CONDITIONAL CHANCE AND WARRANTED CREDENCE

> "… chance claims assert some degrees of belief to be made more reasonable than others by objective empirical features of the world" – (Mellor 1971, xii)

Abstract: A fully adequate interpretation of the concepts of chance and of conditional chance should, *inter alia*, make sense of discourse in which these concepts appear, explain why (conditional) chances are (conditional) probabilities, account for the objectivity of chances and conditional chances and account for their authority with respect to what we believe and what we choose. No extant theory of chance does so. In contrast, these desiderata are met by the interpretation proposed in this paper: that the (conditional) chances at a given time are the (conditional) credences warranted by what is knowable at that time or, more metaphorically, the credences of a perfect inductive reasoner that is as fully informed as it is possible to be.

1. Introduction

Notions of chance and of conditional chance play an important role in many different domains. They are ubiquitous in the natural and social sciences, in both theoretical and applied work, in the methodology of science and statistics, in decision theory and policy evaluation, and of course in philosophy itself. In this paper I aim to provide an interpretation of these notions that make sense of the role they play in the kinds of discourse in which they appear and, in particular, which explains why chance claims bear certain characteristic relationships to claims about what should be believed and what should be chosen.

When I speak of an interpretation of chance or conditional chance, I mean an assignment of semantic content to statements or claims like 'The chance of rain tomorrow is 0.6' or 'The chance of death, conditional on an infection, is 0.1' that explains why (conditional) chances have the properties that they do. In the philosophical literature, the problem of interpreting probability often refers to a closely related project, that of supplying mathematical probability theory with canonical applications. And indeed the two problems of interpretation (of interpretation$_1$ and interpretation$_2$) may be regarded as opposite sides of the same coin. But there are nuances that need to be noted.

In the first place, the term 'chance', as I use it, refers to just one type of probability: the objective kind (I will clarify what I mean by this in due course). So there are perfectly adequate applications of mathematical probability, such as to the degrees of belief of rational agents, that fall outside the scope of the problem of interpretation I am interested in. And secondly, it is possible in principle that the best sense-making account of discourse involving chances and conditional chances allows that they do not satisfy the mathematical laws of probability and/or conditional probability. This possibility will however be ruled out by fiat in this paper: the account of conditional chance I seek is one that will provide an interpretation$_2$ of mathematical probability that is an interpretation$_1$ of chance and conditional chance, i.e. that is at the intersection of the two projects of interpretation.

Since chance talk is to be found in so many different domains and put to many different purposes, it is of course perfectly possible that no interpretation$_1$ will make sense of all chance claims that we are interested in. Indeed, it is recurring theme of philosophical discussion of probability that there are two different kinds of probability – though much less agreement about what the two are! According to Hacking, for instance, probability is:

> *".. Janus faced. On the one side it is statistical, concerning itself with the stochastic laws of chance processes. On the other side it is epistemological, dedicated to assessing the reasonable degrees of belief in propositions quite devoid of statistical background"* – (Hacking, 1975, p. 12).

This view is echoed in Carnap's (1945) distinction between probability as degree of confirmation and as long-run relative frequency, but rejected by Lewis (1980) who held that the primary distinction was between the subjective interpretation of probability as a measure of an agent's degrees of rational belief (her credences) and an objective one of probability as a measure of single case chance or propensity. Hájek (2019) on the other hand identifies *three* different classes of interpretation: the epistemic ones which construe probability as measuring a relation between propositions or judgements, this being either of partial implication (the logical) or of evidential support (the evidential); the subjective one which construes probabilities as subjective degrees of belief (credences), and the physical ones which construe probability as a measure of mind-independent features of the world.

What is indisputable, I think, is that chance claims don't refer to the asserter's degrees of belief. The weather forecaster who says that the probability of rainfall is 0.9 may well believe to this degree that it will rain, but this fact does not exhaust the content of what they claim. If it did then we could not make any sense of the fact that another forecaster could disagree with them and assert that the probability of rainfall is 0.8. For if each were simply reporting their own degree of belief in rain, the two numbers – 0.9 and 0.8 – would measure properties of different objects and so could not serve as rival measures of something. As Mellor puts it:

> *"To admit even the possibility of chances, we must take chance claims to not only express a partial belief, but also or instead to assert that it is in some way objectively more justified than is the expression of some other partial beliefs … ."* – (Mellor, 1971, p. 18)

Mellor is also correct in thinking that what the forecasters disagree about is some objective property of the situation, a property that warrants or merits the adoption of certain degrees of belief, but which is distinct from those held by any particular individual. Now this claim might seem to imply commitment to a physical interpretation of chance: as frequencies, for instance, or, as Mellor held, the propensities that purportedly explain them. But this is not quite right. It is no doubt true that forecasters often do disagree about physical facts of some kind when they disagree about the chances of rain, but it doesn't follow that the chances are identical to these facts. Indeed, as I shall argue, there simply aren't any good candidates for physical probabilities that are sufficiently general in nature as to constitute the objects of chance claims as they are made in very different domains.

The proposal of this paper is that chance claims are not, as, as Mellor argued, physical facts that warrant certain degrees of belief but the *judgements* that are warranted by knowledge of the relevant physical facts. Or, to put it somewhat differently, they are the credences of an ideal reasoner, one who is as well-informed about all relevant facts as it is possible to be. On this account, the objectivity of chance claims lies not in a correspondence between the content they assert and some set of physical facts, but in the fact that they express the best-possible judgements that can be made, given these facts.

My case for this interpretation appeals to five requirements on interpretations of chance and conditional chance that I will argue are not jointly met by any of the prevailing accounts. They are:

1. **Scope**: The interpretation should give content to all chance and conditional chance claims of interest.

2. **Formal Admissibility**. It should follow from the interpretation that chance and conditional chance claims, so interpreted, obey the core axioms of probability and conditional probability.

3. **Objectivity**: It should explain the objectivity of chance claims.

4. **Belief Determination.** It should explain why and how (conditional) chances are authoritative for our (conditional) degrees of belief.

5. **Action Guidance.** It should explain why and how our actions are guided by what we know about the (conditional) chances.

I will proceed as follows. In the next section I will more carefully define the notion of an interpretation of chance, allowing for a precise statement of the scope and admissibility conditions. Section 3 will offer a criterion for identifying interpretations that confer chances with the requisite objectivity and show that while most epistemic ones do not meet it, the interpretation of chance as warranted credence does. Section 4 explains why the various physical interpretations are unable to meet the combination of scope and formal admissibility. Sections 5 and 6 respectively argue for a version of requirement that chances are authoritative for credence and demonstrates that only chance as warranted credence can explain it. Section 7 makes a similar argument for the requirement that chances be action-guiding.

2. Adequacy Conditions on Interpretations

The conditions of Scope and Formal Admissibility jointly serve to require that any assignment of semantic content to a set of chance claims should both give content to all the claims in the set and, in doing so, should serve to explain why chances and conditional chances are, formally speaking, probabilities and conditional probabilities. Given the many domains in which chance talk is to be found, the Scope requirement is ambitious. It is also vague in some respects; but not in ways that matter for my argument and I intend neither to fix precisely what chance talk counts as of interest, nor to preclude partially revisionary interpretations that imply that some chance talk is illegitimate.

