Economic Growth in a Cooperative Economy

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Abstract

We develop and formalize an equilibrium concept for a dynamic economy in which production takes place in worker cooperatives. The concept rules out allocations of workers to cooperatives in which a worker in one cooperative could move to a different cooperative and make both herself and the existing workers in the receiving cooperative better off. It also rules out allocations in which workers in a cooperative would be made better off by some of the other workers leaving. We also provide a minimum-information equilibrium-selection criterion which operationalizes our equilibrium concept. We illustrate the application of our concept and operationalization in the context of an overlapping-generation economy with specific preferences and technology. The cooperative economy follows a dynamic path qualitatively similar to the path followed by a capitalist economy, featuring gradual convergence to a steady state with constant output. Quantitatively, however, the cooperative economy features a static inefficiency, in that, for a given aggregate capital stock, firm size is smaller than what a social planner would choose. On the other hand, the cooperative economy cannot be dynamically inefficient, and could accumulate capital at a rate that is higher or lower than the capitalist economy. We also present an illustrative calibration which quantitatively compares steady-state incomes in a cooperative and in a capitalist economy.

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1 Introduction

For the first time in many decades the capitalist organization of production is under discussion in several Western societies. In the United States, avowed socialists are among the most popular politicians in the country - and one of them has been a leading candidate to be nominated by a major party in the last two Presidential election. Meanwhile, historically unprecedented percentages of opinion-poll respondents express positive views of socialism. Perhaps more significantly for future developments, socialism is viewed more favorably than capitalism among the youngest cohorts.[1] In the United Kingdom, leaders with a Marxist background, and with a recent history of advocating worker ownership of the means of production, have recently led the major opposition party, and might have succeeded in winning power had they not chosen an unpopular stance on Brexit. Disaffection with capitalism is also affecting political dynamics in several other countries.

A similar, vigorous debate is taking place among academics and public intellectuals. New books about the failures of capitalism appear on a monthly basis, and columns on the same topic are featured daily on the major newspapers.[2] Major research programs, involving management scientists, sociologists, political scientists, and economists repudiate Friedmanite shareholder value and attempt to redefine the role and purpose of corporations.[3] Campaigns to redistribute power from shareholders to workers attract support from thousands of academics in social science disciplines.[4]

Macroeconomists have yet to make significant contributions to this important debate, and yet the institutional changes under discussion cry out for rigorous analysis of their general equilibrium and dynamic implications. What do they imply for aggregate productivity? How do they affect economic growth? This paper attempts to take a first step towards filling this gap.[5]

[1]E.g. Pew Research centre, 2019

[2]For a tiny sample of this burgeoning literature, see Block (2018), Cass (2018), Collier (2018), which all appeared within a few months of each other.

[3]E.g. the British Academy's *Future of the Corporation* programme, lead by Colin Mayer.

[4]E.g. the *Democratizing Work* campaign of the Summer of 2020.

[5]Microeconomists have been quicker to the mark, and have produced important normative insights in a partial equilibrium context (e.g. Magill, Quinzii, and Rochet (2015) and Hart and Zingales (2017)). But these contributions cannot substitute for positive assessments of the dynamic and general equilibrium consequences of alternative arrangements.

The alternative to shareholder capitalism that we study in this paper is the worker cooperative. This is a natural starting point for several reasons. First, cooperatives are among the most frequently cited possible remedies to the perceived crisis of capitalism, making an assessment of their growth implications particularly relevant for the ongoing debate.[6] Second, worker cooperatives have existed for nearly 200 years, and continue to exist virtually everywhere in the world.[7] This provides a real-world basis to build the model on, and some confidence that the alternative to capitalism being studied has a chance to survive impact with reality. Third, as we discuss shortly, there is a pre-existing (if largely forgotten) tradition of economic modelling of worker cooperatives which we can relate our work to.

We study a production economy inhabited by two-period lived overlapping generations, where only the young work, while both old and young consume. The capitalist version of this economy, characterized by individual property of capital and profit-maximizing firms, is entirely standard and its dynamic properties are well known. Consistent with real-world arrangements, we conceptualize cooperatives as *labor-managed* entities which allow *no individual ownership* of their assets. In our model this implies that cooperatives, and not any individuals, own their own capital stock, and that young workers come together to produce and collectively choose investment plans to maximize the present value of their (common) lifetime utility. As in the capitalist economy, these cooperatives supply their output on a perfectly competitive product market.

A somewhat non-standard institutional feature of the cooperatives in our model is that old workers continue to participate in the distribution of income of the cooperative to which they were attached when young. As was noted in the early economics literature on labour-managed firms, in traditional cooperatives members' horizon when voting over investment is limited to their expected remaining time with the coop. This tends to depress cooperative investment. There is a minority of cooperatives which obviate to this design defect by letting former workers receive payments from current income. An example is the celebrated Mondragon organization.

[6]For example all the authors cited in Footnote 2 devote several pages to the social benefits of worker cooperatives. Other books are entirely devoted to advocacy for cooperatives, e.g. Ranis (2016) and Mulder (2015).

[7]The worker cooperative movement has its origins in the industrial revolution. Then as now it emerged as a response to the perceived shortcomings of subordinate-labour capitalism.

These are the types of cooperatives that we model.

One of our main goals for the paper is to identify an appropriate equilibrium concept for a dynamic cooperative economy. This is challenging because it is not a priori obvious how young workers will sort themselves into the cooperatives that exist when they join the labor market, and also under what conditions they will decide to form new cooperatives rather than joining an existing one. Furthermore, any worker allocation rule has repercussions for investment, as a cooperative's current workers incentive to invest depends on the expected employment of the cooperative in the future. We solve these challenges by developing a "minimum rationality" constraint on the admissible allocations. Part of this criterion is that workers in one cooperative cannot improve their lifetime utility by attracting a *willing* worker from another cooperative. After establishing the general equilibrium notion, we also provide an equilibrium-selection criterion which minimizes informational requirements.

After developing the framework and the equilibrium concepts for a cooperative economy, we study a couple of examples. In these examples, we characterize the growth path of the cooperative economy and compare it to the growth path of the same economy when production takes place in the "standard" capitalist firms which feature in neoclassical growth theory. Our analysis is based on choices of technology and preferences for which we are able to develop qualitative, or at least quantitative results.

In our examples, the cooperative economy converges to a steady state level of income per efficiency unit of labor - just as the capitalist economy is well known to do. In general, steady state income, consumption, and welfare can be higher or lower in the cooperative economy, depending on parameter values. Still, there are some systematic differences. We uncover a form of *static inefficiency* in the cooperative economy: for a given aggregate capital stock, worker cooperatives are inefficiently small (or, equivalently, there are too many firms in the cooperative economy). On the other hand, in our one fully-solved example the capitalist economy features potential over-accumulation of capital, while the cooperative economy is always dynamically efficient.

We calibrate our model's preference and technology parameters by matching the capitalist version of the model to relevant US data moments. In our baseline calibration the steady state output of the cooperative economy is 73% of what it is in the capitalist economy. All of this output gap is due to the static inefficiency of cooperatives: the aggregate saving rate is in fact slightly higher in the cooperative economy. Needless to say these results are illustrative and more in the nature of a "proof of concept" for the modelling framework. As we discuss in the Conclusions, their robustness will have to be assessed against a number of modelling extensions.

The Golden Era of the theoretical economic analysis of worker cooperatives was the period between the late 1950s and the late 1970s, when some of the stars of the profession took an interest in the topic. Ward (1958), Domar (1966), and Sen (1966) set up static, partial equilibrium models focused on the determination of cooperative labor input (on the extensive and/or intensive margin). Vanek (1970), Dreze (1976, 1989), and Laffont and M. Moreaux (1983) provided general equilibrium analyses, and established conditions for the existence and Pareto optimality of equilibria in economies constituted by worker cooperatives.[8] However, their analyses were still static. Furubotn and Peyovich (1973) and Furubotn (1976) argued that this gave them a blind spot for the anti-investment bias arising from the limited planning horizons of traditional cooperative members, who lose property rights in the cooperative's assets when they leave the firm.[9]

Conceptually, our paper can be understood as a step towards marrying Vanek and Dreze's general equilibrium analysis with Furubotn and Peyovich's dynamic (but partial equilibrium) one - while at the same time proposing a solution to the Furubotn and Peyovich critique (in the form of giving former workers a claim on current distributions). However the modelling framework is completely different and much more in line with recognizable modern macroeconomic practice.

More recent theoretical developments have returned to the microeconomic analysis of the internal workings of the coop, by bringing in strategic considerations. Hence, concerns have been raised with cooperative members' incentives to provide effort (e.g. Holmstroem (1982)), and their

[8] Or "labor-managed economies," as earlier writers prefer to call them. We use "worker cooperative," "producer cooperative," and "labor-managed firm" interchangeably.

[9] Atkinson (1972) is another early attempt to inject dynamic considerations in the Ward (1958) model.

possible exacerbation when workers have heterogeneous abilities (e.g. Kremer (1997), Levin and Tadelis (2005)). In order to keep the focus on the macroeconomic implications, in this paper we abstract from the intensive margin of effort. We do however note that, as pointed out by Bonin, Jones, and Putterman (1993), and confirmed in many successive surveys, shirking by workers is virtually never reported as a concern in studies of real-world producer cooperatives. This is not surprising, as cooperative workers have much greater incentives to monitor each other's effort than subordinate employees on a fixed salary.[10]

In the last two or three decades the focus of the research effort on worker cooperatives (and more generally of forms of worker participation in profit and/or management) has shifted from the development of theoretical models to the mobilization of empirical evidence. Excellent recent surveys of this large literature, which collectively covers a large variety of countries and industries, can be found in Bonin and Putterman (2013), Pencavel (2013), and Jones (2018). Generally speaking, the evidence suggests that worker cooperatives tend to be (somewhat) more productive than conventional firms, to afford their workers greater job satisfaction, and to display comparable exit and investment rates. It must be acknowledged, however, that only rarely are these empirical results immune from concerns regarding selectivity.

The paper is organized as follows. Section 2 describes the physical environment, including technology, demographics, and preferences. Section 3 describes the institutional setup with which we represent the "capitalist" system, and the maximization problems and equilibrium conditions that derive from it. These are familiar to all economic students. Section 4 sets out institutions, maximization problems, and equilibrium conditions for a cooperative economy. This is the main conceptual and methodological contribution of the paper. Section 5 solves the model, both under capitalist and under cooperative institutions, for the case in which individuals derive log utility from consumption and production is Cobb-Douglas. For this example we are able to

[10]Always on the theory side, we should mention a healthy parallel literature on other types of cooperatives. For example, Rey and Tirole (2007) study cooperative investment by groups of firms, and Hart and Moore (1996, 1998) study consumer cooperatives. We should also cite an important 1980s research program on profit sharing (e.g. Weitzman (1984, 1985), Meade (1986)), which had a particular focus on its potential role in dealing with stagflation.

develop closed from solutions and make a number of general statements about the comparative growth paths of the two economies. Section 6 presents a calibration of the model with slightly more realistic preferences and derive the main quantitative results. Section 7 discusses some of the many directions in which we hope to take this project in future work, both to probe the robustness of our preliminary results and to investigate additional issues, such as inequality.

