CHANCE AS A GUIDE TO LIFE: THE OTHER PRINCIPAL PRINCIPLE*

Bishop Butler's claim that "Probability is the very guide in life" is much quoted, but like many good slogans has multiple possible meanings.[1] The subjective interpretation of probability as rational degree of belief, or *credence*, provides one sense in which it might be true: you should, on pain of irrationality, prefer courses of action which have the best expected consequences relative to your beliefs. The logical interpretation provides another: the degree of confirmation or support conferred upon a claim by the evidence measures the extent to which it should serve as a basis for action.[2] Some have argued that neither is viable as an interpretation of the slogan because, unless you act on probabilities that are accurate as well as consistent and well-confirmed, your act might fail to maximise expected benefit.[3] Whether this is so or not, my concern will be with a different sense in which the slogan might be true and which seems immune to this objection: the claim that *objective chance* is a guide to life.

It is often said that we should set our degree of belief in any proposition to what its chance of truth is. This thought, of which its best-known expression is David Lewis' Principal Principle,[4] plausibly provides part of the content of Butler's claim. But only part of it, and perhaps not even the most important one. For chance also provides practical guidance by constraining how we should form preferences and what choices we should make. When choosing between alternative courses of action, for instance, we should do so in the light of how each alternative will affect the chances of obtaining the outcomes we desire. In particular, if all available actions have the same possible outcomes we should choose the one that confers higher chance on the outcomes we prefer more. For example:

1. You can either bet on rainfall exceeding 5mm next month or bet on it not. Both bets cost $5 and pay $10 if you are correct. Suppose that the chance of rainfall in excess of 5mm next month is greater than 0.5. Which bet should you take? Answer: The first.
2. An urn contains 1 red ball, 10 black balls and 89 white balls. You can either buy a lottery ticket that pays $1000 dollars if a red ball is drawn, $10 dollars if a black one is and nothing otherwise or one that pays $1000 dollars if a red ball is drawn, $10 dollars if a white one is and nothing otherwise. Which should you buy, assuming you will buy one and not both? Answer: The second.
3. A population faces an epidemic of a disease which is expected to infect 80% of the population, of which the majority will only suffer moderate effects but 10% will die. A vaccine eliminates

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[1] The slogan, although usually attributed to Joseph Butler, *Analogy of religion, natural and revealed, to the constitution and nature* (London, England: Knapton, 1736), is apparently taken by him from Marcus Tullius Cicero, *De natura deorum: academica* 5, 12 (London: Heinemann, 1951)

[2] See Rudolf Carnap, "Probability as a Guide in Life," *The Journal of Philosophy* XLIV, No. 6 (Mar. 13, 1947): 141-148.

[3] See David H. Mellor "Chance and Degrees of Belief," in R. McLaughlin, ed., *What? Where? When? Why?* (Dordrecht: Reidel, 1982): 49-65. See also Helen Beebee and David Papineau, "Probability as a Guide to Life," *The Journal of Philosophy* XCIV, 5 (May, 1997): 217-243.

[4] Introduced in David Lewis "A Subjectivist's Guide to Objective Chance," in Richard C. Jeffrey, ed., *Studies in Inductive Logic and Probability*, vol. II. (Berkeley: University of California Press), p. 266.

the risk of infection, but at a 15% risk of side-effects that are worse than the disease for those who survive. Should you be vaccinated against the disease? Answer: Yes

The reasoning underpinning the prescription in the first of these examples is simple and compelling. There are only two possible outcomes of your choice of action: you either lose $5 or you gain $5. The chance of the latter (gaining $5) is greater if you take the first bet than if you take the second. So you should prefer the first bet to the second. It then follows that if you make choices in the light of your preferences and if you must take one or other of the two bets (but not both), then you should choose to take the first.

The principle at work here is the one that will be the focus of this paper. It can be stated in a more general form as follows:

> **Choice from Chance:** Let A and A^* be any two actions and let *G* and *B* be two mutually exclusive possibilities that jointly exhaust the possible outcomes of choosing either of these actions and which are such that *G* is strictly preferred to *B*. (Intuitively *G* is the good outcome and *B* the bad one.) Suppose that you know what the chances of *G* and *B* are, conditional on the performance of each of the two acts. Then you should choose action A over action A^* iff the conditional chance of *G* given that A is greater than the conditional chance of *G* given that A^*.

Things are less simple in the second example, where there are three possible outcomes, and in the third, where the two actions do not even share the same consequences. Nonetheless, similar arguments can be made in support of the prescriptions I proposed for each. In the second example, the two lottery tickets on offer confer the same chance (0.01) of winning $1000, but the first confers a much lower chance of obtaining the second-best outcome (winning $10) than the second, and therefore a much higher chance of obtaining the third and worst outcome. In the third example, if we rank the possible consequences of the two choices in descending order of desirability – death, vaccinated with side-effects, infected and survive, not infected and, if vaccinated, no side-effects – we find that for each consequence *C*, vaccinating results in a consequence that is at least as good as *C* with a higher chance than not vaccinating. So, in both of these examples, the prescribed alternative confers higher chances of the better outcomes than the alternative.

These arguments implicitly draw on a natural generalisation of Choice from Chance known as Stochastic Dominance, a principle that is the cornerstone of the modern theory of rational decision making under risk. Informally it says that if, for any possible consequence *C*, the chance of getting a consequence at least as good as *C* conditional on one act is greater than getting one conditional on another, then the former should be chosen over the latter. I will discuss Stochastic Dominance further in section III, where it will be shown to be implied by Choice from Chance together with some uncontroversial assumptions about the nature of rational choice. This latter fact suggests that Choice from Chance captures much about the role that chance plays in guiding our decision making.

In this paper I will argue for an even stronger claim: that Choice from Chance is another 'Principal Principle' governing the concept of chance and arguably *the* principal one. To establish it I will proceed as follows. In the next section, I will consider various formulations of the idea that chance is authoritative for credence. I will then show (in section II) that, given some modest assumptions about rational preference, the most promising of these is implied by Choice from Chance (but not vice versa). Section III, as indicated before, sees the derivation of Stochastic Dominance. In section IV, I consider whether Choice from Chance itself can be derived from some more elementary principle of preference for chances. Finally, in sections V and VI, I consider the implications of the acceptance of Choice from Chance for the theory of instrumental rationality and the debate between causal and evidential decision theory.

All the conditions to be studied will be stated as constraints on the attitudes of a rational agent (sometimes called 'you') and especially on her attitudes to propositions about the chances. Before stating them precisely, let me introduce the formal framework that will be used throughout. Let S be a background set of possible states of the world with *u* a utility or value function on them that, for simplicity, is assumed to take only a finite number of values.[5] Let Ω and Ω^* be Boolean algebras of sets of such states, called propositions, and such that $\Omega^* \subseteq \Omega$. Let Δ and Δ^* be the set of all probability functions Ω and Ω^* respectively. Intuitively Ω^* is the set of propositions with respect to which chances and conditional chances are defined, while Ω is the wider set of propositions serving as the objects of belief. I will need to assume that Ω contains, amongst other things, propositions about what the chances are, but not that such propositions also belong to Ω^*. For technical convenience, I will assume throughout that the set of propositions Ω forms a complete, atomless Boolean algebra.[6]

I will assume throughout that both an agent's degrees of beliefs (or credences) and her degrees of belief under the supposition that some proposition is true are probabilities and independent of the value function *u*. If $P \in \Delta$ measures the degrees of belief of a rational agent at some fixed time *t* then the measure of her degrees of belief at *t* under the supposition that some $A \in \Omega$ is true is denoted by P_A. No more about the suppositional probability function P_A needs be assumed other than that if $A = \Omega$, then $P_A = P$ and that $P_A(A) = 1$.[7]

Let *V* be the desirability function on $\Omega - \emptyset$ measuring the agent's degrees of preference or desire at *t* defined by *u* and P, with $V(X)$ being the conditional expectation of utility, relative to *u* and P, given the truth of *X*. It follows that for any finite partition $\{E_i\}$ of the set of possible worlds, $V(X) = \sum_i V(X, E_i).P(E_i|X)$, where (X, E_i) is the intersection of the two propositions X and E_i.[8] Finally, let $\succcurlyeq$ be the ranking over the propositions in $\Omega - \emptyset$ that V induces or represents (that is, such that $V(X) \geq$

[5] So, in particular, *u* is bounded above and below.

[6] A Boolean algebra of events is atomless just in case every event in the algebra (other than the empty set) has a strict subset that belongs to the algebra and complete if every subset has a supremum (or upper bound). Many familiar domains have this structure, for instance, the set of intervals of rainfall. If rainfall is a real-valued quantity then for every possible interval of rainfalls there is a possible narrower interval, and so this set is atomless.

[7] These are respectively the Anchoring and Certainty conditions of Richard Bradley, *Decision theory with a human face* (Cambridge: Cambridge University Press, 2017), p. 93, both consequences of the Regularity condition on suppositional probability proposed by James Joyce, *The Foundations of Causal Decision Theory* (Cambridge University Press: Cambridge UK, 1999).

[8] For further details, see Richard C. Jeffrey, *The Logic of Decision*, 2nd ed. (Chicago: University of Chicago Press, 1965); Joyce, *The Foundations of Causal Decision Theory*, *op. cit.*; or Bradley, *Decision theory with a human face*, *op. cit.*

$V(Y) \Leftrightarrow X \succcurlyeq Y$). Intuitively $\succcurlyeq$ represents the agent's preferences for the propositions being true at *t*. Let $\succ$ and $\approx$ be the corresponding strict preference and indifference relations on $\Omega - \emptyset$.

Let us call any partition of the set of worlds S in accordance with their utilities, a value-level partition of S. Elements of such partitions will serve as canonical consequences of actions, and when I speak of a value consequence of a particular action, I simply mean an element of the value-level partition that has non-zero probability on the supposition that the action is performed. Note that if *C* is an element of such a partition, then $V(C)$ equals the common utility of the *C*-worlds and hence for any proposition *X*, $V(X, C) = V(C)$, provided that $P(C|X) \neq 0$. (Note also that the composition of the value-level partition depends on the underlying utility function *u* on worlds and hence that by varying the latter we can construct different value-level partitions.)