The condition of formal admissibility, in contrast, needs clarification. Since chances and conditional chances are related via the standard definition of conditional probability in terms of a ratio of unconditional ones, it might seem sufficient to provide an interpretation of chance, from which one for conditional chance can be derived. But this is not enough for present purposes, for it also needs to be shown that the proposed interpretation endows the derived notion of conditional chance with the properties required for it to play the explanatory roles that it does in the various different domains in which it is employed. And simply defining conditional chances in terms of unconditional chances will not suffice for this. This is so for a number of reasons.

Firstly, as Hájek (2003) has persuasively argued, the ratio definition is not fully adequate as an analysis of conditional chance as that notion is employed in some domains. In particular, it leaves conditional chances undefined relative to events that have zero probability (or to those with indeterminate or infinitesimal probability). Secondly, we sometimes make claims of conditional chance without commitment to the unconditional chances being defined. For instance, before deciding what shellfish to buy, I might want to consider the conditional chance of vibrio vulnificus poisoning from oysters, given that it is January, without thinking that there is anything like a chance of it being January. Finally, and most importantly, claims of conditional chances are not simply assertions to the effect that the ratios of chances have some or other value. Consequently it has to be explained by an interpretation why this ratio equation holds.

This last point can be illustrated by consideration of the subjective interpretation of probability as developed by Ramsey (1926). Ramsey shows not only that partial beliefs, defined in terms of their causal effect on our choices, must be probabilities if our choices are rational, but also introduces a conceptually distinct notion of conditional partial belief given that E, defined in terms of dispositions to choose actions that would be useful were E true. It is a theorem, not an assumption, of his account that conditional degrees of belief so defined are related via the ratio formula to degrees of belief as he defines them. An adequate objective interpretation of probability should similarly imply, rather than presuppose, that objective conditional probabilities are ratios of unconditional ones.

To make all of this more precise, let $S = \{s\}$ be a set of possible states of the word and $T = \{t\}$ a set of possible times. A complete history is just a sequence of states, one for each moment of time, i.e. a function from times to states. Let $H = \{h\}$ be the set of all logically possible histories and H and H^{ch} be Boolean algebras of sets of such histories with the latter being a sub-algebra of the former. Intuitively, H is the set of all events or propositions of interest and H^{ch} the set of those to which chances can be meaningfully ascribed. For instance, it might be that H contains the proposition that the chance of rain in London tomorrow is greater than 0.8, but that H^{ch} does not (if there is no such a thing as the chance of the chance of rain being some value).

Let Ch_t be a real-valued random variable on H^{ch} that specifies the chances of H^{ch} propositions at time t and, for any proposition $Y \in \mathrm{H}^{ch}$ let Ch_t^Y be a real-valued random variable on H^{ch} that specifies the conditional chances given Y at time t. Let corresponding propositions as to the chances and conditional chances of particular propositions at time t be denoted by expressions of the form

'$Ch_t(X) = x$' and '$Ch_t(X|Y) = y$'. For any history h and time t, let $h_t := h(t)$, $Ch_{h,t} := Ch_t(h)$ and $Ch^Y_{h,t} := Ch^Y_t(h)$.

Now $Ch_{h,t}$ and $Ch^Y_{h,t}$ can respectively serve as the truth-conditions (sets of histories) for propositions concerning the chances and conditional chances given *Y* at t. '$Ch_t(X) = x'$, for example, will be true at the set of histories *h* such that $Ch_{h,t}(X) = x$ while '$Ch_t(X|Y) = y$' will be true at the set of histories *h* such that $Ch_{h,t}(X|Y) = y$. So we can treat $Ch_{h,t}$ and $Ch^Y_{h,t}$ as picking out the properties of history *h* that determine the chances and conditional chances at time *t* of the H^{ch} propositions. An interpretation of chance is just an identification of these properties, of the chance facts at the $h(t)$ that make it the case that the chances at time *t* in history *h* are as measured by $Ch_{h,t}$. Likewise an interpretation of conditional chance is an identification of the properties of any history *h* that determine the H^{ch} conditional chances at time t, i.e., of the conditional chance facts at the $h(t)$ that determine the values of the $Ch^Y_{h,t}$.

The condition of scope and of formal admissibility on an interpretation of chance and conditional chance can now be rendered as follows:

> **Scope**: The interpretation explains, for all propositions $X, Y \in \mathrm{H}^{ch}$, histories *h* and times *t*, why the functions $Ch_{h,t}(X)$ and $Ch^Y_{h,t}(X)$ take the values that they do.[1]
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> **Admissibility**: For any history *h* and time t, the function $Ch_{h,t}$, so interpreted, is a probability function on H^{ch}. Furthermore, for any proposition $Y \in \mathrm{H}^{ch}$, $Ch^Y_{h,t}$ is a probability function on H^{ch} such that if $Ch_{h,t}(Y) \neq 0$, then $Ch^Y_{h,t}(Z) = \frac{Ch_{h,t}(Z,Y)}{Ch_{h,t}(Y)}$.

For well-documented reasons, the two main physical interpretations of probability fail to meet this pair of very basic conditions. Frequentism fails to offer an interpretation of single-chance claims, such as that there is a high chance of some particular tropical storm striking a particular location. The propensity theory on the other hand fails to offer an interpretation of non-causal conditional chances, such as the conditional chance that it rained yesterday, given that the lawn is wet (I will return to these issues in more detail in section 3). In contrast, the subjective interpretation does meet them. But, as I suggested before, it fails to confer chance claims with the requisite objectivity. It is time to make this claim more precise.

Objectivity and Knowledge

An interpretation of chance should, I claimed, distinguish it from subjective interpretations of probability and explain the sense in which chance claims are objective. The language of 'subjective' versus 'objective' is not particularly helpful here however. Probability statements interpreted as claims about someone's degrees of belief can be construed as factive: the person either has those credences or they don't. So what makes this interpretation 'subjective' is thus not that claims about credences are not truth-apt, but that their truth depends on characteristics of some person (the 'subject'). It is this feature that we want to rule out: adequate interpretations of chance claims should

[1] Note that the requirement remains vague in that I have not said what H^{ch} must contain.

not allow that there be both 'your chance of rain' and 'my chance of rain', for this would, as noted before, make disagreement about the chance of rain *simpliciter* senseless.

To make this rough idea more precise I propose to exploit an idea of List and Pivato (2015): that while the truth of a chance claim depends on what the state of the world is at the time of the claim, and possibly on its history, an agent's degrees of belief depend on her information state at some time t, i.e. on what she knows (or believes) at t about the state of the world and its past. Consequently, conditionalizing a probability measure of the t-chances on any true proposition concerning the past or present state of the world will not change it at all, while conditionalizing a credence measure on this information typically will.