2 Physical Environment

As noted in the Introduction, a critical economic feature of producer cooperatives is the finite planning horizon of self-managing workers. These workers know that benefits accruing to the coop after they have left may escape them, potentially leading to severe under-investment (and failure to implement other choices with back-loaded returns). These considerations need to be taken into account when choosing the appropriate modelling of demographic. It is clear that a model with infinitely-lived individuals who do not move across cooperatives would fail to capture this central aspect of cooperative economics.[11] On the other hand, an infinite-lives model with worker turnover would be exceedingly complex, as we would have to track each worker's employment history, and allow workers to optimize over employment transitions stretching into the infinite future.

A much more manageable option is to opt for a simple, two-period overlapping-generations framework. In this framework agents only work when young (for a profit-maximizing firm in the capitalist economy; as members of a cooperative in the cooperative economy), but consume when both young and old. This means that young cooperative workers make decisions which will affect cooperative outcomes after they have stepped down from their membership.

Formally, we endow agents with utility function

$$U(c^Y, c^O),$$

[11]Furthermore such a model would be absolutely trivial, as it would essentially reduce to a Robinson-Crusoe economy.

where c^Y (c^O) is consumption of the economy's final good when young (old). All agents in a generation are identical, and each young agent supplies one unit of labour inelastically. For simplicity, we assume that the population is constant and denote L the mass of each generation.

There exists a technology that uses capital and labour as inputs to produce the final good according to the production function:

$$F(k, l).$$

The capital used for production fully depreciates across periods, while investment of the final good generates new capital on a one-for-one basis.

3 Capitalist Economy

Our capitalist benchmark is a standard competitive equilibrium where profit-maximizing firms can enter and exit freely; young workers supply labour, consume and save in the form of capital; old workers rent out the capital they saved and use the proceeds to finance consumption; and all agents are price takers.

The prices of labour and capital at time t are denoted w_t and r_t, and they are in terms of the final good, which acts as numéraire. Conditional on entry, individual firms maximize profits taking current prices as given:

$$\pi(r_t, w_t) = \max_{k,l}\{F(k, l) - r_t k - w_t l\},$$

with factor demands denoted: $k(r_t, w_t)$ and $l(r_t, w_t)$. We assume that these factor demands are single-valued so that all active firms behave symmetrically, and we can omit firm subscripts.

Capital is owned by individuals. We assume that each period-0 old agents is endowed with some initial capital stock κ_0.[12] In each subsequent period, old workers can sell their savings in the form of capital stock κ_t at the market price r_t. At the same time, young workers become

[12] Heterogeneous capital endowments among the initial old could trivially be allowed for, but all heterogeneity would immediately disappear with the first young generation.

old capitalists by saving some of their labor income. In particular, the young solve the following program:

$$\max_{c^Y, c^O, \kappa_{t+1}} U(c^Y, c^O)$$
$$\text{s.t. } c^Y + \kappa_{t+1} = w_t$$
$$c^O = r_{t+1}\kappa_{t+1}.$$

The solution to this problem defines the optimal capital investment as a function of the prices w_t and r_{t+1}:

$$\kappa_{t+1} = \mathfrak{K}(w_t, r_{t+1}).$$

In equilibrium, markets for capital, labour and the final good clear, and free entry and exit drive the firms' profits to zero. Denoting N_t the equilibrium measure of operating firms, the competitive equilibrium in each period is characterised by the following system:

$$\pi_t(r_t, w_t) = 0$$
$$N_t l(r_t, w_t) = L$$
$$N_t k(r_t, w_t) = L\kappa_t.$$

A solution to this system defines the equilibrium prices and number of firms as functions of the state variable κ_t: $r_t = r(\kappa_t)$, $w_t = w(\kappa_t)$, $N_t = N(\kappa_t)$. It follows that the dynamics in this economy are characterised by the following capital accumulation equation:

$$\kappa_{t+1} = \mathfrak{K}\big(w(\kappa_t), r(\kappa_{t+1})\big).$$

4 Cooperative Economy

This section starts with a description of the key ingredients of a general equilibrium model of a cooperative economy. Namely, we clarify what we mean by 'a cooperative,' what decisions cooperatives make, and how these decisions interact at the aggregate level. In particular, we discuss the allocation of labour in the absence of a wage rate.

4.1 Concept of a Cooperative

Our conceptualization of cooperatives stresses two features which seem to most clearly distinguish this mode of organization from standard, externally-owned corporations: collective decision making by workers (labour management) and the non-tradability of productive assets. Self-management implies that decisions concerning the cooperative's size and investment are made collectively by the current workers of the cooperative. In our simplified context, where all workers are identical, this means that the objective function of the cooperative is the maximization of the present value of the lifetime utility of its current workers. Non-tradability means that capital is directly owned by the cooperative.

Any period t begins with a set of *incumbent* cooperatives, indexed by i. An incumbent cooperative i is characterized by an inherited capital stock k_{it} and a set of former workers l_{it-1}. Each incumbent cooperative is allocated a set of workers l_{it} via a mechanism which we describe later (Section 4.3). These workers produce output $y_{it} = F(k_{it}, l_{it})$. A share τ of this output is immediately distributed to the former workers. Next, the current-period workers decide how much of the cash flow (net of payments to the old) should be invested to put in place capital to be used in the next period, k_{it+1}. All non-retained earnings are distributed equally among current-period workers. These assumptions result in the following consumption levels for a representative young worker of incumbent cooperative i in period t:

$$
\begin{aligned}
c_{it}^{Y} &= \frac{(1-\tau)y_{it} - k_{it+1}}{l_{it}}, \\
c_{it+1}^{O} &= \frac{\tau y_{it+1}}{l_{it}}.
\end{aligned}
$$

The *sharing rule* τ between current and former workers of the cooperative requires further discussion. We treat τ as a constitutional principle of the cooperative, which entitles former workers to keep sharing in the coop's distributions. A practical reason for introducing a sharing rule is that, absent such a rule, old workers would typically have no consumption. This follows from the fact that our physical environment only allows for capital as a store of value. Coupled with the non-tradability of assets, this means that workers have no means to save other than through the implicit *within cooperative* pay-as-you go pension system represented by the sharing rule. It is hard to imagine that a form of social organization which provides no consumption to those who are retired could thrive.

More importantly, though, the sharing rule τ provides young workers with an incentive to agree on the retention of earnings for the purposes of investment. It should be clear from the equations above that if $\tau = 0$ young workers will wish to set k_{it+1} to 0 as well. This is the Furubotn-Peyovich critique of traditional cooperatives as it manifests itself in our model. In our view this critique largely explains why a majority of real world cooperatives tend to remain small over their life cycle. Hence, the cooperatives in our model are "enhanced coops," which have a built-in mechanism to overcome the anti-investment bias of traditional cooperatives.[13]

We focus on symmetric equilibria in which cooperatives adopt a perfectly equalitarian pay structure (within current workers and within former workers). We conjecture that standard arguments used in the context of capitalist economies could still be deployed to rule out equilibria with inequality within generations. In particular, workers receiving below-average pay in one cooperative could offer to undercut workers receiving above-average pay at another cooperative.

4.2 Continuation, Entry and Exit

The previous subsection describes the consumption of workers allocated to a *continuing* incumbent cooperative, which in our framework is an incumbent cooperative which is allocated some positive young membership l_{it}.

Our model also allows for entry and exit of cooperatives. Entering cooperatives have no capital

[13] In future work we plan to endogenize τ. It is known that pay-as-you-go social-security systems can emerge as time consistent equilibria in infinite-horizon intergenerational games (e.g. Kandori (1992), Cooley and Soares (1999)) and we conjecture that similar results could be obtained for our *within firm* pay-as-you go system.

stock, so they produce with labour only. They also have no former workers. Hence, young-worker consumption in an entering cooperative is $c_{0t}^{Y} = \frac{F(0,l_{0t})-k_{0t+1}}{l_{0t}}$ - where we use the subscript 0 for workers belonging to or inputs and outputs of entering cooperatives.

An exiting cooperative at time t is an incumbent which is assigned no workers by the worker-allocation mechanism. Such a cooperative produces zero output and its capital stock is left idle. Because of full depreciation this cooperative does not continue to period $t+1$. Note that the consumption of old workers attached to exiting cooperatives is 0.

4.3 General Equilibrium Concept for Cooperative Economies

We now discuss, jointly, how labour is allocated to cooperatives and how cooperatives make their investment decisions. Informally, we have in mind a decentralized mechanism in which workers are able to move freely into cooperatives, as long as these are willing to accept them. Therefore, workers sort into the cooperatives which generate highest utility levels until the market clears. This process takes into account the possibility that groups of workers might create a new cooperative without any initial capital. On the other hand, any remaining cooperative without any worker willing to join exits. Once workers have been allocated to cooperatives and production has taken place, workers collectively decide on the amount of earnings that should be retained to put in place as capital for the next period. In making this decision workers take into account the implications of the worker-allocation mechanism for the number of young workers joining the cooperative in that period.

Formally, in each period, the economy is characterised by a set of incumbent cooperatives I_t, and by a distribution of initial capital stocks: $\{k_{it}\}_{i \in I_t}$. For convenience, we assume the set of cooperatives is located on a continuum, and that in each period the set of incumbents I_t is a subset of the real line with finite Lebesgue measure. A worker allocation mechanism is a stationary mapping taking as given the current state of the economy and defining a measurable set of future incumbents I_{t+1} by assigning workers to them:

$$\mathbb{L} : \big(I_t, \{k_{it}\}_{i\in I_t}\big) \mapsto \{l_{it}\}_{i\in I_{t+1}},$$

such that $i \mapsto l_{it}$ is integrable and satisfies:

$$\int_{I_{t+1}} l_{it} di = L,$$

with:

$$I_{t+1} = (I_t \setminus X_t) \cup E_t,$$

where $X_t \subseteq I_t$ is the set of exiting cooperatives, and E_t the set of entrants. Note that we impose full employment, motivated by the idea that any set of unemployed workers could create a new entering cooperative.

For $j \in I_t$, denote $\mathbb{L}_j\big(I_t, \{k_{it}\}_{i\in I_t}\big)$ the labour input assigned to j by the mechanism (thus $\mathbb{L}_j\big(I_t, \{k_{it}\}_{i\in I_t}\big) = 0$ if $j \in X_t$). For ease of notation, we sometimes use $\mathbb{L}_j\big(I_t, \{k_{it}\}_{i\in I_t}\big) = \mathbb{L}_j$, when the dependence is not ambiguous. Given such a mechanism and any current allocation of workers, the continuation of the economy is characterised by optimal investment decisions taking as given the behaviour of other cooperatives, and we denote $\mathcal{U}_j^{I_{t+1}}(\{l_{it}\}_{i\in I_{t+1}})$ the utility to workers in cooperative $j \in I_{t+1}$ under the current allocation of workers. That is, if $j \in I_t \setminus X_t$:

$$\mathcal{U}_j^{I_{t+1}}(\{l_{it}\}_{i\in I_{t+1}}) = \max_{k_{j,t+1}} U\left(\frac{(1-\tau)F(k_{jt}, l_{jt}) - k_{j,t+1}}{l_{jt}}, \frac{\tau F(k_{j,t+1}, \mathbb{L}_j\big(I_{t+1}, \{k_{i,t+1}\}_{i\in I_{t+1}}\big))}{l_{jt}}\right),$$

while for $j \in E_t$:

$$\mathcal{U}_j^{I_{t+1}}(\{l_{it}\}_{i\in I_{t+1}}) = \max_{k_{j,t+1}} U\left(\frac{F(0, l_{jt}) - k_{j,t+1}}{l_{jt}}, \frac{\tau F(k_{j,t+1}, \mathbb{L}_j\big(I_{t+1}, \{k_{i,t+1}\}_{i\in I_{t+1}}\big))}{l_{jt}}\right).$$

The last two expressions define the utility level workers can rationally expect by joining the various cooperatives in the economy. At the same time, they provide information to workers who have joined a particular cooperative about the consequences of allowing further workers to join in. Hence, we can use these objects to define an equilibrium as one in which there exist no reallocation in which the transfer of a worker to a different cooperative makes *both* this worker *and* the original members of this cooperative better off.