All of this is, for the most part, purely definitional as I have not said anything about the significance of the attitudes of an agent defined above for the choices that she makes. Most decision theorists take a broadly consequentialist view of the justification of action and hold, in particular, that when the true state of the world is known to an agent, then she should choose one action over another just in case the consequence of performing it is more desirable than that of performing the other. Where there is controversy is over how to extend such consequentialist reasoning to situations of uncertainty about the outcome of acting. The 'evidential' decision theory of Richard Jeffrey is strongly consequentialist in that it identifies the choice-worthiness of an act with the expected goodness or desirability of its outcomes, conditional on its performance, in all circumstances and however the act is represented.[9] Causal decision theorists reject this and allow that it can be desirable that something be true without it being advisable that one seek to make it so. The choice-worthiness of an act depends, they say, on whether it is efficacious in achieving desired ends, not in whether it serves as evidence for them obtaining.

What is at stake is perhaps clearest in the so-called 'medical Newcomb' cases. Here is one of the earliest, due to Robert Nozick.[10] You do not know whether your father was Prof. S or Mr. T. If Prof. S is your father, you will die soon of a disease inherited from him; if Mr. T is, you will live a long life. It is known that there is a tendency for those who have the disease to choose an academic profession as measured by the statistical correlation between the two. You must decide whether to become an academic or an athlete. You prefer the life of an academic to that of an athlete whether or not you have the disease (though, of course, you much prefer to have a long life than a short one) and your choice of career cannot affect whether you have the disease. So it seems obvious that you should choose to be an academic. But evidential decision theory could deem the athletic life more choice-worthy because the probability of you living a short life, conditional on choosing to be an academic, is higher than the probability of you living a short life, conditional on choosing to be an athlete, and you prefer a long life as an athlete to a short one as an academic. So it risks deeming the athletic life choice-worthy in virtue of the fact that choosing it is evidence of not being diseased, even though the latter condition is not in your control. (Whether it does do so deem it will depend on the precise assignment

[9] See Jeffrey, *The Logic of Decision*, *op. cit*.

[10] Robert Nozick, "Newcomb's Problem and Two Principles of Choice", in *Essays in Honor of C. G. Hempel*, in Nicholas Rescher et al., eds. (Dordrecht: Reidel, 1969): 114-146.

of probability and desirability values, but the casual decision theorist's argument only depends on there being an assignment consistent with evidential decision theory that makes it so.)

It is not my intention to take a position on the rather complex debate around whether these kinds of cases refute evidential decision theory.[11] Instead, I will make use of a background principle of choice that is weak enough that it should be acceptable to all sides in the debate. The required assumption is simply that one should choose one action over another just in case the expectation of utility, supposing that the former is chosen, is greater than the expectation of utility supposing that the latter is. More formally, let $\gtrsim$ be a weak ordering of actions in terms of their choice-worthiness. Then:

Expectationalism: For any actions A and A^* and value-level partition $\{C_i\}$ of S:

$$A \gtrsim A^* \Leftrightarrow \sum_i V(C_i).P_A(C_i) \geq \sum_i V(C_i).P_{A^*}(C_i)$$

While most decision theorists accept Expectationalism, they disagree about what probability function induced by the performance of an action should be used in determining its choice-worthiness. Those working in the tradition of Savage[12] will view $P_A(C)$ as the sum of probabilities of the states of world in which act *A* has consequence *C*. Evidential decision theorists, on the other hand, will set $P_A(C)$ to $P(C|A)$, so that the choice-worthiness of the act *A* equals its desirability, $V(A)$. Finally, causal decision theorists will contend that $P_A(C)$ should measure the causal effect on *C* of performing *A*, something that can differ from the probabilistic correlation between the two. They can nonetheless disagree on exactly what determines P_A.[13]

I. Credences from Chance

David Lewis asserted that "all that we take ourselves to know about chance" was implied by what he called the Principal Principle.[14] Informally the principle says that we should set our degrees of belief in any proposition to what we believe the chance is of that proposition being true or, equivalently, that the credibility of any proposition conditional on the truth of some chance hypothesis is just whatever chances that hypothesis accords it. Lewis thought that this idea, that chance was authoritative with respect to credence, fundamentally constrained what chances could be, and that in virtue of playing this role explains why chances are a guide to life.

Although there is widespread agreement that something like the Principal Principle is true of the way in which credence should depend on chance, there is also a lot of disagreement on the details and a

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[11] See Arif Ahmed, *Evidence, decision and causality* (Cambridge: Cambridge University Press, 2014) for an influential defence of evidential decision theory.

[12] Leonard Savage, *The Foundations of Statistics*, 2nd ed. (New York: Dover, 1954/1972).

[13] For different proposals, see David Lewis, "Causal Decision Theory," *Australasian Journal of Philosophy* LIX, 1 (1981): 5-30; Brian Skyrms, *Pragmatics and empiricism* (New Haven: Yale University Press, 1984); Joyce, *The Foundations of Causal Decision Theory*, *op. cit.*; Allan Gibbard and William Harper, "Counterfactuals and Two Kinds of Expected Utility," in Clifford Alan Hooker, James L. Leach, and Edward Francis McClennan, eds., *Foundations and Applications of Decision Theory*, (University of Western Ontario Series in Philosophy of Science, 13a, Dordrecht: D. Reidel, 1978), pp. 125–162

[14] Lewis "A Subjectivist's Guide to Objective Chance," *op. cit.*, p. 266. A similar claim is made in Isaac Levi, *The Enterprise of Knowledge : an Essay on Knowledge, Credal Probability, and Chance* (Cambridge, Mass: MIT Press, 1980).

wide variety of competing principles on offer.[15] To set our course through it, let's start with a more precise statement of Lewis' original condition. To that end, let Ch be a random variable taking probability functions on (non-chance) propositions as its values; that is, for all worlds w, $Ch(w) \in \Omega^*$. For any proposition $A \in \Omega^*$, let Ch^A be a corresponding random variable taking probability functions on Ω* as its values, meeting the condition that at all words w such that $Ch(w)(A) > 0$, $Ch^A(w) = Ch(w)|A$. Intuitively Ch measures the chances at different possible worlds at some time t and Ch^A the corresponding conditional chances given that *A* at these worlds. For simplicity, I will assume that *t* is constant, so that corresponding propositions as to the chances and conditional chances of particular propositions at time *t* can be denoted by (non-temporally indexed) expressions of the form '$Ch(X) = x$' and '$Ch^A(Y) = y$' and propositions as to all such chances and conditional chances by expressions of the form '$Ch = ch$' and '$Ch^A = ch$', where ch is a probability function on the set of (non-chance) propositions. With this simplification, Lewis' proposal reduces to the following:

> **Principal Principle**: Let $P_0 \in \Delta$ be a reasonable initial credence function, *X* be any proposition in Ω* and *E* be any proposition in Ω consistent with *X* that is admissible at time *t* and such that $P_0(Ch(X) = x, E) \neq 0$. Then:
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> $$P_0(X|Ch(X) = x, E) = x$$

The notion that needs most filling out is that of an admissible proposition, as the usefulness of the Principal Principle depends on how restrictive it is. According to Lewis, a proposition is admissible with respect to *X* insofar if it does not include any information that pertains to the truth of *X* other than the information it provides about the chances of X:

> *"Admissible propositions are the sort of information whose impact on credence about outcomes comes entirely by way of credence about the chances of those outcomes."*[16]

Lewis suggested that historical information and information about how possible histories and possible laws bear on chances is admissible, but that information about future chances and information relevant to the truth of *X* that is not encoded in the chances is not. For instance, the proposition that it rained for the last three days is admissible evidence regarding the prospect of it raining tomorrow. But the proposition that everyone will be carrying umbrellas tomorrow is not.

Lewis says little about what a reasonable initial credence function is other than it should be consistent and regular, that is, it should assign non-zero probability to all propositions in the domain of the chance function. But others have argued that the main concern here is that the credence function should not contain any inadmissible evidence that 'trumps' the chances.[17] For instance, if $P(X) = 1$, then $P(X|Ch(X) = x, E)$ should equal one and not x, even if E is admissible. This worry would be

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[15] For a comprehensive survey and discussion see Richard Pettigrew, *Accuracy and the Laws of Credence* (Oxford, United Kingdom: Oxford University Press, 2018).

[16] Lewis "A Subjectivist's Guide to Objective Chance," *op. cit.*, p. 272.

[17] Michael Strevens, "Objective Probability as a Guide to the World," *Philosophical studies* ICV, 3 (1999): 243-275; Christopher J. G. Meacham, "Two Mistakes Regarding the Principal Principle," *The British journal for the philosophy of science* LXI, 2 (2010): 407-431

defused however if whenever we were certain of the truth of *X* we would, on pain of inconsistency, expect the chance of *X* to equal one, that is:

> **Certainty**: If P is a rational credence function at *t*, and X a proposition such that $P(X) = 1$, then for any chance function ch such that $ch(X) < 1$, $P(Ch = ch) = 0$.

My own view is that Certainty should be accepted as a basic constraint on the relationship between rational credence and chance, for we should not be certain at some time *t* that X is true while, at the same time, giving positive credence to the chance of X's truth at t being less than 1. (This view assumes, as does Lewis', that chances are time-indexed and that they entail any truth that has been settled). The philosophical literature invokes crystal balls, oracles and such like to suggest counterexamples, but if such direct access to the future were possible, I think we would then have to accept the sensitivity of the present chances to future facts. Nonetheless it is not essential to my argument that Certainty hold generally. The important point (on which there is general agreement) is that it is rational to be guided by the chances *only insofar* as the chances are at least as well-informed as we are on relevant matters. So any explanation of the sense in which chance serves, in some context, as an epistemic authority must be framed by the presumption that Certainty holds in this context.