Let's spell this out more formally using the framework introduced in the previous section. For any history-world pair, (h, t), let $h(\leq t)$, the inclusive past of $h(t)$, be the sequence of states in h preceding and including $h(t)$. Let $Past(h,t) := \{h^i : h^i(\leq t) = h(\leq t)\}$ be the proposition that *h*'s past at *t* is the true past, i.e. the strongest proposition true at time *t* in all histories sharing *h*'s past. What List and Pivato propose is that a probability function be considered purely epistemic if it becomes degenerate (takes only the values 0 or 1) once it is conditioned on the truth of $Past(h,t)$, i.e. on the entire history of the world up to that point in time. By contrast then the probability function is non-epistemic or objective if it is non-degenerate for at least one proposition. But this does not fully capture what we want. Firstly, it rules out subjective uncertainty about the future when the past is known, so in effect imposes knowledge of the laws of nature. And secondly, non-degeneracy on a single proposition is insufficient for objectivity. Not just because subjective probabilities can be non-degenerate too, but because an objective probability function should not just be degenerate but be invariant under conditionalisation on true propositions about the history of the world. More formally, what objectivity requires therefore is that:[2]

Invariance under Conditionalisation on the Past: Let *Y* be any proposition implied by $Past(h,t)$ that belongs to H^{ch}. If $Ch_{h,t}(X|Y) \neq 0$, then:

$$Ch_{h,t}(\cdot\,|X) = Ch_{h,t}(\cdot\,|X, Y)$$

As one would expect, the Invariance condition is satisfied by the main physical interpretations of probability. The reason is simply that, on these interpretations, whether a chance or conditional chance claim is true at some time *t* is fixed by a history *h*. Hence since $h \in Past(h,t)$, it entails any proposition *Y* entailed by $Past(h,t)$. And so:

$$Ch_{h,t}(\cdot\,|X) = Ch_t(\cdot\,|X, \{h\}) = Ch_t(\cdot\,|X, Y, \{h\}) = Ch_{h,t}(\cdot\,|X, Y)$$

For instance, on the frequency interpretation whether or not it's true that the chance is 0.1 of dying from a heart attack in the next year, depends on the proportion of those dying from a heart attack in

[2] Lewis (1980) requires something stronger, namely that chances be invariant under conditionalization on all true propositions concerning the history up to *t* and all true history-to-chance propositions.

the relevant reference class. And facts about the state of the world and its history are already factored into the determination of this reference class.

In contrast the condition is clearly incompatible with a subjective or Bayesian interpretation of chance, for it is no requirement of rationality that an agent knows the true state and history of the world. Hence there will be times and histories in which conditionalizing on truths about the past changes their credences. This is true even of Objective Bayesianism: the constraints of objectivity that it imposes relate to how evidence (and the lack of it) should be treated, not to the extent of the evidence held by the agent.

The condition is also incompatible with most epistemic interpretations of probability. Consider first the logical interpretations that take the conditional probability of a proposition *Y* given that *X* to measure the degree to which *X* entails that *Y*. Whatever this entailment relation is, it is clear that the degree to which *X* entails that *Y* will not always (or even usually) be the same as the degree to which *X* and $Past(h, t)$ jointly entail that *Y*. For $Past(h, t)$ is a contingent truth and so cannot be part of a logical relationship between *X* and *Y*.

Consider now the evidential interpretations, according to which the probability of proposition *X* measures the degree to which *X* is confirmed or supported by our evidence and the conditional probability of *X* given that *Y* the degree to which *X* is supported by this evidence on the hypothesis that *Y* is true. Degrees of evidential support depend on our state of knowledge (hence the evidential support relation is not purely logical), and so only if this evidence includes $Past(h, t)$ will the Invariance condition hold. Typically, it will not. On the other hand, an evidential probability based on an information set that includes all truths about the past and present state of the world would have the requisite objectivity and would thus be a candidate for an objective interpretation of chance. Chance, I contend, is just such a warranted evidential probability.

To spell out in a bit more detail what this might look like, let's work with a definition of the evidential probability of *X* (given that *Y*) relative to information E as the (conditional) degree of a belief in *X* (given that *Y*) warranted by information *E*. Then the contention is that the chances and conditional chances of truth of propositions are, respectively, the unconditional and conditional rational degrees of belief warranted by an information set that includes all truths concerning the past and current state of the world. But the credences warranted by a body of evidence are just those that a perfect inductive reasoner would adopt if they held just this body of evidence. So I shall say that the chances are the rational degrees of belief of a perfect inductive reasoner (call them 'Chance') who is fully informed of all relevant facts. Such credences unproblematically respect the scope condition on interpretation and, courtesy of Ramsey's demonstration, their rationality ensure that they are admissible. And by construction they meet our requirement of objectivity.

Making this 'Warranted Credence' view more precise requires spelling out what is meant by a 'perfect inductive reasoner' and by 'fully informed'. By the former, I mean no more than that the reasoner makes no mistakes in reasoning: she draws all those inferences from what she knows that she should and no inferences that she should not (i.e., all and only those she is warranted in making!). To do this she must possess a set of (potentially context-dependent) inference rules and apply them correctly in each circumstance. There can be legitimate philosophical disagreement over what rules, beyond those ensuring probabilistic consistency, this set will contain. But it will not matter to my argument what they are.

The more important question is what information Chance should be assumed to hold because, in doing so, we identify which of the many evidential probabilities meeting the objectivity requirement counts as the chances. It seems to me that the answer to this may depend on the context in which the concept of chance is being applied and that different sciences and other forms of enquiry might work with somewhat different restrictions on what Chance knows. Such context-dependence can be handled in our framework in a natural way by specifying the sets of propositions H and H^{ch} (to which credences and chances respectively attach), in such a way as to fit the informational basis of Chance to the kinds of evidence used in the domain of interest. So we can ignore this complication in what follows.

The basic challenge for the Warranted Credence view is to endow the ideal reasoner with sufficient resources to determine their judgements about future outcomes (which then constitute the chances) without building into these resources some exogenous concept of chance (for which I claim we lack an adequate interpretation). As a minimum, to secure objectivity, Chance should be apprised of all facts relating to the state of the word and its past. On the other hand, if they were not just apprised of the history of the world but also the laws that govern it, this would risk eliminating non-trivial chances altogether. There are natural intermediate positions however that allow for some information about the future but not so much as to render the chances trivial. Frequentist or propensity-friendly versions could include all relevant frequency and/or propensity facts in the information set of Chance, for instance, thereby potentially reaping the benefits of the interpretations from which they derive, without suffering the problems of scope and/or formal admissibility.

The base-line version of the Warranted Credence interpretation that I propose is that the chances at some time *t* should be identified with the credences warranted by the facts *knowable* at *t*. Knowability is of course a modal notion that can be specified in less or more demanding ways, but here I read a fact as being knowable at *t* as meaning that there exists a physically feasible method of enquiry by which its truth can be established at *t*. It follows that facts about the past are knowable in principle, ensuring that the objectivity condition is met (at least relative to a potentially coarse-grained specification of H and H^{ch}). On the other hand, the future outcomes of indeterministic processes clearly fall outside of the scope of what is knowable in this sense. Indeed since, in general, future facts are physically inaccessible, it follows that the laws of nature are not knowable. So too are most counterfacts: facts about what would have happened if something that is false had been true.

This rather abstemious characterisation of the information available to Chance has the implication that it is possible for the chances of future events to change over time. For if the ideal inductive reasoner makes her judgements based on what evidence she holds at the time of judgment, then as her evidence changes over time, so too could her judgements. Let h and h' be two histories that have shared pasts up till time t_0 but have diverged by later time t_1, i.e., $Past(h, t_0) = Past(h', t_0)$ but $Past(h, t_1) \neq Past(h', t_1)$. Let *X* be any proposition whose truth is not settled at t_0. Now the inductive principles that Chance employs will serve (if sufficiently rich) to fix the truth or falsity of the history-to-chance propositions regarding X at either time and hence, given the true history of the world at that time, any propositions regarding the chances of X (at that time). But since at t_0 it is still open what the true past of the world will be at t_1, it is still open at t_0 what the chances of X will be at t_1. So it can be true that $Past(h, t_1) \rightarrow Ch_{t_1}(X) = x$ and $Past(h', t_1) \rightarrow Ch_{t_1}(X) = x'$ both not true that $x = x'$ and hence not true that $Past(h, t_0) \rightarrow Ch_{t_0}(X) = x$. Of course, this is an implication of many theories for the special case where the truth of X is settled by t_1, but seems equally reasonable in the case where it is not.

