Workshop on

Structure and randomness in hypergraphs

London School of Economics

17–21 December 2018

Cut distance for graphs and hypergraphs

Jan Hladký

Abstract: The cut distance is arguably the most natural distance for graphs. It is also the distance that underlies the theory of dense graph limits. I will survey some alternative ways of expressing the cut distance, and in particular the weak* approach we recently introduced (arXiv:1806.07368). Much less is known about cut distance-like metrics for hypergraphs, but still I will try to make some comments there.

Monochromatic tight cycle partition for 3-graphs

Allan Lo

Abstract: A conjecture of Lehel states that every 2-edge-coloured complete graph can be partitioned into two disjoint monochromatic cycles. This conjecture was confirmed by Bessy and Thomassé. We prove that its analogous result holds for tight cycles in 3-uniform hypergraph, that is, every 2-edge-coloured (large) complete 3-uniform hypergraph can be partitioned into two monochromatic tight cycles. This is joint work with Frederik Garbe, Richard Lang, Richard Mycroft and Nicolás Sanhueza-Matamala.

High-order components in random hypergraphs

Mihyun Kang

Abstract: In this talk we shall discuss recent results on high-order components in random hypergraphs.

The minimum number of triangles in graphs of given order and size

Oleg Pikhurko

Abstract: In 1941 Rademacher (unpublished) asked for the minimum number of triangles in a graph of given order n and size m. This problem was revived by Erdős in the 1950-60s who in particular solved in the case when $m = n^2/4 + o(n)$, just above the threshold when at least one triangle is guaranteed. This problem has attracted much attention since then and, in a major breakthrough, was solved asymptotically by Razborov in 2008. I will discuss an exact solution (joint work with Hong Liu and Katherine Staden) for all large graphs whose edge density is bounded away from one.

Two Erdős-Hajnal-type Theorems in Hypergraphs

Asaf Shapira

Abstract: The Erdős–Hajnal Theorem asserts that non-universal graphs, that is, graphs that do not contain an induced copy of some fixed graph H, have homogeneous sets of size significantly larger than one can generally expect to find in a graph. We obtain two results of this flavor in the setting of r-uniform hypergraphs.

A theorem of Rödl asserts that if an n-vertex graph is non-universal then it contains an almost homogeneous set (i.e one with edge density either very close to 0 or 1) of size $\Omega(n)$. We prove that if a 3-uniform hypergraph is non-universal then it contains an almost homogeneous set of size $\Omega(\log n)$. An example of Rödl from 1986 shows that this bound is tight.

Let $R_r(t)$ denote the size of the largest non-universal r-graph G so that neither G nor its complement contain a complete r-partite subgraph with parts of size t. We prove an Erdős–Hajnal-type stepping-up lemma, showing how to transform a lower bound for $R_r(t)$ into a lower bound for $R_{r+1}(t)$. As an application of this lemma, we improve a bound of Conlon–Fox–Sudakov by showing that $R_3(t) \geq t^{\Omega(t)}$.

Degree conditions for embedding trees

Maya Stein

Abstract: We conjecture that every n-vertex graph of minimum degree at least $k/2$ and maximum degree at least $2k$ contains all trees with k edges as subgraphs. We prove an approximate version of this conjecture for trees of bounded degree and dense host graphs. Our work also has implications on the Erdős–Sós conjecture and the 2/3-conjecture. We prove an approximate version of both conjectures for bounded degree trees and dense host graphs.

Joint work with Guido Besomi, Matías Pavez-Signé.

Rainbow factors in hypergraphs

Liana Yepremyan

Abstract: For any r-graph H, we consider the problem of finding a rainbow H-factor in an r-graph G with large minimum l-degree and an edge-colouring that is suitably bounded. We show that the asymptotic degree threshold is the same as that for finding an H-factor.

Joint work with Matthew Coulson, Peter Keevash, Guillem Perarnau.

VIRTUALLY FIBERING RANDOM RIGHT-ANGLED COXETER GROUPS

Gonzalo Fiz Pontiveros

A group K *virtually algebraically fibers* if there is a finite index subgroup K' admitting a surjective homomorphism $K' \to \mathbb{Z}$ with finitely generated kernel. This notion arises from topology: it was shown by Stallings that a 3-manifold M is virtually a surface bundle over a circle precisely when the fundamental group of M virtually algebraically fibers.

A *Right-Angled Coxeter group* (RACG) K is a group given by a presentation of the form

$$\Big\langle x_1, x_2, \dots x_n \mid x_i^2, [x_i, x_j]^{\sigma_{ij}} \ : 1 \leqslant i < j \leqslant n \Big\rangle$$

where $\sigma_{ij} \in \{0, 1\}$ for each $1 \leqslant i < j \leqslant n$. One can encode this information with a graph Γ_K whose vertices are the generators $x_1, \dots, x_n$ and $x_i \sim x_j$ if and only if $\sigma_{ij} = 1$. Conversely given a graph G on n vertices, we will denote the corresponding RACG by $K(G)$.

We show that the Right-Angled Coxeter group $C = C(G)$ associated to a random graph $G \sim \mathcal{G}(n, p)$ with $\frac{\log n + \log\log n + \omega(1)}{n} \leqslant p < 1 - \omega(n^{-2})$ virtually algebraically fibers with high probability. We also obtain the corresponding hitting time statements, more precisely, we show that as soon as G has minimum degree at least 2 and as long as it is not the complete graph, then $C(G)$ virtually algebraically fibers. The result builds upon the work of Jankiewicz, Norin, and Wise and it is essentially best possible.

Joint work with Roman Glebov and Ilan Karpas.