With this in mind, let us henceforth take our discussion to concern only such contexts in which violations of Certainty are ruled out. This will have the additional benefit of allowing us to dispense with concerns about admissibility. To see this, consider a much simpler expression of the idea that one should defer to chance.

> **Credence from Chance**: [18] For any rational credence function $P \in \Delta$, chance function $ch \in \Delta^*$ and proposition $X \in \Omega^*$, if $P(Ch = ch) \neq 0$ then:
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> $$P(X|Ch = ch) = ch(X)$$

Credence from Chance has the same basic rationale as the Principal Principle: deference to chance implies that conditional on the chances being correctly measured by ch, one should adopt a credence in any proposition equal to what ch says its chance is. Indeed, if the proposition that $Ch = ch$ itself is admissible with respect to X, then Credence from Chance is a consequence of Lewis' principle (with the qualification that P contains no inadmissible information, as implied by Certainty). Now $Ch = ch$ certainly seems to be exactly the sort of proposition that passes Lewis' test for admissibility, in that its impact on the agent's credence in *X* goes via her credences regarding the chances of *X*. And if it does then, by the Principal Principle, $P(X|Ch(X) = ch(X), Ch = ch) = ch(X) = P(X|Ch = ch)$, in virtue of the fact that $Ch = ch$ implies that $Ch(X) = ch(X)$. Furthermore Lewis says that the conjunction of all true propositions about the past and all true history to chance conditionals is admissible.[19] But this conjunction implies a proposition as to the true chances, namely of the form '$Ch = ch$'.

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[18] This principle is often termed 'Miller's Principle' after the discussion of it in David Miller, *Critical rationalism: A restatement and defence* (Chicago: Open Court, 1966).

[19] Lewis "A Subjectivist's Guide to Objective Chance," *op. cit.*, p. 276

Nonetheless, on pain of invalidating the Principal Principle or of trivialising the notion of admissibility, $Ch = ch$ cannot be admissible. For note that Credence from Chance implies that $P(X, E|Ch = ch) = ch(X, E)$ and that $P(E|Ch = ch) = ch(E)$. So, the following condition, first introduced by Biran Skyrms[20], follows directly from it: [21]

> **Conditional Credence from Conditional Chance**: For any rational credence function $P \in \Delta$, $ch \in \Delta^*$ and $X \in \Omega^*$, if $P(E|Ch^E = ch(\cdot\,|E)) \neq 0$ then:
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> $$P(X|Ch^E = ch, E) = ch(X|E)$$

On the face of it, this condition gives different prescriptions to the Principal Principle. Suppose your chance of survival from an operation is 95%, but only 50% conditional on being disposed to an allergic response to one of drugs being administered. Then Conditional Credence from Conditional Chance will dictate that your conditional degrees of belief in survival given the allergic response should be 0.5 while the Principal Principle will dictate a degree of belief of 0.95, assuming that the information that you will have an allergic reaction is admissible. And it certainly does not seem inadmissible by the lights of Lewis' criterion: the impact on credence concerning survival of the possibility of a disposition to allergic reaction does go via credence about the chances of survival. Moreover, information about the acquisition of the allergic disposition is historical.

In any case, Conditional Credence from Conditional Chance clearly gives the right answer here. So, either the Principal Principle is false or, contrary to appearances, it makes the same prescription in virtue of the notion of admissibility being a good deal more restrictive than Lewis suggests. How restrictive? For the Principal Principle to be consistent with Credence from Chance it must be that case that E is admissible with respect to X iff $ch(X|E) = ch(X)$, that is, iff the chances of X and E are independent of one another.[22] But this makes the Principal Principle simply a special case of Credence from Chance and renders the notion of admissibility entirely uninteresting, contrary to the many claims in the literature that admissibility is somehow deeply connected to the notion of chance. In any case it is the latter principle that will serve in the ensuing sections as the preferred expression of the authority of chance with respect to credence.

II. The Principle of Choice from Chance

Is Lewis correct in claiming that principles expressing the epistemic authority of chance with respect to credence contain everything that we know about chance? In this section I will argue that the answer is 'no' and that there are requirements on the relation between our preferences and choices and our beliefs about the chances that are practical in nature, not epistemic. In particular, given the rationality conditions assumed throughout, the practical requirement expressed by Choice from Chance implies Credence from Chance, the condition that (I argued) most adequately expresses the authority of

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[20] Brian Skyrms, "Conditional Chance," in J.H. Fetzer, ed. *Probability and Causality*, Synthese Library (Studies in Epistemology, Logic, Methodology, and Philosophy of Science), vol 192, (Springer: Dordrecht, 1988), pp 161-78.

[21] A formal proof is given in the appendix as Theorem 1(a). In the absence of Certainty, the derived principle would be restricted to any $P \in \Delta$ containing no inadmissible evidence.

[22] See Ittay Nissan-Rozen, "The Principal Principle, Adams' Thesis, the Desire as Belief Thesis and Jeffrey's conditionalization", *British Journal for the philosophy of Science* LXIV (2013): 837-50

chance with respect to our partial beliefs. So it is the role that chance plays in guiding our actions that explains the requirement to set our beliefs in line with the chances, and not vice versa.

Let's begin with a somewhat more formal statement of Choice from Chance. Let A and A^* be any two actions and *G* and *B* be a pair of mutually exclusive propositions that exhaust the possible outcomes of choosing A and A^* and suppose that you strictly prefer *G* to *B*. Suppose that you believe with probability one that the chances are given by probability function ch on Ω*, so that $P(Ch = ch) = 1$. Then you should (weakly) prefer to perform A than to perform A^* iff the conditional chance of *G* given that A is at least as great as that given that A^*. Formally:

Choice from Chance: $A \gtrsim A^*$ iff $ch(G|A) \geq ch(G|A^*)$[23]

Although the field of decision making under risk is characterised by lively debate over what rationality requires of us under such circumstances, the Choice from Chance principle is implicit in all the main theories. Indeed, it has the status in this field of an unstated platitude about how preferences and choices should be related to the (conditional) chances.[24] In this respect, Choice from Chance can be considered to express part of the core meaning of the concept of chance as it appears in decision theory. I will say more in the section 5 in defence of Choice from Chance, but for the moment I will take this as sufficient grounds for accepting it.

A number of preliminary remarks and clarifications. Firstly, the proffered definition of Choice from Chance implicitly assumes that the actions A and A^* belong to the domain of ch. But this is just for simplicity. If you do not like the idea of actions having unconditional chances then just suppose that you know that the conditional chances given that A and that A^* are respectively given by chance functions $ch(\cdot\,|A)$ and $ch(\cdot\,|A^*)$ defined on Ω*.

Secondly, Choice from Chance asserts a relation between an ordering over actions and a conditional chance ordering over their consequences. The intended interpretation of both is subjective: '$A \gtrsim A^*$' expresses the agent's judgement that the former action is a better choice than the latter and '$ch(G|A) \geq ch(G|A^*)$' her judgement that the conditional chances are so related. On this interpretation, therefore, Choice from Chance is a consistency condition on judgement. A more objective interpretation is also possible, whereby it would express an equivalence between an objective betterness relation over actions and the true conditional chances given the actions. But although I think the principle is true on this interpretation too, it will not figure in subsequent discussion.

Finally, and most importantly, the relation $\gtrsim$ ranks actions in terms of how worthy they are of being chosen, not how desirable it would be if it were true that they will be chosen. In other words, $A \gtrsim A^*$ expresses the agent's preference for *making* A true rather than A^*, instead of her preference for it *being* true that A rather than that A^*. (Recall that preferences of the latter kind are captured by the relation $\succcurlyeq$ and numerically measured by the desirability function V.) The difference, although

[23] Note that the principle suffices to generate a preference ordering over sets of alternatives of any size, since the transitivity of preference is induced by that of the numerical ordering of chances.

[24] In Michael D. Resnick, *Choices : An Introduction to Decision Theory.* (Minneapolis : University of Minnesota Press, 1987) it is called the Better Chances principle and is used in the derivation of expected utility theory.

seemingly obscure, is at the heart of causal decision theory's claim that the desirability of an action does not (always) correctly measure how choice-worthy it is.

Since Choice from Chance regulates the choice-worthiness of actions it must be read as a condition of *instrumental* rationality; true, if it is, in virtue of the fact that the conditional chance of some outcome given some action measures how efficacious the action is in bringing about that outcome, that is, that the conditional chance of *G* given that A being greater than given that A^* means that *A* is instrumentally more effective for obtaining *G* than is A^*. (In section 5, I will argue that the reason we should choose the more efficacious act is that it is better to have a higher chance of a desirable outcome than a lower one.)

That instrumental rationality requires that one should choose A^* over *A* when the conditional chance of *G* given that A is greater than that given A^* is not, on the face of it, inconsistent with *A* being more *desirable* than A^* under these circumstances. On the contrary, the latter claim also seems plausible. Knowing that the conditional chance of *G* is greater given that A than given that A^*, means that one is in an epistemic situation in which getting the news that A is true is better than getting the news that A^*. This is because A's truth is stronger evidence for the truth of *G* than is A^*'s. And hence stronger evidence of the obtaining of the consequence one prefers. For if one knows the conditional chances given acts, then learning which act will be performed enables one to infer the probability with which *G* will obtain. And the higher that probability the better, given that *G* is a better outcome than *B*.

The distinction between the two claims under consideration, respectively about the requirements of instrumental and evidential rationality, can be made more precise by comparing Choice from Chance with the following principle. Suppose that A and A^* are the two actions described before. Then you should prefer that A be true rather than that A^* be iff the conditional chance of *G* given that A is greater than the conditional chance of G given that A^*. More precisely, given that preferences for truth are measured by desirability function V:

Desire from Chance: If $P(Ch = ch|A) \neq 0 \neq P(Ch = ch|A^*)$, then:

$$V(A|Ch = ch) \geq V(A^*|Ch = ch) \text{ iff } ch(G|A) \geq ch(G|A^*)$$

Desire from Chance closely resembles Credence from Chance. While the latter requires alignment between the credibility of a proposition and its chance of truth, when this chance is known, the former requires alignment between the desirability of a proposition and the conditional chance, given its truth, of the good. In fact, this resemblance is more than just appearance: Credence from Chance implies Desire from Chance. (The proof of this claim is given in the appendix as Theorem 1.)

