**Catastrophe Models and Insurance Decision Making Under Ambiguity**

**Abstract**

Insurers draw on sophisticated models of natural hazards to provide probability distributions for possible catastrophic events and associated losses in order to price catastrophe insurance. But prevailing pricing methods don’t factor in the uncertainty around model-based projections that derive from the relative paucity of data about extreme events. This paper proposes a pricing method that is responsive the consequent ambiguity faced by the insurer. Its centrepiece is the use of a confidence grading of sets of probability distributions and a cautiousness index for the insurer to determine the set of probability distributions to serve as the input to a decision rule for ambiguity.

**Key words**

Catastrophe models, ambiguity, insurance decisions, pricing, confidence, capital reserves

**Introduction**

Catastrophe (cat) models are increasingly used by insurers and reinsurers to make crucial decisions about what policies to offer customers and how to price them. Three will be of particular concern here:

1. What price to put on different policies,
2. What capital reserves to hold,
3. How much exposure to accept to any particular hazardous event.

Although the use of cat models has greatly increased the ability of insurers to give precise answers to these questions, doubts remain as to whether the models capture all relevant uncertainty. When they don’t, insurers face ambiguity and standard techniques for producing answers to them cannot be applied. In this note I will propose a way of embedding models within a framework for decision-making under ambiguity that overcomes this problem.

Standard theory treats insurers as attempting to maximise profit subject to a survival constraint (Stone 1973). To spell it out more formally what this entails, consider a state space consisting of all possible states of the world relevant to the performance of the insurer’s book.[[1]](#footnote-2) The book can then be viewed simply as a mapping from each state to a monetary gain or loss, determined by the difference in that state between the premiums collected and the claims paid out plus other costs. To calculate an expected return on the book, the insurer draws on a probability measure *P* defined on a Boolean algebra of payoff-relevant events. For any book *b*, let us denote by *x* the event of *b* paying out *x* currency units to settle claims and let be expected pay-out of the book. Now we can define an associated probability measure on the Borel σ-algebra of pay-out events by:

Let the probability of the book *b* paying out more than *x* be denoted by , a measure known as the exceedance probability for the book *b* for *x*. Then standard theory says that the insurer will, given book *b*, set its capital holding to:

where is a benchmark level that depends on the caution or conservatism of the insurer or regulator. Note that this threshold is a probability of survival and independent of the absolute losses and benefits at stake, something we return to later.

Now suppose that the insurer is considering whether to sell another contract *c*, a transaction that will leave her with a book *b+c*, where this book is defined by, for all states s, *b+c*(s) = *b*(s) + *c*(s). The sale will require an increase in capital holding from to , so if the new contract is competitively priced then the expected profit from it cannot be less than the opportunity cost of the additional capital – denoted by - ) – required in order to mitigate the risk of ruin. Now the expected profit from the sale of the new contract is just the difference between its price and the expected losses associated with it: . So, it follows that:

 (1)

It is common in catastrophe reinsurance to set this price according to Kreps’s formula (Kreps 1990):

 (2)

Here is the standard deviation of the new contract *c* and is a risk parameter on the cumulative density for losses corresponding to the benchmark level determined by the insurer’s attitude to ruin and which depends on the difference in the standard deviations of the books *b+c* and *b*. This means that will depend on the degree of correlation in the losses associated with the new contract *c* and those of current book held by the insurer.

This entire theoretical edifice depends on the availability of a probability measure on the set of states that the insurer can use to compute the expected payoffs of possible contracts and the exceedance probabilities from which capital requirements and premiums can be derived. For this they rely on the projections coming from models of natural hazards and vulnerabilities that are typically constructed by others. But a combination of sparse historical data and the complexity of the processes determining hazard and exposure characteristics means that the precise probabilistic outputs of these models do not capture all uncertainty potentially relevant to the insurer. The problem has two characteristic manifestations: in the persistence of multiple rival models of the natural hazard (model disagreement) and residual uncertainty amongst scientists and those drawing on model projections about the reliability of the models themselves (model uncertainty). Both arise because the available data is not sufficient in quantity and quality either to uniquely identify the set of relevant causal factors responsible for the properties of the natural hazard or to fix the precise functional relationships between those that have been identified.