Formally, we define the equilibrium as one in which the allocation mechanism $\mathbb{L}$ is such that, given the current state of the economy $\left(I_t, \{k_{it}\}_{i\in I_t}\right)$, for any feasible reallocation $\left(\hat{I}_{t+1}, \{l_{it}\}_{i\in \hat{I}_{t+1}}\right)$, if $i \in I_{t+1}$ and $j \in \hat{I}_{t+1}$ satisfy:

$$l_{it} < \mathbb{L}_i \quad \textit{and} \quad l_{jt} > \mathbb{L}_j$$

then

$$\mathcal{U}_i^{I_{t+1}}(\{\mathbb{L}_\iota\}_{\iota\in I_{t+1}}) \geq \mathcal{U}_i^{\hat{I}_{t+1}}(\{l_{\iota t}\}_{\iota\in \hat{I}_{t+1}}) \tag{1}$$

and

$$\textit{either} \ \ \mathcal{U}_i^{I_{t+1}}(\{\mathbb{L}_\iota\}_{\iota\in I_{t+1}}) \geq \mathcal{U}_j^{\hat{I}_{t+1}}(\{l_{\iota t}\}_{\iota\in \hat{I}_{t+1}}) \tag{2}$$

$$\textit{or} \ \ \mathcal{U}_j^{\hat{I}_{t+1}}(\{l_{\iota t}\}_{\iota\in \hat{I}_{t+1}}) < \mathcal{U}_j^{I_{t+1}}(\{\mathbb{L}_\iota\}_{\iota\in I_{t+1}}). \tag{3}$$

In words, we are considering a feasible reallocation of workers from cooperative i to cooperative j. Condition (1) says that in an equilibrium this reallocation must not be beneficial to the remaining workers of cooperative i (or these workers would wish to reduce the membership). Furthermore, either the reallocation does not make the reallocated workers better off [condition (2)], or it makes the workers of the receiving cooperative worse off [condition (3)]. Note that the subscripts i and j can equally apply to continuing, entering, and exiting cooperatives.

One may legitimately wonder whether it is possible to conceive of a decentralized mechanism which will bring about the allocation criterion we propose. In the context of our economy, a possible narrative which accomplishes this is the following. At the beginning of every period the new generation begins to spread itself in the landscape populated by incumbent cooperatives, each of which is a bundle of (empty) capital: picture a structure with equipment inside. each time a worker enters through the gate of a cooperative, she and the workers already there determine whether they would like further workers to join, or whether they have reached an optimal size. In the latter case, the gate is shut and no further workers are let in. Workers who find all gates shut, or who do not wish to join any of the coops whose gates are open, form a new cooperative. When there are no further workers roaming the landscape, firms whose gates are still open post vacancies, and workers who have entered other coops still have a chance to compare their lifetime utility where they are with their prospective utility if they switch, which they can do frictionlessly if they choose to.

4.4 Operational Equilibrium Concept for Cooperative Economies

The general concept of equilibrium in the previous section is inspired by minimal requirements of rationality and efficiency. Needless to say, these general principles are hardly sufficient as a basis for a study of economic growth in a cooperative economy. What is needed is a more operational refinement allowing us to focus on a subset of equilibria which are tractable for the modeller, and do not impose unrealistic information requirements on the agents on the model. In particular, the generic decentralization of the equilibrium definition in the previous section requires knowledge by each agent of the strategies of all agents in all future generations. This is in sharp contrast to the equilibrium in the capitalist economy where agents need only know current wages and interest rates.

The particular restriction we impose on our equilibria is as follows: the worker allocation mechanism assigns to each incumbent cooperative a number of workers which depends only on that cooperative's capital stock k_{it}. Formally, we only consider equilibria in which, for $t > 0$, there is a mapping $\mathcal{L}(k_{jt})$ such that $\mathbb{L}_j\big(I_t, \{k_{it}\}_{i \in I_t}\big) = \mathcal{L}(k_{jt})$ for $j \in I_t$. Note that the above is a statement about the allocation of workers only to incumbent cooperatives on path. We do not impose restrictions on the allocation of workers to entering cooperatives.

It can easily be seen that if $\mathcal{L}(k_{jt})$ is an allocation mechanism in an equilibrium as defined in the previous section, then each cooperative has an investment policy rule which also depends only on that cooperative's capital stock, $\mathcal{K}(k_{jt})$. Furthermore, in Appendix A.1 we establish the following hugely useful property of $\mathcal{L}(k_{jt})$ and $\mathcal{K}(k_{jt})$:

$$\big(\mathcal{L}(k_{jt}), \mathcal{K}(k_{jt})\big) \in \arg\max_{l,k} U\left(\frac{(1-\tau)F(k_{jt}, l) - k}{l}, \frac{\tau F\big(k, \mathcal{L}(k)\big)}{l}\right). \tag{4}$$

In words, focusing only on equilibria in which an incumbent's worker allocation depends only on that incumbent's initial capital stock is equivalent to focusing on equilibria in which each incumbent cooperative chooses current employment and investment so as to maximize the utility of current young workers, taking as given the fact that all future generations will follow the same strategy. Importantly, this maximization is unconstrained. Therefore, we term equilibria which satisfy this equilibrium-selection principle "unconstrained-cooperative equilibria."

It is important to stress some implications and limitations of the unconstrained-cooperative equilibrium concept. Equation (4) implies that, for $t > 0$, incumbent cooperatives are never constrained in the number of members they can attract. This rules out growth paths along which some cooperatives are forced to exit by their inability to attract workers. This is certainly a limitation, in that it is possible to conceive of economies where this kind of exit is necessary to achieve an efficient allocation of workers in the sense of our more general worker-allocation mechanism. It is one reason why we cannot provide a generic result of existence of unconstrained-cooperative equilibria. Nevertheless, in Sections 5 and 6 we show by example that under standard growth-theoretic assumptions about preferences and technology unconstrained-cooperative equilibria do exist.[14]

A redeeming feature of the unconstrained-cooperative equilibrium concept is that the independence of the labour allocation from the full distribution of capital stocks to incumbents only applies for $t > 0$. Hence, a burst of exit due to insufficient labour supply is allowed at time 0. This is useful because it makes the existence of unconstrained-cooperative equilibria independent of the initial distribution and size of the capital stock. More importantly, it makes it potentially possible to study "MIT-type" shocks: unanticipated permanent changes in endowments or

[14]In our examples it will also turn out that $\mathcal{L}(k_{jt}) > 0 \; \forall i \in I_t$, so not only there is no exit due to cooperatives being unable to attract workers but also there is no optimal (or voluntary) exit - resulting in no exit whatsoever for $t > 0$.

technology which require a burst of exit.

Another important feature of the unconstrained-cooperative equilibrium concept is that it is fully consistent with entry and imposes no restriction on the allocation of workers to cooperatives - other than the restriction imposed by the aggregate labour supply. In particular, it must be the case that:

$$\forall t, \quad \int_{I_t} \mathcal{L}(k_{it}) di \leq L.$$

Therefore, in any period, once incumbents have been allocated workers, new entering cooperatives are created and allocated workers. This allocation of workers to new cooperatives follows the restrictions imposed in section 4.3, and in particular takes into account the economy's resource constraint in terms of labour supply. Importantly though, cooperatives may be constrained only upon entry, but expect to be allocated workers as incumbents in future periods according to the mapping $\mathcal{L}$.

In an unconstrained-cooperative equilibrium all behaviour is pinned down by an initial distribution of capital stocks and the mappings $\mathcal{L}$ and $\mathcal{K}$. The capital accumulation dynamics within a cooperative are pinned down by the equilibrium mapping:

$$k_{jt+1} = \mathcal{K}(k_{jt}).$$

As cooperatives in the economy may differ only in their capital stock, we can then easily study aggregate dynamics as resulting from the sum of individual independent decisions using the same mapping $\mathcal{K}$. The precise algorithm we follow to solve for unconstrained-cooperative equilibria is detailed in Appendix A.2.

5 An Example with Closed Forms

In this section, we use specific functional forms for preferences and the production technology which allow us to characterise analytically the capital accumulation dynamics both in the capitalist and cooperative economies. We use these results to compare the two economies in terms of efficiency.

The production function for production units with positive inputs $(k, l > 0)$ takes the form

$$F(k, l) = Ak^{\alpha}(l - \underline{l})^{\beta}, \tag{5}$$

where $A > 0$, $\underline{l} \in (0, L)$, $\alpha > 0$ and $\beta > 0$ are constant parameters.

Relative to the familiar neoclassical growth model, this production function features the slightly unusual property that there is a fixed cost, in the form of a minimum of $\underline{l}$ units of labor which are required independently of the scale of operation. This assumption is a direct legacy of the older static literature on cooperatives, which showed that in the absence of a fixed cost of production there is no equilibrium with positive cooperative size.[15] The intuition will be apparent below. Needless to say the assumption that production involves fixed costs is entirely realistic.

Since fixed costs of production introduce a form of increasing returns to scale, in order for the model to have an equilibrium under the capitalist form of organization we need decreasing returns to scale in the variable inputs, i.e.

$$\alpha + \beta < 1.$$

The assumption of decreasing returns to variable inputs is also realistic, and it is usually motivated by span-of-control considerations.

Since the cooperative model features potential entry, we will also need an assumption for production in production units with $k = 0$. However, it will turn out that we do not need a specific functional form. Hence, for now we simply assume that $F(0, l) > 0$. We will add some restrictions to this below.

[15] More accurately the existence of cooperatives requires that at low levels of membership the marginal product of labour exceeds average income. In models of capitalist economies the omission of fixed costs of production is without loss of generality due to the replication argument. This is not the case in modelling cooperatives.

As for preferences, in order to derive closed form results we assume that agents obtain log-utility from consumption, with a discount factor $\delta \in (0, 1]$:

$$U(c^Y, c^O) = \log c^Y + \delta \log c^O.$$

5.1 Capitalist Economy

Using these functional forms, we can solve for the capitalist equilibrium as outlined in section 3. The procedure to find the equilibrium is entirely standard and hence we relegate the details to Appendix A.3. Here we only discuss the main aspects of the equilibrium.

The only sightly unfamiliar feature of the capitalist equilibrium is that, because of the fixed production cost, it features an optimal firm size:

$$l_{cap} = \frac{1-\alpha}{1-\alpha-\beta}\underline{l}, \tag{6}$$

where the subscript *cap* will be helpful later to distinguish firm size in a capitalist equilibrium from firm size in the cooperative economy. This contrasts with standard neoclassical textbook models where firm size is undetermined.

Even with a fixed cost, the optimal firm size would generally depend on state variables, such as the aggregate capital stock. This will be the case in the example in the next section. However, under the particular combination of functional forms in this section, the optimal firm size is constant over time both under capitalist and under cooperative arrangements. It is this constancy that allows us to solve the model in closed form, and hence it is a valuable simplification.