One might take this fact to show that the role of chance in providing practical guidance derives from its role in providing epistemic guidance. In particular, the thought might go, it is the fact that we are required to defer to chance in forming our beliefs, along with the usual rationality constraints on our desires, that explains why we should prefer actions that confer higher chances on the consequences that we desire. As David Lewis puts it:

> *"The greater chance you think the ticket has of winning, the greater should be your degree of belief that it will win; and the greater is your degree of belief that*

it will win, the more, ceteris paribus, it should be worth to you and the more you should be disposed to choose it … . "[25]

But this argument is not quite right. While the requirement of epistemic deference to chances might explain why actions that confer higher chances on preferred consequences are more desirable than those that do not, this does not explain why we should *choose* them. At least it does not do so if we accept the causal decision theorist's point that desirability is not generally adequate as a guide to efficaciousness. So, we still need to show why it is that when our degrees of desire for actions reflect what we know about the chances, they do so in virtue of tracking their instrumental value.

We can put this claim a bit more precisely as follows. If both Desire from Chance and Choice from Chance are true then it must be the case that when the chances are known the choice-worthiness of an action goes by its desirability (as the evidentialist claims is generally true), that is, that $A \succsim A^*$ iff $A \succcurlyeq A^*$ whenever $P(Ch = ch) = 1$ for some function ch on Ω^*. Likewise, it is sufficient that the latter be true that Desire from Chance implies Choice from Chance. But desirability and choice-worthiness (respectively the evidential and instrumental values of acting) coincide only when the probability of a desirable consequence on the supposition of performing an act is accurately measured by its conditional probability given the act's performance. And this coincidence is *not* entailed by Credence from Chance, or indeed, for the same reason, by any of the Principal Principle-like conditions that have been proposed as expressions of the epistemic authority of chance. So, unless the causal decision theorists have it wrong, these epistemic principles do not exhaust all that we know about chance.[26]

In contrast, in the presence of the background assumption of Expectationalism, Choice from Chance entails Credence from Chance. This fact is a consequence of a more general result proven in the appendix as Theorem 3. Here I will simply give a relatively informal demonstration of why it holds. Let *u* be such that it induces a value partition $\{G, B\}$ on states, with G states having utility 1 and B states utility 0. For any event $E \in \Omega^*$, let A^E be an action that makes it true that the chance of G equals that of E and which is instrumentally independent of *E* in the sense that $P_{A^E}(E) = P(E)$. We can think of A^E as a bet on the truth of E that pays (the monetary equivalent of) 1 util when E is true and nothing otherwise. For this reason, I will call A^E a betting act. Now suppose that the chances are given by function ch on Ω^* and let E and F be any two propositions in Ω^*. Then by the definition of *u* and of betting acts, $ch(G|A^E) = ch(E)$ and $ch(G|A^F) = ch(F)$. And so by Choice from Chance, $A^E \succsim A^F \Leftrightarrow ch(E) \geq ch(F)$. But since $P_{A^E}(E) = P(E)$ and $P_{A^F}(F) = P(F)$, it follows from Expectationalism that $A^E \succsim A^F \Leftrightarrow P(E) \geq P(F)$. But since $P(Ch = ch) = 1$, $P(\cdot) = P(\cdot\,|Ch = ch)$ it follows that:

$$P(E|Ch = ch) \geq P(F|Ch = ch) \Leftrightarrow ch(E) \geq ch(F)$$

But this is just a relational version of the Credence from Chance principle. Indeed, given our structural assumption the algebra of propositions is atomless, it is equivalent to it.[27]

There are two different views one might take to the significance of this result. A pragmatist might see it as showing that it is the requirement of instrumental rationality expressed by Choice from Chance

[25] Lewis "A Subjectivist's Guide to Objective Chance," *op. cit.*, p. 288.
[26] Ironically, Lewis was a proponent of causal decision theory, so necessarily he was wrong about something.
[27] This follows from Corollary 1.23 of Ethan Bolker, "Functions Resembling Quotients of Measures," *Transactions of the American Mathematical Society* CXXIV, 2 (1966): 292-312

that explains the authority of chance with respect to both credence and desire. That is, it is because chance guides what is choice-worthy that we should, qua rational seekers of what is good, set our beliefs to the chances and hence form our desires in the light of them. An evidentialist, in contrast, is likely to regard the fact that instrumental rationality requires conformity to the Choice from Chance principle to be explained (in part) by the requirement that credences respect the Credence from Chance principle. Explained by this requirement because it is a necessary condition for instrumental rationality, but only in part because instrumental rationality makes stronger demands on our relationship to chance than does any purely epistemic requirement.

While the facts about the logical relationship between these principles cannot settle the question of the explanatory relations between them, they do strongly support the claim that Lewis was wrong in thinking that any epistemic principle could exhaust all that we know about chance. They also make Choice from Chance a strong candidate for being another 'Principal Principle' regulating our relation to the chances. It does not follow of course that it exhausts all that we know about chance. But in the next section, I will give support to this latter claim as well, by showing that it entails the core theory of rational choice under risk.

Let me end this one with an important qualification. Unstated platitude of mainstream decision theory though it might be, Choice from Chance is no logical truth. Here is a putative counter-example due to Arif Ahmed[28]. Suppose that armour provides good protection so that the chances of you surviving an upcoming battle conditional on wearing armour are higher than those conditional on you not. Choice from Chance says that if your choice is between wearing the armour and not, then you should do so. Now suppose that an oracle tells you that will either survive the battle without armour or die wearing it. The oracle is never wrong so it looks like you should forego the armour despite the better chances of survival that it confers, contrary to the prescription of Choice from Chance.

It is easy enough to diagnose the problem: it lies in the failure, implicitly assumed in the counterexample, of the Certainty condition. For in the case it describes, Certainty implies that the only chance functions you should consider possible once you have heard from the oracle are those that confer *zero* chance of survival conditional on wearing armour. In which case, Choice from Chance gives the correct prescription to forego armour in battle. For the counterexample to work against this principle, it must be assumed that the oracle holds information upon which the current chances are not conditioned, so that you are both certain (in virtue of the oracle's testimony) that you will die, conditional on wearing armour, and that there is some chance that you will survive, conditional on wearing armour. Such violations of Certainty seem unacceptable to me. But if they are not, then there will be cases is which rationality steers us into conflict with Choice from Chance and hence with all mainstream normative theories of decision making under risk (as we shall now see).

III. Stochastic Dominance

Most theories of decision making under risk endorse a principle, known as Stochastic Dominance, that is less trivial than Choice from Chance. Informally the idea is that when actions have many possible consequences (not just a good and bad one) then we should prefer actions which confer higher conditional chances, given their performance, on the more preferred consequences. In formal expressions of the principle however, it is the lotteries over consequences induced by actions, and the

[28] In private communication.

not actions themselves, that serve both as the objects of preference and/or choice. So let's start with lotteries and then return to actions later.

Let $\mathcal{C} = \{C_i\}$ be an *n*-fold value-level partition of consequences such that $C_1 \succcurlyeq C_2 \succcurlyeq \cdots \succcurlyeq C_n$. A *lottery* L on $\mathcal{C}$ is just a chance distribution over its elements. Let $L(C)$ be the chance that lottery L yields consequence C and let $L(\succcurlyeq C)$ be the chance that lottery L yields a consequence weakly preferred to C, that is, the sum of the $L(C_j)$ such that $C_j \succcurlyeq C$. Then we say that lottery L *weakly stochastically dominates* lottery L^* just in case, for all $i \in \{1, n\}$, $L(\succcurlyeq C_i) \geq L^*(\succcurlyeq C_i)$. And we say that it does so *strongly* if, in addition, there exists an outcome C_{i^*} such that $L(\succcurlyeq C_{i^*}) > L^*(\succcurlyeq C_{i^*})$.

In the theories populating the field of decision making under risk, lotteries are treated both as objects of choice (as in the von Neumann and Morgenstern theory[29]) and as consequences of actions (as in the Anscombe and Aumann theory[30]). None of these theories make a formal distinction between the choice-worthiness and the desirability of a lottery however and all of them implicitly assume that the difference is of no consequence in this context. But since the distinction is central to the dispute between causal and evidential decision theory, we need to attend to it. With that in mind, let's start with a version of the condition of stochastic dominance that applies to preferences over lotteries.

> **Stochastic Dominance (Lotteries)**: If L weakly (strongly) stochastically dominates lottery L^* then L is weakly (strongly) preferred to L^*.

This rationality condition can be extended to actions in a natural way by treating the latter as instruments for inducing a chance distribution over consequences. In the second of the examples that we began with, for instance, the choice of which lottery ticket to buy amounts to a choice between the different chance distributions over monetary outcomes that the tickets secure for you. And in the third, the choice of whether to vaccinate or not, is a choice between the lotteries over the possible outcomes (death, survival with side-effects, and so on) that these actions induce. In both examples, the prescribed choice of action is justified by the fact that the lottery it induces stochastically dominates the one induced by the alternative.

To state a version of the stochastic dominance condition appropriate for acts, we must identify the lottery induced by a choice of action. Now this will depend on features of the state of the world in which it is performed and, in particular, what the conditional chances given its performance are. So let us define, for any chance function ch and action A, a corresponding lottery $L = L(A, ch)$ as the lottery over the C_i which assigns to each the chance $ch(C_i|A)$. Then:

> **Stochastic Dominance (Acts)**: Suppose that the chances are given by probability function ch. Let $L(A, ch)$ and $L^*(A^*, ch)$ be the lotteries determined by acts A and A^* when this is the case. If L weakly (strongly) stochastically dominates lottery L^* then $A \succsim A^*$ ($A \succ A^*$).