The modelling of the impact of hurricanes provides a useful example. It is striking, firstly, how many models of hurricane formation and of associated landfall rates are to be found in the scientific literature. Guin (2010) reports that the Florida Commission on Hurricane Loss Projection Methodology 2007 assessment of the modelling industry used an ensemble of 972 models, while Risk Management Solutions, a leading modelling firm, uses an ensemble of 13 models to generate the “Medium-Term Rate,” their preferred prediction of hurricane landfall frequency (Sabbatelli and Waters, 2015). These models differ both in their methodology - some use statistical extrapolations from historical landfalling rates, while others are physical models of hurricane formation; some identify periods of greater and lesser hurricane activity based on the hypothesized Atlantic Multidecadal Oscillation, others don’t (Shome et al, 2018) - and in the causal factors they incorporate, e.g. whether the influence of Indian and Pacific ocean sea-surface temperatures are incorporated in models of hurricane formation in the Atlantic. (See also Bender et al., 2010; Knutson et al., 2008; Ranger and Niehoerster, 2012.)

Secondly there is considerable model uncertainty for a number of reasons. The historical dataset used to score these models is small, as large hurricanes are infrequent. HURDAT2, the standard database for hurricanes hitting the Atlantic coast of the USA, is moderate in size, with ~300 storms to date and only 1/3 of those counting as “major hurricanes”. If we split the dataset by region the numbers drop well below what is typically regarded as sufficient to form a reliable predictive statistical model and modellers frequently resort to creating “statistical storms” to expand and “fill in” the dataset. Model confirmation is further complicated by the fact that scientists expect climate change to affect hurricane generation, which implies that in the future key climate variables which drive hurricane formation will be outside of their historical ranges. Finally, there is general recognition that existing models omit potentially relevant facts such as the effects of aerosols and pollution. Hazard metrics exclude many characteristics known to be relevant such as duration of inundation, flow velocity and pollution levels.

Similar problems arise in assessing the vulnerability of communities to a hurricane hit and of the financial losses associated with it. Claims experience is insufficient for risk estimation in cases of catastrophic loss because of the paucity of claims data and because of trends that make the past an inadequate guide to the future. These include changes in exposure characteristics of populations due to factors like urbanisation, in vulnerability characteristics such as infrastructure (e.g. flood defences) and regulation (e.g. building standards), and in the processes determining the frequency and severity of the natural hazards themselves because of climate change.

In a nutshell, catastrophe insurers must make decisions not just under risk but under *ambiguity*, i.e. in circumstances in which they should not have full confidence in any single probability measure of the uncertainty they face. This fact seems to be at least partially recognised by insurers. There is growing empirical evidence for instance that insurers and (particularly) reinsurers charge an ‘ambiguity premium’ when selling coverage against catastrophic events (Hogarth 1985, 1989; Kunreuther et al, 1995; Cantabous, 2007), and some evidence that insurers are reluctant to supply coverage in these conditions (Kunreuther et al, 1993), both expressions of less than full confidence in model-based expected loss projections and an aversion to the uncertainty regarding their reliability. On the other hand, there is little evidence of *explicit* modelling of ambiguity, nor of procedures for managing it within insurance companies (beyond the kind of averaging techniques described later). This in turn may partially reflect the sparsity of theoretical work on insurance decision making under ambiguity and of evaluations of the suitability of the various proposals for ambiguity-sensitive decision rules to insurance applications.