Let me finish with a couple of qualifications of the view of chances as warranted credences. Firstly, it is no part of this view that all events have determinate chances associated with them. There may be no well-defined chances for the laws of nature holding or for a free agent acting one way rather than another or of the universe coming into existence. Whether there are will depend on whether there is evidence for them that an ideal inductive reasoner could use to form beliefs about them.

Secondly, I have been speaking as if the probabilistic judgements of an ideal reasoner will be completely determined by the evidence. But this is not at all essential to the view. It may well be the case that even the sum total of all the physical facts that are in principle accessible, together with all valid rules of inference, will not suffice to determine a unique probability for all events. In this case, the fully informed ideal reasoner should make imprecise probabilistic judgements concerning these events. It follows, on this view, that such events will not have precise chances. (Whether, and to what extent, this issue arises will of course depend on how much information Chance is endowed with and what rules of inference should be followed.)

Physical Interpretations

I have argued that the credences of a fully informed perfect inductive reasoner serve as a viable objective interpretation of chance in the sense that it meets the requirements of scope, formal admissibility and objectivity. The physical interpretations of chance also meet the objectivity requirement but, as I shall now argue, none meet both the scope and admissibility requirements.

Let's start with the frequency interpretation which falls at first hurdle (of scope) in virtue of failing to explain the role that chance claims play in much discourse. For we often make claims about single case chances: the chance that I will catch a cold this winter, the chance that a satellite launch will be successful conditional on the weather being fine and so on. What can a frequentist make of these claims when they concern events for which there are no frequencies? They might follow von Mises in saying that, since there are no single case chances, such talk is either meaningless or just an enunciation of belief. But a more common response amongst working frequentists is that such claims are not really about single case chances at all, but claims that the event in question belongs to a class of such events displaying certain frequency characteristics. When I say that there is a 40% chance that I will catch a cold this winter, what I am saying is that of the people like me, 40% will catch a cold this winter, or that if I, with my actual characteristics, were to endure a large number of winters, I would catch a cold 40% of the time. In what respects must the individuals or winters belonging to the class in question be similar and what is it about them that makes this the right class to employ? This is the reference class problem that represents what is perhaps the most fundamental challenge for frequentism. For the moment, however, I will set it aside and assume that some solution can be offered to it. As we shall see, the problems for frequentism do not end there.

The propensity interpretation is explicitly designed to deal with single-case chances and so does not suffer from these limitations. The main problem for it is that propensities are not probabilities at all in the strict sense: this is what Humphreys' paradox teaches us (Humphreys 1985). A propensity is a disposition of a set-up to produce certain kinds of outcomes: of spins of roulettes wheels to cause balls to land on even numbers, of weather systems to produce snow, of levels of sugar consumption to result in diabetes, etc. Such talk is causal in nature and causation by its nature is typically one-directional, while probabilities are always two-directional. The chance of a window shattering if a stone is thrown at it might sensibly be viewed as a physical propensity of a set-up involving flying stones and windows, but the chance of the stone being thrown at some time, given that the window shattered a few moments later, cannot. But the two conditional probabilities are equally well-defined.

A simple example will serve to illustrate related problems for the two main physical interpretations. A fair coin is to be tossed four times and then destroyed. What is meant by fair depends, of course, on the interpretation that is given to chances. According to the propensity theorist, the coin will have a certain disposition to land heads whose magnitude will depend on the set-up: the properties of the coin, the manner in which it is tossed and various environmental factors. A coin is fair therefore when the set-up is such as to ensure that the coin is equally disposed to land heads as to land tails. Frequency theorists, on the other hand, will say that the coin is fair because on half the tosses in the relevant reference class of tosses of this coin it lands heads and on half it lands tails, though they may disagree amongst themselves as to the relevant class.

Let's consult our intuitions on some basic cases. What is the chance of the coin landing heads on the first toss? One-half is the only reasonable answer in view of the stipulated fairness of the coin. And the chance of it landing heads on the last (fourth) toss, given that it has landed heads on the first three tosses? Again, the answer is one-half, absent any grounds for thinking that the tossing of the coin has undermined the conditions for its fairness. Finally, what is the chance of it landing heads on the last toss given that it has landed heads on the first three tosses and that it will land heads in only three out of the four tosses? The answer must be zero. For we cannot accommodate this information about the proportion of heads landings without drawing this conclusion.

Let H_i be the event of the *i*'th coin toss landing heads and *ch* be a chance function on the Boolean algebra based on the events $\{H_1, H_2, H_3, H_4\}$. Let *E* be the event of exactly three out of the four tosses landing heads (note that it is an element on the algebra just defined). Now the description of the set-up plus our answers to the three questions constrain *ch* as follows:

1. $ch(H_1) = 0.5$
2. $ch(H_1H_2H_3) = (0.5)^3 = 0.125$
3. $ch(H_4|H_1H_2H_3) = 0.5$
4. $ch(H_4|H_1H_2H_3, E) = 0$

The propensity view cannot give an interpretation of the (fragment of) a chance function just defined on the algebra of coin landing events. For the fact that the coin has landed heads on the first three tosses and that it will land heads in only three out of the four tosses does not give us grounds for thinking that the coin is not fair. One should expect that frequencies in small classes of events will (frequently) diverge from propensities and so such divergences are slim evidence for a change in the dispositional facts. So, on the propensity view, the chance of a fair coin landing heads on the fourth toss is still one half, no matter what the sequence of coin landings being conditioned on.

The root problem here, it seems to me, is that the conditional chance of the last toss landing heads, conditional on three out of the four tosses landing heads, is not plausibly a propensity at all. It is a judgement that is made in the light of knowledge of the relative frequencies, knowledge that in this case overrides anything that we know about the physical propensities of the coin. Many propensity theorists seem to accept this either explicitly or implicitly. Fetzer (1982) for instance argues that in virtue of the causal directedness of propensities, they cannot be formalized as conditional probabilities, while Mellor (1971) insists that propensities are not probabilities. Both Miller (1994) and Gillies (2000), on the other hand, interpret the conditional probability of some event *E* given a condition *C* as the (respectively short and long run) propensity for the set-up to produce *E*, given that the set up will produce *C*. The connection between *C* and *E* is therefore epistemic, not causal. There is a propensity for our coin tossing set up to produce a heads landing on the fourth toss and a propensity for it to produce exactly three heads landings in four tosses, but no causal propensity for latter to bring about the former. Consequently, they offer no account *in terms of propensities* for why conditional chances should equal ratios of unconditional ones.

Frequentism also cannot give an interpretation of these chances, though for somewhat different reasons. The finite frequentist must deny 2., because it follows from their construal of what it means for a coin to be fair that $ch(H_1H_2H_3) = 0$. Similarly, they can make little sense of the conditional probability of the fair coin landing heads, given that its frequency of landing heads is greater than one-half. So for the finite frequentist constraint 3. is meaningless. Not necessarily so for frequentists who look to hypothetical limiting relative frequencies within infinite sequences of events for their probabilities and those within relevant sub-sequences for the conditional probabilities. In particular, to find the conditional probability of H_4 given $H_1H_2H_3$ they will look at the infinite subsequence of four coin landings of which the first three are heads-landings and determine the limiting relative frequency of heads of the fourth landing within this collective. So constraint 3. is meaningful for them.