[29] John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior*. 2nd ed. (Princeton, N.J: Princeton University Press, 1947).
[30] F. J. Anscombe and R. J. Aumann, "A Definition of Subjective Probability," *The Annals of mathematical statistics* XXXIV, 1 (1963): 199–205

That rationality requires choice between acts to satisfy this condition follows directly from the requirement that preferences between lotteries respect the corresponding condition of Stochastic Dominance (Lotteries) together with the claim that the choice-worthiness of an act depends on the desirability of the lottery that it induces. The latter claim, implicitly assumed in most applications of theories of decision making under risk to problems of action or policy choice, is rendered by:

> **Distribution**: Suppose that the chances are known to be given by probability function ch. Let $L(A, ch)$ and $L^*(A^*, ch)$ be the lotteries determined by acts A and A^* when this is the case. Then:
>
> $$A \succsim A^* \Leftrightarrow L(A, ch) \succcurlyeq L^*(A^*, ch)$$

Distribution expresses a claim about what instrumental rationality requires of us and provides a link the between the epistemic authority of chance with respect to credence and its pragmatic authority with respect to choice. For, in the presence of Expectationalism, Choice from Chance is equivalent to the conjunction of Credence from Chance and Distribution. (This claim is proved as Theorem 5 in the appendix.) So Distribution must precisely express the non-epistemic content of Choice from Chance; that is, the part of its content missing from Credence from Chance and which invalidates Lewis' claim that the latter exhausts all that we know about chance.

We are now in position to state the central facts about the relationship between Choice from Chance, the choice-worthiness of actions and the property of stochastic dominance, in the presence of the background assumption of Expectationalism. Firstly, Credence from Chance suffices for rational preference between lotteries to respect the condition of Stochastic Dominance (Lotteries). This is proved as Theorem 2 in the appendix. As a corollary, Distribution and Credence from Chance jointly imply Stochastic Dominance (Acts). Thirdly since, as noted above, Choice from Chance implies both Distribution and Credence from Chance, the former suffices for the choice-worthiness of acts to governed by the principle of Stochastic Dominance (Acts).

This completes my argument that Choice from Chance is *the* principal condition expressing the authority of chance with respect to both to our choices and our attitudes of belief and desire. It rests on the demonstration that, given Expectationalism, Choice from Chance implies both Credence from Chance and Desire from Chance but is not implied by them, its additional content being captured by the (pragmatic) principle of Distribution. Furthermore, given Expectationalism, Choice from Chance requires that rational choice between lotteries and rational choice between acts respect the corresponding conditions of Stochastic Dominance for lotteries and acts. So it entails what is commonly regarded as the uncontroversial core of the theory of rational choice under risk and its implication for rational choice under uncertainty. *Choice from Chance, it would seem, fully explicates the sense in which chance is the guide to life.*

IV. Consequentialism and Instrumental Efficacy

Why should our actions be guided by the chances in the manner claimed by Choice from Chance? The answer, I suggest, lies predominantly in the fact that *ceteris paribus* it's better to have a higher chance of something you desire than a lower one. So, one should choose actions that confer higher chances on the outcomes one desires over those that confer lower ones. In the first of our opening examples, for instance, one should choose to bet on rainfall next month exceeding 5mm over a bet on it not doing so, because the former bet confers a higher chance of winning the $5 than the latter and having a higher chance of winning is better than having a lower one.

This informal justification of Choice from Chance is consequentialist in form. By this I mean that it takes the efficacy of an action in producing a desirable consequence to serve as the grounds for choosing it; in this case the desirable consequence being a higher chance of having what one wants. To make this thought more precise, let's start with the most basic consequentialist thesis: that of two actions the one that in fact has the more desirable consequence is the more choice worthy. More precisely:

> **State Consequentialism**: Let A and A^* be two any actions and C and C^* be any two consequences such that $C \succcurlyeq C^*$. Suppose that world being in state *s* entails that if A were performed then C would be the outcome and that if A^* were performed than C^* would be. Then if you know that *s* is the case, you should regard A to be at least as choice worthy as A^*.

State Consequentialism only has implications for choice-worthiness when the known state of the world fully determines the consequences of the actions under consideration. But when there are non-trivial objective chances attached to more than one consequence, then it will not apply. In these circumstances the most that we can know about the effect of acting is what chances our doing so induces over possible consequences. And so a different consequentialist principle applies:

> **Chance Consequentialism**: Let A and A^* be any two actions and *G* and *B* be a pair of mutually exclusive propositions that exhaust the possible outcomes of choosing A and A^* and such that $G \succ B$. Suppose that the world being in state *s* entails that if A were performed then the chance of G would be α and that if A^* were performed then the chance of G would be α^*. Then if you know that *s* is the case, you should regard A as at least as choice worthy as A^* iff you weakly prefer that the chance of G be α than that it be α^*.

Chance Consequentialism should be fairly uncontroversial at least amongst decision theorists. For note, firstly, that nothing in this statement of it precludes that specification of what makes outcomes desirable requires reference to intrinsic features of the actions themselves: that they are fair or honourable or risky, for instance. So, it imposes no substantial constraint on how actions should be valued. And secondly, it leaves open the question of what makes for instrumental efficacy, so it is neutral with respect to the different accounts of this favoured by different decision theories, both causal and evidential.

A second, more controversial principle is required for the truth of Choice from Chance. Suppose that the conditional chance of *G* given that *A* equals α and that of *B* given that *A* equals $1-\alpha$. Then it must be the case that performing action *A* brings it about with chance α that *G* and with chance $1-\alpha$ that *B*. If the conditional chance of being eaten by a shark, conditional on swimming after dark, is 0.01%, for instance, then my deciding to swim after dark should make it the case that I have a 0.01% chance of being eaten by a shark. More generally:

Efficacy: Let A be any action and C be any consequence. Suppose that the conditional chance of C given that A equals α. Then making A true brings it about that the chance of C is α.

Efficacy underpins an instrumentalist justification of Choice from Chance, because it entails that the conditional chance of a consequence *C*, given an action *A*, provides the correct measure of the *instrumental* efficacy of *A* in bringing it about that *C* and hence, in the presence of Chance Consequentialism, of degree to which the desirability of a chance of this consequence weighs in favour of the action. But this would seem to present us with a problem. According to causal decision theory this will only be the case if this conditional chance measures the *causal* efficacy of *A* in bringing it about that *C*. In contrast, according to evidential decision theory, this will only be the case if the conditional chance in question measures the *evidential* relevance of the *A* to the truth of *C*. Since causal and evidential relevance can diverge, it seems that at least one of these camps will want to deny Efficacy.

Let's consider the possible grounds decision theorists of either kind might have for taking this view. For an evidentialist to reject Choice from Chance, they must allow that the truth of *A* be evidentially relevant to *C* in ways not captured by the conditional chance of *C* given that *A*. Now if this were so, then knowledge of the conditional chances would not screen *C* off from *A* and so, contrary to the Credence from Chance principle, $P(C|A, Ch = ch)$ might not equal $ch(C|A)$. This is certainly possible on some interpretations of what chance measures. For example, suppose that the coin that is to be tossed is a fair one (so the chance of it landing heads, conditional on being tossed, is one-half), but someone else is able to control the flight of the coin and make it land tails when they observe you tossing it. Then, supposing that they always do this, it will be true that, conditional on you tossing the coin, it will land tails, whatever the chances are. But the natural thing to say in this case, even if one is an evidentialist, is that the chance of the coin landing heads, conditional on you tossing it, is not one-half, but one, because these chances depend on the global situation and not just the local properties of the coin. So, cases like this do not provide compelling evidentialist grounds for rejecting either Credence from Chance or Choice from Chance.

For a causal decision theorist to reject Choice from Chance, on the other hand, they must allow that the truth of *A* be *causally* relevant to *C* in ways not captured by the conditional chance of *C* given that *A*. In which case knowing the conditional chance of *C* given that *A* would not tell us how probable *C* would be if *A* were performed. This would be true, for instance, if chances were construed as frequencies, for statistical correlation is notoriously deficient as a guide to causation. Likewise, Lewis' Best Systems account of chance allows that the best system of probabilistic laws attributes conditional chances to individual events that do not align with the causal relationships between them in all cases (though presumably they must do so in the majority of them).[31] So a causal decision theorist who accepts such interpretations of objective probability might be led to reject the instrumentalist case for Choice from Chance.

On the other hand, both evidential and causal decision theorists have very strong reasons for accepting Choice from Chance. For Choice from Chance has the status of a platitude in the normative theory of decision making under risk. So too does the principle of Stochastic Dominance, which it

[31] David Lewis, "Humean Supervenience Debugged," *Mind*, CIII, 412 (1994): 473–90.

implies in conjunction with Expectationalism (and which implies it). So, one denies Choice from Chance as a general principle of rational preference under uncertainty at the cost of having to reject the mainstream view about choice under risk or to posit a discontinuity between what rationality requires in situations of (objective) risk and what it requires in situations of (subjective) uncertainty, when the true chances are not known with certainty. I think neither evidentialists nor causalists are likely to be willing to pay this cost. But they must then accept Choice from Chance as a constraint on the interpretation of chances and reject interpretations of objective probability, such as typical versions of the frequentist and Best Systems theories, that conflict with it.

I suggested above that it would seem that either evidentialists or causalists would have to deny Choice from Chance, on the grounds that both could endorse it only if the causal and evidential value of an action coincide – which they do not. But this is not, in fact, correct. All that Choice from Chance requires is that, *when we know what the chances are*, then our estimation of the evidential relevance of the performance of action to the truth of any consequence must be the same as our estimation of its causal relevance. This more restricted claim is something that both evidentialists and causalists should be willing to accept and, indeed, must if Choice from Chance is true (more on this in the next section).