**Decision Making under Ambiguity**

Although the decision theoretic literature on ambiguity is quite diverse, there is wide recognition of the need in such circumstances to look at more than just a single probabilistic estimate and instead to consider sets of probabilities and the corresponding range of expected benefits and losses that they induce. This implies that decisions about pricing and capital holdings should be based on the characteristics of this range. Proposals as to how to do so can be broken into two broad classes, identifiable by the decision consideration they attempt to track.[[2]](#footnote-3)

1. *Caution*: One thought is that in situations of ambiguity an agent is justified in choosing cautiously by giving greater weight to the downside risks of alternative actions than the upside opportunities. The most popular version of this prescribes choice of the action that maximises the minimum expected benefit (Levi 1974, Gilboa & Schmeidler 1989), but more sophisticated ones recommend choice based on a ‘pessimism’-weighted average of the minimum and maximum expected benefit associated with each action (Binmore 2017), of the best and minimum estimates of expected benefit (Ellsberg 1961), or even of all of expected benefit estimates (Klibanoff et al 2005).
2. *Robustness*: Another thought is that agents should look for actions or policies that achieve pregiven goals robustly in the sense that they can be expected to reach these goals under all assumptions. More precisely, an action is robust if the expected benefit of performing it is over a required threshold when calculated relative to every probability function in the set of those qualifying for consideration (Gärdenfors and Sahlin 1982; Nehring 2009; Ben Haim 2010).

There is a lot to be said about the relationship between these different proposals and about their relative merits but, for present purposes, it will suffice to consider one illustrative rule for the determination of capital reserves and pricing of premiums under ambiguity. Let be the set of exceedance probabilities for a book *b* associated with *n* candidate cat models. For any and threshold k, let be defined as the minimum amount *x* such that . Then a maximally cautious approach to capital reserves would be to require that they be set at the minimum holding such that the probability of a loss greater than this amount is lower than the chosen threshold on every probability function in the set; i.e. that for book *b*:

 (3)

In practice this rule may dictate capital holdings that are so large as to make it impossible to stay in business. Other, less cautious, rules are less likely to have this implication: see, for instance, the proposal of Walker and Dietz (2017) that capital holdings be set to the minimum amount such that a weighted average of the maximum and minimum probability that losses exceed this amount is below the threshold.

The main challenge for all of the proposed rules for making decisions under ambiguity is the same, namely to explain what determines the set of probability functions that are to serve as inputs to the decision making. This is a question that gets surprising little attention in the theoretical literature, perhaps because it is assumed that it will be addressed in concrete applications. In this context, it is natural to focus on the class of hazard and loss models that deserve consideration and the range of estimates that they produce. Such a class might be generated in a number of different ways. Where there is a model available that is known or commonly believed to best represent the underlying physical processes generating the catastrophic events, then a salient class is the one produced by varying the assumptions about parameter values and initial conditions. But when there is not, then the set should include all candidate causal and statistical models as well as the variations obtained by perturbing parameter values and initial conditions.

The obvious problem with this approach is that the range of estimates generated by a process like this is likely to be large, especially in the second case. Many of the rules for decision making under ambiguity will then recommend setting premiums and capital reserves at levels that are not commercially viable and which encode ambiguity loads well in excess of those reported in the empirical studies mentioned before. Moreover, there are variety of reasons why both cat model vendors and insurers purchasing them prefer relatively precise probabilities, not least of which are the requirements imposed by regulators.

The prevailing working solution to this problem amongst vendors of cat models, and some users of them, is to achieve the required precision by averaging the outputs of the different models under consideration, weighting the models in terms of skill (typically using hindcasting to determine skill weights). There are however a number of limitations to this method (see Roussos et al 2020). In the first place, it is only sensible to average model outputs under very specific conditions, such as when the structural assumptions underlying them are sufficiently similar. This condition is not met in much catastrophe modelling (Philp et al. 2019). Secondly, the historical dataset used to score these models is typically small because the events that matter most (the ones that cause the most damage) are rare. Consequently, hindcasting against this dataset does not significantly distinguish models. Thirdly, the range of scoring rules on offer is so diverse that almost any reasonable answer could be selected by *one* of them (Stainforth et al. 2007). So, the question remains of which one to select. Finally, in practice it doesn’t entirely solve the problem for the insurer since the projections based on such averaging techniques still often differ from vendor to vendor and so the insurer is still confronted with a range of estimates.