Despite this slightly unfamiliar feature the dynamics of the economy are qualitatively the ones we have come to expect from standard growth models. In particular, individual capital holdings evolve according to

$$\kappa_{t+1} = \frac{\delta}{1+\delta} A (1-\alpha)^{\alpha} \beta^{\beta} \left(\frac{1-\alpha-\beta}{\underline{l}} \right)^{1-\alpha-\beta} \kappa_t^{\alpha}, \tag{7}$$

where, recall, κ_t is the savings decision by a member of the period t young. It follows from this functional form that κ_t converges to a steady-state value.

5.2 Cooperative Economy

We study the cooperative economy following the approach presented in section 4.4. We start with an analysis of the "unconstrained" incumbent cooperative under our assumed functional forms for preferences and production technology. We follow a "conjecture and verify" strategy. The conjecture is that in the unconstrained-cooperative equilibrium, if one exists, cooperative firm size is constant, or $\mathcal{L}(k) = l_{coop}$. If this is so, then an unconstrained cooperative solves the problem:

$$\max_{l_t, k_{t+1}} \log\left(\frac{(1-\tau)Ak_t^{\alpha}(l_t - \underline{l})^{\beta} - k_{t+1}}{l_t}\right) + \delta \log\left(\frac{\tau A k_{t+1}^{\alpha}(l_{coop} - \underline{l})^{\beta}}{l_t}\right). \tag{8}$$

The necessary and sufficient first-order conditions for this problem are:

$$-\frac{1}{(1-\tau)Ak_t^{\alpha}(l_t - \underline{l})^{\beta} - k_{t+1}} + \frac{\alpha\delta}{k_{t+1}} = 0, \tag{9}$$

$$\frac{\beta(1-\tau)Ak_t^{\alpha}(l_t - \underline{l})^{\beta-1}}{(1-\tau)Ak_t^{\alpha}(l_t - \underline{l})^{\beta} - k_{t+1}} - \frac{1+\delta}{l_t} = 0. \tag{10}$$

Equation (9) describes the optimal reinvestment of earnings. The first term is the marginal utility loss from diminished current consumption from an extra unit of investment, while the second term is the marginal utility gain from the extra output that investment will deliver next period. First order condition (10) determines the optimal current employment level l_t. Here the trade-off is that an extra worker has a positive marginal impact on current output (first term) but also a negative marginal impact on the share of other workers both in the current period and in the next period, both of which effects are captured in the second term.

This system is easy to solve and yields:

$$\begin{aligned} l_t &= \frac{1+\delta}{1+\delta-\beta(1+\alpha\delta)}\underline{l} \equiv l_{coop}, \\ k_{t+1} &= \frac{\alpha\delta}{1+\alpha\delta}(1-\tau)Ak_t^{\alpha}(l_t - \underline{l})^{\beta}. \end{aligned} \tag{11}$$

The first of these two equations shows that, when expecting a constant labour input in the next period, cooperatives choose a constant labour input in the current period. This both verifies our conjecture and defines the equilibrium cooperative size, l_{coop}. The second equation characterizes the investment policy of cooperatives. This policy inherits the conventional proportionality to current income associated with log utility. Plugging in the form of $l_t = l_{coop}$, we obtain the capital accumulation equation for a single cooperative:

$$k_{t+1} = \frac{\alpha\delta}{1+\alpha\delta}(1-\tau)A\left(\frac{\beta(1+\alpha\delta)}{1+\delta-\beta(1+\alpha\delta)}\underline{l}\right)^{\beta} k_t^{\alpha}, \tag{12}$$

which has the same qualitative features as those derived for the capital accumulation process of individuals in the capitalist economy. We define k^*_{coop} the steady state cooperative capital implied by (12). For later reference we also define $\mathcal{U}(k_{it})$ as the maximized value of (8). It is trivial (but important) to see that $\mathcal{U}(k_{it})$ is an increasing function: workers prefer joining incumbents with larger capital stocks.

To move now to a full characterization of the dynamics of the economy, as well as to complete the argument that the unconstrained-cooperative equilibrium sketched thus far exists, we must now consider the possibility of entry. It is easy to see that, in the unconstrained-cooperative equilibrium we are constructing, the allocation of labour to an entrant and the entrant's investment policy must maximize the objective

$$\log\left(\frac{F(0,l)-k}{l}\right) + \delta\log\left(\frac{\tau A k^{\alpha}\left(l_{coop}-\underline{l}\right)^{\beta}}{l}\right). \tag{13}$$

Note that this problem is time invariant, so both entry size and the utility afforded to a young worker who helps forming a new cooperative are also time invariant. To facilitate the discussion of dynamics we label $\mathcal{L}_e$ the size of an entrant, $\mathcal{K}_e$ its investment policy, and $\mathcal{U}_e$ the utility experienced by a worker joining an entrant.

At any time t, it may conceivably be the case that $\mathcal{U}_e > \mathcal{U}(k_{it})$ for some incumbents i with sufficiently low capital stock. In this case, these incumbents will not be able to attract any workers and will have to exit. We define as $I_t^+ \subseteq I_t$ the set of incumbents at time t such that $\mathcal{U}_e \leq \mathcal{U}(k_{it})$. We can think of I_t^+ as the set of *viable* incumbents. As we will soon see, the key assumption we need to make to insure the existence of an unconstrained cooperative equilibrium is that $\mathcal{U}_e \leq \mathcal{U}(k^*_{coop})$. In other words, an incumbent endowed with the steady state level of capital is viable. We refer to this as Assumption 1.[16]

Define

$$N_{coop} \equiv \frac{L}{l_{coop}}$$

as the measure of incumbent cooperatives consistent with full employment when each cooperative operates at its optimal size l_{coop}. The dynamics of the economy, as well as the further assumptions (if any) required to establish the existence of the equilibrium, are slightly different in the case in which N_{coop} is smaller or larger than the initial endowment of viable cooperatives, $|I_0^+|$, where we use $|x|$ for the measure of set x.

Case 1: $N_{coop} \leq |I_0^+|$

In this case the N_{coop} incumbents with the largest capital stocks will scoop up all the workers in the economy at time 0, and each of them will employ l_{coop} workers. The $|I_0| - N_{coop}$ coops with the *smallest* capital stock (including some viable ones) will exit. No entry will occur as all continuing incumbents afford workers more utility. Moving to period 1, there are no non-viable incumbents. Those incumbents which had capital stock less than k^*_{coop} have experienced capital growth, so they are a fortiori viable in period 1. Even those incumbents which started in period 0 with capital in excess of k^*_{coop} are still viable in light of Assumption 1. Furthermore, since the existing viable incumbents are exactly N_{coop}, there are no workers left out and forced to create a new cooperative. Hence, there is neither entry nor exit, and the same is true in all subsequent periods. Hence, each coop's capital stock evolves according to (12), and eventually, *the entire measure N_{coop} of cooperatives converge to the identical steady state level k^*_{coop}*. Note that no further assumptions on $F(0, l)$ were required.

[16]This is an assumption that $F(0, l)$ is not too productive. If $F(0, l) = BG(l)$ one can always choose B low enough that Assumption 1 is verified.

Case 2: $N_{coop} > |I_0^+|$

In this case the economy is not initially endowed with a measure of viable incumbents sufficient to absorb the entire young-worker population. Hence, while each viable incumbent will be assigned l_{coop} workers, there will have to be entry to employ the remaining $L - |I_0^+| l_{coop}$ workers. To fully describe the dynamics and establish existence of the equilibrium we then need further restrictions on $F(0, l)$. The first restriction (Assumption 2) is that the size of entrants is no less than the size of incumbents, or $\mathcal{L}_e \geq l_{coop}$. The second restriction (Assumption 3) is that entrants become viable incumbents in the period after entry, or $\mathcal{U}(\mathcal{K}_e) \geq \mathcal{U}_e$.[17]

With these assumptions, consider first the special case in which $\mathcal{L}_e = l_{coop}$. In this case there will be exactly a measure $N_{coop} - |I_0^+|$ of entrants at time 0. From there, just as in Case 1, there is no further entry or exit, and each coop once again converges to the capital stock k^*_{coop}. If instead $\mathcal{L}_e > l_{coop}$, the size of period-0 entrants will drop to l_{coop} in period 1, necessitating a further round of entry in that period to insure full-employment. This pattern of residual entry and subsequent shrinkage will continue until the measure of entrants shrinks to 0. From then on no further entry or exit occurs and once again we converge to a steady state with N_{coop} identical cooperatives, all with capital k^*_{coop}.

To understand why Assumption 2 is needed consider the consequences of entrants having scale smaller than l_{coop}. These entrants would have to *grow* in size to satisfy the conjectured equilibrium property that all incumbents are allocated l_{coop} workers. But this is clearly incompatible with the labor resource constraint, because there are not enough workers in the economy to allow all entrants to grow to size l_{coop}.[18] Similarly, a violation of Assumption 3 would imply re-exit at time 1 of cooperatives which entered at time 0, but this violates the equilibrium requirement that all

[17] A sufficient condition for Assumption 2 is that $F(0, l) = B(l - \underline{l}_e)^\gamma$, $\gamma \in (0, (1+\alpha)/(1+\alpha\delta))$, and $\underline{l}_e \geq [1 + \delta - \gamma(1+\alpha\delta)] / [1 + \delta - \beta(1+\alpha\delta)] \, \underline{l}$. This can be verified by substituting these assumptions into (13) and solving the maximization problem. If $\gamma = \beta$ and $\underline{l}_e = \underline{l}$ then $\mathcal{L}_e = l_{coop}$. As regards Assumption 3, if $F(0, l) = BG(l)$ one can always find a B small enough that the assumption is verified. Notice that since $\mathcal{L}_e$ does not depend on B there is no possible tension between Assumptions 1, 2, and 3.

[18] It may seem counter-intuitive to have entrants which are larger than incumbents, but our intuitions are based on observations of capitalist economies. There is no empirical basis to form a prior on whether entering cooperatives would be larger or smaller than incumbent ones.

incumbents have membership l_{coop} for $t > 0$.[19]

5.3 Comparison

In this section, we compare economic performance in the two models along two dimensions: (i) static organization of production and efficiency, and (ii) capital accumulation and dynamic efficiency. We also include a quantitative comparison as a prelude to the subsequent quantitative section which uses more realistic preferences.

5.3.1 Firm Size and Static Efficiency

As we have seen capitalist firms always opt for a constant firm size l_{cap}, while (incumbent) cooperatives always choose firm size l_{coop}. A first important observation (Appendix A.4) is that

$$l_{coop} < l_{cap}.$$

This also means that there are more cooperatives than capitalist firms in equilibrium. The intuition for this result is more transparently conveyed after discussing the output-maximizing choice of firm size.

Consider the choices of a planner intent on maximizing aggregate output, *for a given aggregate stock of capital* K. We can loosely refer to this exercise as identifying the firm size which would make the economy *statically efficient*. Of course the welfare significance of static efficiency is limited, because overall efficiency also depends on the amount of capital in the economy, which in turn depends on dynamic considerations (which we take up in the next sub-section). Indeed the relevance of the analysis of static efficiency is mostly as an input to the subsequent analysis of dynamic efficiency.

[19] In the text we have implicitly assumed that incumbent firms do not have access to technology $F(0, l)$. This is clearly immaterial for Case 1. In Case 2 one could wonder whether non-viable incumbents might be able to avoid exit by switching to the labor-only technology. The answer is no, as any young worker joining an incumbent "inherits" the incumbent's stock of former workers and is thus subject to the sharing rule. She is thus always better off striking out with a new venture.