The interpretation of constraint 4. poses more a serious difficulty however. Some frequentists (e.g., von Mises 1957) would say that *E* does not pick out an admissible attribute of the collective because it uses the outcome (heads or tails) at a particular place in the sequence to determine whether that coin tossing event can be a member of the corresponding subsequence. In which case 4. is meaningless. We could however take *E* to be a collective consisting of sequences of four coin landings of which only one is a tails-landing. But in this collective the chance of heads is not one-half, so this is not a collective in which the coin is fair. So we don't have a single collective in which both 1. and 4. are satisfied.

There is a simple reason why frequentism gets into a tangle even on its home ground, where the events under consideration are repeatable and the relevant reference class apparently easy to identify. It is that information about particular events can trump what we infer about them from the characteristics of the reference class to which they belong. Because the frequentist attaches a chance to a coin landing heads on a particular toss only as part of some reference class of landings, any such information can only be used by changing the reference class for the chance attribution - at the cost of consistency. Such confusion is avoided by saying that the chances of particular events are *inferred* from the frequencies rather than that they *are* frequencies. That coins in the relevant reference class with a frequency of one-half grounds the judgement that its chance of landing heads is one-half. But then chances are the judgements mandated by the frequency facts, they are not themselves frequencies.

Let me finish by saying something about the best-systems interpretation of chance, according to which chances are simply the probabilities determined by the best system of probabilistic laws, where 'best' means that it optimally balances considerations of simplicity, strength and fit (Lewis 1994, see also Loewer 2004). Setting aside questions about how these standards are to be interpreted and balanced against each other, the obvious problem with this account is that it seems to be circular. For what interpretation is to be given to the probabilities appearing in the probabilistic laws? Schwartz (2014) suggests that no answer is required: we can evaluate systems of laws for fit, simplicity and strength without interpreting the probability claims they make, i.e., without knowing what they say! But the notion of best fit with actual outcomes suggests that the probabilities are to be interpreted as either frequencies or as the dispositional properties that explain them, making the best-systems account either a version of frequentism or of the propensity theory and hence subject to the objections already voiced.

There is another possibility however. One can take the best-systems theory to be an epistemological, rather than a metaphysical, theory of chance, i.e., not as an account of what the chances are, but of what values they take. As such it is naturally viewed as a rival version of the warranted credence view; one on which the best system of laws provides the inductive principles by which the fully informed ideal reasoner forms degrees of belief about the future from her knowledge of the past and present and so furnishes the warrant for her credences. This reading of the best-systems theory allows it to bypass the problems I have raised in this section. Our task now is to assess whether, so construed, it or any other version of the warranted credence view adequately explains the role chance plays in fixing our beliefs and guiding our actions.

The Authority of Chance for Credence

It is widely held that chances are authoritative for belief, in the sense that a rational agent should set her credence in any proposition that she takes to have a chance of truth to what she believes this chance of truth to be. If I know that the chance of rain tomorrow is 0.7, this is the degree to which I should believe it; if I know that the chance of rain given a red sky at night is just 0.2, then my credence in it raining tomorrow, conditional on a red sky at night, should be 0.2. In other words, we should treat Chance as what Haim Gaifman (1988) called a 'probability expert'; someone to whom we should defer in probabilistic judgements.

This informal idea – that we should treat Chance as an expert – has been given a wide variety of different expressions in the literature (see, for instance, van Fraassen 1989, Joyce 2007 and Hall 1994). Here I will defend one rather minimal version of it, before comparing it to the well-known formulation of Lewis (1980). Informally it says that rationality requires us to set our conditional credence in any proposition (to which chances attach), given a hypothesis as to the true chances, to whatever that hypothesis says that chance of the proposition is. More formally, let $\Delta^{ch} = \{ch\}$ be the set of possible chance functions on H^{ch} and probability *P* be a reasonable credence function on the set of propositions H (I will say more later about what 'reasonable' means). Let Ch_t be a real-valued random variable on H^{ch} that specifies the chances at time t. Then $P(Ch_t = ch) = \sum P(h: Ch_{h,t} = ch)$ measures the degree to which the agent believes that the t-chances of truth of the H^{ch} propositions are as given by probability function ch. Then deference to the epistemic authority of Chance requires that:

Credence from Chance: For any proposition $X \in \mathrm{H}^{ch}$ and $ch \in \Delta^{ch}$ such that $P(Ch_t = ch) \neq 0$:

(**CFC**) $P(X|Ch_t = ch) = ch(X)$

Credence from Chance is rich in consequences. Firstly, it implies that the conditional chances are authoritative for our conditional credences in the same way that the chances are for our unconditional ones. That is, a rational agent should set her conditional credences to what the thinks the conditional chances are. More formally:

Conditional Credence from Conditional Chance: For any propositions $X, Y \in \mathrm{H}^{ch}$ and $ch \in \Delta^{ch}$ such that $P(Y|Ch_t = ch) \neq 0$:

(**CCFCC**) $P(X|Ch_t = ch, Y) = ch(X|Y)$

[Secondly, suppose that the agent's credences P at time t are obtained by conditionalising a prior probability function P_0 on the total evidence *E* she holds at *t*. Then her current credences should be set to what she thinks the conditional chances are given her total evidence. More formally:

Credence from Conditional Chance: if $P(Ch = ch) \neq 0$ and $P(E) = 1$ then:

(**CFCC**) $P(X|Ch_t = ch) = ch(X|E)$]

All these expressions of the epistemic authority have their proponents in the philosophical literature (see Pettigrew (2018) for details) and viable interpretations of chance should explain why the posited relationships between chance and credence hold (or at least why they do, when they do). But first we need to consider an influential objection to them. Here is one version (Strevens 1999, Meacham 2010). Suppose that $ch(X) < 1$ but that you know that *X* is true. Then since $P(X) = 1$, consistency requires that $P(X|Ch_t = ch)$ equal one and not $ch(X)$, contrary to Credence from Chance. Another version. Suppose you know that *E* is true but not whether X is. Then by CFC, $P(X|Ch_t = ch) = ch(X)$ and by CCFCC, $P(X|Ch_t = ch) = P(X|Ch_t = ch, E) = ch(X|E)$. Since CFC implies CCFCC, it follows that $ch(X|E) = ch(X)$, i.e., that the chance of any such proposition X is independent of whatever you know to be true. But it would be irrational to give positive credence only to chance functions satisfying this condition.

Both of these arguments implicitly assume that you are not certain of the falsity of the chance hypothesis $Ch_t = ch$. The objection they raise would be defused therefore if it were true that whenever a rational agent was certain of the truth of a proposition, they should expect its chance to equal one; that is:

> **Certainty**: If P is a rational credence function at t, and X a proposition such that $P(X) = 1$, then for any chance function ch such that $ch(X) < 1$, $P(Ch_t = ch) = 0$.

Certainty says that we should not be certain at some time *t* that X is true while, at the same time, giving positive credence to the chance of X's truth at *t* being less than 1. This must be right if it is rational to defer to Chance on the question of X, for it would not be so if Chance were not at least as well-informed about X as we are. (Note that this is not the same as saying that the truth of *X* implies that the chance of X equals one, something that would imply that non-trivial chances are incompatible with determinism.)