An alternative strategy to averaging over the space of all models is to restrict the set of models to be considered to those meeting some criterion, e.g. to those of reliability greater than some threshold (as in Gärdenfors and Sahlin, 1982) or those that lie within some specified distance from the ‘best’ one, relative to some metric on the space of models (as in Hansen and Sargent, 1982). To implement this strategy, we need to be able to say what the criterion for inclusion should be: how reliable a model must be, for instance, or how close it must be to the reference one in order to be considered. Without this, there is a risk of introducing an ad hoc filter on decision inputs.

Let us step back and consider what is at stake here. Any choice of set of probability distributions amounts in effect to a compromise between robustness and specificity. Suppose a decision depends on some parameter (say rainfall) and consider the set of all probability distributions over its values (see Figure I below). Any subset of them corresponds to a set of claims about, or estimates of, these values, namely those that are supported by all distributions in that subset. Small sets determine fine-grained, precise claims such as that (E) the probability of flooding is 0.25; larger ones claims that are either more coarse-grained or less precise, such as that (F) the probability of flooding is between 0.2 and 0.3. Basing a decision on a more precise estimate serves the goal of optimisation: this is what makes information valuable to decision makers. On the other hand, basing the decision on a less precise set confers robustness on it in the sense that it will have acceptable consequences over a wider range of possible contingencies. If too little specificity is sought then either no action will be sanctioned (if drawing on the first class of rules for decision making under ambiguity) or only very cautious ones will (if drawing from the second). If too much specificity is sought, then confidence in the correctness of the decision must be sacrificed.

[Insert Figure I here]

This trade-off between specificity and robustness can be represented by a confidence ranking of sets of probability distributions of the kind illustrated in Figure II below, where the inner, darkly-filled set represents the ‘best’ probability distributions and each of the outer, lighter-filled sets contains a sufficiently expanded set of distributions to confer greater confidence on the judgements that it supports than any set of distributions contained within it. (Only three confidence levels are exhibited in this figure, but in principle the confidence ranking can be as fine-grained as the evidence allows.) Any projection supported by a set of probability distributions containing a confidence level is held with confidence equal to or greater than that level. For example, we can read off from this figure that the projection that the probability of flooding is 0.25 is held at low confidence only, but that the projection that it will be between 0.2 and 0.3 is held with medium confidence.

[Insert Figure II here]

Such a representation of uncertainty helps us see the limitations of the ones standardly adopted. To measure uncertainty by a single probabilistic projection is to focus exclusively on the inner set (indeed on an inner *point*), thereby ignoring all second-order model uncertainty. To measure it instead by a set of probabilities is to fix on one of the level-sets of the confidence ranking, thereby implicitly making a choice for the decision maker of what level of confidence they should seek in the projections they draw on. Only by looking at the full set of sets of distributions does one gets a sense of the trade-off between precision and robustness in the projections engendered by the prevailing level of scientific understanding.

A representation of the ambiguity a decision maker faces by a confidence ranking of decision relevant projections does not by itself determine what action should be taken. The decision maker also needs to settle on the level of confidence she requires in her choice; that is, how robust she requires the chosen action to be in achieving her goals in the light of the ambiguity she faces. Let us call the characteristic of the agent that determines her confidence requirement in a particular decision problem, her *cautiousness*. Intuitively cautiousness is a subjective attitude that can vary between decision makers: a bold agent will require less confidence in her choice of action in any given decision problem than a more cautious one. It is also reasonable to expect, as Hill (2013, 2016) argues, that how cautious an agent is will depend on what is at *stake* for her in the decision problem she faces: what the range of possible outcomes are of any choice of action and how much she values (or disvalues) these possible consequences, perhaps paying particular attention to the worst and best possible outcomes. Both possibilities are allowed by a formal representation of cautiousness as function of an agent and a decision problem that picks out a set of probabilistic projections, intuitively the small set of projections meeting the confidence requirement that her cautiousness dictates.