Because of the concavity of the production function the output-maximizing planner will still choose the same size for all firms (and hence the same capital and labor inputs K/N and L/N). So the problem boils down to characterising the optimal firm size l_{eff}. Then a mass $N_{eff} = L/l_{eff}$ of production units each produce with $K/N = Kl_{eff}/L$ units of capital and l_{eff} workers. The optimal firm size is solution to the following problem:

$$\max_{l} \quad \frac{L}{l} F\Big(\frac{K}{L} l, l\Big).$$

Using our functional assumption, the problem rewrites:

$$\max_{l} \quad A\frac{(l-\underline{l})^{\beta}}{l^{1-\alpha}} K^{\alpha} L^{1-\alpha}.$$

As a result, the aggregate variables K and L do not affect the maximisation problem, and we can define:

$$Z(l) = A\frac{(l-\underline{l})^{\beta}}{l^{1-\alpha}},$$

as a measure of static efficiency associated with any arbitrary firm size. Indeed, the social planner's objective is simply to maximise $Z(l)$ with respect to l. The larger $Z(l)$, the more statically efficient the economy. The socially optimum firm size trades off the following considerations: smaller firms allows the economy to spread variable inputs across more units, thereby reducing the impact of diminishing returns to variable inputs. On the other hand, the larger the measure of firms, the larger the amount of labor "wasted" because of the fixed cost $\underline{l}$.

Taking the first-order condition of this problem we find

$$l_{eff} = l_{cap},$$

where recall that l_{cap} is the equilibrium size of firms in the capitalist economy (equation (6)). Hence, the capitalist economy is *statically efficient* (in the sense of output maximization taking the capital stock as given). Since the cooperative economy features smaller firm sizes than the capitalist economy, a corollary is that the *cooperative economy is statically inefficient.* In particular, cooperatives are (statically) inefficiently small.

There are two reasons why cooperatives are inefficiently small. First, unlike the social planner cooperatives take their *current* capital stock as given. When they consider adding extra workers they only perceive the impact on the average product of labor. Instead, the social planner also takes into account that an extra worker increases the marginal product of capital, and that he can therefore counter the decline in the marginal product of labor by reallocating some extra capital to the production unit. The same happens in the capitalist economy, because extra workers induce the firm to rent extra capital.

The second reason why cooperatives are inefficiently small (in a static sense) is the existence of the sharing rule. An extra worker today is an extra claimant to the payments that will accrue to old workers tomorrow. This is why the firm size in the cooperative economy is decreasing in the weight agents give to old-age consumption, δ.

We can confirm these intuitions by considering the case $\alpha = \delta = 0$, i.e. when labor is the only input and agents discount old age completely. In this case, we can readily check that $l_{coop} = l_{cap} = l_{eff} = \underline{l}/(1-\beta)$. Firm size in all scenarios depends exclusively on how rapidly diminishing returns to labor set in (the more so, the smaller the firm size). If $\delta = 0$ but $\alpha > 0$, firm size in the cooperative economy is still $\underline{l}/(1-\beta)$, which maximizes firm output per worker keeping firm capital constant, but the efficient and capitalist firm size is the larger expression we have derived above, and is increasing in α. Finally, if $\alpha = 0$ but $\delta > 0$ the efficient size is $\underline{l}/(1-\beta)$, but the cooperative size drops to $\underline{l}(1+\delta)/(1+\delta-\beta)$.

Of course one consequence of the fact that the output-maximizing firm size equals the equilibrium firm size in the capitalist economy is that aggregate output for a given capital stock K is also the same:

$$Y_{eff}(K, L) = Y_{cap}(K, L),$$

which will be useful below.

5.3.2 Capital Accumulation and Dynamic Efficiency

Aggregating the capitalist law of motion (7) over individuals and the cooperative law of motion (12) over cooperatives, and making the appropriate substitutions, we easily see that both economies have laws of motion for the aggregate capital stock K_t of the form

$$K_{t+1} = sF(K_t, L),$$

with the corresponding aggregate savings rates

$$s_{cap} = \frac{\delta}{1+\delta}(1-\alpha),$$

and

$$s_{coop} = \frac{\alpha\delta}{1+\alpha\delta}(1-\tau).$$

It is worth discussing the qualitative similarities and differences between these two saving rates.

In both economies, a higher preference for the future increases the saving rate – which is hardly surprising. However, a higher elasticity of output to capital reduces the saving rate in the capitalist economy, while it increases it in the cooperative economy. In the capitalist economy all savings are financed out of labor income, so a larger capital share reduces resources available for saving. In the cooperative economy, the share of income received by young workers is $1-\tau$, independent of α. However, unlike young savers in the capitalist economy, who believe they face a linear return, workers in the cooperative economy internalize the concavity of the production function. The more steeply the marginal product of capital declines with the capital stock (i.e. the lower is α) the less they wish to invest.[20] This means that the larger the capital elasticity of output, the more likely it is that the cooperative economy has a higher saving rate than the capitalist economy.

[20] The following stylized version of the problems faced by workers in the two economies clarifies this point. In the capitalist economy workers essentially maximize $\log(w-k)+\delta\log(rk)$, which of course means that r is irrelevant to the chosen level of k. Instead, if workers maximize $\log(w-k)+\delta\log(A*k^{\alpha})$ the solution will still depend on α and indeed it is clear that in particular the term $\alpha\delta$ will be critical.

It is well established that capitalist OLG economies can exhibit dynamic inefficiency, in the sense that a reduction in saving can improve the consumption and hence the welfare of all generations. This of course applies to the capitalist version of the OLG economy studied here. But can the cooperative economy also be dynamically inefficient? This is the question with which we conclude this subsection.

The standard analysis of dynamic efficiency begins by establishing a *golden rule* level of the capital stock (or, equivalently, of the saving rate) which maximizes total consumption (the sum of the consumption of the young and of the old) subject to enough output being reinvested to keep the total capital stock constant. In our context this problem can be stated as

$$\max_{K,N} c^Y + c^O$$

subject to

$$NF\left(\frac{K}{N}, \frac{L}{N}\right) = L(c^Y + c^O) + K.$$

Now it is clear that, for any K, the optimal N in the problem just stated must be the output-maximizing one which we identified in the previous subsection. Hence, the golden rule problem can be rewritten as

$$\max_{K} c^Y + c^o$$

subject to

$$Y_{eff}(K, L) = L(c^Y + c^O) + K.$$

Using the functional form for $Y_{eff}(K, L)$ from above, and taking the first order condition, we find the familiar Cobb-Douglas golden rule $K = \alpha Y$. Hence, the *dynamically* consumption maximizing saving rate is α.

It follows from comparison with (5.3.2) that *the cooperative economy can never be dynamically inefficient* as $s_{coop} < \alpha$. In contrast, the capitalist economy can of course be potentially dynamically inefficient. The intuition is closely linked to our discussion of saving in the two economies in the earlier part of this subsection. As is (now) well understood, in the capitalist economy the potential dynamic inefficiency is due to a pecuniary externality: the young do not internalize the fact that by increasing saving they lower the return to capital for everyone.[21] In contrast, as we have seen, young cooperative members fully take into account the consequences of their accumulation decision on the marginal product of capital, and this prevents them from over-accumulating.

5.3.3 Steady State Output

It is easy to see that, if an economy with the technology we have been working with in this section (equation (5)) features a steady state in which all firms are identical and operate with constant inputs k^* and l^*, steady state aggregate output per worker can be written as

$$\frac{Y^*}{L} = (s^*)^{\frac{\alpha}{1-\alpha}} (Z^*)^{\frac{1}{1-\alpha}} \,, \tag{14}$$

where Z is the measure of *static efficiency* we derived in Section 5.3.1, and $s^* = K^*/Y^*$ is the saving rate in steady state.[22].

The interpretation is straightforward after the discussions in the last two subsections. An economy's steady-state output per worker is driven by two factors: how efficiently it produces in a static sense, and how much it saves.

Under our log-utility assumption both the capitalist and the cooperative economy have constant saving rates and constant firm sizes l, the latter implying constant Zs. We have seen that we cannot sign the difference between s_{coop} and s_{cap}, and that $Z_{coop} \leq Z_{cap}$. Despite this disadvantage, because cooperative economies could potentially save at a higher rate, which economy has a higher output per worker is a quantitative matter.

[21] Acemoglu (2009, pp. 338-339) discusses the evolution of thinking about the sources of dynamic inefficiency in OLG economies.

[22] To see this write aggregate output as $Y^* = \frac{L}{l} F(k^*, l^*)$. The capital input of each individual firm is given by: $k^* = \frac{l^*}{L} K^* = \frac{l^*}{L} s^* Y^*$. Plugging this into Y^* and using the functional form for F we get the decomposition in the text

Table 1: Calibrated parameters

Preferences				log	IES =2
Concept	Parameter	Target	Data	Value	Value
Capital share	α	rK/Y	0.33	0.33	0.33
Variable labor elasticity	β	$\underline{l}/L$	0.18	0.55	0.55
Discount rate	δ	K/Y	3/25	0.22	0.13
Sharing Rule	τ	Max U		0.12	0.15

5.3.4 Quantification

In this subsection we calibrate the log-utility economy for a first set of quantitative insights on the comparison between capitalist and cooperative economies. In the next section we quantify an example with more realistic preferences (but no closed-form solutions).

We do not observe a cooperative-based economy but we do observe economies which are broadly organized according to capitalist principles. Hence, we calibrate the parameters of the model so that the capitalist economy in steady-state matches corresponding moments of the US economy in recent decades. Table 1, column "log" summarises the values chosen for the parameters of the model. The parameter α maps as usual into the share of capital in national income. Given α, a choice of β in (6) uniquely determines the share of fixed labour in total firm employment, $\underline{l}/l_{cap}$. We match this to the share of non-production workers in the economy, from the Bureau of Labor Statistics. Finally, given α a choice of δ uniquely determines the saving rate in the capitalist economy, and in turn this saving rate equals the capital-output ratio in steady state, for which we use the standard value of 3 (with an adjustment for a putative 25-year duration of a model period.) The parameter τ is unique to the cooperative economy and thus cannot be calibrated on any kind of data. Hence, we select the value of τ that maximises steady-state lifetime utility of the representative consumer in the cooperative economy. Simple calculations show that this value is:

$$\tau = \frac{\delta}{1+\delta}(1-\alpha).$$

(This happens to also be the saving rate in the capitalist economy - but we do not have a

Table 2: Numerical results

	log	IES = 2
$\left(\frac{Z_{coop}}{Z_{cap}}\right)^{\frac{1}{1-\alpha}}$	0.84	0.69
$\left(\frac{s_{coop}}{s_{cap}}\right)^{\frac{\alpha}{1-\alpha}}$	0.70	1.05
$\left(\frac{Y_{coop}}{Y_{cap}}\right)$	0.60	0.73

compelling intuition for this coincidence.) The fixed cost $\underline{l}$ cancels out in all the ratios of capitalist-to-cooperative outcomes we wish to present, so it does not need to be calibrated here. Similarly, there is no need to choose values for the size of the population L and the productivity factor A.