Those who accept this response will require that any 'reasonable' credence function satisfies Certainty. Lewis and others have responded to the problem for CFC in a different way, however, noting that it seems to arise when the agent holds information of the kind that Lewis termed 'inadmissible', where admissible information is of the sort:

> *"… whose impact on credence about outcomes comes entirely by way of credence about the chances of those outcomes." – (Lewis 1980, p. 92)*

Evidence that people will be carrying umbrellas tomorrow is inadmissible with respect to the proposition that it will rain tomorrow, for instance, because this fact gives us grounds for believing that it will rain that are independent of those afforded by the chances (unless the chances too incorporate this information). In contrast, evidence regarding past weather patterns is admissible since it will be factored into the chances of future rainfall. In general, Lewis suggested, historical information prior to *t* and information about how possible histories and laws bear on the chances is admissible, but information about future chances and information relevant to the truth of *X* that is not encoded in the chances is not.

The concept of admissibility is central to the Principal Principle, Lewis' well-known formulation of idea that chance is authoritative for credence. It can be stated in our framework as follows:[3]

> **Principal Principle**: Let P_0 be a reasonable initial credence function on H, *t* be any time, *X* be any proposition in H^{ch} and *E* be any proposition in H consistent with *X* that is admissible at *t* and such that $P_0(Ch_t = ch, E) \neq 0$. Then:
>
> $$P_0(X|Ch_t = ch, E) = ch(X)$$

Lewis says little about what a reasonable initial credence function is other than that it should be consistent and regular, i.e. it should assign non-zero probability to all propositions in the domain of the chance function. But it is natural to regard as reasonable any credence function that is obtained from a reasonable initial credence function P_0 by conditioning on admissible evidence only. With this reading of the notion, the Principal Principle implies CFC because if E is admissible evidence and

[3] This formulation is slightly more general than the one initially presented in Lewis (1980) but is implied by his reformulation of the principle later in the same paper.

$P = P_0(\cdot|E)$, then it follows from the Principal Principle that $P(X|Ch = ch) = P_0(X|Ch = ch, E) = ch(X)$ in accordance with CFC.

This fact creates a mystery. For, on the face of it, CCFCC gives different prescriptions to the Principal Principle. Suppose your chance of survival from an operation is 95% but only 50% conditional on being disposed to an allergic response to one of drugs being administered. Then Conditional Credence from Conditional Chance will dictate that your conditional degrees of belief in survival given the allergic response should be 0.5 while the Principal Principle will dictate a degree of belief of 0.95, assuming that the information that you will have an allergic reaction is admissible. And it certainly doesn't seem inadmissible by the lights of Lewis' criterion: the impact on credence concerning survival of the possibility of a disposition to allergic reaction does go via credence about the chances of survival. And information about the acquisition of the allergic disposition is historical.

In any case, CCFCC clearly gives the right answer here, an answer that is consistent with Principal Principle only if the admissibility of some evidence with respect to a proposition implies the probabilistic independence of the two. To see this, let $ch \in \Delta^{ch}$ be such that $P_0(Ch_t = ch) \neq 0$ and note that it then follows from the Principal Principle that for any $X \in \mathrm{H}^{ch}$, $P_0(X|Ch_t = ch) = ch(X)$ and hence that for any $X, E \in \mathrm{H}^{ch}$:

$$P_0(X|Ch_t = ch, E) = \frac{P_0(X, E|Ch_t = ch)}{P_0(E|Ch_t = ch)} = \frac{ch(X, E)}{ch(E)} = ch(X|E)$$

Now suppose that E is admissible. Then it also follows from the Principal Principle that if $P_0(E|Ch_t = ch) \neq 0$, then $P_0(X|Ch_t = ch, E) = ch(X)$. Hence if E is admissible then either $P_0(E|Ch_t = ch) = 0$ or $ch(X|E) = ch(X)$; hence either $ch(E) = 0$ or X and E are independent under the hypothesis that $Ch_t = ch$. But then the Principal Principle is simply a special case of Credence from Chance and the notion of admissibility is redundant (see Nissan-Rozen 2013 for a similar argument).

It would seem then that Credence from Chance is the better expression of the requirement of deference to Chance. But is such deference required in all circumstances? Clearly not if it is possible to acquire information about the future that is not encoded in the chances. But how could one have evidence about whether it will rain tomorrow, for instance, that doesn't also shed light on what the chances of rain are? The philosophical literature invokes crystal balls, oracles, time travel and such like to illustrate such a possibility.[4] In these cases, purportedly, Certainty is violated because information is gained about the occurrence of future events that currently have a non-zero chance of not occurring. Such information is therefore inadmissible.

Contrary to this, I claim that such pronouncements of oracles, crystals and time travellers are either admissible information or uninformative. Consider a case in which you learn by such means that it will rain tomorrow and suppose that there is a non-zero objective chance of it not raining tomorrow, i.e., the future is, in some relevant sense, open. Let's first give 'open' a metaphysically robust reading

[4] Healthy scepticism about the utility of drawing conclusions about the concept of chance as it is used in our world from thought experiments involving other-worldly mechanisms seems appropriate here, but I set this aside here.

by which there are (at least) two metaphysically possible futures for our single present: one in which it rains and one in which it does not. In this case, what does it mean to have epistemic access to the (open) future? Consider the time traveller. If it is open today as to whether it will rain tomorrow or not, then it is open today whether the time traveller will land up in the rainy future or the dry future. Suppose that they land up in the rainy one and return with news about what happened during their travels. Could we conclude from their testimony of rain that it is certain that it will rain tomorrow? We cannot. For the time traveller could have landed up in the dry future and returned to testify to the absence of rain. And we do not know now which of these two futures is ours (we can't know, because the future is open). So we cannot know that it will rain tomorrow on the basis of the testimony of the time traveller. The same holds for crystal balls, oracles, etc.,

Now consider an epistemic reading of the openness of the future by which there is only one future, although we don't know which it is. In this case the time traveller visits the only (metaphysically possible) future there is and brings back news of it. But in this case the information they bring *is* relevant to the current chances. In particular, any chance hypothesis assigning non-zero chance to rain tomorrow must, in the light of the knowledge of the future afforded by the time traveller's testimony, be given zero credence. On this reading, therefore, the testimony of the time traveller is admissible. And irrespective of whether the openness of the future is understood metaphysically or (merely) epistemically, no violation of Certainty is required.

The Source of Authority

What is it about chances that makes adherence to Credence from Chance a requirement of rationality? Van Fraassen (1989) called this the "fundamental question about objective chance", before arguing that no answer could be given to it! Strevens (1999) makes a similar claim with respect to the Principal Principle. In contrast others (e.g. Hall 2004, Levi 1980) have followed Lewis (in his more Bayesian register) in arguing that the Principal Principle is an analytic truth, so that chances just are the entities that satisfy it (which of course leaves open the question of whether such entities exist). Neither position is terribly attractive, since they both give up on the ambition of answering the 'fundamental question', albeit in different ways.

It is possible to do better. On the Warranted Credence theory that I am proposing, for instance, the answer is simply that it is rational to defer to expertise. In the first place, the judgements of Chance, since they are based on as much information as it is possible to have, contain more information than one's own. And, in the second place, because Chance never makes mistakes in its reasoning with this information, any verdict that it reaches on the basis on this information will be more reliable than one's own. So one should defer to the judgements of Chance because they are better than one's own; indeed they are the best available.

What about the other main interpretations of chance? This question, or the corresponding one for the Principal Principle, has received some attention in the literature. Mellor (1971), and Howson and Urbach (1990) offer derivations from different versions of hypothetical frequentism (arguments anticipated by Venn (1866)), for instance, while Loewer (2004), Schwarz (2014) and Hoefer (2007) offer ones based on the best-systems theory of Lewis. But let's start by looking at how a finite frequentist might address the question since its limitations are illuminating.