If the level required is independent of the decision problem she faces then she can simply adopt the smallest set of probabilities that meets this confidence threshold and apply one of the rules for decision making under ambiguity mentioned before (in this case the standard representation of ambiguity is sufficient for decision purposes). Plausibly however the level of confidence she will require will depend on what is at stake for her: the greater the stakes the more confidence required. So the set of probabilities functions that serves as the input to a decision rule will vary with the decision problem.

We will return to the implications for insurance decision making in due course, but first let us consider the question of what determines the confidence ranking itself. While the question of how much confidence is required for a decision is something that depends on the decision maker’s aims and values, the trade-off between specificity and robustness captured by a confidence ranking of probabilistic projections is a matter for science to determine. Scientists achieve specificity in their findings by balancing the evidence for and against different claims obtained from running models, taking measurements, conducting laboratory and field experiments and so on. They acquire confidence in these findings by obtaining more evidence and evidence of higher quality, garnered from more diverse sources.

These two considerations are quite distinct. Suppose that I want to know the probability that it will rain tomorrow. At the outset I might do no better than use an estimate of the frequency of rainy days. But, given the opportunity, I could improve this judgement by drawing on state-of-the-art meteorological models and up-to-the-minute data about prevailing conditions, consulting experts in the field, and so on. All this activity could of course leave me with exactly the same probability judgement as I started with. But something would clearly have changed as result; not the projected probability for rain, but the confidence I am entitled to have in the projection. While the probability of rain tomorrow reflects the balance of evidence for and against this possibility, confidence reflects what Keynes (1973/1921) called the weight of evidence, something which depends on how much evidence there is, its quality and consistency, and perhaps the diversity of its sources (see Joyce 2005).

Much of the scientific modelling of hazards has focused on the delivery of probabilistic projections through assessment and improvement of models. But modelling is equally important for determining the robustness of projections and thereby the confidence with which they can be held. This can have significant implications for decision making. For instance, contrast a case in which exploration of the space of reasonable models reveals that they make projections that, while different, all lie within a fairly narrow range, from one in which they make projections that are scattered all over the place. (This is the sort of contrast that would be represented by Figures II and III, for instance.) It could be that while the balance of evidence supports the same precise projection in the two cases, in the former the loss of specificity entailed by adopting an imprecise projection supported by most models is not significant from the decision maker’s point of view, while in the latter it is. So, in the former the gain in confidence obtained by consulting a wide range of model projections outweighs the loss of specificity, but in the latter it does not.

**Insurance Decisions**

Let us turn now to how confidence rankings of projections - in particular, of exceedance probabilities - can support insurance decision making. Consider first the problem of setting capital reserve requirements for a book. The decision maker must decide not only what threshold they wish to apply but also the level of confidence they require that this threshold will not be exceeded. In principle this level can vary from decision to decision as a function of the stakes. But for the moment let us treat it as a constant and suppose that the decision maker fixes values for a pair of parameters where , as before, is the threshold for an acceptable probability of ruin and is the level of confidence required. The insurer can then compute capital reserve requirements using the threshold for each of the exceedance probabilities that fall within the smallest set of such functions meeting the confidence requirement.

More formally, let be the smallest set of probability functions on Boolean algebra of payoff-relevant events sufficient to achieve confidence and be the corresponding set of probability measures on payoffs induced by book *b*. For any let and be the associated expected loss and standard deviation of book *b*. Then an insurer who seeks to set her capital reserves at a level at which she can be sufficiently confident that the risk of ruin is below the threshold, will set them according to:

 (4)

In other words, she will choose the smallest capital sum such that the probability of ruin falls below threshold with confidence .

To determine the price of any new contract *c* the insurer will need to consider a range of (changes in) expected losses and standard deviations associated with *c* that is sufficiently broad as to meet her confidence requirements. She can then apply equation (1) using the calculation of capital reserves suggested above or, more directly, by applying Kreps’s pricing formula (2), in both cases using each of the exceedance probabilities induced by *c*. More formally, let be the set of probability measures on payoffs induced by the smallest set of probabilities sufficient to achieve confidence and the new contract *c*. Then the highest of the resultant range of prices calculated using each of the members of should be selected. In particular, if the Kreps formula is used for pricing, then she should require:

 (5)

At this price the insurer can expect *with sufficient confidence* to make a profit and avoid ruin.