I began the section by suggesting that Choice from Chance is compelling as a principle of instrumental rationality in virtue of the fact that actions that confer higher chances on desirable outcomes should be preferred to those that confer lower ones. This thought, we have seen, can be broken into two separate claims. The first is that the instrumental efficacy of an action in bringing about consequences is measured by their conditional chances given its performance. The second claim is that *ceteris paribus* it's better to have a higher chance of something desirable than a lower one. This latter thought can be expressed more precisely by the following principle. Let $Ch(G=\alpha, B=\beta)$ be the proposition that the chance of *G* is α and the chance of *B* is β. Then:

> **Monotonicity**: Let *G* and *B* be a pair of mutually exclusive and exhaustive propositions such that $G \succ B$. Suppose that both $Ch(G=\alpha, B=1-\alpha)$ and $Ch(G=\alpha^*, B=1-\alpha^*)$ have non-zero subjective probability. Then it is more desirable that the chance of *G* be α and of *B* be $1-\alpha$ than that the chance of G be α^* and of B be $1-\alpha^*$ iff α is at least as large as α^*, that is:
>
> $$Ch(G=\alpha, B=1-\alpha) \succcurlyeq Ch(G=\alpha^*, B=1-\alpha^*) \Leftrightarrow \alpha \geq \alpha^*$$

We now have all the pieces in place for the derivation of Choice from Chance. As we have seen Efficacy says the conditional chance of a consequence given one action being higher than given another means that making the former true confers a higher chance on the consequence than making the latter true. But Monotonicity says that having a higher chance of a good consequence is better than having a lower chance of it. It then follows from Chance Consequentialism that it's better to choose actions with higher conditional chances of good consequences over those with lower ones. So it must be the case that one action is more choice-worthy than another just in case the conditional chance of the good consequence given the performance of the former is higher than given the performance of the latter. More exactly, given Chance Consequentialism and Efficacy, the two principles of Monotonicity and Choice from Chance are equivalent. (This fact is proven in the appendix as Theorem 5.)

V. Interventional Supposition

I have been arguing that Choice from Chance expresses a fundamental norm of instrumental rationality, true (if it is) in virtue of the fact that it is better to have a higher chance of a good consequence (than a lower one) and the fact that the conditional chance of a consequence given the performance of an action measures the instrumental efficacy of the action in bringing the consequence about. In this section and the next, I will put some flesh on this latter claim and then explore the implications for determining the instrumental value of acting and hence for the kind of expectational decision theory assumed in this paper.

Let's start by examining the claim that the conditional chances serve as adequate measures of instrumental efficacy. It is an axiom of instrumental rationality that an action is worthy of choice insofar (and only insofar) as it is effective at bringing about desired consequences. It follows that the desirability of any possible consequence of an action contributes to its instrumental value to the degree (and only to the degree) that the performance of the action secures that consequence. Now according to Expectationalism the degree to which the desirability of any consequence C contributes to the overall choice-worthiness of an action A depends on the magnitude of the quantity $P_A(C)$. On the other hand, Choice from Chance entails that when the chances are known to be given by probability function ch on Ω^*, then its contribution depends on the magnitude of $ch(C|A)$. So when the chances are known these two quantities must be equal. But if $P_A(C)$ measures the efficacy of A-ing with regard to bringing it about that C and $ch(C|A) = P_A(C)$, then the instrumental efficacy of A in bringing about C must also be measured by $ch(C|A)$. *It follows that the chances conditional on available actions are a guide to choice in virtue of tracking the instrumental efficacy of acting in bringing about desired consequences.*

The equality on which this argument depends holds is an instance of a more general and key implication of Choice from Chance in the presence of Expectationalism, namely that the conditional chances are authoritative, not just with respect to our conditional degrees of belief, but with respect to our suppositional degrees of belief more generally. For, as is proven in the appendix as Theorem 3, Choice from Chance and Expectationalism jointly entail:

Principal Suppositional Principle: If $P(Ch^A = ch(\cdot\,|A)) = 1$, then for all $X \in \Omega^*$:

$$P_A(X) = ch(X|A)$$

The Principal Suppositional Principle has significant implications Firstly, Credence from Chance is derivable from it by substitution of the tautology for A, because if $P(Ch = ch) = 1$, then $P = P(\cdot\,|Ch = ch)$ and so, by the Principal Suppositional Principle, $P(X|Ch = ch) = ch(X)$. Hence so too is Conditional Credence from Conditional Chance (indeed this latter condition is simply the evidential version of the Principal Supposition Principle).

Secondly, it follows from the Principal Suppositional Principle that $P(Ch^A = ch(\cdot\,|A)) = 1$ if, and only if, $P_A(Ch^A = ch(\cdot\,|A)) = 1$; that is, that the known conditional chances given that A are the known conditional chances given A *on the supposition that* A. For by the law of total probability:

$$P_A(X) = \sum_{ch^j \in \Delta} P_A\left(X \middle| Ch^A = ch^j\right) \cdot P_A\left(Ch^A = ch^j\right)$$

But by Conditional Credence from Conditional Chance, $P_A(X|Ch^A = ch^j) = ch^j(X)$. Hence:

$$P_A(X) = \sum_{ch^j \in \Delta} ch^j(X) \cdot P_A(Ch^A = ch^j)$$

Now suppose that it is known that the conditional chances given that A are measured by function $ch(\cdot|A)$ on Ω^* and that, were one to A, then the conditional chances given that A would be measured by function $ch^*(\cdot|A)$. Then by Principal Suppositional Principle $P_A(X) = ch(X|A)$. But from the above it follows that $P_A(X) = ch^*(X|A)$. So $ch(\cdot|A) = ch^*(\cdot|A)$.

That supposing that A is in fact true should not alter one's certainty about the conditional chances given that A is a familiar feature of Bayesian conditionalization which, in general, does not allow for retreats from certainty. It is more significant that the same holds for other (non-evidential) forms of supposition. Suppose for example that one has sufficient information to fix the current conditional chances of survival given some surgical intervention. Then in deliberation about what would happen were one to make just such an intervention, one can (in the light of the Principal Suppositional Principle) use just these conditional chances to determine the probabilities of the various possible consequences of the surgical intervention. No further question arises as to what the conditional chances would be if one were to make the intervention.

Thirdly, the Principal Supposition Principle implies the claim made in the previous section, that the causal and evidential significance of an action for a consequence is the same, given the chances. For, on the one hand, the evidential relevance of the performance of action A for the truth of proposition X is measured by $P(X|A)$ and, on the other, the instrumental relevance of A for X is measured by $P_A(X)$. But by the Principal Suppositional Principle both these quantities equal $ch(X|A)$, when it is known that the chances are measured by ch (in the former case via Conditional Credence from Conditional Chance). So, it follows that:

Reconciliation: If $P(Ch^A = ch(\cdot|A)) = 1$, for some $ch \in \Delta$, then for all $X \in \Omega^*$:

$$P_A(X) = P(X|A)$$

The truth of Reconciliation reflects another important fact: that the objective (conditional) chances do not confuse causation and correlation in the way that subjective (conditional) credences can. Recall Nozick's example involving a correlation between the inheritance of a fatal disease and choice of profession. While your conditional credences given the choice of profession may reflect this correlation, the conditional chances given them will not. This is because, at the time of your choice, it is already settled whether or not you have inherited the disease and so the chances of living a long life, conditional on becoming an academic (or an athlete), equals the chances of living a long life, conditional on these choices *and* the fact that you do (or do not) have the disease. And, in Nozick's example, you having the disease (or not) screens out the relation between your longevity and your choice of profession. It follows that the evidential conditional probabilities for lifespan determined by the chances reflect the causal relationship between the disease and lifespan and not the statistical relation between choice of profession and lifespan. This explains why, in this case, the evidential conditional probabilities, given an action A, that are fixed by knowledge of the corresponding conditional chances given A, adequately measure the causal effect of A-ing.[32]

A final noteworthy implication of the Principal Suppositional Principle: in the presence of Expectationalism, it implies Choice from Chance (the formal proof of this claim is given in the appendix as Theorem 6). This fact requires us to reconsider the claim made in section 3, that no epistemic condition relating degrees of belief to chances plausibly contains all that we know about chance. For the Principal Suppositional Principle appears to be exactly a condition of this kind: a constraint on partial belief that, given Expectationalism, implies Choice from Chance and hence Stochastic Dominance, the pragmatic conditions that express the core significance of chance for rational decision making. This means that we are free, formally speaking, to take either the Principal Suppositional Principle or Choice from Chance as our basic expression of the authority of chance.

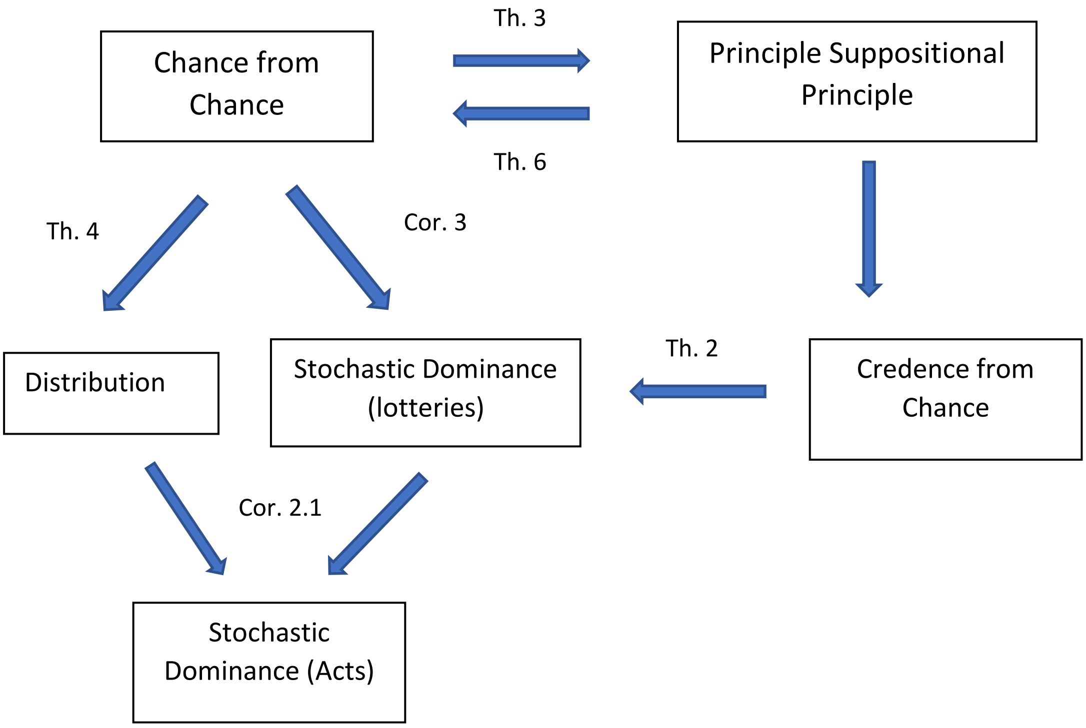
Which of the two principles should be regarded as fundamental from an explanatory and/or justificatory perspective? In this paper the motivation for accepting the Principal Suppositional Principle has come from Choice from Chance and the fact that, for an Expectationalist, conformity with this latter (pragmatic) principle entails conformity with the epistemic constraints expressed by the former. It is an open question however whether independent epistemic grounds can be given for the Principal Suppositional Principle in virtue of which it might serve as the basis for an *epistemic* explanation for why our actions should be guided by Choice from Chance. It's a question well worth addressing, but its answer will not I think challenge my claim that no purely epistemic condition exhausts what we know about chance. For the principle of Monotonicity, a key constituent of the content of Choice from Chance, is quite transparently a constraint on our desires that is independent of any requirements on our beliefs. That Monotonicity may be recovered from the Principal Suppositional Principle in the presence of Expectationalism, simply reflects the fact that it is partially built into the latter. More precisely, given the authority of chance with respect to belief, Expectationalism is reasonable as a principle of choice-worthiness only if it better to have a higher chance of what one desires most, than a lower one.