The implications of this calibration are presented in Table 2 (column "log"). First, cooperatives are only half as large as capitalist firms, or $l_{coop}/l_{cap} = 0.51$. This large size difference implies a significant disparity in static efficiency: $(Z_{coop}/Z_{cap})^{1/(1-\alpha)} = 0.84$. Second, the cooperative economy also saves half as much as the capitalist one, as we have $s_{cap} = 0.12$ and $s_{coop} = 0.06$. (Note that the capitalist economy is dynamically efficient). Hence, the contribution of saving to the output gap is $(s_{coop}/s_{cap})^{\frac{\alpha}{1-\alpha}} = 0.70$. When combined, the static inefficiency and the lower saving rate of the cooperative economy imply that steady state output per worker is 60% of steady state output per worker in the capitalist economy.

The model also allows us to study the transition dynamics following the introduction of a cooperative organisation of production. Suppose that we begin with a capitalist economy initially in steady-state. At some initial period, all capital is seized and distributed equally to $N = L/l_{coop}$ cooperatives. Old agents in the initial period receive a share τ of output. From then on, the economy evolves according to our model of cooperative economy. Figure 1 plots the dynamic evolution of output (normalized by the capitalist benchmark) in this setting.

6 An Example with Numerical Computations

The assumption of log preferences in the previous section was extremely useful in deriving a closed-form characterisation of the equilibrium, and analytical formulas to compare steady states in the

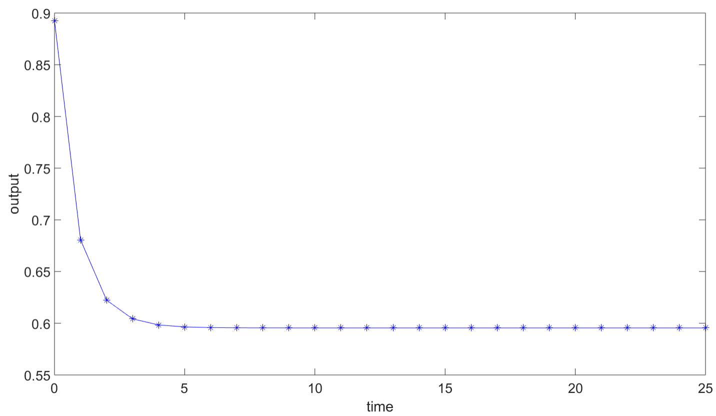

Figure 1: Dynamic evolution of $\frac{Y_{coop}}{Y^*_{cap}}$ in the cooperative economy under log utility.

cooperative and capitalist economies. However, it makes the quantitative exercise somewhat limited. In this section we assume a more general specification for preferences with constant intertemporal elasticity of substitution:

$$U(c^Y, c^O) = \frac{(c^Y)^{1-\sigma}}{1-\sigma} + \delta \frac{(c^O)^{1-\sigma}}{1-\sigma}.$$

If we focus directly on the case where $\sigma = 2$, which is well within the range that is usually considered plausible, we can still produce analytical solutions for the capitalist steady state, which is extremely useful for calibration purposes. We do not change the assumptions on technology.

The competitive equilibrium in the capitalist economy is unaffected by the assumption on preferences, which only affects the saving rule. As a result, the derivation of equilibrium prices and number of firms in the previous section is still valid. Importantly, this implies that firm size is still constant and takes the same value derived above, l_{cap}. Among other things this means that α and β do not need to be re-calibrated.

In Appendix A.5 we study the consumption-saving decision of young workers in the capitalist economy. We show that the capitalist economy converges to a steady state, and, for the case

$\sigma = 2$, we the aggregate steady state saving rate is

$$s^*_{cap} = \frac{4(1-\alpha)}{\left(\left(\frac{\alpha}{\delta(1-\alpha)}\right)^{1/2} + \left(4 + \frac{\alpha}{\delta(1-\alpha)}\right)^{1/2}\right)^2}.$$

As in the previous example, $s^* = K^*/Y^*$, and as we already have a calibration for α, this equation can be used to re-calibrate δ, as reported in the last column of Table 1.

While we have closed form characterizations of the (steady state of the) capitalist economy, for the cooperative economy we must proceed numerically. We begin as before with the problem of an unconstrained incumbent. In particular, we use policy function iteration to find a (numerical) fixed point for the mapping $\mathcal{L}(k)$ which solves problem (4). This is done using the already calibrated α, β, and δ, as well as normalized values for $\underline{l}$, A and L. (Appendix A.6 shows that comparison of steady state values among the two economies is independent of $\underline{l}$. That A and L can be normalized is obvious.) We also re-calibrate τ, to maximise, as before, the steady-state lifetime utility of the representative agent under the new preferences (and the new value of δ).

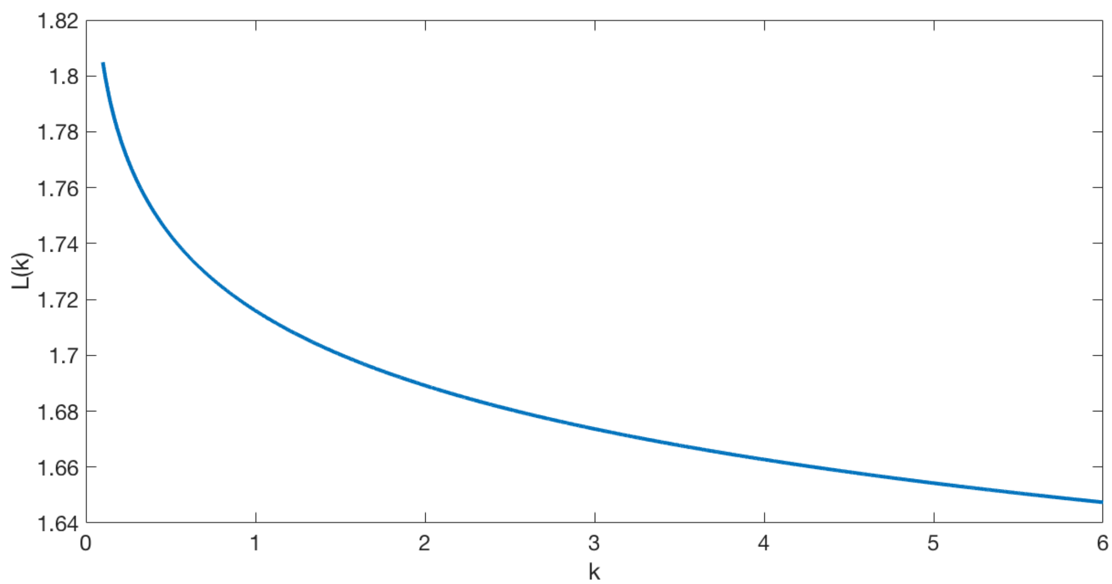

Figure 2: Cooperative labour input as a function of initial capital stock – numerical solution to problem (4).

The policies $\mathcal{L}(k)$ and $\mathcal{K}(k)$ implied by our calibration are plotted in Figures 2 and 3. $\mathcal{L}(k)$

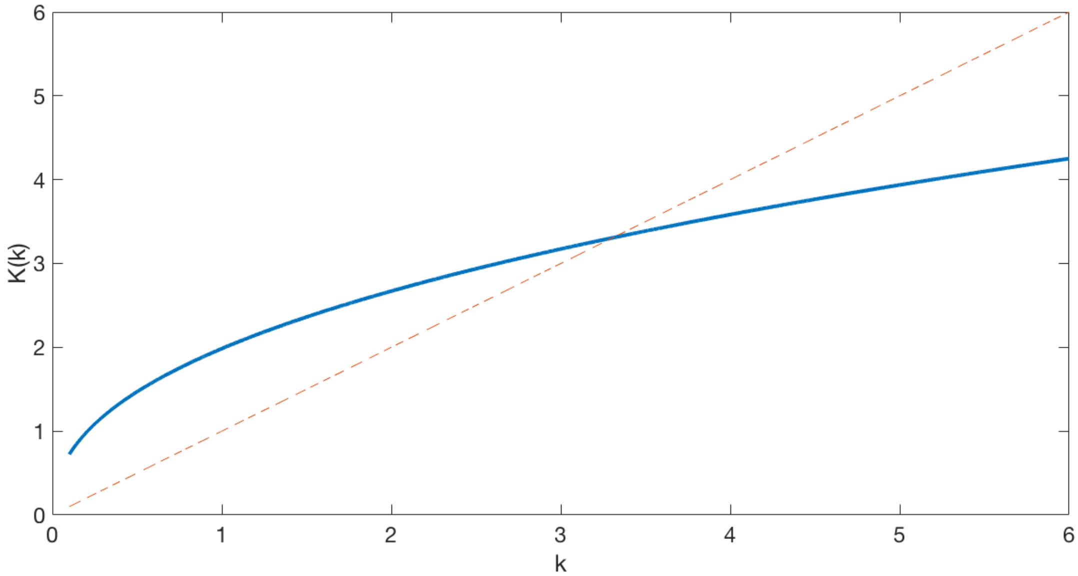

Figure 3: Cooperative capital investment as a function of initial capital stock – numerical solution to problem (4).

is decreasing in k, while $\mathcal{K}(k)$ is increasing and concave. This last property implies that there exists a steady state for the cooperative economy in which all coops have the same capital stock k^*, the same membership $\mathcal{L}(k^*)$, and their measure is $L/\mathcal{L}(k^*)$.[23]

Given the existence of a steady state where identical incumbents maximize the unconstrained-cooperative problem, and given the allocation criterion $\mathcal{L}(k)$ and the investment function $\mathcal{K}(k)$, sufficient conditions for convergence to this steady state can be identified using a reasoning similar to the one we used in Section 5.2. In particular, if (i) all initial incumbents start with a capital stock $k_{i0} \leq k^*$; (ii) in any period, entrants' worker allocation l_e and optimal invest k_e satisfy (a) $l_e \geq \mathcal{L}(k_e)$ and (b) $k_e \leq k^*$; and (iii) one-period-old coops are viable, then every coop's capital stock grows over time towards the steady-state level, while every coop's labour input decreases over time towards the steady-state, generating entry but no exit.

Recall now that that decomposition (14) is valid for any economy featuring a steady state with identical firm sizes, and is thus still valid – with the same interpretation – in the current

[23]The existence of this steady state is established only numerically via the numerical properties of the policy function. Uniqueness of the unconstrained-cooperative steady state, much less of unconstrained-cooperative equilibria, is not established in any formal sense. All we can say is that our policy function iteration converges to the same fixed point from a wide variety of initial guesses we have attempted.

example. The terms of the decomposition are reported in the last column of Table 2. With the alternative choice of preferences the cooperative economy features an even stronger bias towards small firms, meaning that its static inefficiency cause an even greater disadvantage relative to the capitalist economy: the term in Z drops to 0.69. On the other hand, the higher elasticity of intertemporal substitution boosts the relative savings rate of the cooperative economy, which is now 10 per cent higher than the capitalist one (resulting in a 5 per cent higher term in s). As a consequence of this latter feature, the relative output of the cooperative economy rises to 0.73.