Suppose as before that a coin will be tossed four times and will land heads on exactly one of them. Suppose the tosses are indistinguishable. Then for the finite frequentist the chance of the coin landing heads is just one quarter. Now what odds should one accept for a bet on the coin landing heads on any of the tosses? Any odds better than 3:1 are favourable, it would seem, because if you bet on all four tosses at odds of exactly 3:1 you would win once (gaining 3 units) and lose three times (losing 3 units in total), with no net gain or loss. But your credences should match your betting odds (because this is what defines them or because to accept odds that did not match your credences would be irrational), so your degree of belief in the coin landing heads conditional on the frequency of heads being some number should equal that number.

A lot of work is being done in this argument by the supposition that the coin tosses are indistinguishable. Were they not, a reference class problem would arise whenever the relative frequency of heads from tosses with some characteristic not shared by all of them was different from the frequency of heads from tosses without it. In such cases the Credence from Chance principle, on a finite frequentist interpretation of chance, yields inconsistent prescriptions, and so would have to be rejected. But in very few cases where we want to ascribe chances to events do they belong to a class of events that are indistinguishable in all respects that matter to what odds we would accept on bets on their occurrence. So, the finite frequentist's argument for the authority of chance is of very limited scope (see Loewer 2004).

Howson and Urbach (1993) argue that this problem can be circumvented within the framework of von Mises' version of frequentism. The first step of their argument is to show, in similar fashion to above, that if your betting odds on an attribute of some collective (e.g. that of tossed coins landing heads) diverges from its limiting relative frequency, then you would be willing either to buy or to sell a bet on a particular occurrence of the attribute at odds which you regard as disadvantageous. To avoid this you must set your credence in this single-case occurrence to the limiting frequency of the corresponding attribute in the relevant collective. This would not always be true of course if the limiting relative frequency of the attribute was different in some identifiable subsequence of the collective, but such a possibility is ruled out by von Mises' axiom of Randomness, which requires that the limit of the relative frequencies of any attribute in any infinite *sub*sequence of a collective is the same as in the collective itself. Thus, they claim, von Mises' version of frequentism has the resources to explain why relative frequencies are authoritative for credences in single-case events, thereby avoiding the problem of scope bedevilling the finite frequentist.

Unfortunately, their argument is not convincing. Suppose that tosses of a coin belong to a collective in which the limiting relative frequency of heads is one half, but that in actual fact the coin will be tossed just four times and land heads once. What should your credence be for a coin toss landing heads? By the finite frequentist argument above, if you know all of this then it should be one quarter. Nonetheless, argue Howson and Urbach, if your knowledge is restricted to the characteristics of the collective to which it belongs, then you should set to one-half your degree of belief in the next toss landing heads. These two claims are consistent only if your expectation of the finite frequency of heads landings equals the limiting relative frequency of a heads landings in the associated collective. But this isn't ensured by the axiom of Randomness which only constrains expectations regarding the frequencies of *infinite* sub sequences of the collective.

Consider a coin whose limiting relative frequency of landing heads in the relevant collective is *p*. Suppose, say Howson and Urbach, that your degrees of belief in the coin landing heads, conditional (only) on it belonging to this collective, is some $p^* \neq p$. Suppose that in n tosses, the coin will in fact land heads m times. Then you are committed to expecting that for stakes S, your net gain in betting on heads equal $nS(\frac{m}{n} - p^*)$, i.e. that the bet is fair if and only if $\frac{m}{n} = p^*$. But since the limit of $\frac{m}{n}$ is *p*, you "can infer that the odds you have stated would lead to a loss (or gain) after finite time, *and one which would continue thereafter*" (Howson and Urbach 1993, p. 228, my emphasis). But you cannot make any such inference, since nothing about the notion of a collective prohibits that in a finite set of tosses the frequency of heads differs arbitrarily from its limiting relative frequency in the collective. After 10 tosses, you could be making a net gain and after 20 a net loss, no matter how big the difference between *p* and p^* (provided they lie strictly between zero and one). So conditioning on information about the limiting relative frequency of heads does not compel you to adopt any specific degree of belief regarding any particular coin landing heads.

While Howson and Urbach eschew single case chances, best-systems theorists avoid the problem of scope by drawing on the probabilistic laws of the best system to determine them. Consequently they do not face the problem of explaining why properties of a set of events should determine credences about individual members. Indeed, several proponents (Loewer (2004), Schwarz (2014) and Hoefer (2007)) claim that something like the Principal Principle is *derivable* within the best-systems theory. But this is clearly not true without important qualifications (as they acknowledge). The reason is simply that the best-systems interpretation allows that considerations of simplicity dictate attributions of chances to events that diverge from their actual frequencies. It may well be that simplicity considerations dictate that the laws governing the tosses of the particular coin referred to in our example have it landing heads 50% of the time. But the credence I should attach to it doing so is just one quarter. Whether cases like this arise or not depends on what standards of simplicity apply and contingent facts about how coins like this one land throughout history. So it cannot be case that on the Best-System approach adherence to the Credence from Chance principle, or indeed even the Principal Principle, is *rationally* required.

Finally, it does not seem to me that the propensity theorist has a satisfactory answer to the fundamental question either. It is plausible that one should set one's unconditional credence in some event occurring to the propensity for it to do so, but one clearly should not always adopt the conditional propensity of the event given some condition as one's conditional credence. Suppose a coin has a propensity of one-half to land heads and consider the possibility of it landing heads on the next toss. Is its truth or falsity relevant to the propensity of the coin to lands heads? Surely not. Is its truth relevant to the credence one should assign to it landing heads? Surely so. While the propensity of the coin to land heads is independent of how the coin will in fact land, one's conditional credence in it landing heads given that it will in fact do so exactly three times in four tosses is independent of its propensity to do so. In cases like this, facts about the outcome trump facts about the propensity for the outcome to occur. So we should not respect Credence from Chance under a propensity interpretation of conditional chances.

Now of course the information about how the coin will land is inadmissible (in Lewis' sense) and so the propensity interpretation may well be consistent with the Principal Principle. But, as we have seen, the Principal Principle only partially captures the requirement to defer to the chances, so this fact gives little support to the propensity interpretation (though it perhaps explains why the Principal Principle has been so popular).

Action Guidance

Chances and conditional chances serve not only as guides to the degrees of belief that we should adopt but also the choices we should make. When deciding what course of action to pursue we should prefer actions that confer higher chances on the outcomes we seek over those that confer lower ones. In particular, if the available actions have the same two consequences, one better than the other, then we should choose the one that maximises the chance of the better consequences. This principle, that I dub the Choice from Chance principle, is intrinsic to all the main theories of rational decision making under risk, including not just mainstream expected utility in both its evidential and causal variants, but also the various rival theories that allow for kinds of sensitivity to risk and/or uncertainty disallowed by expected utility theory. It follows that any interpretation of chance must, if it is to do justice to the role of chance in our practical life, explain why this principle holds.