VI. Action Guidance

I have argued that within the very general decision-theoretic framework of Expectationalism, the Choice from Chance principle has powerful implications: both epistemic ones, such as Credence from Chance and the more general Principal Suppositional Principle, and pragmatic ones, such as Desire from Chance and the stochastic dominance conditions for both lotteries and the acts that induce them. On these implications (illustrated in Figure 1 below) rests my argument that Choice from Chance expresses the core grounds for taking chance as our guide in life. In this section I briefly consider the upshot of this for the choice-worthiness of actions.

[32] In general, however, the relationship between conditional chance and causation is more complicated since the downstream effects of an action may be correlated in virtue of the presence of other causal factors co-determining (along with the action) these effects. (I am grateful to a referee for this point.)

Figure 1: Logical relationships between principles under the assumption of Expectationalism



Let's begin by defining a function $\mathcal{D}$ that maps any action A and chance function ch to the conditional expectation of utility given that A, according to ch, that is, such that, for any value-level partition $\{C_i\}$ of S:

$$\mathcal{D}(A, ch) \coloneqq \sum_i V(C_i) \cdot ch(C_i|A)$$

Now the quantity $\mathcal{D}(A, ch)$ serves as a measure of the choice worthiness of action A, when the chances are known to be measured by function ch. For, as is proved in the appendix as Theorem 7 (b), it follows from Expectationalism and Choice from Chance that for any actions A and A^* and value-level partition $\{C_i\}$ of S, if $P(Ch^A = ch) = 1$, then:

$$A \gtrsim A^* \Leftrightarrow \mathcal{D}(A, ch) \geq \mathcal{D}(A^*, ch)$$

In the light of this, it is reasonable to call $\mathcal{D}(A)$ the *decision value* of A. (Formally it is a random variable mapping chance functions to the corresponding conditional expectation of utility given A.) With decision value so defined, Expectationalism then (further) requires that choice of action be determined by the expectation of decision value under the supposition of the performance of A. So:

Chance-sensitive Expectationalism: For any value-level partition $\{C_i\}$ of S:

$$A \gtrsim A^* \Leftrightarrow \sum_{ch^j} P_A(Ch^A = ch^j) \cdot \mathcal{D}(A, ch^j) \geq \sum_{ch^j} P_{A^*}(Ch^{A^*} = ch^j) \cdot \mathcal{D}(A^*, ch^j)$$

Within the framework, there can no disagreement on the choice worthiness of actions when the conditional chances are given. Where differences emerge is over the relation between a choice of action and the probabilities of the various conditional chance hypotheses expressed by the propositions $Ch^A = ch^j$). Choice from Chance, as we have seen, implies that $P_A(Ch^A = ch) = 1 \Leftrightarrow P(Ch^A = ch) = 1$, but this does not fully constrain the values of the $P_A(Ch^A = ch^j)$. There are however two salient views on choice-worthiness consistent with it.

1. *Chance-sensitive Evidentialism*: For evidentialists, $P_A(Ch^A = ch) = P(Ch^A = ch|A)$. Hence choice-worthiness goes by the conditional expectation of decision value given that A is performed, that is:

$$V(A) = \sum_{ch^j} P\left(Ch^A = ch^j|A\right) \cdot \mathcal{D}\left(A, ch^j\right)$$

2. *Chance-sensitive Causalism*: For 'pure' causalists, such as Biran Skyrms, the conditional chance given that A is (a measure of) an act-independent property of the state of the world defined functionally in terms of its causal propensity to bring about various possible consequences conditional on the performance of A[33]. In virtue of the independence of this property from the chosen act, $P_A(Ch^A = ch) = P(Ch^A = ch)$. Hence choice-worthiness simply goes by the (unconditional) expectation of decision value, that is, by:

$$\mathbb{E}(\mathcal{D}(A)) = \sum_{ch^j \in \Delta^*} P\left(Ch^A = ch^j\right) \cdot \mathcal{D}\left(A, ch^j\right)$$

Both Evidentialism and Causalism face significant challenges that are well-documented in the literature and it is fair to say that the jury is still out as to whether either holds in all circumstances. But that only goes to reinforce the case for saying that Choice from Chance is the *strongest* principle expressing the authority of chance with respect to choice that should command general assent amongst those who accept Expectationalism. That is to say, Choice from Chance is not just *an* expression of the sense in which chance is our guide in life, but *the* principal expression of it.

VII. Appendix: Proofs

We assume throughout that the set of propositions Ω forms a complete, atomless Boolean algebra.

Theorem 1: (a) Credence from Chance is equivalent to Conditional Credence from Chance and (b) Credence from Chance implies Desire from Chance.

Proof: (a) Credence from Chance is a direct implication of Conditional Credence from Chance. Now assume the former is true. Then for any propositions $E, F \in \Omega^*$ such that $P(Ch = ch, F) \neq 0$:

$$P(E|F, Ch = ch) = \frac{P(E, F|Ch = ch)}{P(F|Ch = ch)} = \frac{ch(E, F)}{ch(F)} = ch(E|F)$$

[33] Brian Skyrms, "Causal Decision Theory," *The Journal of Philosophy* LXXIX, 11 (1982): 695-711; Skyrms, *Pragmatics and empiricism*, *op. cit*.

In accordance with Conditional Credence from Chance. So they are equivalent.

(b) Let A and A^* be any two actions and G and B be a pair of mutually exclusive events that respectively exhaust the possible outcomes of choosing A and A^* and such that $G \succ B$. Then for all ch such that $P(Ch = ch|A) \neq 0 \neq P(Ch = ch|A^*)$, since $AG = G = A^*G$ and $AB = B = A^*B$:

$$V(A|Ch = ch) = V(G|Ch = ch) \cdot P(G|A, Ch = ch) + V(B|Ch = ch) \cdot P(B|A, Ch = ch)$$

$$V(A^*|Ch = ch) = V(G|Ch = ch) \cdot P(G|\ A^*, Ch = ch)\ + V(B|Ch = ch) \cdot P(B|A^*, Ch = ch)$$

Hence $V(A|Ch = ch) \succsim V(A^*|Ch = ch)$ iff $P(G|A, Ch = ch) \geq P(G|A^*, Ch = ch)$. Assume Credence from Chance. Then by (a):

$$P(G|A, Ch = ch) = ch(G|A)$$

$$P(G|A^*, Ch = ch) = ch(G|A^*)$$

It follows from that $V(A|Ch = ch) \geq V(A^*|Ch = ch)$ iff $ch(G|A) \geq ch(G|A^*)$ in accordance with Desire from Chance. ■

Theorem 2: Assume Credence from Chance and Expectationalism. Then rational preference between lotteries respects the condition of Stochastic Dominance (Lotteries).

Proof: Let L and L^* be two lotteries on $\mathcal{C}$, respectively determining chances α_i and α_i^* for the C_i. Then by the definition of the desirability function V:

$$L \succsim L^* \Leftrightarrow \sum_{i=1}^{n} V(C_i). P(C_i|L) \geq \sum_{i=1}^{n} V(C_i) \cdot P(C_i|L^*)$$

But by Theorem 1(a), Credence from Chance implies that:

$$P(C_i|L) = \sum\nolimits_j P\big(C_i|Ch = ch^j, L\big) \cdot P\big(Ch = ch^j|L\big) = \sum\nolimits_j ch^j(C_i|L) \cdot P\big(Ch = ch^j|L\big)$$

But $L(C_i) = \alpha_i$ and so $P\big(Ch = ch^j|L\big) > 0$ only if $ch^j(C_i|L) = \alpha_i$. It follows that $P(C_i|L) = \alpha_i$, and hence that:

$$L \succsim L^* \Leftrightarrow \sum_{i=1}^{n} V(C_i) \cdot \alpha_i \geq \sum_{i=1}^{n} V(C_i) \cdot \alpha_i^*$$

But if lottery L weakly stochastically dominates lottery L^*, then for all $j \in \{1,..,n\}$, $\sum_{i=1}^{j} \alpha_i \geq \sum_{i=1}^{j} \alpha_i^*$. So the right-hand side of this equation must be satisfied. It follows that $L \succsim L^*$. Furthermore, if lottery L^1 strongly stochastically dominates lottery L^*, then $\sum_{i=1}^{n} V(C_i) \cdot \alpha_i > \sum_{i=1}^{n} V(C_i) \cdot \alpha_i^*$ and so $L \succ L^*$. ■

Corollary 2.1: Assume in addition Distribution. Then rational preference between lotteries respects the condition of Stochastic Dominance (Acts).

