1: Duration of Deals The duration (in years) of the buyout deals in our sample.

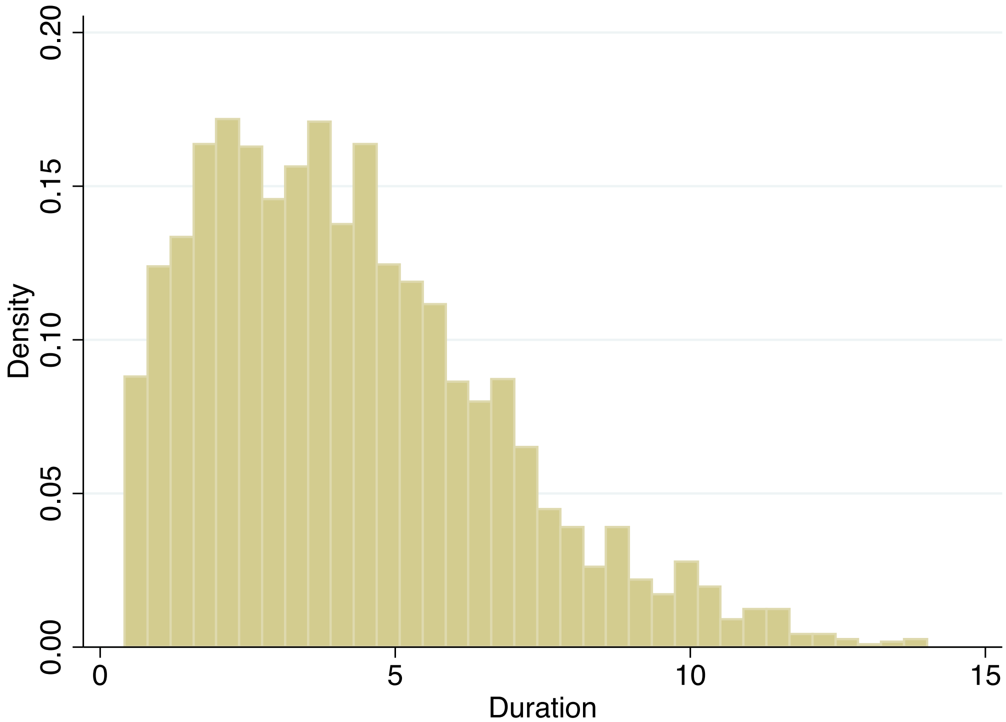

Figure 2: Market Return (1990-2011) This figure plots the cumulative market return normalized to 100 in January 1990.

Figure 3: Total Variance and Deal Duration The figure has the predicted variance as a function of the duration of a deal. The total (not annualized) variance is on the vertical axis. The duration (in years) is on the horizontal axis. The broken (green) line is the predicted variance estimated without an intercept. The solid (blue) line is the predicted variances implied by the estimated covariance structure of BO deals with an intercept.