Let's start with a more formal statement of the principle. Suppose that you believe with probability one that the chances are given by probability function ch on H^{ch}, i.e. that $P((Ch_t \ = \ ch) = 1)$. Let A and A^* be any two actions and *G* and *B* be a pair of mutually exclusive propositions that exhaust the possible outcomes of choosing A and A^* and suppose that you strictly prefer *G* to *B*. (Intuitively *G* is the good outcome and *B* the bad one.) Then you should (weakly) prefer to perform A than to perform A^* iff the conditional chance of *G* given that A is at least as great as that given that A^*, i.e.:

Choice from Chance: $A \gtrsim A^*$ iff $ch(G|A) \geq ch(G|A^*)$

If Choice from Chance is central to our understanding of the role of chance in practical deliberation, then an interpretation of chance should explain why this is so. On the account of chance as warranted credence, the explanation lies in the fact that actions that conform with the principle are those that would be chosen by someone acting on our behalf or sharing our preferences and who is as well-informed about the consequences of the choice as it is possible to be. Consequently, they are the choices that would be made by someone to whom we should defer in our practical reasoning in virtue of them being better informed about our interests.

Other interpretations offer much less satisfactory explanations of the authority of chance with respect to our choices. Some of reasons for this are similar to those discussed in earlier sections and will not be repeated in this one. Instead, I will concentrate on a challenge that is peculiar to the explanation of the practical significance of chance. To illustrate it, suppose that Alma is deliberating as to whether to attempt a direct ascent of a mountain or to take a longer but safer path to the summit with the risk that she will not have enough time to make it all the way. Suppose that the good for her is contained in reaching the mountain summit without injury; the bad then being to either suffer injury or fail to reach it. Suppose also that Alma's prospects for a successful direct ascent, as well as her assessment of them, depend on her state of health. It follows that her health state is a common causal factor in both her decision and the success (or otherwise) of her chosen action and, in particular, that a decision to attempt the direct ascent may serve as evidence of good health and hence for the ascent being successful.

In decision situations in which someone's choice of action is both instrumental in achieving their goals and evidence for how successful for they will be in doing so, the agent should take care not to confound the two. Alma for instance should not treat the fact that the decision to attempt the direct ascent is evidence of her good health as a reason to attempt it. Any account of why chance is authoritative for choices of action must thus include an explanation of why the conditional chances track the causal efficacy of action and why, in calibrating our actions to these conditional chances, we do not risk confounding their causal and evidential value. The problem this poses for the subjective theory will be familiar from the debate between causal and evidential decision theory. But it also afflicts a wide variety of logical and evidential interpretations of chance. For these interpretations to explain the authority of chance for choice they must show that the relevant logical or evidential relation that holds between an action and its consequences tracks the instrumental value of the former in bringing about the latter. But this is not generally the case. Clearly Alma's attempt of the direct ascent can be instrumental in reaching the summit without their being any kind of logical relation between the two. And although there is an evidential relation between the two, the evidential relation between the two may fail to track the causal one when they have a common cause. Thus, Alma's ascent may be good evidence that she will reach the summit in virtue of it being evidence of a good state of health. But if she is in fact ill then her chances of reaching the summit are poor. Only evidential probabilities conditioned on Alma's state of health avoid this problem. But this is precisely what the warranted credence view entails for, on this account, the chances are fully informed about the prevailing state of the world, including Alma's health characteristics. So the chances of outcomes conditional on actions, understood to be the conditional credences warranted by full information about the history of the world up to the present, will never confound causation and correlation. This allows the conditional chances to measure the instrumental value of actions in bringing about these outcomes and thereby explain why practical deliberation should be guided by the Choice from Chance principle.

In this scenario Choice from Chance prescribes attempting the direct ascent just in case it confers a greater chance of reaching the summit injury free than does taking the longer route. For the frequency interpretation this is so when, in the relevant reference class, the relative frequency of injury-free attainments of the summit conditional on attempting the direct ascent is greater than those conditional on taking the indirect route. But since attempting the direct ascent is correlated with its success in virtue of sharing a common cause (Alma's heath state), there is risk that, on the frequency interpretation, Choice from Chance will prescribe the direct ascent even in cases when Alma's health state is poor. So the frequency interpretation fails to adequately account for the authority of chance with respect to choice.

The root problem here for the frequentist is that it is Alma's single-case conditional chance of succeeding in the direct ascent (given her actual state of health) that matters to her decision, not the success rate for such attempts, unless the class is restricted to attempts made under exactly the conditions Alma faces. The best-systems theorist, in contrast, is able to give an interpretation of this single-case chance. The problem is that the measure it puts on it is the wrong one. For it possible that the system of laws that optimally balances considerations of simplicity, strength and fit, yields chances that do not accurately measure Alma's single-case prospects for success e.g. if the laws belonging to this system ignore details of health states for reasons of simplicity. So the best-systems theory too is unable to explain why she should be guided by Choice from Chance.

The propensity interpretation of single-case chances looks much more promising, with the propensity of an action to bring about different possible outcomes being a plausible measure of its instrumental value. But the propensity theorist faces a problem. For it is not the propensity of her action to bring it about that she reaches the summit that Alma should be tracking, but the propensity of her action to do so, *conditional on her actual health state*. But then it might be wrong for her to respect Choice from Chance. This problem does not plague the interpretation of chances as warranted credences for, on this account of them, Alma's chances of success are already conditioned on her state of health in virtue of being fully informed about the current state of the world.

Conclusion

I have argued that the main extant interpretations of objective chance fail to meet one or more of the conditions of scope, admissibility, objectivity, belief determination and action guidance.

1. The subjective, or Bayesian, interpretation fails to confer chances with the requisite objectivity, as expressed by the requirement that chances be invariant under conditionalization on the past. Most other epistemic interpretations, both logical and evidential, fail for the same reason.
2. The propensity interpretation fails to meet the admissibility condition because it affords no natural interpretation of non-causal conditional chances and hence cannot give an account rooted in propensities of why conditional chances satisfy the axioms of conditional probability.
3. The frequency account offers no interpretation of single case chances and so fails to meet the scope condition. This manifests itself in difficulties in explaining the authority of frequencies for our credences and actions.
4. The best-systems' account fails to explain why we should defer to the chances in setting our degrees of belief and in choosing our actions, because the chances implied by the best system of probabilistic laws may not be the most accurate or track the instrumental value of actions.

In contrast the warranted credences interpretation of chance satisfies all of desiderata. Firstly, because the notion of a warranted credence has very wide scope, the interpretation it offers extends across all the domain in which chance talk is found. Secondly, in virtue of being consistent, warranted credences satisfy the axioms of (conditional) probability. Thirdly, in virtue of being as informed as it is possible to be, they are objective. This feature also explains why it is rational to defer to the chances both in what we believe and what we choose to do.

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Appendix: Determinism

For any history h, let h_t denote $h(t)$ and $h(< t)$ and $h(> t)$, respectively the past and the future of h_t, be the sequence of states in h respectively preceding or succeeding $h(t)$. Let $h(\leq t)$ and $h(\geq t)$ be the same but including $h(t)$ (called the inclusive past and inclusive future of $h(t)$). Let's say that time is *branching* if the (inclusive) past of $h(t)$ is metaphysically unique and is *rooting* if the (inclusive) future of $h(t)$ is. *Determinism* is the thesis that time is both branching and rooting. Note that these metaphysical conditions partition H into sets of histories that jointly satisfy them.

List and Pivato (2015) claim that it follows from the assumption of determinism and the Credence From Chance principle that chances are incompatible with determinism. Here is their argument. Let Ch(X) = x be the proposition that the chance if X is x, where 0 < x < 1. Then by CFC, for any admissible proposition H, $Pr(X|Ch(X) = x, H) \; = \; x$. Suppose that H is the history of the world up that moment of time. Since determinism implies that there is only one continuation of this history into the future. So conditional on H, X must have probability 0 or 1. So . But it does not follow from determinism that the agent knows what the continuation of a history. She only knows that there is only one metaphysically possible one.