In Figure fig:Robustness we present a series of plots showing the numerical dependence of the output ratio $\frac{Y^*_{coop}}{Y^*_{cap}}$, as well as its two components, on the parameters α, β, γ, and τ. These plots can be interpreted as robustness checks on our benchmark numerical results or, perhaps more usefully, as numerical comparative-static results for our models of cooperatives.[24]

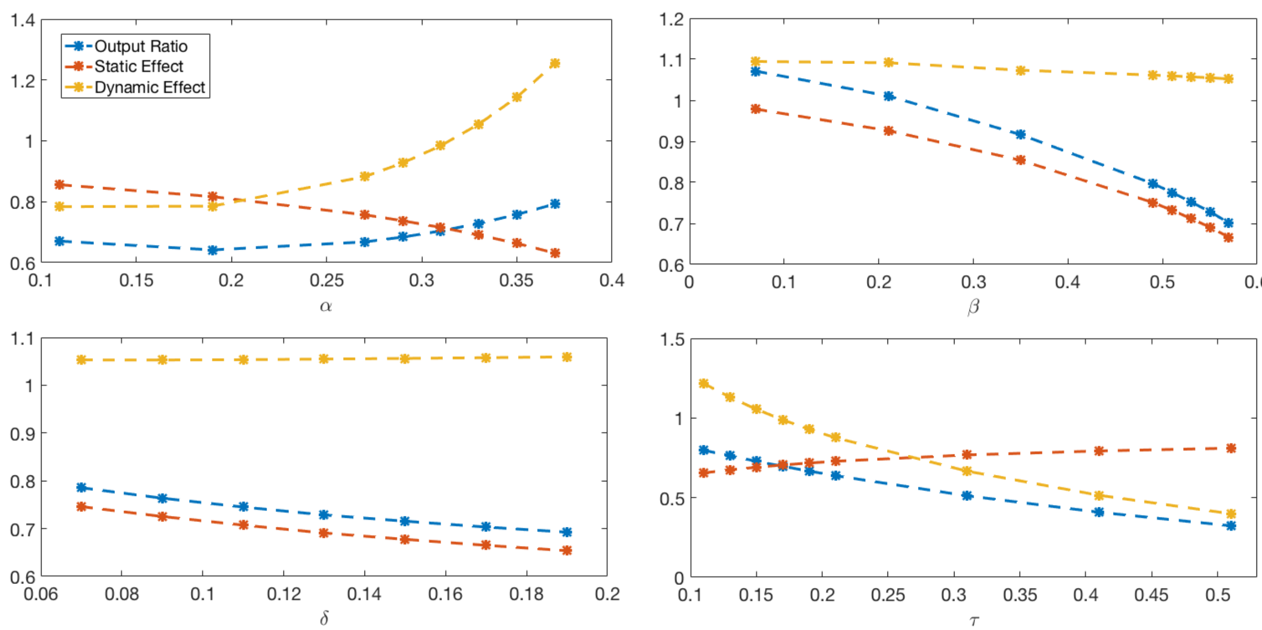

Figure 4: Decomposition of the output ratio $\frac{Y^*_{coop}}{Y^*_{cap}}$ (blue) into static effect $\left(\frac{Z_{coop}}{Z_{cap}}\right)^{\frac{1}{1-\alpha}}$ (orange) and dynamic effect $\left(\frac{s_{coop}}{s_{cap}}\right)^{\frac{\alpha}{1-\alpha}}$ (yellow) for different values of α (top-left), β (top-right), δ (bottom-left) and τ (bottom-right).

The top-left plots reveal that relative steady state cooperative output is first decreasing and

[24]This is particularly the case as we do not re-optimize with respect to τ as we vary the other parameters, i.e. unless the robustness check is with respect to τ itself, this parameter is held constant at the level of the benchmark calibration.

then increasing in the elasticity of output to capital α. Looking at the two sub-components reveals why: static efficiency steadily declines with α, while the saving rate increases with it - for the reasons we discussed in Section 5.3. Clearly the former effect dominates at low level of the output elasticity of capital, while the latter dominates for larger values.

The top-right panels shows relative cooperative output to be monotonically decreasing in the elasticity to variable labour, β. Inspection of equations (6) and (11) shows that a higher value of this parameters reduces the size of cooperative firms relative to capitalist firms, leading to an exacerbation of the static inefficiency of cooperative economies. Quantitatively this is clearly the main driver of the decline of the output ratio with β, though the graph shows that the relative saving rate is also slightly decreasing in this parameter. Another interesting feature of this panel is that it confirms that there actually exist *combinations of parameter values such as steady state output in the cooperative economy is higher than in the capitalist economy.* In this particular case, this happens when the static inefficiency is minimized (through a very low value of β) so that the entire difference in incomes is due to the higher saving rate of the cooperative economy.

The static inefficiency also dominates the dependence of relative output on the discount factor δ. As seen in Section 5.3.1, the more importance workers give to the future, the more they wish to limit current employment. This negative effect is quantitatively much stronger than the positive effect of δ on relative saving, which goes in the opposite direction.

Finally, a larger share of output devoted to former workers, τ, directly reduces the cooperative economy's saving rate, leading again to a reduction in relative cooperative-economy output. This is despite the effect that an increase in τ improves somewhat the cooperative economy's static allocative efficiency.

7 Conclusions

In light of the current crisis in the perceived legitimacy of corporation-based capitalism it is important to investigate the macroeconomic consequences of alternative institutional arrangements for the production of goods and services. This paper has taken a first step towards developing a theoretical and quantitative framework towards this goal, with a particular focus on worker cooperatives as the engine of economic activity. We have also provided quantitative examples of comparisons of macroeconomic outcomes under corporation-based capitalism and under labor-management.

Much more work needs to be done for a proper qualitative and quantitative comparison of capitalist economies and cooperative-based economies. In the rest of these Conclusions we outline the agenda for future research.

Our cooperative economy differs from the capitalist economy in the following main respects: (i) there is a non-wage mechanism which assigns workers to firms in a manner that is collectively rational and yet decentralized; (ii) former workers retain rights to the distribution of the cooperative's income; (iii) investment decisions are made by worker collectives to maximize the lifetime utility of current workers; (iv) the capital used in production by each cooperative is the result of past cooperative investments from retained earnings.

We don't think there is much scope to investigate alternatives to (i) if the productive units in our economy must continue to be recognizable as worker cooperatives. Indeed we think of the conceptualization of the worker-allocation mechanism in a cooperative economy as perhaps the key contribution of the paper. Similarly, dropping (ii) while leaving (iii) in place would trivially lead to an economy with zero investment.

A more feasible alternative might seem to be to drop (iii) and return the investment decision to the individuals. In particular, we could have young workers save in the form of capital, and cooperatives renting capital from old individuals. It is apparent, however, that such an alternative would be isomorphic to the capitalist model – at least in our OLG framework. This is because the rental rate on capital would be the marginal product of capital, so young workers would be the residual claimants of the same share of income as in the capitalist version.

This leaves us with (iv), and it is here that a truly important and fruitful alternative could potentially lie. In particular, it would be useful to investigate the consequences of opening up a market on which cooperatives could rent capital from each other. We have noted earlier that one reason for the inefficiently small size of cooperatives is that they take their capital stock as given. The existence of a rental market for capital might therefore lead to different decisions. Unfortunately, extending our framework to feature an inter-cooperative rental market for capital is challenging, as the worker-allocation rule for each cooperative would have to depend on the indefinite future history of rental rates. Hence, we leave this task for future work.

From a quantitative perspective, future work should also assess the implications of canonical variants of the OLG framework, such as economies with alternative stores of value, like money, or with social security. Always within the current setup, it would also be interesting to identify strategies to study the possible coexistence of capitalist and cooperative firms - or whether one type of firm would necessarily drive the other from the market.

However, the true, long-term payoff of this research agenda will only come from much richer qualitative and quantitative descriptions of the economy. A more complex demographic structure is only a minor aspect of this quest. Introducing realistic distortions to the capitalist economy (monopoly power, monopsony power, short-termism in decision-making, etc.) would put the comparison of efficiency and production on a more even playing field. Considerations of externalities (e.g. pollution) would similarly be informative on the relative welfare properties of the two systems. Most important of all, introducing realistic sources of heterogeneity (in skills, in initial wealth, in access to schooling and high-return assets) would allow to compare corporation-based capitalism and cooperative-based alternatives not only on their implications for aggregate productivity but also on their implications for income and wealth inequality. Since it is aversion to the consequences of extreme inequality which has fostered much of the current push back against capitalism, it is essential that efficiency losses associated with a cooperative-based system (if any) be evaluated against the likely benefits in terms of lower inequality. We hope that our paper will prove to be a first step on this (long) road.

A Appendix

A.1 Proof of Unconstrained Cooperative property

In this appendix we prove the correctness of the statement in equation (4).

For $k > 0$ and $l > 0$, denote:

$$\mathcal{U}(k,l) = \max_{k'} U\left(\frac{(1-\tau)F(k,l) - k'}{l}, \frac{\tau F\big(k', \mathcal{L}(k')\big)}{l}\right).$$

The result we wish to prove is that

$$\mathcal{L}(k) \in \arg\max_{l} \mathcal{U}(k,l).$$

We will prove this by contradiction.

Consider an incumbent $\bar{i} \in I_t$, and suppose that $\mathcal{L}(k_{\bar{i}t})$ does *not* coincide with the argmax. Condition (1) implies that if $l \leq \mathcal{L}(k_{it})$ then $\mathcal{U}(k_{it}, l) \leq \mathcal{U}\big(k_{it}, \mathcal{L}(k_{it})\big)$, so the argmax must be strictly greater than $\mathcal{L}(k_{\bar{i}t})$.

Now apply conditions (2) and (3) to $j = \bar{i}$ and $i \in I_{t+1}$ an arbitrary cooperative. If there is a feasible reallocation in which $\bar{i}$ is allocated $l > \mathcal{L}(k_{\bar{i}t})$ workers and i fewer workers than in the original allocation $\mathbb{L}_i$, then either:

$$\mathcal{U}(k_{\bar{i}t}, l) < \mathcal{U}(k_{\bar{i}t}, \mathcal{L}(k_{\bar{i}t})),$$

or:

$$\mathcal{U}(k_{it}, \mathbb{L}_i) \geq \mathcal{U}(k_{\bar{i}t}, l).$$

Since $\mathcal{L}(k_{\bar{i}t})$ is strictly less than the argmax, there must exist $l > \mathcal{L}(k_{\bar{i}t})$ such that the first condition is violated. It follows that the second condition must hold for any other cooperative

$i \in I_{t+1}$. In particular, since we take l such that $\mathcal{U}(k_{\bar{i}t}, \mathcal{L}(k_{\bar{i}t})) < \mathcal{U}(k_{\bar{i}t}, l)$, it follows that:

$$\forall i \neq \bar{i} \in I_{t+1}, \quad \mathcal{U}(k_{it}, \mathbb{L}_i) > \mathcal{U}(k_{\bar{i}t}, \mathcal{L}(k_{\bar{i}t})).$$

So any incumbent that is not allocated its optimal labour input must be the cooperative that provides the lowest utility level to its workers among all cooperatives in the economy. But being such a cooperative depends not only on that cooperative's own capital stock, but on the capital stock of all other cooperatives. This then contradicts the premise that the allocation of workers to incumbent cooperatives depends exclusively on each cooperative's own initial capital.

A.2 Algorithm to solve for unconstrained-cooperative equilibrium

In practice, we solve for equilibria as follows. The first step is to obtain the stationary Markov perfect equilibrium choices of labour input and capital investment of an unconstrained cooperative:

$$\big(\mathcal{L}(k), \mathcal{K}(k)\big) \in \arg\max_{l,k'} U\left(\frac{(1-\tau)F(k,l) - k'}{l}, \frac{\tau F\big(k', \mathcal{L}(k')\big)}{l}\right),$$

as well as the optimal capital investment level of an entering cooperative with an arbitrary labour input l:

$$\mathbf{K}(l) \in \arg\max_{k'} U\left(\frac{(1-\tau)F(0,l) - k'}{l}, \frac{\tau F\big(k', \mathcal{L}(k')\big)}{l}\right).$$

The value of this problem is denoted $\mathcal{U}_0(l)$. Then, given any initial distribution of capital $\{k_{i0}\}_{i \in I_0}$, one can construct the growth path of the economy and check for feasibility.