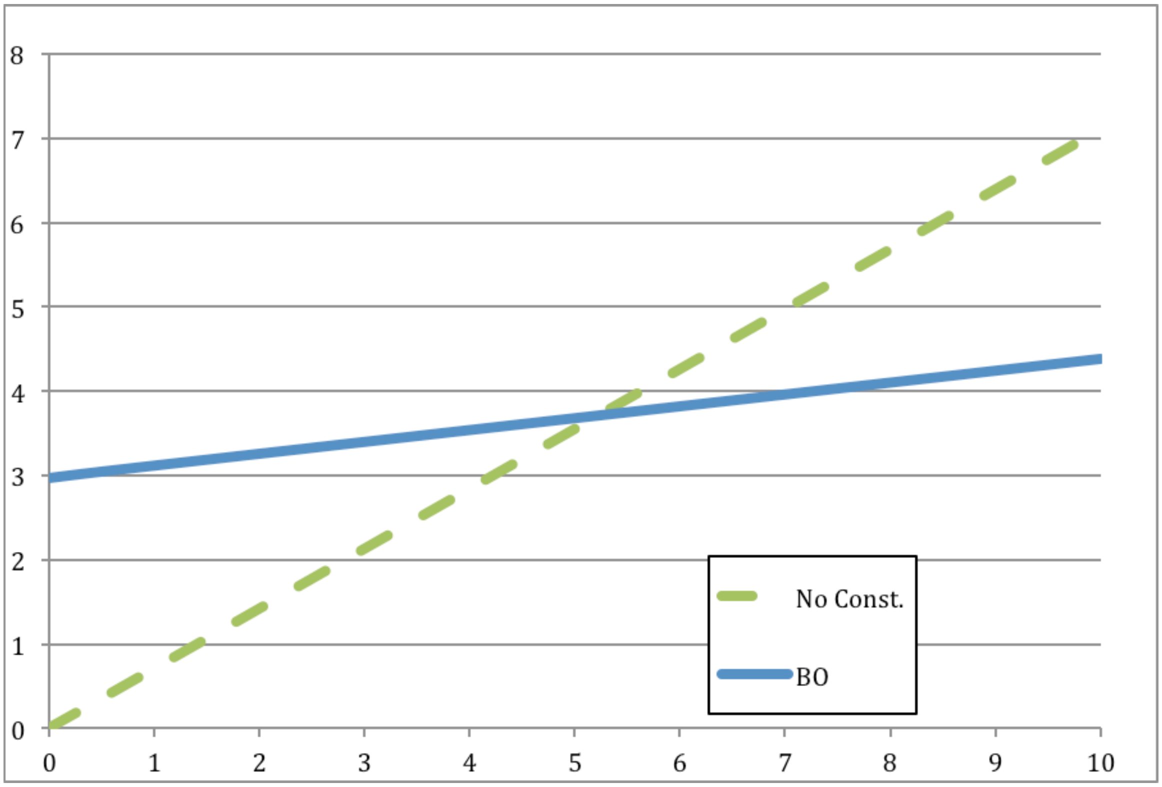

Table I: Buyout deals per year

Year	Buyout
1994	5
1995	12
1996	26
1997	110
1998	128
1999	241
2000	398
Sub-Total (1994-00)	920
2001	151
2002	127
2003	159
2004	225
2005	207
2006	170
2007	116
Sub-Total (2001-07)	1,155
Total	2,075

Table II: Absolute and Excess Annualized Returns The table presents absolute and excess returns for individual buyout transactions. Column (1) presents average returns (IRR) of the transactions. Columns (2) and (3) show annualized excess returns, calculated as the IRR minus the annualized market return and the risk-free rate, respectively; with the market return and risk-free rate calculated over the period of the corresponding transaction. Column (4) shows the excess return on the market, calculated as the market return minus the risk-free rate. The number of deals in parentheses.

	(1) R	(2) $R-R_M$	(3) $R-R_F$	(4) R_M-R_F
Large Cap	12.1%	10.4%	9.0%	-1.4%
	(774)	(774)	(774)	(774)
US Mid-Market	10.9%	8.5%	7.9%	-0.6%
	(352)	(352)	(352)	(352)
EU Mid-Market	18.7%	14.0%	15.7%	1.6%
	(489)	(489)	(489)	(489)
NTM	5.8%	2.6%	2.8%	0.2%
	(460)	(460)	(460)	(460)
Average Early (1994-00)	-1.2%	-1.4%	-4.9%	-3.5%
	(920)	(920)	(920)	(920)
Average Late (2001-07)	22.6%	17.6%	20.1%	2.4%
	(1,155)	(1,155)	(1,155)	(1,155)
Average All	12.1%	9.2%	9.0%	-0.2%
	(2,075)	(2,075)	(2,075)	(2,075)

Table III: Static CAPM This table presents OLS estimates of the static CAPM regression. The dependent variable is the annualized excess arithmetic return of each deal. The independent variable is the annualized excess market return (RMRF), calculated over the same period. Beta is the coefficient on the market return. Year FE indicates investment-year fixed effects. Robust standard errors are in parentheses. Statistical significance at the 10%, 5%, and 1% levels are indicated with *, **, and ***, respectively.

	All		Early (1994-00)		Late (2001-07)	
	(1)	(2)	(3)	(4)	(5)	(6)
Constant	9.55***	9.53***	7.56	8.14	14.0***	14.65***
	(3.30)	(3.31)	(7.65)	(9.71)	(2.18)	(4.94)
Beta	3.14***	3.01***	3.58***	3.75**	2.48***	2.21***
	(0.47)	(0.90)	(0.96)	(1.59)	(0.49)	(0.74)
Year FE	No	Yes	No	Yes	No	Yes
Obs.	2,075	2,075	920	920	1,155	1,155

Table IV: Variance Structure This table presents OLS coefficients of the variance structure for the FGLS estimator. The dependent variable is the squared residual from the first stage regression. The independent variable is the length of the deal (in years). Robust standard errors are in parentheses. Statistical significance at the 10%, 5%, and 1% levels are indicated with *, **, and ***, respectively.