Theorem 3: Assume Expectationalism. Then Choice from Chance implies the Principal Suppositional Principle.

Proof: Suppose that $P(Ch^A = ch(\cdot\,|A)) = 1$. Let A, E and F be any propositions in Ω^* such that $ch(A) > ch(AE) > 0$ and $ch(A) > ch(AF) > 0$. Now consider any proposition A^*, disjoint from A, such that $ch(A^*) > 0$ and $ch(F|A^*)$ lies between $ch(E|A)$ and $ch(F|A)$. (The existence of such a proposition is assured by the assumption that Ω is complete and atomless.) Choose a utility assignment that induces a value partition $\{G, B\}$ on states, with G states having utility 1 and B states utility 0 and such that A has value consequence *G* whenever *E* is the case and value consequence *B* otherwise and A^* has value consequence *G* whenever *F* is the case and *B* otherwise. Now by Expectationalism:

$$A \succsim A^* \Leftrightarrow P_A(G) \geq P_{A^*}(G) \Leftrightarrow P_A\,(E) \geq P_{A^*}(F)$$

But by Choice from Chance:

$$A \succsim A^* \Leftrightarrow ch(G|A) \geq ch(G|A^*) \Leftrightarrow ch(E|A) \geq ch(F|A^*)$$

Hence:

$$P_A(E) \geq P_{A^*}(F) \Leftrightarrow ch(E|A) \geq ch(F|A^*)$$

Now choose a similar utility assignment but such that both A and A^* have value consequence *G* whenever *F* is the case and value consequence *B* otherwise. By the same reasoning as before it follows that:

$$P_A(F) \geq P_{A^*}(F) \Leftrightarrow ch(F|A) \geq ch(F|A^*)$$

Suppose that $ch(E|A) \geq ch(F|A)$. Then by assumption $ch(E|A) \geq ch(F|A^*) \ \geq ch(F|A)$. It follows that $P_A(E) \geq P_{A^*}(F) \geq P_A(F)$. Now suppose instead that $ch(F|A) > ch(E|A)$. Then by assumption $ch(F|A) \geq ch(F|A^*) \ \geq ch(F|A)$. It follows that $P_A(F) \geq P_{A^*}(F) \geq P_A(E)$. Hence:

$$ch(E|A) \geq ch(F|A) \Leftrightarrow P_A(E) \geq P_A(F)$$

So $ch(\cdot\,|A)$ and $P_A\,(\cdot)$ are ordinally equivalent over Ω*. And in particular, $ch(E|A) = 0.5 \Leftrightarrow P_A(E) = 0.5$ But since, by assumption, Ω* is atomless, any probability measure on it is determined by the elements of probability measure one-half (Bolker 1966, Corollary 1.23). So, in accordance with the Principal Suppositional Principle, if $P(Ch^A = ch(\cdot\,|A)) = 1$, then $ch(\cdot\,|A)$ and $P_A(\cdot)$ are identical. ■

Corollary 3.1: Assume Expectationalism. Then Choice from Chance implies Credence from Chance.

Proof: Since $P(Ch = ch|Ch = ch) = 1$, it follows immediately from Theorem 3, by setting $A = \Omega$, that $P(\cdot\,|Ch = ch) = ch(\cdot)$. ■

Theorem 4: Assume Expectationalism. Then Choice from Chance implies Distribution.

Proof: Let A and A^* be any two actions and suppose the chances are given by measure ch. Let $L = L(A, ch)$ and $L^* = L^*(A^*, ch)$ be the lotteries determined by A and A^* when this is the case. Assume Expectationalism. Then:

$$A \succsim A^* \Leftrightarrow \sum_i V(C_i) \cdot P_A(C_i) \geq \sum_i V(C_i) \cdot P_{A^*}(C_i)$$

And by the definition of V:

$$L \succcurlyeq L^* \Leftrightarrow \sum_{i=1}^{n} V(C_i).\,P(C_i|L) \geq \sum_{i=1}^{n} V(C_i).\,P(C_i|L^*)$$

Now by Theorem 3, Choice from Chance implies the Principal Suppositional Principle and so, since $P(Ch = ch) = 1$:

$$P_A(C_i) = ch(C_i|A)$$
$$P_L(C_i) = P_L(C_i|L) = ch(C_i|A)$$

in virtue of the fact that, by definition, $L(C_i) = ch(C_i|A)$. Hence:

$$A \gtrsim A^* \Leftrightarrow \sum_i V(C_i) \cdot ch(C_i|A) \geq \sum_i V(C_i) \cdot ch(C_i|A^*) \Leftrightarrow L \succcurlyeq L^*$$

in accordance with Distribution. ■

Corollary 4.1: Given Expectationalism, Choice from Chance is equivalent to Stochastic Dominance (Acts).

Proof: Assume Expectationalism. By Corollary 3.1, Choice from Chance implies Credence from Chance and by Theorem 4 it implies Distribution. Hence by Corollary 3.1, it implies Stochastic Dominance (Acts). On the other, Choice from Chance follows immediately from Stochastic Dominance (Acts) in the special case where the two acts have just two consequences. ■

Theorem 5: Assume Chance Consequentialism and Efficacy. Then Chance Monotonicity is equivalent to Choice from Chance.

Proof: Let A and A^* be any two actions and G and B be a pair of mutually exclusive propositions that exhaust the possible outcomes of choosing A and A^*. Suppose that the conditional chance of G given A equals α and that the conditional chance of G given A^* equals α^*. By Efficacy, making A true brings it about both that $Ch(G) = \alpha$ and that $Ch(B) = (1 - \alpha)$ and making it true that A^* true brings it about that $Ch(G) = \alpha^*)$ and that $Ch(B) = (1 - \alpha^*)$. Note that $B = \neg G$ and $B^* = \neg G^*$ and so it then follows by Chance Consequentialism that:

(*) $A \gtrsim A^* \Leftrightarrow V(Ch(G) = \alpha) \geq V(Ch(G) = \alpha^*))$

But by Chance Monotonicity, $V(Ch(G) = \alpha) \geq V(Ch(G) = \alpha^*) \Leftrightarrow \alpha \geq \alpha^*$. And by Choice from Chance, $A \gtrsim A^* \Leftrightarrow \alpha \geq \alpha^*$. And so it follows from (*) above that Chance Monotonicity holds iff Choice from Chance does. ■

Theorem 6: Assume Expectationalism. Then the Principal Suppositional Principle implies Choice from Chance

Proof: Let A and A^* be any actions and $\{G, B\}$ be a value partition on states with $G \succ B$. Suppose that the chances are known to be given by the function ch, that is, that $P(Ch = ch) = 1$. Assume that the Principal Suppositional Principle is true. Then:

$$P_A(G) \geq P_{A^*}(G) \Leftrightarrow ch(G|A) \geq ch(G|A^*)$$

But by Expectationalism:

$$A \gtrsim A^* \Leftrightarrow P_A(G) \geq P_{A^*}(G)$$

Hence it follows that:

$$A \gtrsim A^* \Leftrightarrow ch(G|A) \geq ch(G|A^*)$$

in accordance with Choice from Chance. ■

Theorem 7: Assume Expectationalism and the Principle Suppositional Principle. Let $\mathcal{D}(A,ch):=\sum_i V(C_i)\cdot ch(C_i|A)$. Then (a):

$$A \succsim A^* \Leftrightarrow \sum_j P_A(Ch^A = ch^j)\cdot\mathcal{D}(A,ch^j) \geq \sum_j P_A(Ch^{A^*} = ch^j)\cdot\mathcal{D}(A^*,ch^j)$$

Furthermore (b) if the chances are given by probability ch, that is, $P(Ch = ch) = 1$, then:

$$A \succsim A^* \Leftrightarrow \mathcal{D}(A,ch) \geq \mathcal{D}(A^*,ch)$$

Proof: Let A and A^* be any actions. Then by Expectationalism, for any value-level partition $\{C_i\}$ of S:

$$A \succsim A^* \Leftrightarrow \sum_i V(C_i)\cdot P_A(C_i) \geq \sum_i V(C_i)\cdot P_{A^*}(C_i)$$

$$\Leftrightarrow \sum_i V(C_i)\left(\sum_j P_A(C_i|Ch^A = ch^j)\cdot P_A(Ch^A = ch^j)\right)$$
$$\geq \sum_i V(C_i)\left(\sum_j P_{A^*}(C_i|Ch^{A^*} = ch^j)\cdot P_A(Ch^{A^*} = ch^j)\right)$$

$$\Leftrightarrow \sum_j P_A(Ch^A = ch^j)\left(\sum_i V(C_i)\cdot P_A(C_i|Ch^A = ch^j)\right)$$
$$\geq \sum_j P_A(Ch^{A^*} = ch^j)\left(\sum_i V(C_i)\cdot P_{A^*}(C_i|Ch^{A^*} = ch^j)\right)$$

$$\Leftrightarrow \sum_j P_A(Ch^A = ch^j)\left(\sum_i V(C_i)\cdot ch^j(C_i|A)\right)$$
$$\geq \sum_j P_A(Ch^{A^*} = ch^j)\left(\sum_i V(C_i)\cdot ch^j(C_i|A^*)\right)$$

in virtue of Credence for Chance. So (a) by the definition of $\mathcal{D}(A,ch)$:

$$A \succsim A^* \Leftrightarrow \sum_j P_A(Ch^A = ch^j)\cdot\mathcal{D}(A,ch^j) \geq \sum_j P_A(Ch^{A^*} = ch^j)\cdot\mathcal{D}(A^*,ch^j)$$

(b) Now suppose that the chances are known to be measured by function ch, that is, that $P(Ch = ch) = 1$. Then from the Principal Suppositional Principle it follows that $P_A(Ch^A = ch) = 1$. Hence from above it follows that:

$$A \succsim A^* \Leftrightarrow \mathcal{D}(A,ch) \geq \mathcal{D}(A^*,ch)$$

■

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