Specifically, in each period, all incumbents $i \in I_t$ are allocated $\mathcal{L}(k_{it})$ workers. If $\int_{I_t} \mathcal{L}(k_{it}) di > L$, feasibility is violated so the initial distribution does not lead to an unconstrained equilibrium. If $\int_{I_t} \mathcal{L}(k_{it}) di = L$, then $I_{t+1} = I_t$ and $k_{i,t+1} = \mathcal{K}(k_{it})$. If $\int_{I_t} \mathcal{L}(k_{it}) di < L$, define:

$$l^* \in \arg\max_{l} \mathcal{U}_0(l).$$

Then a set of entrants E_t of measure $|E_t| = \frac{L - \int_{I_t} \mathcal{L}(k_{it}) di}{l^*}$ is created. As long as it is feasible, new cooperatives are created with $l_i = l^*$ workers. Finally, if there remain workers, one extra cooperative is created with these workers. Then, $I_{t+1} = I_t \cup E_t$ where E_t is the set of newly created cooperatives, and for $i \in I_t$, $k_{i,t+1} = \mathcal{K}(k_{it})$, while for $i \in E_t$, $k_{i,t+1} = \mathbf{K}(l_i)$.

A.3 Derivation of capitalist equilibrium with log utility

Conditional factor demands from individual firms take the form:

$$\begin{aligned} k(r_t, w_t) =& \left[A\Big(\frac{\alpha}{r_t}\Big)^{1-\beta} \Big(\frac{\beta}{w_t}\Big)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}}, \\ l(r_t, w_t) =& \underline{l} + \left[A\Big(\frac{\alpha}{r_t}\Big)^{\alpha} \Big(\frac{\beta}{w_t}\Big)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}}, \end{aligned}$$

while profits write:

$$\pi(r_t, w_t) = (1-\alpha-\beta)\left[A\Big(\frac{\alpha}{r_t}\Big)^{\alpha} \Big(\frac{\beta}{w_t}\Big)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}} - w_t \underline{l}.$$

As a result, we can solve the system of equilibrium conditions to derive:

$$\begin{aligned} r(\kappa_t) &= A\frac{\alpha}{(1-\alpha)^{1-\alpha}}\beta^{\beta}\Big(\frac{1-\alpha-\beta}{\underline{l}}\Big)^{1-\alpha-\beta}\kappa_t^{\alpha-1}, \\ w(\kappa_t) &= A(1-\alpha)^{\alpha}\beta^{\beta}\Big(\frac{1-\alpha-\beta}{\underline{l}}\Big)^{1-\alpha-\beta}\kappa_t^{\alpha}, \\ N(\kappa_t) &\equiv N = \frac{1-\alpha-\beta}{1-\alpha}\Big(\frac{L}{\underline{l}}\Big). \end{aligned}$$

Note that the number of firms is constant over time and hence independent of the size of the capital stock, or equivalently, capitalist firms have the constant size given in equation (6).

The solution to the Young's consumption-saving problem leads to the well known log-utility saving rule

$$\kappa_{t+1} = \frac{\delta}{1+\delta} w_t.$$

Substituting from the equations above this delivers the capital accumulation equation (7).

A.4 Proof that cooperatives are smaller than capitalist firms

Capitalist firms have $\frac{1-\alpha}{1-\alpha-\beta}\underline{l}$ employees, while cooperatives have $\frac{1+\delta}{1+\delta-\beta(1+\delta\alpha)}\underline{l}$ workers. Since $\alpha \in (0,1)$, it must be the case that:

$$1+\delta \geq (1-\alpha)(1+\delta\alpha).$$

It follows that $\frac{1}{1-\alpha} \geq \frac{1+\delta\alpha}{1+\delta}$, which implies that:

$$1-\frac{\beta}{1-\alpha} \leq 1-\frac{\beta(1+\delta\alpha)}{1+\delta}.$$

Therefore:

$$\frac{1-\alpha}{1-\alpha-\beta}\underline{l} \geq \frac{1+\delta}{1+\delta-\beta(1+\delta\alpha)}\underline{l}.$$

A.5 Capitalist dynamics with IES = 2

Solving the consumption-saving problem of young agents yields the following saving rule:

$$\kappa_{t+1} = \frac{\delta^{\frac{1}{\sigma}} r_{t+1}^{\frac{1-\sigma}{\sigma}}}{1+\delta^{\frac{1}{\sigma}} r_{t+1}^{\frac{1-\sigma}{\sigma}}} w_t.$$

As a result, capital accumulation dynamics are characterised by the following equation:

$$\begin{aligned}\left[1+\delta^{-\frac{1}{\sigma}}\left(A\frac{\alpha}{(1-\alpha)^{1-\alpha}}\beta^{\beta}\left(\frac{1-\alpha-\beta}{\underline{l}}\right)^{1-\alpha-\beta}\kappa_{t+1}^{\alpha-1}\right)^{-\frac{1-\sigma}{\sigma}}\right]\kappa_{t+1} \\ = A(1-\alpha)^{\alpha}\beta^{\beta}\left(\frac{1-\alpha-\beta}{\underline{l}}\right)^{1-\alpha-\beta}\kappa_t^{\alpha}.\end{aligned} \tag{15}$$

If $\sigma > 1$ and $\alpha \in (0,1)$, equation (15) defines κ_{t+1} as an increasing and concave function of κ_t, with a first-order derivative which is infinite at 0 and vanishes at infinity.

To simplify notations, denote:

$$a = \delta^{-\frac{1}{\sigma}} \left(A \frac{\alpha}{(1-\alpha)^{1-\alpha}} \beta^{\beta} \Big(\frac{1-\alpha-\beta}{\underline{l}} \Big)^{1-\alpha-\beta} \right)^{-\frac{1-\sigma}{\sigma}},$$

$$b = A(1-\alpha)^{\alpha} \beta^{\beta} \Big(\frac{1-\alpha-\beta}{\underline{l}} \Big)^{1-\alpha-\beta},$$

$$\theta = \alpha + \frac{1-\alpha}{\sigma}.$$

Then equation (15) rewrites:

$$\kappa_{t+1} + a\kappa_{t+1}^{\theta} = b\kappa_t^{\alpha},$$

or equivalently:

$$\kappa_{t+1} = f^{-1}(\kappa_t),$$

where $f(x) = \left(\frac{1}{b}\right)^{\frac{1}{\alpha}} \left(x + ax^{\theta}\right)^{\frac{1}{\alpha}}$ is strictly increasing and strictly convex on $(0, \infty)$, and satisfies:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} f'(x) = 0,$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} f'(x) = \infty,$$

if we restrict attention to the case $\sigma > 1$, so that $\theta \in (\frac{1}{\sigma}, 1)$.

Indeed, these properties are easily derived from differentiating twice, which yields:

$$f'(x) = \Big(\frac{1}{b}\Big)^{\frac{1}{\alpha}} \big(1 + a\theta x^{\theta-1}\big)\big(x + ax^{\theta}\big)^{\frac{1-\alpha}{\alpha}},$$

and:

$$f''(x) = \Big(\frac{1}{b}\Big)^{\frac{1}{\alpha}} \big(x + ax^{\theta}\big)^{\frac{1-\alpha}{\alpha}-1} \Big[\frac{1-\alpha}{\alpha} + \theta\big(\theta - 3 + \frac{2}{\alpha}\big)ax^{\theta-1} + \theta\big(\frac{\theta}{\alpha} - 1\big)\big(ax^{\theta-1}\big)^2\Big],$$

where $\frac{\theta}{\alpha} - 1 = \frac{1-\alpha}{\alpha\sigma} > 0$, and $\theta - 3 + \frac{2}{\alpha} = (1-\alpha)\big(\frac{2-\alpha}{\alpha} + \frac{1}{\sigma}\big) > 0$.

It follows that capital accumulation follows standard dynamics with a unique strictly positive attractive steady-state.

In the special case where $\sigma = 2$, the steady-state capital stock per old worker takes a simple algebraic form:

$$\kappa^* = \left(\frac{\left(4A(1-\alpha)^\alpha \beta^\beta \left(\frac{1-\alpha-\beta}{\underline{l}}\right)^{1-\alpha-\beta}\right)^{1/2}}{\left(\frac{\alpha}{\delta(1-\alpha)}\right)^{1/2} + \left(4 + \frac{\alpha}{\delta(1-\alpha)}\right)^{1/2}} \right)^{\frac{2}{1-\alpha}}.$$

It follows that the discount factor δ can still be identified from the targeting of the capital-output ratio:

$$\frac{K}{Y} = \frac{4(1-\alpha)}{\left(\left(\frac{\alpha}{\delta(1-\alpha)}\right)^{1/2} + \left(4 + \frac{\alpha}{\delta(1-\alpha)}\right)^{1/2}\right)^2}.$$

A.6 Fixed-cost Normalization

We use the following functional form for the production technology:

$$F_{\underline{l}}(k, l) = Ak^\alpha (l - \underline{l})^\beta.$$

In this appendix, we show that the specific value of the parameter $\underline{l}$ does not affect the quantitative comparison between the two economies.

For any given $\underline{l}$, we can implement the following change of variables. Any quantity of labour l can be renormalized as:

$$\tilde{l} = \frac{l}{\underline{l}},$$

while any quantity of capital k can be renormalized as:

$$\tilde{k} = \underline{l}^{-\frac{\beta}{1-\alpha}} k.$$

It follows that the production output is also renormalized as:

$$F_{\underline{l}}(k, l) = \underline{l}^{\frac{\beta}{1-\alpha}} F_1(\tilde{k}, \tilde{l}).$$

In our model of capitalist economy, the problem of the firm is completely unchanged as long as the wage is suitably renormalized to:

$$\tilde{w} = \underline{l}^{1-\frac{\beta}{1-\alpha}} w.$$

The consumer's problem is unchanged either (note that each consumer now supplies $1/\underline{l}$ units of labour). The renormalization implies that consumption level c is to be renormalized as: $\tilde{c} = \underline{l}^{-\frac{\beta}{1-\alpha}} c$. Given that preferences are of the form:

$$U(c^Y, c^O) = \frac{(c^Y)^{1-\sigma}}{1-\sigma} + \delta \frac{(c^O)^{1-\sigma}}{1-\sigma},$$

the renormalization amounts to multiplying the utility function by a positive constant, thus does not affect choices. Note also that in the log case, the renormalization simply corresponds to adding a constant to the utility function, so the argument is also valid.

Similarly, in the cooperative model, consumption levels per consumer write:

$$c^Y = \underline{l}^{\frac{\beta}{1-\alpha}} \frac{(1-\tau)F_1(\tilde{k}, \tilde{l}) - \tilde{k}'}{\tilde{l}},$$

$$c^O = \underline{l}^{\frac{\beta}{1-\alpha}} \frac{\tau F_1(\tilde{k}', \tilde{l}')}{\tilde{l}}.$$

Therefore, choices are unaffected by the same renormalization. Since the normalization affects the levels of relevant quantities in the same way in the two models, no quantitative comparison is affected by the level of $\underline{l}$.

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