	(1) All	(2) No Const.	(3) Early (1994-00)	(4) Late (2001-07)
Duration	0.141***	0.665***	-0.027	0.516***
	(0.038)	(0.020)	(0.045)	(0.065)
Constant	2.967***		3.361***	1.757***
	(0.187)		(0.248)	(0.282)
Obs.	2,075	2,075	920	1,155

Table V Log-return CAPM: This table shows regression coefficients from the second step of the FGLS estimator. Panel A shows standard FGLS estimates where the second step is estimated using OLS. Panel B shows a modified FGLS estimator where the second step is estimated using a Tobit regression that is robust to the left-truncation of the log-return. The dependent variable is the normalized excess log-return of each deal. The independent variables are a constant term (Gamma), a normalized constant term (Constant), and the normalized excess log-return on the market (Beta). Robust standard errors are in parentheses. Statistical significance at the 10%, 5%, and 1% levels are indicated with *, **, and ***, respectively.

Panel A: FGLS Specification

	(1) All	(2) All	(3) Early (1994-00)	(4) Late (2001-07)
Constant	-0.046***	0.041**	0.107***	-0.013
	(0.009)	(0.017)	(0.022)	(0.029)
Gamma		-0.471***	-0.934***	-0.103
		(0.070)	(0.128)	(0.106)
Beta	2.417***	2.278***	2.258***	1.873***
	(0.160)	(0.157)	(0.235)	(0.250)
Obs.	2,075	2,075	920	1,155

Panel B: Tobit Specification

	(1) All	(2) All	(3) Early (1994-00)	(4) Late (2001-07)
Constant	-0.089***	0.048**	0.145***	-0.047
	(0.011)	(0.022)	(0.029)	(0.036)
Gamma		-0.738***	-1.353***	-0.213
		(0.103)	(0.160)	(0.144)
Beta	2.934***	2.758***	2.750***	2.342***
	(0.203)	(0.204)	(0.311)	(0.305)
Obs.	2,075	2,075	920	1,155

Table VI: Beta Estimates Using Simulated Deal Data For each alpha and beta choice, we construct 100 datasets. Each dataset contains 150 funds, each of which has 20 deals. The performance of each deal is simulated using the jump-CAPM model with the remaining parameters equal to their estimated values ($\gamma = -0.5$, $\sigma^2 = 0.141$, and $\sigma_J^2 = 2.967$). The length of each deal is drawn from the empirical distribution in Figure 2 (conditional on the deal exiting prior to the termination of the fund). The vintage year of each fund is drawn uniformly from 1980 to 2000. The market return and risk-free rates are actual historical rates. For each of the 100 datasets, we estimate the beta coefficients using five different empirical models: Gross-IRR, Net-IRR, Static-CAPM, log-return CAPM (log-CAPM), and jump-CAPM, as described in the text. The table reports the average and standard deviation of the resulting 100 estimated beta coefficients. The case with alpha and beta parameters closest to their empirical counterparts is highlighted in bold.

True Alpha	True Beta	Gross IRR	Net IRR	Static CAPM	log-CAPM	Jump-CAPM
0%	0.0	-0.163	-0.613	-0.203	-0.015	-0.008
		(2.37)	(4.99)	(0.11)	(0.13)	(0.12)
10%	0.0	-0.044	0.025	0.098	0.001	0.009
		(1.37)	(1.45)	(0.11)	(0.12)	(0.12)
20%	0.0	-0.085	-0.078	0.374	-0.009	-0.003
		(1.64)	(1.42)	(0.11)	(0.12)	(0.12)
0%	1.0	0.379	-1.452	0.805	1.012	1.018
		(2.81)	(5.02)	(0.11)	(0.12)	(0.12)
10%	1.0	1.055	0.785	1.090	0.993	1.001
		(1.12)	(1.45)	(0.11)	(0.12)	(0.12)
20%	1.0	1.082	0.898	1.369	0.978	0.985
		(2.30)	(2.26)	(0.12)	(0.12)	(0.12)
0%	2.0	1.120	-1.450	1.777	2.092	2.101
		(3.47)	(4.90)	(0.11)	(0.12)	(0.12)
10%	**2.0**	**1.800**	**1.336**	**2.064**	**2.081**	**2.089**
		(1.84)	**(2.41)**	**(0.11)**	**(0.12)**	**(0.12)**
20%	2.0	2.236	2.076	2.351	2.045	2.054
		(1.75)	(1.46)	(0.12)	(0.12)	(0.12)