In practice market competition makes individual insurers price takers and the significant decision is whether to write policies at the market price and how much exposure to accept, in the light of the ‘technical’ price obtained by application of their pricing formula. Confidence considerations should play an important role here as well. Consider, for example, a very simple case in which an insurer can decide whether to write a certain quantity of business in two different markets for protection against losses deriving from events uncorrelated with her current book (e.g. hurricane insurance in Florida and earthquake insurance in Pakistan). Suppose that the best estimate of the exceedance probabilities is the same for both contracts but that the weight of evidence supporting those for the first (say the hurricane projections) is much greater than those for the second (the earthquake projections). The situation is then as illustrated by Figures II and III in which for any confidence level the set of probabilities required to achieve that level is larger for the earthquake projections (given by Figure II) than the hurricane ones (given by Figure III). Application of pricing equation (5) will then yield higher minimum prices for the insurance against earthquake damage than hurricane damage. The insurer should therefore enter the first market in preference to the second if market prices for insurance are the same in both. More generally, they should prefer the first in case the difference in price required to achieve the requisite confidence of ruin avoidance exceeds the difference in the price for insurance contracts in the two markets.

[Insert Figure III here]

The argument of the previous paragraph implicitly rests on the assumption that the insurer’s exposure to the two events (the hurricanes and earthquakes) is roughly the same. When this is not the case consideration must also be given to the opportunity to hedge risks afforded by diversifying one’s portfolio of business. To keep things simple, suppose that the insurer has already written a good deal of hurricane insurance but none for earthquakes and must now choose between writing more contracts for hurricanes or writing the same volume of business in insurance against earthquake damage. Now two considerations will need to be balanced: the fact that writing earthquake insurance affords a hedging of the risks and the fact that projections of earthquake-caused losses are more ambiguous. We can do this by applying the Kreps pricing formula to marginal increases in business in both markets and identifying the apportioning of business that equalises the differences between market and technical prices.

Let us turn finally to the possibility of reducing exposure through reinsurance. Figure IV below shows three loss exceedance curves deriving from different models of the underlying hazards and of the vulnerability of insured assets. Suppose that the insurer’s confidence requirement dictates that they consider all three curves. Application of equation (4) with a threshold of 0.2% yields a relatively high capital holding requirement of around 10 million dollars. To avoid this the insurer could seek to reinsure against the losses associated with the 5% - 0.2% probability range with a less ambiguity averse reinsurer. For instance, suppose the reinsurer is ambiguity neutral and uses only the grey loss exceedance curve so that application of the 0.2% threshold would imply capital holdings of 8 million dollars. Then while the insurer must set aside an additional seven million dollars to take the risk of ruin from below 5% to below 0.2%, the reinsurer can achieve this by setting aside only an additional five million dollars. The difference in the opportunity costs of a capital holding of seven and five million represents the potential gains from reinsurance.

[Insert Figure IV here]

**Concluding Remarks**

On the analysis given here, the price of catastrophe insurance depends on three factors:

1. The ambiguity profile of projections of the insured hazard,
2. The risk attitudes of insurer as measured by the probability of ruin threshold and the confidence requirement ,
3. The exposure characteristics of the insurer’s book; in particular its size and diversity, as captured by and .

This suggests three corresponding ways in which the price of insurance can be reduced. This first is through improvements in scientific understanding of the hazard. While new research may of course lead to higher estimates of the probability of the hazard, the increase in confidence that improvements in scientific understanding justify will serve to offset this to some degree (and magnify the effect on the price of a reduced probability estimate).

The second path is through the optimisation of exposure characteristics of the insurer’s book; for instance, through hedging against exposure to one kind of peril in one region by selling contracts for different perils or in other regions. The benefits of hedging under risk are well-understood, but the analysis here shows that they have to be balanced against increases in ambiguity that may result from selling contracts in perils or for regions for which the level of scientific uncertainty understanding is lower.

The third and final way in which prices can be reduced is by risk transfer e.g. through reinsurance or partial socialisation of the risk or government take-up of layers of the exposure. Again, there is nothing new about this, but the presence of ambiguity offers additional need and opportunity for transferring exposure from the ambiguity averse to agencies that are less so. Indeed, the very high levels of ambiguity that are characteristic of the rare but extremely dangerous catastrophic events may make it impossible to insure against them (Charpentier, 2008). If so, developing mechanisms for transfer of exposure to the public sector or international bodies is all the more pressing.

**Acknowledgements**

Removed for blind refereeing

**Figures**

Figure I: Specificity of Probability Claims

**F**: Pr(flooding) = [0.2,0.3]

**E**: Pr(flooding) = 0.25

Figure II: Confidence Grading of Claims

**F**

**E**

Low Confidence

Medium Confidence

High Confidence

Figure III: Confidence Grading of Claims (High Weight of Evidence)

**F’**

Low Confidence

**E’**

Medium Confidence

High Confidence

Figure IV: Family of Loss Exceedance Curves

**References**

Alary, D., C. Gollier, and N. Treich (2013): “The effect of ambiguity aversion on insurance and self-protection,” Economic Journal, 123, 1188– 1202.

Bender, M.A., Knutson, T.R., Tuleya, R.E., Sirutis, J.J., Vecchi, G.A., Garner, S.T., Held, I.M., (2010) “Modeled impact of anthropogenic warming on the frequency of intense Atlantic hurricanes”. *Science* 327, 454–458.

Ben-Haim, Y. (2010) ***Info-Gap Economics: An Operational Introduction***, Palgrave-Macmillan UK

Bradley, R. (2017). *Decision theory with a human face*. Cambridge: Cambridge University Press.

Bradley, R., C. Helgeson, and B. Hill (2017). Climate change assessments: Confidence, probability, and decision. *Philosophy of Science* 84 (3), 500–522.

Cabantous, L. (2007): “Ambiguity aversion in the field of insurance: insurers’ attitude to imprecise and conflicting probability estimates,” Theory and Decision, 62, 219–240.

Charpentier, L. (2008): “The Insurability of Climate Risks,” The Geneva Papers 33, 91–109

Dietz, S. and O. Walker (2017) “Ambiguity and insurance: capital requirements and premiums”, Journal of Risk and Insurance 86(1), 213-235

Ellsberg, D. (1961). Risk, Ambiguity, and the Savage Axioms. *Quarterly Journal of Economics*, 75(4), 643-669.

 Gärdenfors, Peter and Nils-Eric Sahlin ([1982] 1988), "Unreliable probabilities, risk taking, and decision making", pp. 313-334 in Gärdenfors, Peter and Nils-Eric Sahlin, eds. (1988), Decision, probability, and utility.

Gilboa, I. and M. Marinacci, “[Ambiguity and the Bayesian Paradigm](http://itzhakgilboa.weebly.com/uploads/8/3/6/3/8363317/gilboa_marinacci_ambiguity_and_bayesian_paradigm.pdf)”, Advances in Economics and Econometrics: Theory and Applications, Tenth World Congress of the Econometric Society.  D. Acemoglu, M. Arellano, and E. Dekel (Eds.).  New York: Cambridge University Press, 2013.

Gilboa, I. and D. Schmeidler (1989). “Maxmin Expected Utility with Non-Unique Prior.” *Journal of Mathematical Economics* 18 (2): 141–53.

Gollier, C. (2014): “Optimal insurance design of ambiguous risks,” Economic Theory, 57, 555–576.

Guin, J. (2010). “Understanding Uncertainty.” AIR Worldwide Blog. 2010. http://www.air-worldwide.com/Publications/AIR-Currents/2010/Understanding-Uncertainty/.

Hansen, L. P., and T. J. Sargent (1982). *Robustness*. Princeton: Princeton University Press.

Heal, G. and A. Milner (2014). “Uncertainty and Decision Making in Climate Change Economics.” *Review of Environmental Economics and Policy* 8: 120–37.

Hill, B. (2013). “Confidence and Decision.” *Games and Economic Behavior* 82: 675–92.

Hill, B. (2019). “Confidence in Beliefs and Rational Decision Making.” *Economics and Philosophy* 35 (2): 223–58.

Keynes, J. M. (1973/1921). A treatise on probability. In *The Collected Writings of John Maynard Keynes,* Volume 8. London: Macmillan

Klibanoff P., Marinacci M. and Mukerji S. (2005) A smooth model of decision making under ambiguity. *Econometrica* 73(6):1849–1892

Kreps, R. (1990). "Reinsurer Risk Loads From Marginal Surplus Requirements," *Proceedings of the Casualty Actuarial Society*, 76, 196-203.

Joyce, J. (2005). How probabilities reflect evidence. Philosophical Perspectives, 19(1), 153-178

Knutson,T.R., McBride,J.L., Chan,J., Emanuel,K., Holland,G.,Landsea,C., Held,I., Kossin,J.P., Srivastava,A.K., Sugi,M., (2010). “Tropical cyclones and climate change”. *Nature Geoscience* 3, 157–163

Kunreuther, H., R. Hogarth, and J. Meszaros (1993). “Insurer ambiguity and market failure,” *Journal of Risk and Uncertainty*, 7, 71–87.

Kunreuther, H., J. Meszaros, R. Hogarth, and M. Spranca (1995). “Ambiguity and underwriter decision processes,” *Journal of Economic Behavior & Organization*, 26, 337–352.

Marinacci, M. (2015). “Model Uncertainty.” *Journal of the European Economic Association* 13: 1022–1100.

Nehring, K. (2009). Coping rationally with ambiguity: Robustness versus ambiguity-aversion. *Economics and Philosophy*, 25(03), 303-334.

Philp, T., T. Sabbatelli, C. Roberston, and P. Wilson. (2019). “Issues of Importance to the (Re)Insurance Industry: A Timescale Perspective.” In *Hurricane Risk*, edited by Jennifer Collins and Kevin Walsh. Vol. 1. Hurricane Risk. Springer International.

Ranger, N. and Niehörster, F. (2012) “Deep uncertainty in long-term hurricance risk: scenario generation and implications for future climate experiments”, *Global Environmental Change* 22: 703–712

Roussos, J., Bradley, R and R. Frigg (forthcoming 2020) “Making confident decisions with model ensembles”, *Philosophy of Science*

Sabbatelli, T., and J, Waters. (2015). “We’re Still All Wondering – Where Have All The Hurricanes Gone?” *The RMS Blog* (blog). October 27, 2015. http://www.rms.com/blog/2015/10/27/were-still-all-wondering-where-have-all-the-hurricanes-gone/.

Shome, N., M. Rahnama, S. Jewson, and P. Wilson (2017). “Quantifying Model Uncertainty and Risk.” In Michel, G. (ed.) *Risk Modeling for Hazards and Disasters*, 3–46. Elsevier.

Stainforth, D., M.R. Allen, E.R. Tredger, and L. Smith. (2007). “Confidence, Uncertainty and Decision-Support Relevance in Climate Predictions.” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 365 (June): 2145–61.

Stainforth, D., T. E Downing, R. Washington, A. Lopez, and M. New. (2007). “Issues in the Interpretation of Climate Model Ensembles to Inform Decisions.” *Philosophical Transactions of the Royal Society A* 365 (1857): 2163–77.

J. M. Stone (1973), “A Theory of Capacity and the Insurance of Catastrophe Risks,” *Journal of Risk and Insurance* Vol. XL, No. 2, 1973, p. 23 1 and Vol. XL, No. 3, 1973, p. 339.

1. Here I follow Dietz and Walker (2017). [↑](#footnote-ref-2)
2. See Heal and Milner (2014), Gilboa and Marinacci (2013) and Bradley (2017) for surveys of existing proposals for decision rules for ambiguity. [↑](#footnote-ref-3)